AC Circuits

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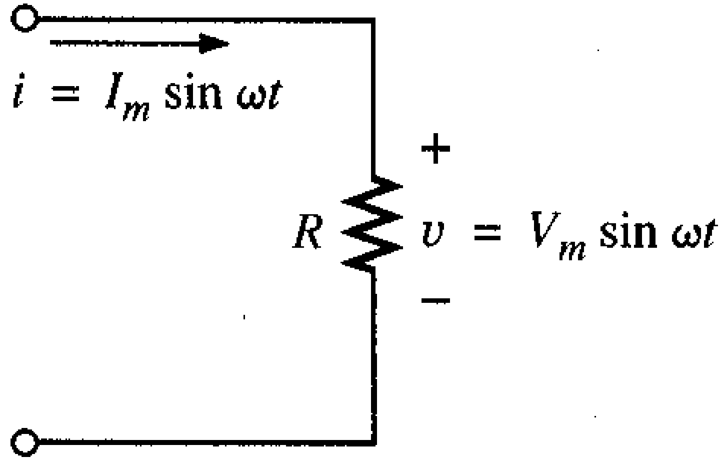
The response of the basic elements R,L, and C to a sinusoidal voltage or current

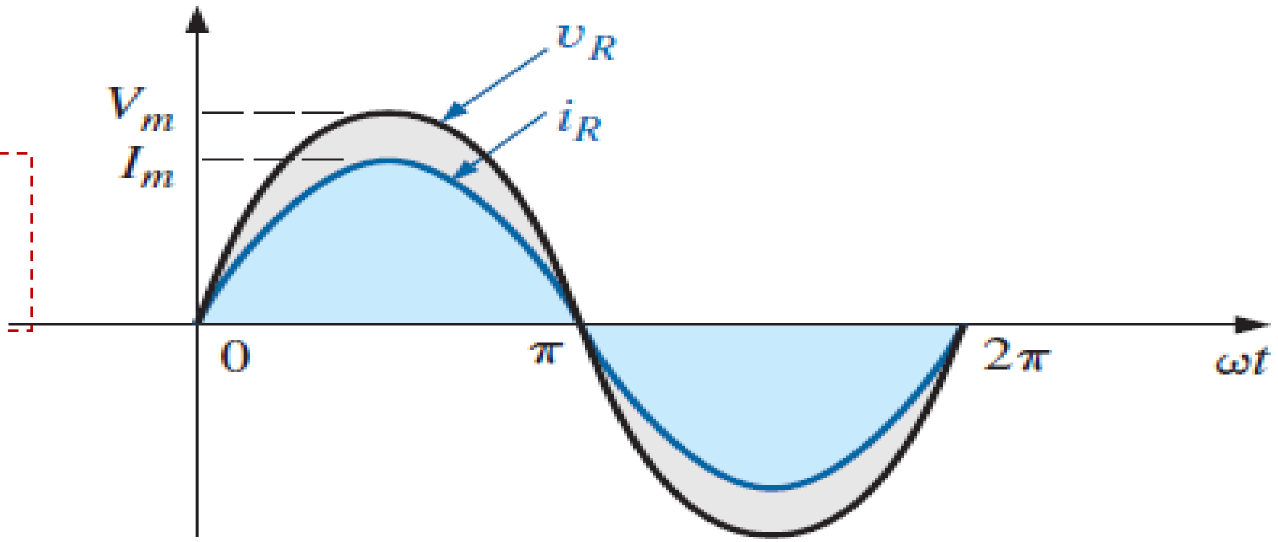
I. Resistor

The **resistive impedance Z_R** is given by;

$$\therefore Z_R = R \qquad (\Omega)$$



The voltage and current of a pure resistive element are in phase.



