XII NEERC Western Subregional Contest

Problem	Supposed difficulty	Solved	Tried
Arrays	impossible	0	14
“Bulls and Cows”	trivial	52	52
Courier	medium	14	18
Dales and Hills	easy	36	44
Extremal Permutations	medium	6	12
Figure and Spots	easy	29	37
Game	medium	5	17
Hotel in Ves Lagos	medium	16	29
Illumination of Buildings	hard	0	4
Journal	hard	1	1
Kids and Prizes	easy	21	22
L-Shapes	easy	46	49

Problem A. Arrays

Idea: Yuri Stepin, Pavel Irzhavski

Solutions: Pavel Irzhavski

Tests: Pavel Irzhavski

Analysis: Pavel Irzhavski

Statement: Serge Kashkevich, Pavel Irzhavski, Ivan Metelsky

Problem A. Arrays

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$$0 \leq X_1 \leq \cdots \leq X_{3 \cdot N}.$$

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Consider $N = 1$.

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Also we can assume that B_i will be sorted in increasing order (otherwise we can rearrange all three sequences to satisfy this condition).

Consider $N = 1$. We have $3! = 6$ ways to get A_1, B_1 and C_1, but taking $A_1 = 3$, $B_1 = 1$ and $C_1 = 2$ we always get value of S that is not less than in any other case.

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$$(X_1 - X_2) \cdot X_3 \le (X_3 - X_1) \cdot X_2, \tag{1}$$

$$(X_1 - X_3) \cdot X_2 \le (X_3 - X_1) \cdot X_2, \tag{2}$$

$$(X_2 - X_1) \cdot X_3 \le (X_3 - X_1) \cdot X_2, \tag{3}$$

$$(X_2 - X_3) \cdot X_1 \le (X_3 - X_1) \cdot X_2, \tag{4}$$

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$$(3) \Leftrightarrow -X_1 \cdot X_3 \le -X_1 \cdot X_2 \Leftrightarrow 0 \le (X_3 - X_2) \cdot X_1.$$

$$(5) \Leftrightarrow X_3 \cdot X_1 \le X_3 \cdot X_2 \Leftrightarrow 0 \le X_3 \cdot (X_2 - X_1).$$

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In truth we have $\frac{6!}{3! \cdot 3!} = 20$ ways to divide set $\{1, \cdots, 6\}$ into two subsets $\{A_1, B_1, C_1\}$ and $\{A_2, B_2, C_2\}$ (for each of this triples we know that we should consider only $A_i > C_i > B_i$).

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It may be noticed that always at least one of two values $(X_6 - X_1) \cdot X_5 + (X_4 - X_1) \cdot X_3$ and $(X_6 - X_1) \cdot X_4 + (X_5 - X_1) \cdot X_3$ is not less than value of S in any other case.

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It may be noticed that always at least one of two values $(X_6 - X_1) \cdot X_5 + (X_4 - X_1) \cdot X_3$ and $(X_6 - X_1) \cdot X_4 + (X_5 - X_1) \cdot X_3$ is not less than value of S in any other case.

We have to prove for each of other eight ways to divide $\{1, \cdots, 6\}$ into two triples that corresponding value of S is not greater than at least one of expressions above.

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$$
\begin{aligned}
(X_3 - X_1)X_2 + (X_6 - X_4)X_5 &\le (X_6 - X_1)X_5 + (X_4 - X_2)X_3,\\
(X_4 - X_1)X_2 + (X_6 - X_3)X_5 &\le (X_6 - X_1)X_5 + (X_4 - X_2)X_3,\\
(X_4 - X_1)X_3 + (X_6 - X_2)X_5 &\le (X_6 - X_1)X_5 + (X_4 - X_2)X_3,\\
(X_5 - X_1)X_2 + (X_6 - X_3)X_4 &\le (X_6 - X_1)X_4 + (X_5 - X_2)X_3,\\
(X_5 - X_1)X_3 + (X_6 - X_2)X_4 &\le (X_6 - X_1)X_4 + (X_5 - X_2)X_3,\\
(X_5 - X_1)X_4 + (X_6 - X_2)X_3 &\le (X_6 - X_1)X_4 + (X_5 - X_2)X_3,\\
(X_6 - X_1)X_2 + (X_5 - X_3)X_4 &\le (X_6 - X_1)X_4 + (X_5 - X_2)X_3,\\
(X_6 - X_1)X_3 + (X_5 - X_2)X_4 &\le (X_6 - X_1)X_4 + (X_5 - X_2)X_3.
\end{aligned}
$$

Skip proof

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In truth $ad + bc \leq ab + cd \Leftrightarrow 0 \leq (d - c)(b - a)$.

