Polynomial regression with regularization

Membrane pressure drop When purifying drinking water, you can use a so-called membrane filtration. In an experiment one wishes to examine the relationship between the pressure drop across a membrane and the flux (flow per area) through the membrane. We observe the following 10 related values of pressure (x) and flux (y):

Use **polynomial regression with regularization** (gradient descent algorithm) to fit the following data to the hypothesis.

$\mathbf{h_\theta (x) = \theta_0 + \theta_1 * x^1 + \theta_2 * x^2 + \theta_3 * x^3 + \theta_4 * x^4 + \theta_5 * x^5 + \theta_6 * x^6}$

Pressure (x)	1.02	2.08	2.89	4.01	5.32	5.83	7.26	7.96	9.11	9.99
Flux (y)	1.15	0.85	1.56	1.72	4.32	5.07	5.00	5.31	6.17	7.04

let initial $\theta_0 = \theta_1 = \theta_2 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0$ **, learning rate** $\alpha = 0.005$**, regularization parameter** $\lambda = 0.5$

calculate two epochs

solution:

1- Normalize features x

$X\_scaled = \frac{\text{x-mean}}{\text{std}}$

2- You are free to regularize θ_0 **or not**

in this example θ_0 **is regularized for simplicity of calculations.**

3- Cost function: $J(\theta) = \frac{1}{2m} \sum err^2 + \frac{\lambda}{2m} \sum \theta^2$

4- Weight update: $\theta_j = \theta_j * \left(1 - \frac{\alpha\lambda}{m}\right) - \frac{\alpha}{m} \sum err * x^j$

1st epoch:

#	x	y	h = ∑θ_i*x^i	err = h-y	err*x	err*x^2	err*x^3	err*x^4	err*x^5	err*x^6	err^2
1	-1.4884	1.1500	0	-1.1500	1.7117	-2.5476	3.7919	-5.6439	8.4003	-12.5030	1.3225
2	-1.1399	0.8500	0	-0.8500	0.9689	-1.1045	1.2590	-1.4351	1.6359	-1.8647	0.7225
3	-0.8736	1.5600	0	-1.5600	1.3628	-1.1906	1.0401	-0.9086	0.7938	-0.6934	2.4336
4	-0.5053	1.7200	0	-1.7200	0.8691	-0.4392	0.2219	-0.1121	0.0567	-0.0286	2.9584
5	-0.0746	4.3200	0	-4.3200	0.3223	-0.0240	0.0018	-0.0001	0.0000	0.0000	18.6624
6	0.0930	5.0700	0	-5.0700	-0.4715	-0.0439	-0.0041	-0.0004	0.0000	0.0000	25.7049
7	0.5632	5.0000	0	-5.0000	-2.8160	-1.5860	-0.8932	-0.5031	-0.2833	-0.1596	25.0000
8	0.7933	5.3100	0	-5.3100	-4.2124	-3.3417	-2.6510	-2.1030	-1.6683	-1.3235	28.1961
9	1.1714	6.1700	0	-6.1700	-7.2275	-8.4663	-9.9175	-11.6173	-13.6085	-15.9410	38.0689
10	1.4608	7.0400	0	-7.0400	-10.2840	-15.0229	-21.9455	-32.0579	-46.8302	-68.4096	49.5616
∑				-38.1900	-19.7767	-33.7667	-29.0966	-54.3816	-51.5038	-100.9236	192.6309

	θ0	θ1	θ2	θ3	θ4	θ5	θ6	∑
	0.0191	0.0099	0.0169	0.0145	0.0272	0.0258	0.0505	
θ^2	0.0004	0.0001	0.0003	0.0002	0.0007	0.0007	0.0025	0.0049