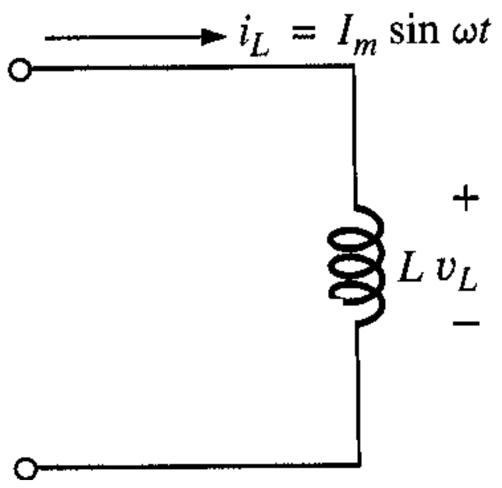
II. Inductor

The quantity ωL, is called the **inductive reactance, X_L**

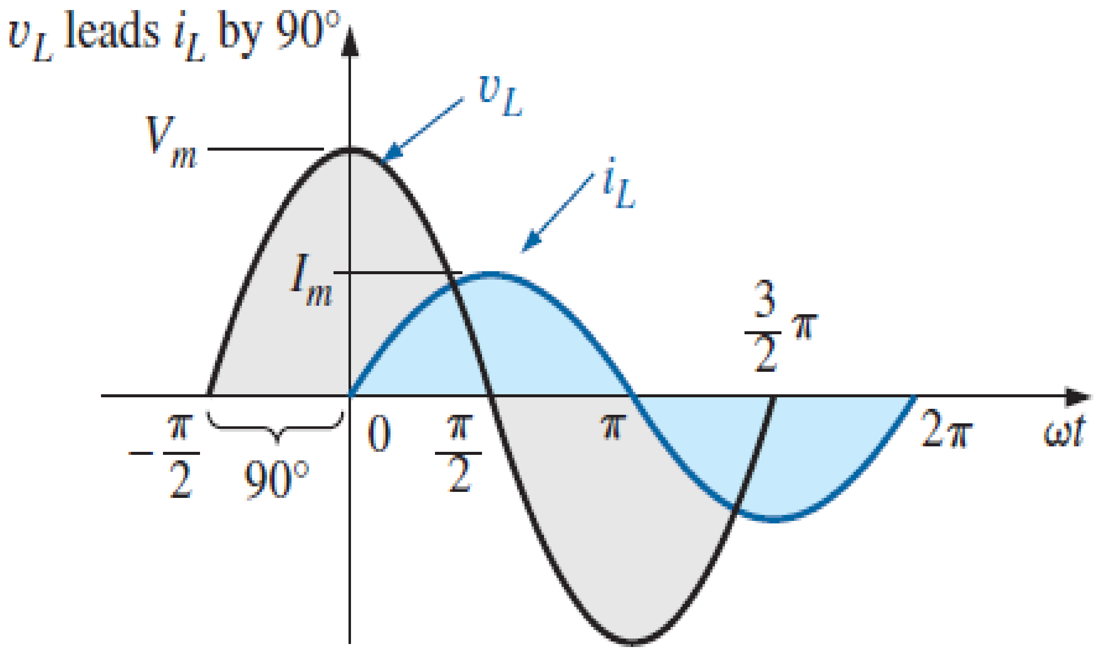
$$X_L = \omega L = 2\pi f L \qquad (\Omega)$$

The **inductive impedance Z_L** is given as:

$$Z_L = jX_L = j\omega L = \omega L\angle 90^\circ \ (\Omega)$$



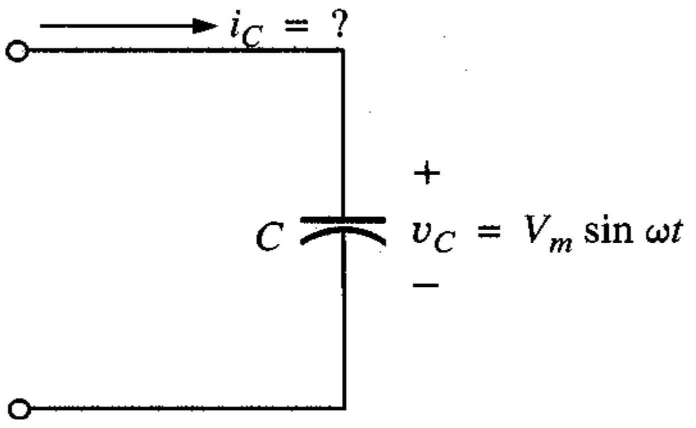
For a pure inductor, the current through the coil lags the voltage across the coil by 90°