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In truth $ad + bc \leq ab + cd \Leftrightarrow 0 \leq (d - c)(b - a)$.

$$(X_3 - X_1)X_2 + (X_6 - X_4)X_5 \leq (X_6 - X_1)X_5 + (X_4 - X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow X_3X_2 - X_1X_2 - X_4X_5 \leq -X_1X_5 + X_4X_3 - X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow 2X_3X_2 + X_1X_5 \leq X_4X3 + X_1X_2 + X_4X_5$$

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In truth $ad + bc \leq ab + cd \Leftrightarrow 0 \leq (d - c)(b - a)$.

$$(X_3 - X_1)X_2 + (X_6 - X_4)X_5 \leq (X_6 - X_1)X_5 + (X_4 - X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow X_3X_2 - X_1X_2 - X_4X_5 \leq -X_1X_5 + X_4X_3 - X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow 2X_3X_2 + X_1X_5 \leq X_4X3 + X_1X_2 + X_4X_5$$

This follows from:

$$X_3X_2 \leq X_3X_4$$
$$X_3X_2 \leq X_4X_2$$
$$X_4X_2 + X_1X_5 \leq X_1X_2 + X_4X_5 \text{ (since } X_1 \leq X_4 \text{ and } X_2 \leq X_5\text{)}$$

Problem A. Arrays

$$(X_4 - X_1)X_2 + (X_6 - X_3)X_5 \le (X_6 - X_1)X_5 + (X_4 - X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow X_4X_2 - X_1X_2 - X_3X_5 \le -X_1X_5 + X_4X_3 - X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow X_4X_2 + X_3X_2 + X_1X_5 \le X_4X_3 + X_1X_2 + X_3X_5$$

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$$(X_4 - X_1)X_2 + (X_6 - X_3)X_5 \le (X_6 - X_1)X_5 + (X_4 - X_2)X_3 \Leftrightarrow$$
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$$\Leftrightarrow X_4X_2 + X_3X_2 + X_1X_5 \le X_4X_3 + X_1X_2 + X_3X_5$$

Follows from:

$$X_4X_2 \le X_4X_3$$
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Problem A. Arrays

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$$\Leftrightarrow X_4X_2 - X_1X_2 - X_3X_5 \le -X_1X_5 + X_4X_3 - X_2X_3 \Leftrightarrow$$
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Follows from:

$$X_4X_2 \le X_4X_3$$
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$$(X_4 - X_1)X_3 + (X_6 - X_2)X_5 \le (X_6 - X_1)X_5 + (X_4 - X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow -X_1X_3 - X_2X_5 \le -X_1X_5 - X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow X_1X_5 + X_2X_3 \le X_1X_3 + X_2X_5, \text{ which is true.}$$

Problem A. Arrays

$$
\begin{aligned}
&(X_5 - X_1)X_2 + (X_6 - X_3)X_4 \leq (X_6 - X_1)X_4 + (X_5 - X_2)X_3 \Leftrightarrow \\
&\quad \Leftrightarrow X_5X_2 - X_1X_2 - X_3X_4 \leq -X_1X_4 + X_5X_3 - X_2X_3 \Leftrightarrow \\
&\quad \Leftrightarrow X_5X_2 + X_1X_4 + X_2X_3 \leq X_3X_4 + X_1X_2 + X_5X_3
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$$\Leftrightarrow X_5X_2 + X_1X_4 + X_2X_3 \le X_3X_4 + X_1X_2 + X_5X_3$$

This is a sum of three inequalities:

$$X_5X_2 + X_1X_3 \le X_1X_2 + X_5X_3$$
$$X_1X_4 + X_2X_3 \le X_1X_3 + X_2X_4$$
$$X_2X_4 \le X_3X_4$$

Problem A. Arrays

$$(X_5 - X_1)X_2 + (X_6 - X_3)X_4 \le (X_6 - X_1)X_4 + (X_5 - X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow X_5X_2 - X_1X_2 - X_3X_4 \le -X_1X_4 + X_5X_3 - X_2X_3 \Leftrightarrow$$
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$$X_1X_4 + X_2X_3 \le X_1X_3 + X_2X_4$$
$$X_2X_4 \le X_3X_4$$

$$(X_5 - X_1)X_3 + (X_6 - X_2)X_4 \le (X_6 - X_1)X_4 + (X_5 - X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow -X_1X_3 - X_2X_4 \le -X_1X_4 - X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow X_2X_3 + X_1X_4 \le X_1X_3 + X_2X_4, \text{ which is true.}$$

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$$(X_5 - X_1)X_4 + (X_6 - X_2)X_3 \le (X_6 - X_1)X_4 + (X_5 - X_2)X_3 \Leftrightarrow$$
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$$(X_6-X_1)X_2+(X_5-X_3)X_4 \le (X_6-X_1)X_4+(X_5-X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow X_6X_2-X_1X_2+X_5X_4-X_3X_4 \le X_6X_4-X_1X_4+X_5X_3-X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow X_6X_2+X_1X_4+X_5X_4+X_2X_3 \le X_6X_4+X_1X_2+X_5X_3+X_3X_4$$

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$$(X_6-X_1)X_2+(X_5-X_3)X_4 \le (X_6-X_1)X_4+(X_5-X_2)X_3 \Leftrightarrow$$
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$$\Leftrightarrow X_6X_2+X_1X_4+X_5X_4+X_2X_3 \le X_6X_4+X_1X_2+X_5X_3+X_3X_4$$

Follows from:

$$X_6X_3 + X_5X_4 \le X_5X_3 + X_6X_4$$
$$X_6X_2 \le X_6X_3$$
$$X_1X_4 + X_3X_2 \le X_1X_2 + X_3X_4$$

