SOME OTHER STATISTICAL MEASURES (MEASURES OF POSITION)

MEASURES OF POSITION

Used to describe the position of a data value **in relation to the rest of the data.**

There are three types:

1. Quartiles
2. Percentiles
3. Deciles

1: QUARTILES:

Values which divide the arranged data into four equal parts, denoted by Q_1, Q_2, Q_3 .

Q_1 : (Lower Quartile)
At most, 25% of data is smaller than Q_1, It divides the lower half of a data set in half.

Q_2 : (Median)
50% of the data values fall below the median and 50% fall above.

Q_3 : (Upper Quartile)

At most, 25% of data is larger than Q_3 . It divides the upper half of the data set in half.

For arranged ungrouped data:

$$Q_i = i\left(\frac{n+1}{4}\right) th\ value, where\, i = 1, 2, 3$$

For grouped frequency distribution:

$$Q_i = l + \frac{h}{f}\left(\frac{i\sum f}{4} - C.f\right), where\, i = 1, 2, 3$$

where,

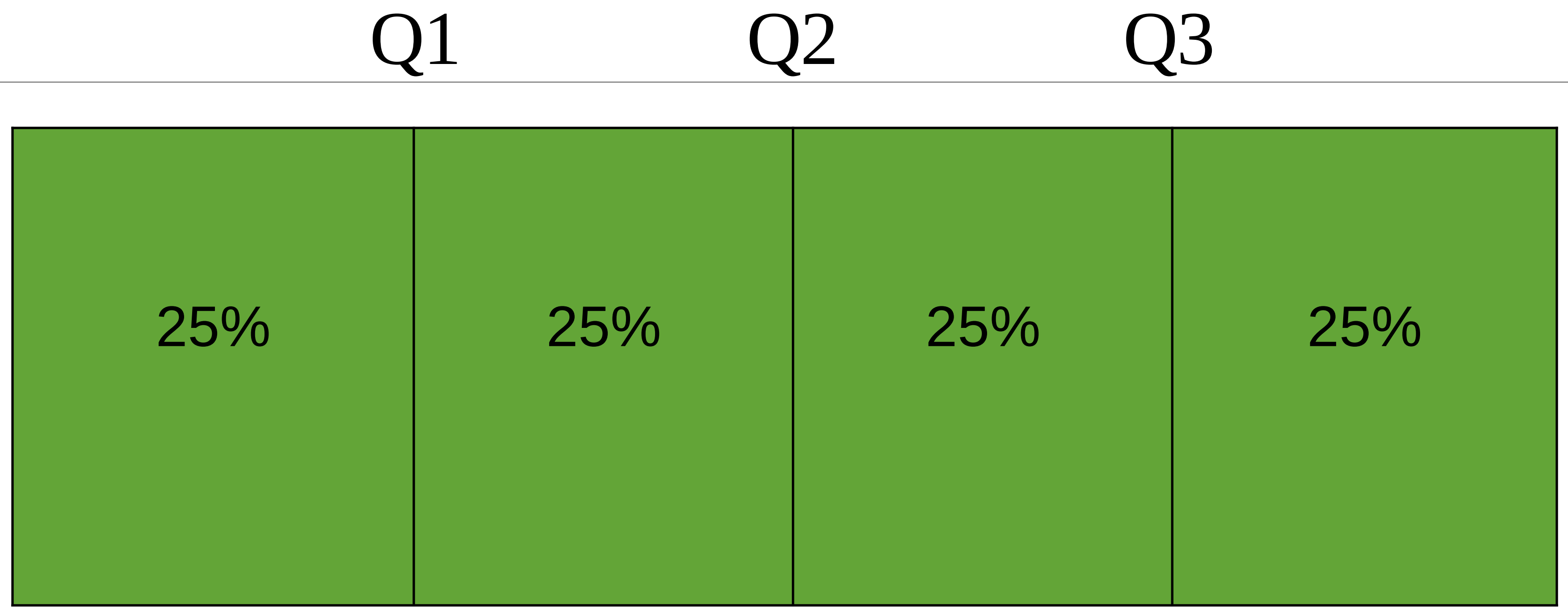
l = lower class boundary of quartile class

h = width of quartile class

f = frequency of quartile class

$\sum f$ = total frequency (n)

$C.f$ = cumulative frequency of preceding the quartile class


Q1
Q2
Q3
25%
25%
25%
25%

Example 1:

Find Q_1, Q_2 and Q_3 for the following data.