III. Capacitor

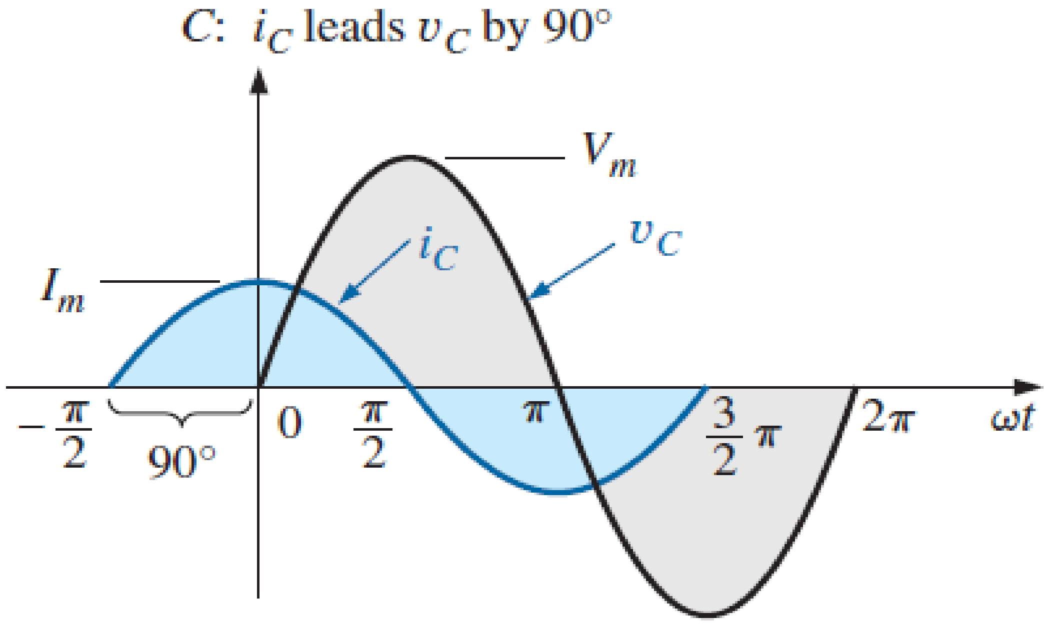
The quantity $1/\omega C$, called the **capacitive reactance, X_C**

$$\mathbf{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad (\Omega)}$$



The **capacitive impedance Z_C** is given as:

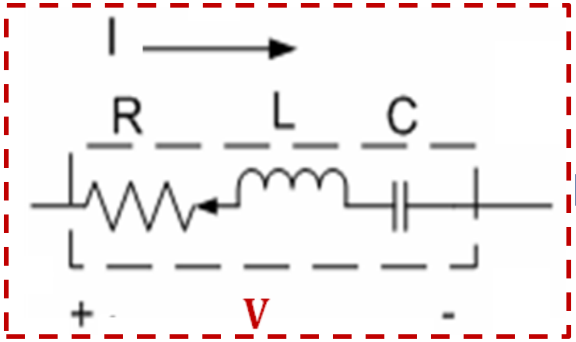
$$\therefore Z_C = -jX_C = -j\frac{1}{\omega C} = \frac{1}{\omega C}\angle -90° \qquad (\Omega)$$



For a pure capacitor, the current through the capacitor leads the voltage across it by 90°.