Problem A. Arrays

And finally:

$$(X_6-X_1)X_3+(X_5-X_2)X_4 \leq (X_6-X_1)X_4+(X_5-X_2)X_3 \Leftrightarrow$$
$$\Leftrightarrow X_6X_3-X_1X_3+X_5X_2-X_2X_4 \leq X_6X_4-X_1X_4+X_5X_3-X_2X_3 \Leftrightarrow$$
$$\Leftrightarrow X_6X_3+X_5X_2+X_1X_4+X_2X_3 \leq X_6X_4+X_5X_3+X_1X_3+X_2X_4$$

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Follows from:

$$X_6X_3 \leq X_6X_4$$
$$X_5X_2 \leq X_5X_3$$
$$X_1X_4 + X_2X_3 \leq X_1X_3 + X_2X_4$$

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$$A_1 = 6, B_1 = 1, C_1 = 5 \;\; A_2 = 4, B_2 = 2, C_2 = 3$$
$$A_1 = 6, B_1 = 1, C_1 = 4 \;\; A_2 = 5, B_2 = 2, C_2 = 3$$

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Consider arbitrary N.

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Consider arbitrary N. Now we have some information about our sequences.

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Let $1 \leq i < j \leq N$.

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$$A_1 = 6, B_1 = 1, C_1 = 4 \;\; A_2 = 5, B_2 = 2, C_2 = 3$$

Consider arbitrary N. Now we have some information about our sequences.

Let $1 \le i < j \le N$. Consider $N' = 2$ and $\{X'_1, \cdots, X'_6\} =$
$= \{X_{A_i}, X_{B_i}, X_{C_i}, X_{A_j}, X_{B_j}, X_{C_j}\}$.

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So there are two ways to form sequences A, B, C that we are interested in:

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$$A_1 = 6, B_1 = 1, C_1 = 4 \;\; A_2 = 5, B_2 = 2, C_2 = 3$$

Consider arbitrary N. Now we have some information about our sequences.

Let $1 \leq i < j \leq N$. Consider $N' = 2$ and $\{X'_1, \cdots, X'_6\} =$ $= \{X_{A_i}, X_{B_i}, X_{C_i}, X_{A_j}, X_{B_j}, X_{C_j}\}$.

Then $S' = (X_{A_i} - X_{B_i})X_{C_i} + (X_{A_j} - X_{B_j})X_{C_j}$, 'cause otherwise we could rearrange $A_i, B_i, C_i, A_j, B_j, C_j$ and get greater value of S.

Problem A. Arrays

So we can rearrange $A_i, B_i, C_i, A_j, B_j, C_j$ without decreasing S in such way that $B_i < B_j < C_j < C_i < A_j < A_i$ or
$B_i < B_j < C_j < A_j < C_i < A_i$.

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Anyway we get:

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So we can rearrange $A_i, B_i, C_i, A_j, B_j, C_j$ without decreasing S in such way that $B_i < B_j < C_j < C_i < A_j < A_i$ or
$B_i < B_j < C_j < A_j < C_i < A_i$.

Anyway we get:

$$B_k < C_l \text{ for any } 1 \leq k, l \leq N, \tag{6}$$

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$B_i < B_j < C_j < A_j < C_i < A_i$.

Anyway we get:

$$B_k < C_l \text{ for any } 1 \leq k, l \leq N, \tag{6}$$

$$B_k < A_l \text{ for any } 1 \leq k, l \leq N, \tag{7}$$

Problem A. Arrays

So we can rearrange $A_i, B_i, C_i, A_j, B_j, C_j$ without decreasing S in such way that $B_i < B_j < C_j < C_i < A_j < A_i$ or $B_i < B_j < C_j < A_j < C_i < A_i$.

Anyway we get:

$$B_k < C_l \text{ for any } 1 \leq k, l \leq N, \tag{6}$$

$$B_k < A_l \text{ for any } 1 \leq k, l \leq N, \tag{7}$$

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This implies that $B_i = i$, $A_1 = 3N$, $C_1 \ge 2N$.

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This algorithm has $O(N!)$ complexity, that is much better than trivial one with complexity $O\big((3N)!\big)$, but it is still rather slow to be used as solution.

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This algorithm has $O\left(N \cdot 2^{2N}\right)$ complexity, that is better, but still not enough to be accepted.

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Running ahead we can say that this algorithm is much better than it seems now. Really it has complexity $O\left(N \cdot 2^N\right)$ which seems to be enough for solving this task. However, memorizing results requires some complicated structures (like `map<long long, int>` or `Map<Long, Integer>`) since mask can be up to $2^{50} - 1$ and this structures are far too slow to keep within given time.

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- To restore mask we can pad this N bits with i ones from the left side and with $N - i$ zeros from the right side.

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Thus we proved that our algorithm operates with at most 2^N different masks (therefore it has complexity $O\left(N \cdot 2^N\right)$) and showed efficient way to store result for each mask.

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There are some more optimizations (avoiding recursion and 64bit numbers and some others), but described algorithm was good enough.

Problem B. “Bulls and Cows”

Idea:	Serge Kashkevich
Solutions:	Pavel Irzhavski, Serge Kashkevich, Vladimir Kerus, Ivan Metelsky
Tests:	Ivan Metelsky
Analysis:	Pavel Irzhavski
Statement:	Serge Kashkevich

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Problem C. Courier

Idea:	Yuri Orlovich
Solutions:	Pavel Irzhavski, Serge Kashkevich, Ivan Metelsky
Tests:	Ivan Metelsky
Analysis:	Pavel Irzhavski
Statement:	Serge Kashkevich
Checker:	Serge Kashkevich

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This solution has $O(N)$ complexity, but less efficient approaches (like $O\left(N^2\right)$) were allowed.