2, 3, 3, 9, 6, 6, 12, 11, 8, 2, 3, 5, 7, 5, 4, 4, 5, 12, 9

Solution :

First, arrange the data in ascending order:

2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 8, 9, 9, 11, 12, 12

5th Q_1 10th Q_2 15th Q_3

Here n = 19, then $Q_1 = \left(\frac{n+1}{4}\right)$th value $= \left(\frac{19+1}{4}\right) =$ 5th value = 3

$Q_1 = 3$

$Q_2 = 2\left(\frac{n+1}{4}\right)$th value $= 2\left(\frac{19+1}{4}\right) =$ 10th value = 5

$Q_2 = 5$

$Q_3 = 3\left(\frac{n+1}{4}\right)$th value $= 3\left(\frac{19+1}{4}\right) =$ 15th value = 9

$Q_3 = 9$

Example 2:

From the following grouped frequency distribution, Calculate Q_1 and Q_3

Wages in Rs.	150 - 170	170 - 190	190 - 210	210 - 230	230 - 250
No. of workers	30	50	80	30	10

Solution :

C.B.	f	C.f.
150 - 170	30	30
170 - 190	50	80
190 - 210	80	160
210 - 230	30	190
230 - 250	10	200
Total	200	

50th value

150th value

Since $Q_1 = \left(\frac{\sum f}{4}\right)$th value $= \left(\frac{200}{4}\right)$th value $=$ 50th value.

Therefore, class of Q_1 is (170 - 190)

$$Q_1 = l + \frac{h}{f}\left(\frac{\sum f}{4} - C.f.\right)$$

$$Q_1 = 170 + \frac{20}{50}(50 - 30) = 178$$

$Q_1 = 178$

Since $Q_3 = 3\left(\frac{\sum f}{4}\right)$th value $= 3\left(\frac{200}{4}\right)$th value $=$ 150th value

Therefore, class of Q_3 is (190 – 210)

$$Q_3 = l + \frac{h}{f}\left(\frac{3\sum f}{4} - C.f.\right) = 190 + \frac{20}{80}(150 - 80) = 207.5$$

$$Q_3 = 207.5$$

Note: Same as we did for finding median, for grouped frequency distribution, first we have to locate our quartile class. For that we find class boundaries to make our data continuous one and the cumulative frequencies to count our number of observations easily then we can determine our quartile class by finding where our $\frac{\sum f}{4}$th value lies in the data. The class interval is called quartile class

2: DECILES:

Values which divide the arranged data into **ten** equal parts, denoted by $D_1, D_2, D_3, ..., D_9$.

Note that D5 = Q2 = Median because all divides data into two equal parts.

For arranged ungrouped data:

$$\mathrm{D_i} = i\left(\frac{n+1}{10}\right) th\ value, where\ i = 1, 2, 3, 4, 5, 6, 7, 8, 9$$

Again remember that for grouped frequency distribution, first we have to locate our decile class. For that we find class boundaries to make our data continuous one and the cumulative frequencies to count our number of observations easily then we can determine our decile class by finding where our $\frac{\sum f}{10}$ th value lies in the data. The class interval is called decile class

For grouped frequency distribution:

$$\mathrm{D_i} = l + \frac{h}{f}\left(\frac{i\sum f}{10} - C.f\right), \mathit{where}\, i = 1, 2, 3, \ldots, 9$$

where,

l = lower class boundary of decile class

h = width of decile class

f = frequency of decile class

$\sum f$ = total frequency (n)

$C.f$ = cumulative frequency of preceding the decile class

Example 1:

Find D_4 and D_6 from the following weights in kg.