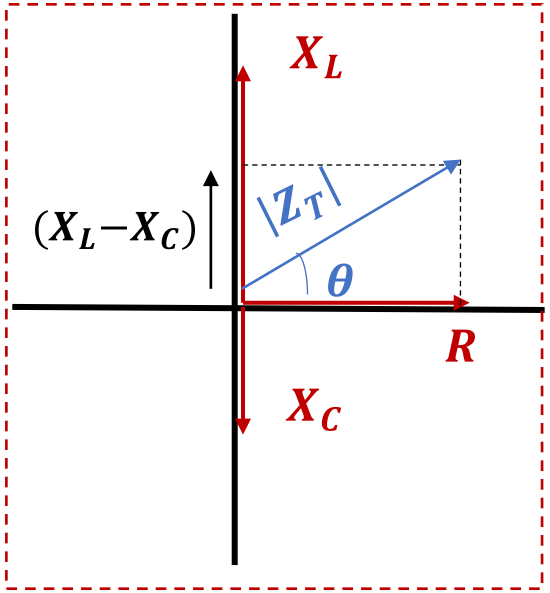
RLC Circuits

$$If\ v = V_m \sin(\omega t + \varphi) \rightarrow\rightarrow\rightarrow \therefore V = V_{rms}\angle\varphi \rightarrow\rightarrow\rightarrow\ I = \frac{V}{Z_T} = \frac{V_{rms}\angle\varphi}{|Z_T|\angle\theta} = |I|\angle(\varphi - \theta)$$

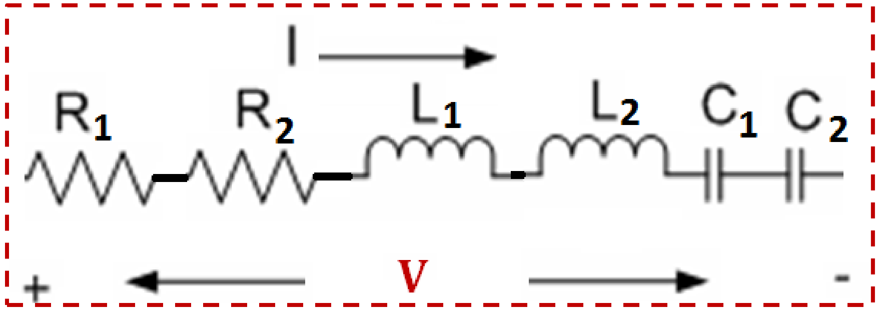


$$Total\ impedance, Z_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

$$Z_T = R + j\left(\omega L - \frac{1}{\omega C}\right) = |Z_T|\angle\theta$$



Impedance diagram

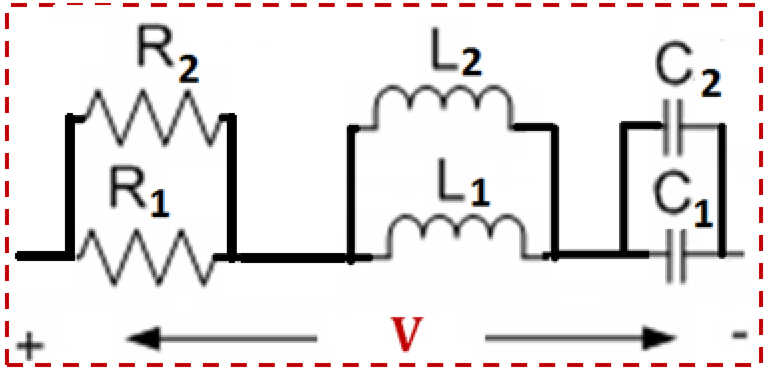


$$R_T = R_1 + R_2$$

$$L_T = L_1 + L_2 \rightarrow\rightarrow X_{L_T} = \omega L_T$$

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2} \rightarrow\rightarrow X_{C_T} = \frac{1}{\omega C_T}$$

$$Total\ impedance, Z_T = R_T + jX_{L_T} - jX_{C_T}$$



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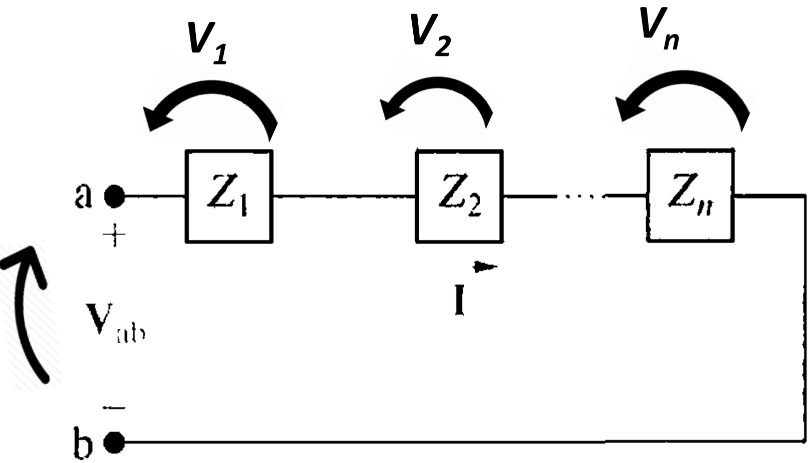
Impedances connections