Problem D. Dales and Hills

Idea:	Serge Kashkevich
Solutions:	Serge Kashkevich, Ivan Metelsky, Vladimir Kerus, Pavel Irzhavski
Tests:	Ivan Metelsky
Analysis:	Pavel Irzhavski
Statement:	Serge Kashkevich

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Let b_j be the least possible $i \geq 1$ such that $a_t < a_{t+1}$ for any $i \leq t < j$. We can calculate all b_j in following way:

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- $c_N = \mathrm{N}$.
- For j moving from $N - 1$ to 1 let $c_j = c_{j+1}$ if $a_j > a_{j+1}$ and $c_j = j$ otherwise.

Let's call a_j a *peak* of a hill $a_i, \cdots, a_j, \cdots, a_k$ if $a_t < a_{t+1}$ for any $i \leq t < j$ and $a_t > a_{t+1}$ for any $j \leq t < k$.

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Now for each a_j we can fastly determine, whether it is a peak of some hill and if so also the height of the highest possible hill with peak a_j. More precisely a_j is peak of some hill if $b_j < a_j < c_j$ and value $\max\{a_j - b_j, c_j - a_j\}$ is the height of the highest hill with peak a_j.

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Now for each a_j we can fastly determine, whether it is a peak of some hill and if so also the height of the highest possible hill with peak a_j. More precisely a_j is peak of some hill if $b_j < a_j < c_j$ and value $\max\{a_j - b_j, c_j - a_j\}$ is the height of the highest hill with peak a_j. Thus we can calculate the height of the highest hill in linear time.

Problem D. Dales and Hills

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The depth of the deepest dale can be calculated similarly. To reduce code size we may let $a'_i = -a_i$. Then the height of the highest hill for sequence $a'_1, \cdots, a'_N$ is equal to the depth of the deepest dale for $a_1, \cdots, a_N$.

Problem E. Extremal Permutations

Idea: Pavel Irzhavski

Solutions: Pavel Irzhavski, Ivan Metelsky, Vladimir Kerus

Tests: Pavel Irzhavski

Analysis: Pavel Irzhavski

Statement: Pavel Irzhavski

Problem E. Extremal Permutations

Number of such permutations that $p_1 < p_2 > p_3 < \cdots$ is equal to number of permutations with $p_1 > p_2 < p_3 > \cdots$.

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We can find number of good permutations and multiply this number by 2 to get number of extremal permutations (this doesn't hold only when $n = 1$).

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We can find number of good permutations and multiply this number by 2 to get number of extremal permutations (this doesn't hold only when $n = 1$).

Let $F(n, k)$ be number of good permutations with $p_1 = k$.

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For fixed k there is one-to-one correspondence between good permutations of $1, \cdots, n$ with $p_1 = k$ and good permutations of $1, \cdots, n-1$ with $p_1 \geq n-k$.

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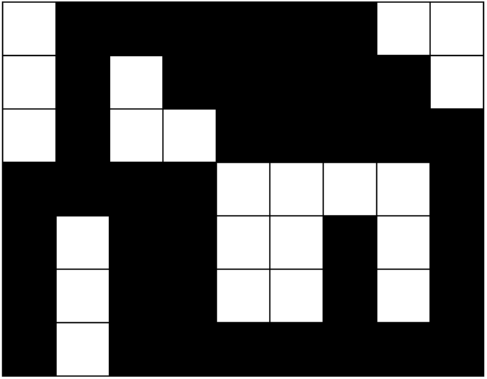
Taking into account that $F(n,n) = 0$ if $n > 1$ and $F(1,1) = 1$ we get full solution with $O\left(N^2\right)$ complexity.
There are more approaches with $O\left(N^2\right)$ complexity, but some of them use modulo multiplication instead of modulo addition and thus require about 10-15 times more time. Such approaches were rejected.

Problem F. Figure and Spots

Idea:	Ivan Metelsky
Solutions:	Ivan Metelsky, Pavel Irzhavski, Serge Kashkevich
Tests:	Ivan Metelsky
Analysis:	Ivan Metelsky
Statement:	Ivan Metelsky
Checker:	Ivan Metelsky

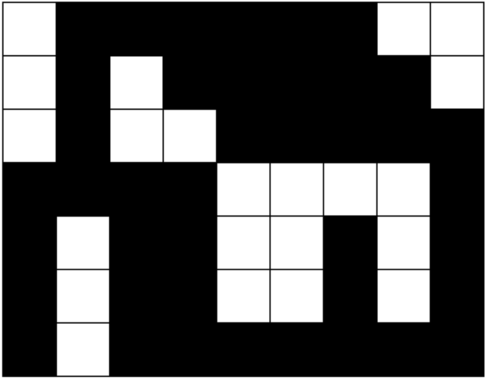
Problem F. Figure and Spots

First of all, we describe two transformations that, given a figure, allow to simplify it greatly, but preserving the figure's width, height and number of spots.