19, 27, 24, 39, 57, 44, 56, 50, 59, 67, 62, 42, 47, 60, 26, 34, 57, 51, 59, 45

Solution :

First we array the data. i.e. 19, 24, 26, 27, 34, 39, 44, 44, 45, 47, 50, 51, 56, 57, 57, 59, 59, 60, 62, 67. Here n = 20

$$D_4 = 4\left(\frac{n+1}{10}\right) \text{th value}$$

$$D_4 = 4\left(\frac{20+1}{10}\right) \text{th value} = 8.4\text{th value} = 8\text{th value} + 0.4[\,9\text{th value} - 8\text{th value}]$$

$$= 44 + 0.4\,[45 - 44] = 44 + 0.4 = 44.4 \text{ kg.}$$

$$D_4 = 44.4 \text{ kg}$$

$$D_6 = 6\left(\frac{n+1}{10}\right) \text{th value} = 6\left(\frac{20+1}{10}\right) \text{th value} = 12.6 \text{ th value}$$

$$D_6 = 12\text{th value} + 0.6\,[\,13\text{th value} - 12\text{th value}] = 51 + 0.6\,[\,56 - 51] = 54 \text{ kg.}$$

Example 2:

Calculate D_2 and D_3 from the following data:

x	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
f	7	18	25	30	20

Solution :

C.B.	f	C.f.
0 - 5	7	7
5 - 10	18	25
10 - 15	25	50
15 - 20	30	80
20 - 25	20	100
Total	100	

20th value

50th value

Since $D_2 = 2\left(\frac{\sum f}{10}\right)$th value $= 2\left(\frac{100}{10}\right)$th value $=$ 20th value.

Therefore, D_2 lies in the class $(5-10)$, then

$$D_2 = l + \frac{h}{f}\left(\frac{2\sum f}{10} - C.f.\right) = 5 + \frac{5}{18}(20-7) = 8.6$$

$$D_2 = 8.6$$

Since $D_3 = 3\left(\frac{\sum f}{10}\right)$th value $= 3\left(\frac{100}{10}\right)$th value $=$ 30th value .

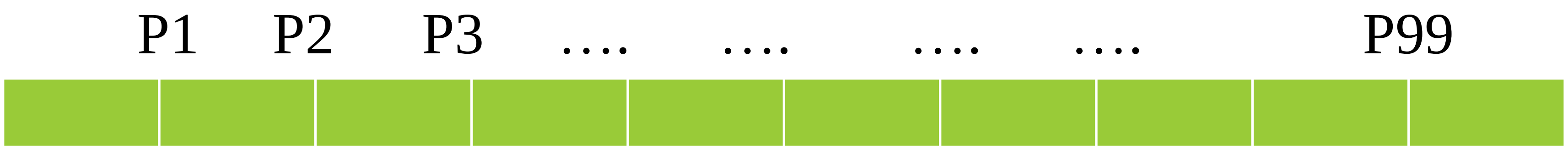
Therefore, D_3 lies in the class $(10-15)$, then

$$D_3 = l + \frac{h}{f}\left(\frac{3\sum f}{10} - C.f.\right) = 10 + \frac{5}{25}(30-25) = 11.0$$

$$D_3 = 11.0$$

3: PERCENTILES:

Values which divide the arranged data into **hundred** equal parts, denoted by $P_1, P_2, P_3, \ldots, P_{99}$.



Note that P50 = D5 = Q2 = Median because all divides data into two equal parts.

For arranged ungrouped data:

$$\mathrm{P}_{\mathrm{i}} = i\left(\frac{n+1}{100}\right) th\ value,\ where\ i = 1, 2, 3, \ldots, 99$$

Again remember that for grouped frequency distribution, first we have to locate our percentile class. For that we find class boundaries to make our data continuous one and the cumulative frequencies to count our number of observations easily then we can determine our percentile class by finding where our $\frac{\sum f}{100}$ th value lies in the data.