I. Series connection



Applying K.V.L.

$$V_{ab} = V_1 + V_2 + \cdots + V_n$$

$$\mathbf{V}_{ab} = Z_1\mathbf{I} + Z_2\mathbf{I} + \cdots + Z_n\mathbf{I}$$
$$= (Z_1 + Z_2 + \cdots + Z_n)\mathbf{I}.$$

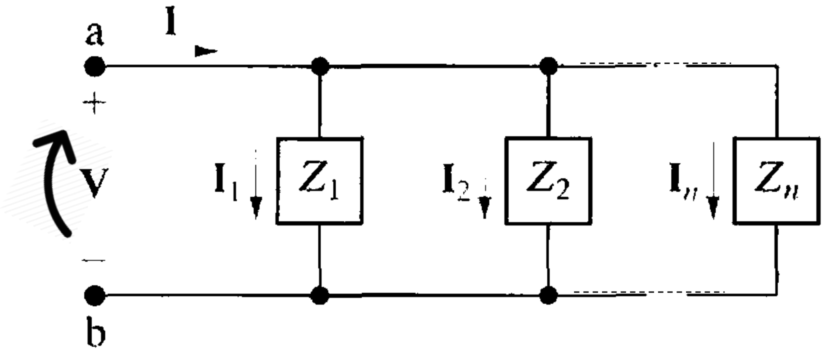
The equivalent impedance between terminals a,b is

$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}} = Z_1 + Z_2 + \cdots + Z_n$$

Applying voltage divider

$$V_1 = V_{ab}\frac{Z_1}{Z_{ab}}, V_2 = V_{ab}\frac{Z_2}{Z_{ab}} \dots\dots\dots, V_n = V_{ab}\frac{Z_n}{Z_{ab}}$$

II. Parallel connection



Applying K.C.L.

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n,$$

$$\frac{\mathbf{V}}{Z_{ab}} = \frac{\mathbf{V}}{Z_1} + \frac{\mathbf{V}}{Z_2} + \cdots + \frac{\mathbf{V}}{Z_n}.$$

The equivalent impedance between terminals a,b is

$$\frac{1}{Z_{ab}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

For only 2 impedances in parallel

$$Z_{ab} = \frac{Z_1 Z_2}{Z_1 + Z_2},$$

Applying current divider

$$I_1 = I\frac{Z_{ab}}{Z_1}, I_2 = I\frac{Z_{ab}}{Z_2} \dots\dots\dots I_n = I\frac{Z_{ab}}{Z_n}$$

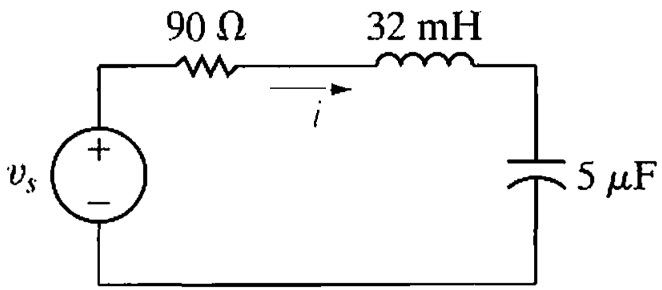
For only 2 impedances in parallel

$$I_1 = I\frac{Z_2}{Z_1 + Z_2}$$

$$I_2 = I\frac{Z_1}{Z_1 + Z_2}$$

Problem 1

The steady-state expression for the source voltage v_s is 750 cos (5000 t + 30°), get i



Solution

From the expression for v_s ,ω= 5000 rad/s

$$Z_L = j\omega L = j(5000)(32 \times 10^{-3}) = j160\ \Omega.$$

$$Z_C = j\frac{-1}{\omega C} = -j\frac{10^6}{(5000)(5)} = -j40\ \Omega$$

$$\begin{aligned} \mathbf{Z}_T &= 90 + j160 - j40 \\ &= 90 + j120 = 150\underline{/53.13^\circ}\ \Omega \end{aligned}$$