Problem F. Figure and Spots

If you consider a figure within its bounding rectangle, you see several white components that could be spots, but aren't really spots because they touch the rectangle's borders. On the picture below there are 3 of them. As they are not spots, we can fill them with black color without changing any important parameters of the figure. This is our first transformation.



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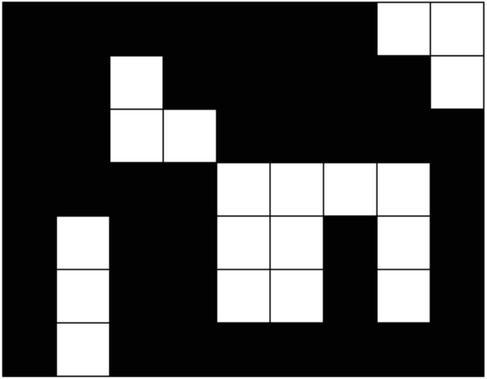

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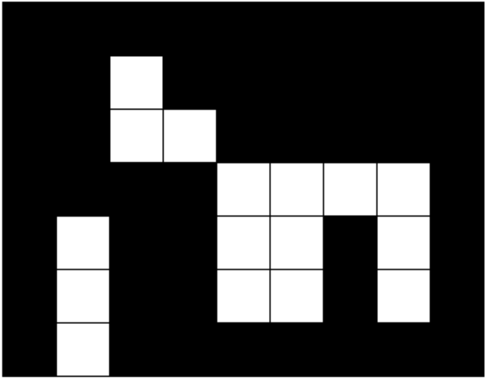

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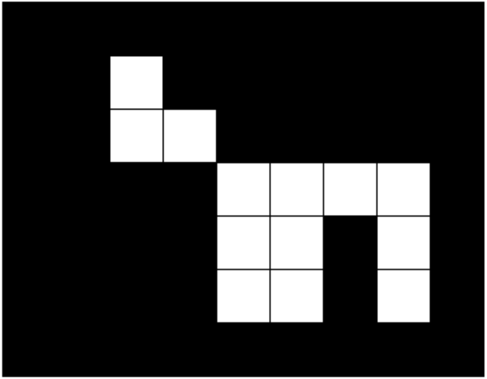
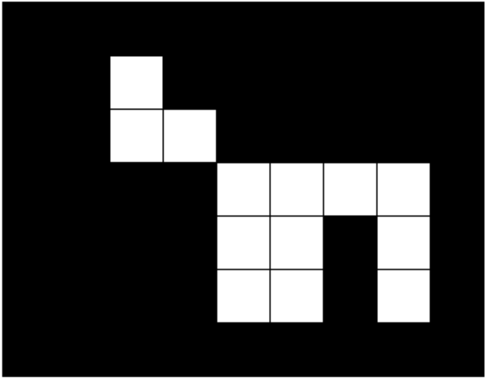


Problem F. Figure and Spots

Now comes the second transformation. Let's consider our spots and replace each spot by a single white cell (among all cells of a spot we choose any one and leave only this cell). It's easy to see that this transformation also leaves the important parameters of the figure unchanged.



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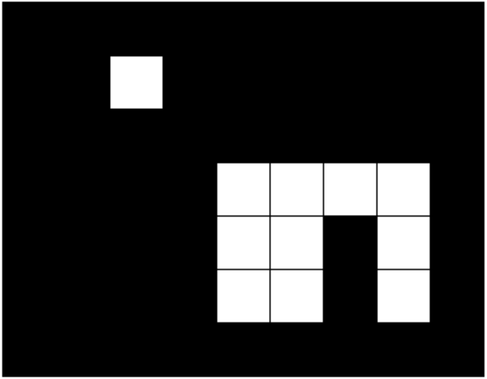

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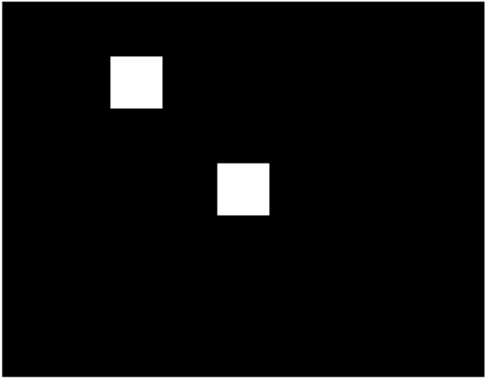
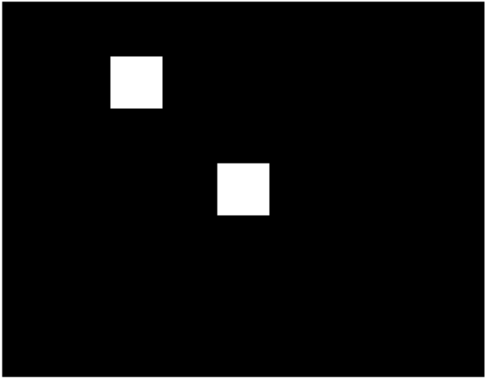


Problem F. Figure and Spots

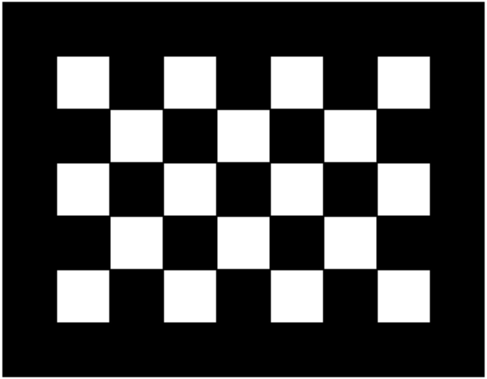
So without loss of generality we can consider only the figures with the following two properties:

1. The bounding rectangle's border is completely black.
2. Each spot consists of a single white cell.



Problem F. Figure and Spots

Under these conditions, it's easy to see how to fit the maximum number of spots into a figure with the given width and height. The border must be completely black and the inner cells must be filled according to a chessboard pattern shown below.