The class interval is called percentile class.

For grouped frequency distribution:

$$P_i = l + \frac{h}{f}\left(\frac{i\sum f}{100} - C.f\right), where\, i = 1, 2, 3, \ldots, 99$$

where,

l = lower class boundary of percentile class

h = width of percentile class

f = frequency of percentile class

$\sum f$ = total frequency (n)

$C.f$ = cumulative frequency of preceding the percentile class

For the following data calculate P_{25}, P_{59} and P_{80}

10, 11, 15, 16, 10, 12, 13, 14, 15, 14, 16, 17, 20, 14, 13 and 10

Solution :

First we arrange the data i.e.

10, 10, 10, 11, 12, 13, 13, 14, 14, 14, 15, 15, 16, 16, 17, 20

Here $n = 16$

then $P_{25} = 25\left(\frac{n+1}{100}\right)$th value

$$P_{25} = 25\left(\frac{16+1}{100}\right)\text{th value} = 4.25\text{th value}$$

$$= 4\text{th value} + 0.25\,[\,5\text{th value} - 4\text{th value}]$$

$$= 11 + 0.25\,[\,12 - 11] = 11.25$$

$$P_{59} = 59\left(\frac{n+1}{100}\right)\text{th value}$$

$$P_{59} = 59\left(\frac{16+1}{100}\right)\text{th value} = 10.03\text{th value}$$

$$= 10\text{th value} + 0.03\ [\ 11\text{th value} - 10\text{th value}]$$

$$= 14 + 0.03\ [\ 15 - 14] = 14.03$$

$$\text{Now } P_{80} = 80\left(\frac{n+1}{100}\right)\text{th value}$$

$$P_{80} = 80\left(\frac{16+1}{100}\right)\text{th value} = 13.6\text{th value}$$

$$= 13\text{th value} + 0.6\ [14\text{th value} - 13\text{th value}]$$

$$= 16 + 0.6\ [\ 16 - 16] = 16$$

Example 2:

For grouped frequency distribution, I picked the following data.

Find P_{20} , P_{60} and P_{75} from the following grouped frequency distribution.

Gr.	2-12	12-22	22-32	32-42	42-52	52-62	62-72	72-82	82-92	92-102
f	2	5	8	12	15	20	16	14	10	8

Solution :

Groups	*f*	*C.f.*
2 - 12	2	2
12 - 22	5	7
22 - 32	8	15
32 - 42	12	27
42 - 52	15	42
52 - 62	20	62
62 - 72	16	78
72 - 82	14	92
82 - 92	10	102
92 - 102	8	110
Total	110	

22nd value

66th value

Since, $P_{20} = 20\left(\frac{\sum f}{100}\right)$th value $= 20\left(\frac{110}{100}\right)$th value $= 22$nd value

Therefore, P_{20} lies in the group (32 – 40).

$$\text{Hence; } P_{20} = l + \frac{h}{f}\left(\frac{20\sum f}{100} - C.f.\right)$$

$$P_{20} = 32 + \frac{10}{12}(22 - 15) = 37.83$$

$$P_{20} = 37.83$$

Since $P_{60} = 60\left(\frac{\sum f}{100}\right)$th value $= 60\left(\frac{110}{100}\right)$th value $=$ 66th value.

Therefore, P_{60} lies in the group $(62 - 72)$

$$\text{Hence; } P_{60} = l + \frac{h}{f}\left(\frac{60\sum f}{100} - C.f.\right)$$

$$P_{60} = 62 + \frac{10}{12}(66 - 62) = 64.50$$

$$P_{60} = 64.50$$

TASK:

You can choose any grouped frequency data or you may use your previous data which you used for your last task, calculate Q1, Q3, D4, D8, P15 and P75.