The phasor transform of v_s is

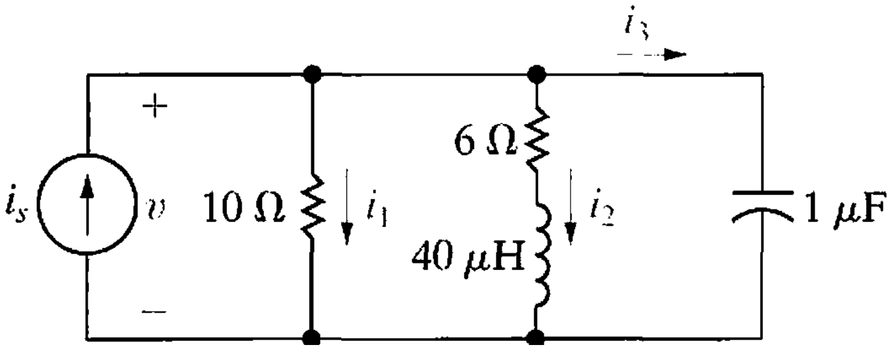
$$\mathbf{V}_s = 750\underline{/30^\circ}\ \text{V}.$$

$$\mathbf{I} = \frac{750\underline{/30^\circ}}{150\underline{/53.13^\circ}} = 5\underline{/-23.13^\circ}\ \text{A}$$

$$i = 5\cos(5000t - 23.13^\circ)\ \text{A}$$

Problem 2

The steady-state expression for the source current, i_s is 8 cos (200,000 t) A get v, i_1, i_2, *and* i_3



Solution

ω= 200,000 rad/s

$$Z_L = j\omega L = j8\,\Omega$$

$$Z_C = j\frac{-1}{\omega C} = -j5\,\Omega$$

$$\frac{1}{Z} = \frac{1}{10} + \frac{1}{6 + j8} + \frac{1}{-j5}$$

$$= 0.2\angle 36.87^\circ \text{ S}$$

$$Z = 5\angle -36.87^\circ\ \Omega.$$

The phasor transform of i_s is

$$\mathbf{I} = 8\angle 0^\circ$$

The Voltage **V** is

$$\mathbf{V} = Z\mathbf{I} = 40\angle -36.87^\circ \text{ V}.$$

Hence

$$\mathbf{I}_1 = \frac{40\angle -36.87^\circ}{10} = 4\angle -36.87^\circ = 3.2 - j2.4 \text{ A}$$

$$\mathbf{I}_2 = \frac{40\angle -36.87^\circ}{6 + j8} = 4\angle -90^\circ = -j4 \text{ A},$$

and

$$\mathbf{I}_3 = \frac{40\angle -36.87^\circ}{5\angle -90^\circ} = 8\angle 53.13^\circ = 4.8 + j6.4 \text{ A}$$

The corresponding steady-state time expressions are

$$v = 40\cos(200{,}000t - 36.87^\circ) \text{ V},$$

$$i_1 = 4\cos(200{,}000t - 36.87^\circ) \text{ A},$$

$$i_2 = 4\cos(200{,}000t - 90^\circ) \text{ A},$$

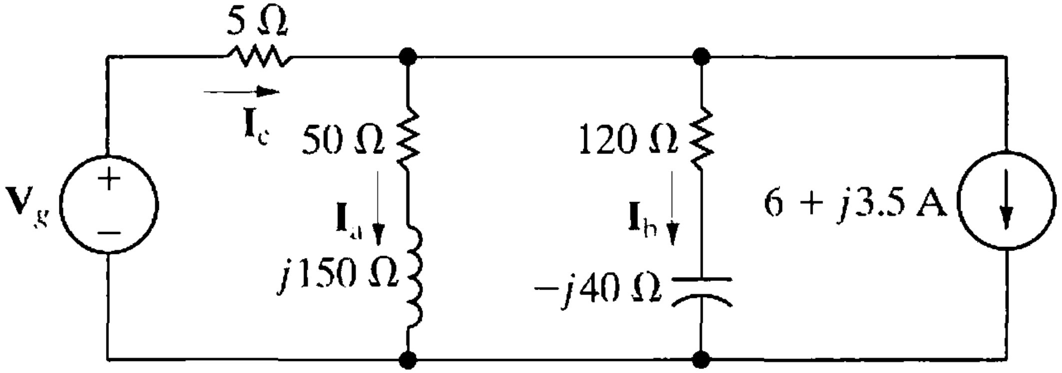
$$i_3 = 8\cos(200{,}000t + 53.13^\circ) \text{ A}.$$