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If $N = M$, the solution looks like described on the previous slide.

If $N > M$, there's no solution at all.

If $N < M$, the solution can be constructed as follows. First, take a solution with M spots and then remove any $M - N$ of them.

Problem F. Figure and Spots

The solution has linear complexity $O(W \cdot H)$.

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The constraints were deliberately set to low value in order to make it a little bit less obvious that the problem can be solved with a simple linear algorithm.

Problem G. Game for Little Johnny

Idea:	Ivan Metelsky, Pavel Irzhavski
Solutions:	Ivan Metelsky, Pavel Irzhavski
Tests:	Pavel Irzhavski
Analysis:	Ivan Metelsky
Statement:	Ivan Metelsky

Problem G. Game for Little Johnny

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The fastest way to do it is modified Eratosthenes Sieve algorithm.

Let $DivList_i$ be the list of all divisors for integer i. Initially all these lists are empty.

For each $i = 1, 2, ..., N$ we iterate through all j such that i is a divisor of j. These j are i, $2i$, $3i$, $\cdots$, $k \cdot i$, where $k = \left\lfloor \frac{N}{i} \right\rfloor$. For all these j we add i to the end of $DivList_j$.

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For a given A, what values of B result in valid pairs (A, B)? Suppose that (A, B) is a valid pair. Then there are some $x, y \ge 1$ such that

$$A \cdot x + B \cdot y = N$$

Let's express B from this equation:

$$B = \frac{N - A \cdot x}{y}$$

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In order to calculate the number of B values for a given A, we check numbers $N - A$, $N - 2A$, $\cdots$, $N - k \cdot A$, where $k = \left\lfloor \frac{N}{A} \right\rfloor$, and select all their divisors that are greater than A.

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The number of distinct divisors gives the number of valid (A, B) pairs for this A and must be added to the result.

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In order to select only distinct divisors efficiently, we can use an array of integers $last[1..N]$. The idea is to store in $last[i]$ the maximum value of A found such that $(last[i], i)$ is a valid pair.

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When we are iterating through the divisors for a given value of A and have a divisor d, we need to check the value of $last[d]$. If $last[d] < A$, we have another pair (A, d), so $last[d]$ must be set to A and result must be increased by 1. However, if $last[d] = A$, we have already considered such value of d for this A, so it must be ignored.

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Estimating complexity of this solution is not trivial. It's not hard to see that we check divisors for $O(N \cdot \log N)$ numbers (argumentation is the same as we had when analyzing the Eratosthenes Sieve algorithm).

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When $N \leq 100\,000$, an integer between 1 and N can have at most 128 divisors, so we certainly have to check at most $128 \cdot N \cdot (\ln N + 1) < 150\,000\,000$ divisors and this is already not too much. Or course, the real working time is much smaller because most of numbers have much smaller amount of divisors.

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In average an integer between 1 and N has $O(\log N)$ divisors, and our solution tends to check smaller integers more frequently than larger integers (and of course, smaler integers have less divisors in average), so we expect the real working time for the solution to be $O\left(N \cdot \log^2 N\right)$.

Problem H. Hotel in Ves Lagos

Idea:	Rihards Opmanis
Solutions:	Rihards Opmanis, Pavel Irzhavski, Ivan Metelsky, Vladimir Kerus
Tests:	Ivan Metelsky
Analysis:	Pavel Irzhavski
Statement:	Ivan Metelsky

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Suppose $K = A \cdot 10^{i+2} + B \cdot 10^{i} + C,\ B < 100,\ C < 10^{i}$ and $X = D \cdot 10^{i+2} + 13 \cdot 10^{i} + E,\ E < 10^{i}$.

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Problem H. Hotel in Ves Lagos

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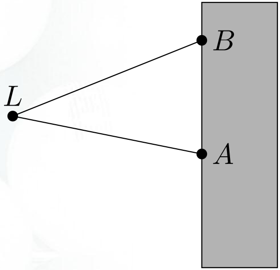
There also exists an $O(\log N)$ approach.

Problem I. Illumination of Buildings

Idea:	Vladimir Kotov
Solutions:	Pavel Irzhavski, Ivan Metelsky
Tests:	Ivan Metelsky
Analysis:	Pavel Irzhavski
Statement:	Ivan Metelsky

Problem I. Illumination of Buildings



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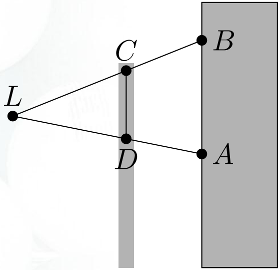

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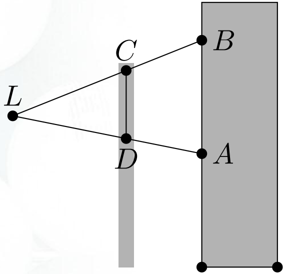

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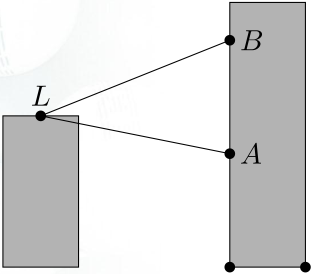

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Now it is obvious that light source located inside the top edge is useless since it can't illuminate any bottom point.