cost	9.6317

2^{nd} epoch

#	x	y	h = ∑θ_i*x^i	err = h-y	err*x	err*x^2	err*x^3	err*x^4	err*x^5	err*x^6	err^2
1	**-1.4884**	**1.1500**	**0.4878**	**-0.6622**	**0.9857**	**-1.4670**	**2.1836**	**-3.2500**	**4.8373**	**-7.1998**	**0.4385**
2	**-1.1399**	**0.8500**	**0.1153**	**-0.7347**	**0.8375**	**-0.9547**	**1.0883**	**-1.2405**	**1.4141**	**-1.6119**	**0.5398**
3	**-0.8736**	**1.5600**	**0.0388**	**-1.5212**	**1.3289**	**-1.1609**	**1.0142**	**-0.8860**	**0.7740**	**-0.6762**	**2.3140**
4	**-0.5053**	**1.7200**	**0.0183**	**-1.7017**	**0.8599**	**-0.4345**	**0.2195**	**-0.1109**	**0.0561**	**-0.0283**	**2.8958**
5	**-0.0746**	**4.3200**	**0.0184**	**-4.3016**	**0.3209**	**-0.0239**	**0.0018**	**-0.0001**	**0.0000**	**0.0000**	**18.5034**
6	**0.0930**	**5.0700**	**0.0202**	**-5.0498**	**-0.4696**	**-0.0437**	**-0.0041**	**-0.0004**	**0.0000**	**0.0000**	**25.5007**
7	**0.5632**	**5.0000**	**0.0384**	**-4.9616**	**-2.7944**	**-1.5738**	**-0.8864**	**-0.4992**	**-0.2811**	**-0.1583**	**24.6172**
8	**0.7933**	**5.3100**	**0.0763**	**-5.2337**	**-4.1519**	**-3.2937**	**-2.6129**	**-2.0728**	**-1.6444**	**-1.3045**	**27.3920**
9	**1.1714**	**6.1700**	**0.3156**	**-5.8544**	**-6.8578**	**-8.0333**	**-9.4102**	**-11.0231**	**-12.9124**	**-15.1256**	**34.2740**
10	**1.4608**	**7.0400**	**0.9004**	**-6.1396**	**-8.9687**	**-13.1015**	**-19.1387**	**-27.9579**	**-40.8408**	**-59.6603**	**37.6948**
∑				**-36.1606**	**-18.9096**	**-30.0871**	**-27.5449**	**-47.0409**	**-48.5974**	**-85.7650**	**174.1703**

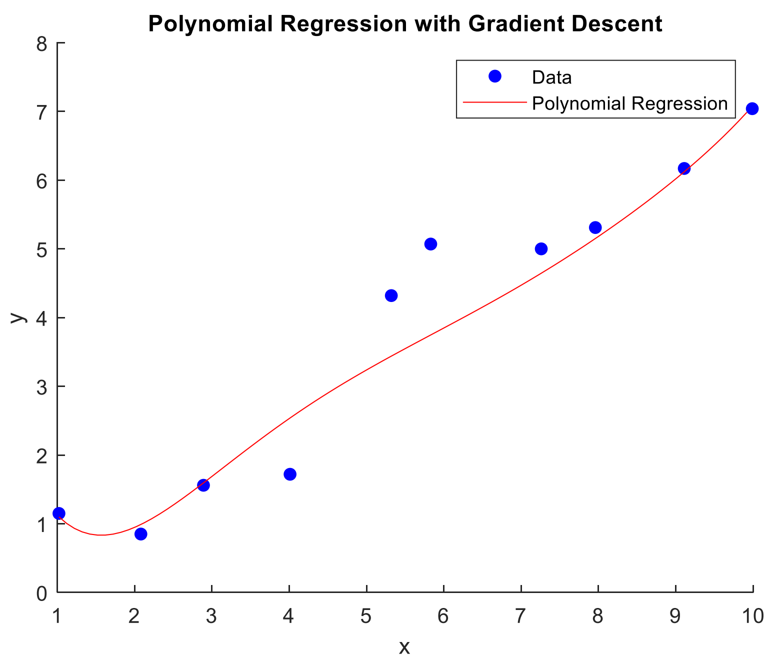
	θ0	θ1	θ2	θ3	θ4	θ5	θ6	∑
	0.0181	**0.0095**	**0.0150**	**0.0138**	**0.0235**	**0.0243**	**0.0429**	
θ^2	**0.0003**	**0.0001**	**0.0002**	**0.0002**	**0.0006**	**0.0006**	**0.0018**	**0.0038**

cost	**8.7086**

Cost 2 < cost 1

After 100000 iterations

θ0	**3.5773**
θ1	**1.8358**
θ2	**-0.1659**
θ3	**0.5668**
θ4	**-0.0559**
θ5	**-0.2135**
θ6	**0.1109**



At $\lambda = 0$ (no regularization)