

Back propagate algorithm Q3 solution

two layer neural network
three inputs - two neuron in hidden layer - two output
hidden function = sigmoid - output function = linear

initial weight hidden layer $\theta_{ij}(1) = [0.3 \ 0.3 \ 0.7; 0.9 \ 0.6 \ 0.1]$
initial weight output layer $\theta_{ij}(2) = [0.9 \ 0.4; 0.2 \ 0.1]$
learning rate $\eta = 1$

iteration (1)

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input $x(1)$ = $[0.8; 1; 0.9]$
Target $y(1)$ = $[1; 1]$

step(1): Forward (calculate the output of NN)

$Z(2) = \theta(1) * x_i$ = $[1.17; 1.41]$
 $a(2) = \text{sigmoid}(Z(2))$ = $[0.76; 0.8]$
 $Z(3) = \theta(2) * a(2)$ = $[1; 0.23]$
 $a(3) = Z(3)$ = $[1; 0.23]$

step(2): find the error

$er = y - a(3)$ = $[0; 0.77]$

step(3): propagate the error to the output layer

$\delta(3) = er * dg(Z(2))$ = $[0; 0.77]$
 $D\theta(2) = \eta * \delta(3) * a(2)$ = $[0 \ 0.59; 0 \ 0.62]$

step(4): propagate the error to the hidden layer

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er2 =  $\theta(2)^T * \delta(3)$           =    [0.15;0.08]
dg(Z(2)) = a(2).(1-a(2))        =    [0.18;0.16]
 $\delta(2)$  = er2*dg(Z(2))          =    [0.03;0.01]
D $\theta(1)$  =  $\alpha * \delta(2) * x$     =    [0.02 0.01;0.03 0.01;0.03 0.01]

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step(5) cost =      0.30
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step(6): Update Wiegths :
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 $\theta(1)$  =  $\theta(1) + D\theta(1)$       =    [0.32 0.91;0.33 0.61;0.73 0.11]
 $\theta(2)$  =  $\theta(2) + D\theta(1)$       =    [0.9 0.79;0.4 0.72]

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iteration (2)
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    input x(1)          =    [0.8;1;0.9]
Target y(1)             =    [1;1]

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step(1): Forword (cal the output of NN)
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Z(2) =  $\theta(1) * x_i$               =    [1.24;1.44]
a(2) = sigmoid(Z(2))      =    [0.78;0.81]
Z(3) =  $\theta(2) * a(2)$             =    [1.03;1.2]
a(3) = Z(3)                =    [1.03;1.2]

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step(2): find the error
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er = y - a(3)            =    [-0.03;-0.2]

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step(3): propagate the error to the output layer
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 $\delta(3)$  = er*dg(Z(2))          =    [-0.03;-0.2]

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$$D\theta(2) = \alpha * \delta(3) * a(2) = [-0.02 \ -0.16; -0.02 \ -0.16]$$

step(4): propagate the error to the hidden layer

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$$\begin{aligned} \text{er2} &= \theta(2)^T * \delta(3) &= [-0.19; -0.16] \\ \text{dg}(Z(2)) &= a(2) * (1 - a(2)) &= [0.17; 0.15] \\ \delta(2) &= \text{er2} * \text{dg}(Z(2)) &= [-0.03; -0.02] \\ D\theta(1) &= \alpha * \delta(2) * x &= [-0.02 \ -0.02; -0.03 \ -0.02; -0.03 \ -0.02] \end{aligned}$$

$$\text{step(5) cost} = 0.02$$

step(6): Update Wiegths :

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$$\begin{aligned} \theta(1) &= \theta(1) + D\theta(1) &= [0.3 \ 0.89; 0.3 \ 0.59; 0.7 \ 0.09] \\ \theta(2) &= \theta(2) + D\theta(1) &= [0.88 \ 0.63; 0.38 \ 0.56] \end{aligned}$$

iteration (3)

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$$\begin{aligned} \text{input } x(1) &= [0.8; 1; 0.9] \\ \text{Target } y(1) &= [1; 1] \end{aligned}$$

step(1): Forward (cal the output of NN)

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$$\begin{aligned} Z(2) &= \theta(1) * x_i &= [1.17; 1.38] \\ a(2) &= \text{sigmoid}(Z(2)) &= [0.76; 0.8] \\ Z(3) &= \theta(2) * a(2) &= [0.97; 0.93] \\ a(3) &= Z(3) &= [0.97; 0.93] \end{aligned}$$

step(2): find the error

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$$\text{er} = y - a(3) = [0.03; 0.07]$$

step(3): propagate the error to the output layer

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δ(3) = er*dg(Z(2))          = [0.03;0.07]  
Dθ(2) = α*δ(3)*a(2)         = [0.02 0.05;0.02 0.06]
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step(4): propagate the error to the hidden layer

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er2 = θ(2)T * δ(3)          = [0.07;0.05]  
dg(Z(2)) = a(2).(1-a(2))    = [0.18;0.16]  
δ(2) = er2*dg(Z(2))         = [0.01;0.01]  
Dθ(1) = α*δ(2)*x            = [0.01 0.01;0.01 0.01;0.01 0.01]
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step(5) cost = 0.00

step(6): Update Wiegths :

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-----  
θ(1) = θ(1) + Dθ(1)         = [0.31 0.9;0.31 0.6;0.71 0.1]  
θ(2) = θ(2) + Dθ(1)         = [0.9 0.68;0.4 0.62]
```

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