Problem I. Illumination of Buildings

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- And finally we place light source into C_i or B_{i+1} (into any of them) if D_i or A_{i+1} isn't illuminated.

Problem I. Illumination of Buildings

Let's prove algorithm is correct.

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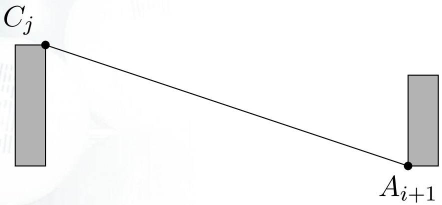

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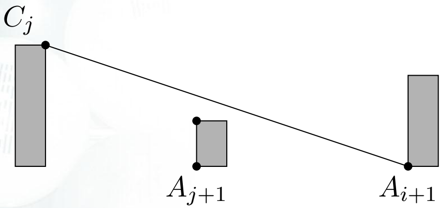

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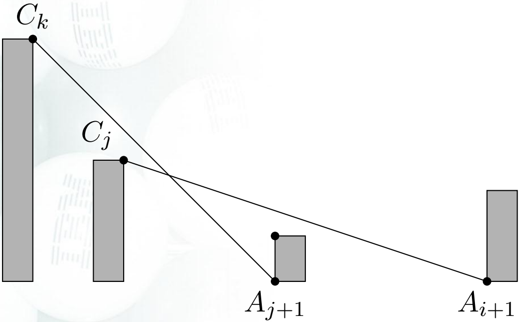

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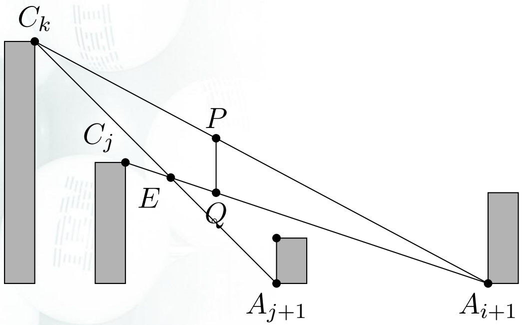

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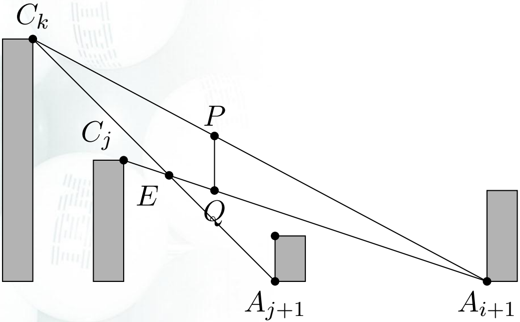

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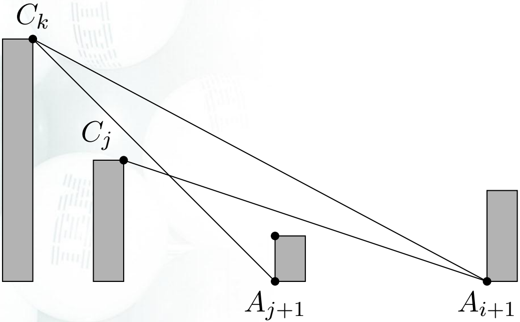

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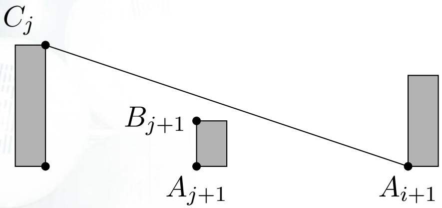

A_{i+1} can be illuminated from C_j so $H_{j+1} < H_j$ and light source placed in B_{j+1} can illuminate only two points while light source placed in C_j can illuminate the same two points and point A_i.

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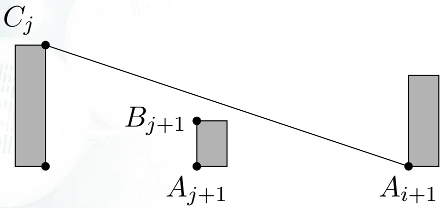

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Similarly we get that for each $1 \leq i \leq N-1$ light source should be placed in the rightmost position where D_i can be illuminated from if this position is not equal to B_{i+1}.