Problem 3

The phasor current $\mathbf{I}_a$ in the circuit shown in Fig. P9.33 is $2\angle 0^\circ$ A.

a) Find $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{V}_g$.

b) If $\omega = 800$ rad/s, write the expressions for $i_b(t)$, $i_c(t)$, and $v_g(t)$.



Solution

[a] $$\mathbf{V}_a = (50 + j150)(2\angle 0^\circ) = 100 + j300 \text{ V}$$

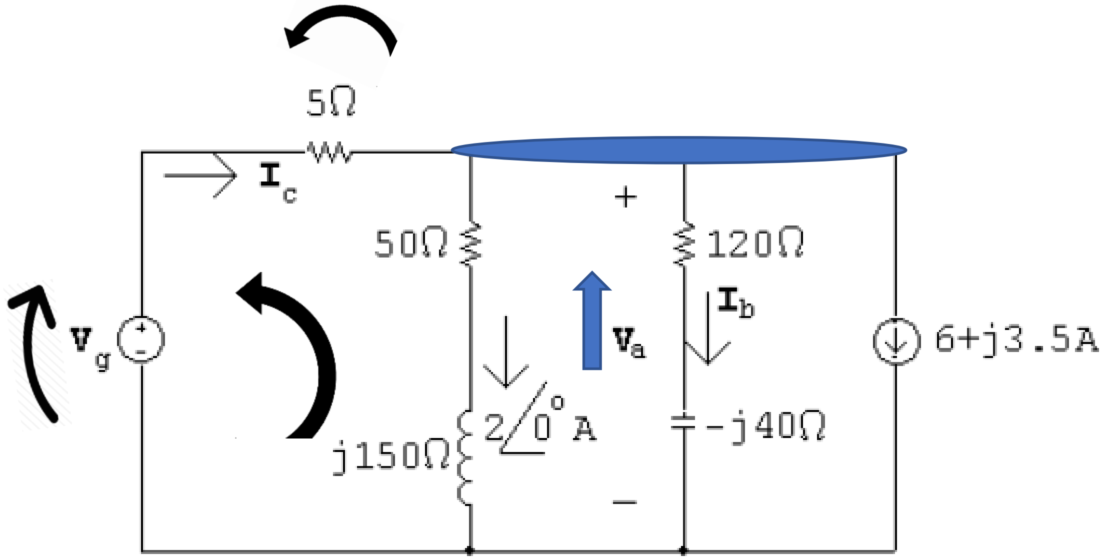
$$\mathbf{I}_b = \frac{100 + j300}{120 - j40} = j2.5 \text{ A}$$

K.C.L

$$\mathbf{I}_c = 2\angle 0^\circ + j2.5 + 6 + j3.5 = 8 + j6 \text{ A}$$

K.V.L.

$$\mathbf{V}_g = 5\mathbf{I}_c + \mathbf{V}_a = 5(8 + j6) + 100 + j300 = 140 + j330 \text{ V}$$



[b] $$i_b = 2.5\cos(800t + 90^\circ) \text{ A}$$

$$i_c = 10\cos(800t + 36.87^\circ) \text{ A}$$

$$v_g = 358.47\cos(800t + 67.01^\circ) \text{ V}$$

Report

Find $\mathbf{I}_b$ and $\mathbf{Z}$ in the circuit shown in Fig. P9.32 if $\mathbf{V}_g = 25\angle 0^\circ$ V and $\mathbf{I}_a = 5\angle 90^\circ$ A.