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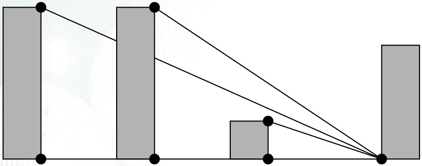

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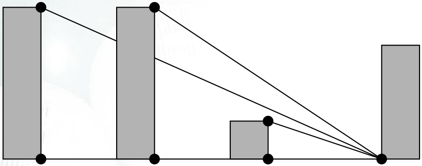

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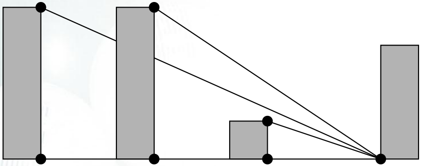

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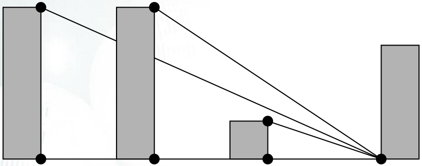

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Complexity of this algorithm is $O\left(N^2\right)$.

Problem J. Journal

Idea:	Uldis Barbans
Solutions:	Uldis Barbans, Pavel Irzhavski, Ivan Metelsky, Vladimir Kerus
Tests:	Uldis Barbans, Ivan Metelsky
Analysis:	Ivan Metelsky
Statement:	Uldis Barbans, Ivan Metelsky

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But first let's calculate some data that will be useful for both these solutions.

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Of course, no real manipulation with words are made. We just need to store at each moment the total length of words currently in the line (including spaces between them) and modify this length accordingly.

Problem J. Journal

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Let's define K as the smallest k such that $BeginSkip(k) = N + 1$. K is the number of lines required to print the article without image and obviously $K \leq N$.

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$EndSkip(pos)$ is also easy to calculate:

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$$EndSkip(pos) = EndSkip\big(skip(pos, W)\big) + 1, k \leq N$$

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3. For each x and k calculate the number of lines required to locate the image and the text. Choose the best out of all layouts.

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3. At this point we are at word $curWord$ and are required $EndSkip(curWord)$ lines in order to place the rest of the article. So the total number of occupied lines is $k + R + EndSkip(curWord)$.

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It allows to calculate $next^R$ in $O(N \cdot \log R)$ time and thus achieve an $O(W \cdot N \cdot \log R)$ solution.

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$next^R(i)$ in terms of this graph is the R-th grandparent of the vertex i (or the root of the corresponding tree if i is located at a height less than R).

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Calculation of $next^R$ becomes linear and therefore we have an $O(W \cdot N)$ algorithm.

Problem K. Kids and Prizes

Idea:	Pavel Irzhavski
Solutions:	Pavel Irzhavski, Ivan Metelsky
Tests:	Pavel Irzhavski
Analysis:	Pavel Irzhavski
Statement:	Ivan Metelsky

Problem K. Kids and Prizes

Let E_K be desired expectation for N prizes and K people.

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Problem K. Kids and Prizes

$$E_{K+1} = \frac{N-1}{N}\sum_{i=0}^{N} i p_K(i) + \sum_{i=0}^{N} p_K(i) = \frac{N-1}{N}E_K + 1.$$

This equation is enough to get E_M using $O(M)$ time.
But we still can improve this result.

$$E_{K+1} = \frac{N-1}{N}E_K + 1.$$

$$E_{K+1} - N = \frac{N-1}{N}(E_K - N) = \left(\frac{N-1}{N}\right)^2 (E_{K-1} - N) =$$

$$= \cdots = \left(\frac{N-1}{N}\right)^K (E_1 - N) = -\left(\frac{N-1}{N}\right)^K (N-1)$$

$$E_M = N - \left(\frac{N-1}{N}\right)^{M-1} (N-1) = N\left(1 - \left(\frac{N-1}{N}\right)^M\right).$$

So E_M might be found using $O(1)$ time.

Problem L. L-Shapes

Idea:	Serge Kashkevich, Ivan Metelsky
Solutions:	Pavel Irzhavski, Serge Kashkevich, Vladimir Kerus, Ivan Metelsky
Tests:	Pavel Irzhavski
Analysis:	Ivan Metelsky
Statement:	Serge Kashkevich, Ivan Metelsky, Pavel Irzhavski

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Problem L. L-Shapes

The first thing to note is that the number of pairs of segments is relatively small.

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Therefore we can simply test all of them for being L-Shapes.

Problem L. L-Shapes

Let's consider a pair of segments AB and CD, where $A = (x_1, y_1)$, $B = (x_2, y_2)$, $C = (x_3, y_3)$, $D = (x_4, y_4)$.

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How to check whether AB and CD form an L-Shape?

There are two conditions to check:

1. They share an endvertex.
2. The angle between them is 90°.

Problem L. L-Shapes

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Checking the second condition is a bit harder. There are several ways to do it, but the easiest one is probably using the scalar product of vectors.

Problem L. L-Shapes

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The angle between the segments is 90° if and only if their scalar product is zero:

$$\overrightarrow{AB} \cdot \overrightarrow{CD} = 0.$$

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The scalar product of vectors $\overrightarrow{u}(x_u, y_u)$ and $\overrightarrow{v}(x_v, y_v)$ can be calculated as follows:

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So the angle between the given segments is 90 degrees if and only if

$$(x_2 - x_1) \cdot (x_4 - x_3) + (y_2 - y_1) \cdot (y_4 - y_3) = 0.$$

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There is also a solution with complexity $O(N \cdot \log N)$. However it's harder to find and to implement this solution, so using it during the contest would be simply a waste of time.

Special thanks to:

Ahto Truu
for statements translation

Denis Kanonik
for testing system management and support