Supplement to "Extended Comparisons of Best Subset Selection, Forward Stepwise Selection, and the Lasso"

Trevor Hastie Robert Tibshirani Ryan J. Tibshirani

This supplementary document contains plots from the simulation suite described in the paper "Extended Comparisons of Best Subset Selection, Forward Stepwise Selection, and the Lasso". The plots in Section A precisely follow the simulation format described in the paper. Those in Section B follow an analogous format, except that the tuning has been done using an "oracle", rather than a validation set as in Section A. Specifically, the tuning parameter for each method in each scenario is chosen to minimize the average risk over all of the repetitions. Sections A and B compare best subset selection, forward stepwise regression, the lasso and the relaxed lasso. In Sections C and D we have added three more methods to the comparisons: L0Learn-1, L0Learn-2 and SparseNet.

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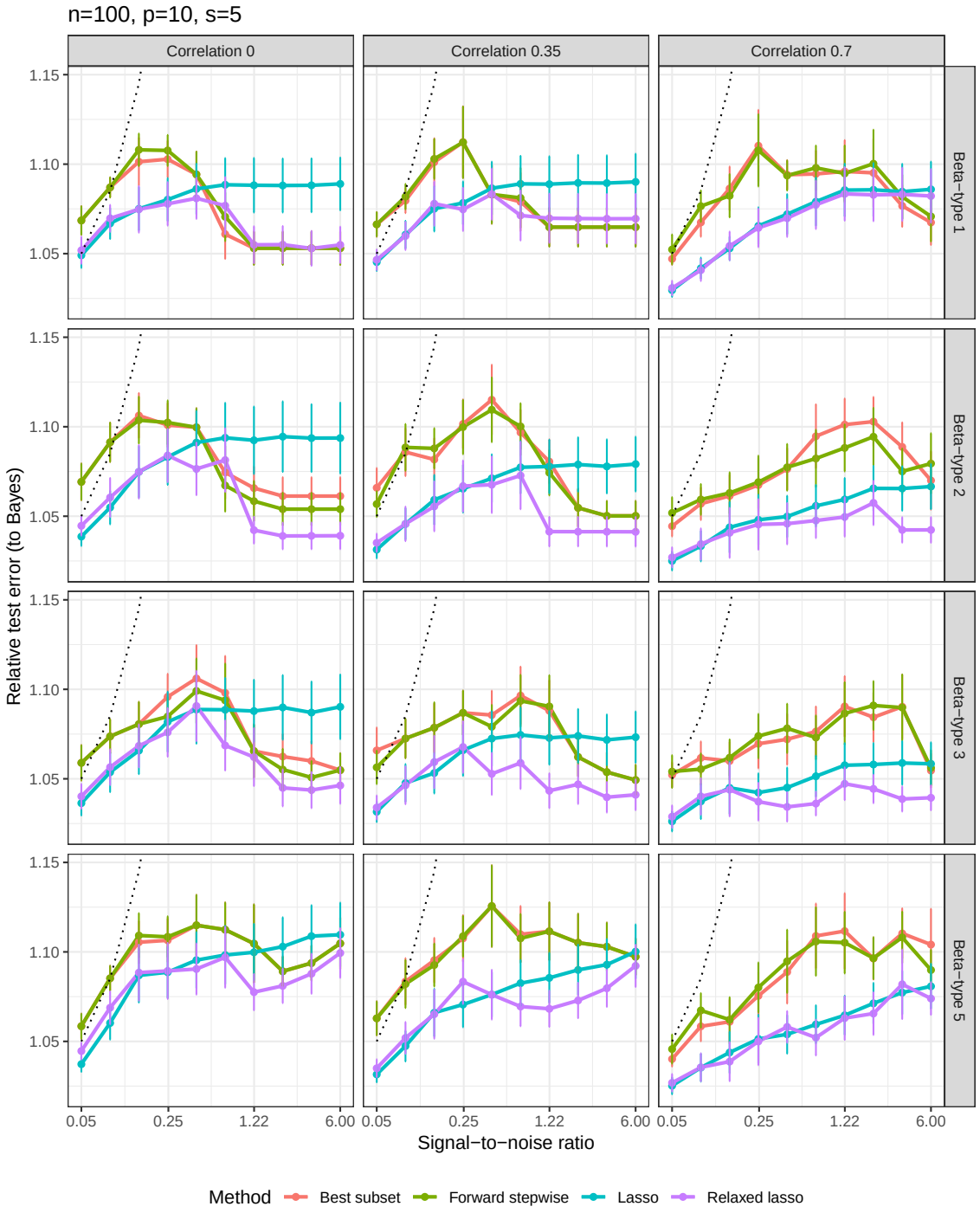
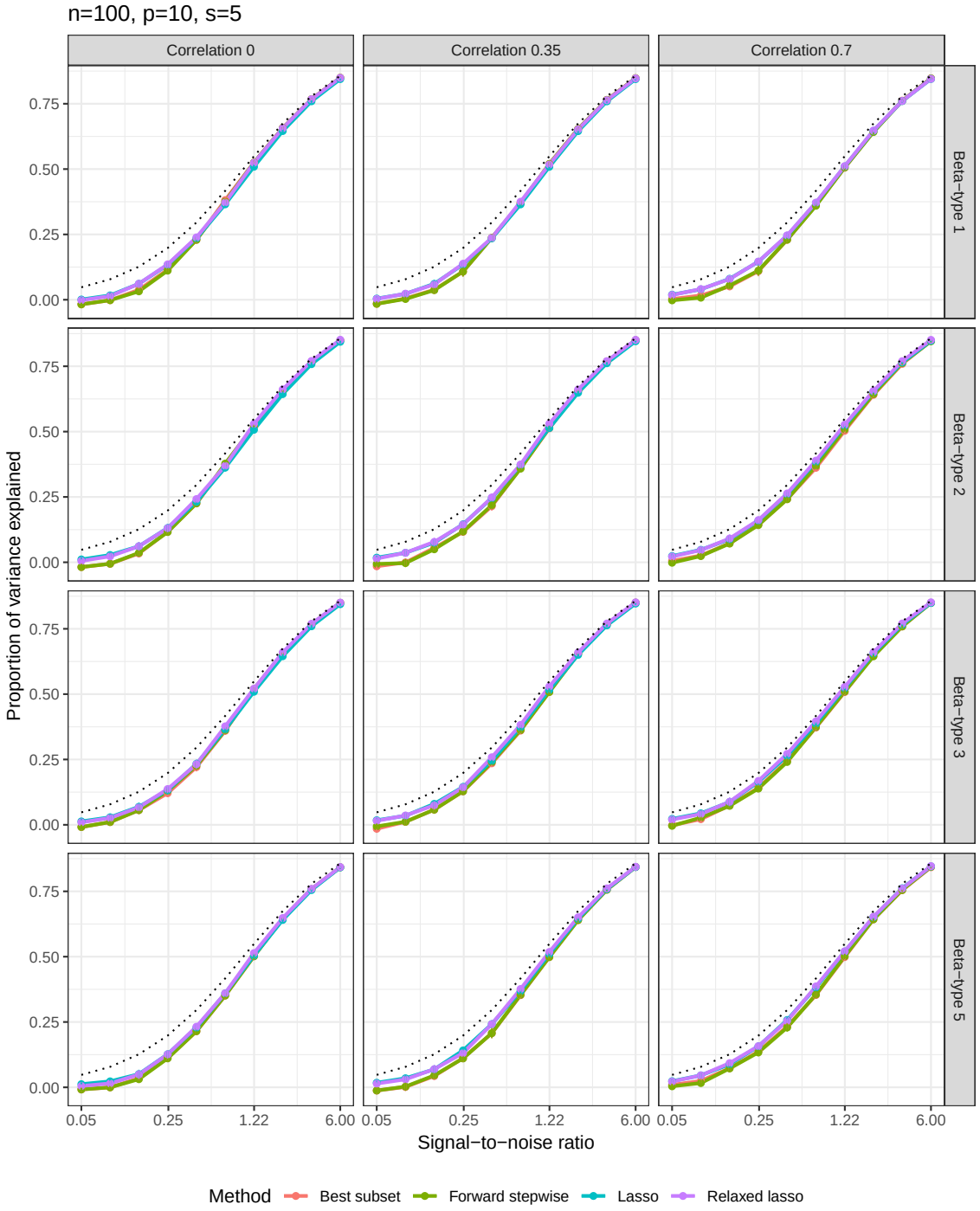
A Validation tuning

A.1 Low setting: $n = 100$, $p = 10$, $s = 5$

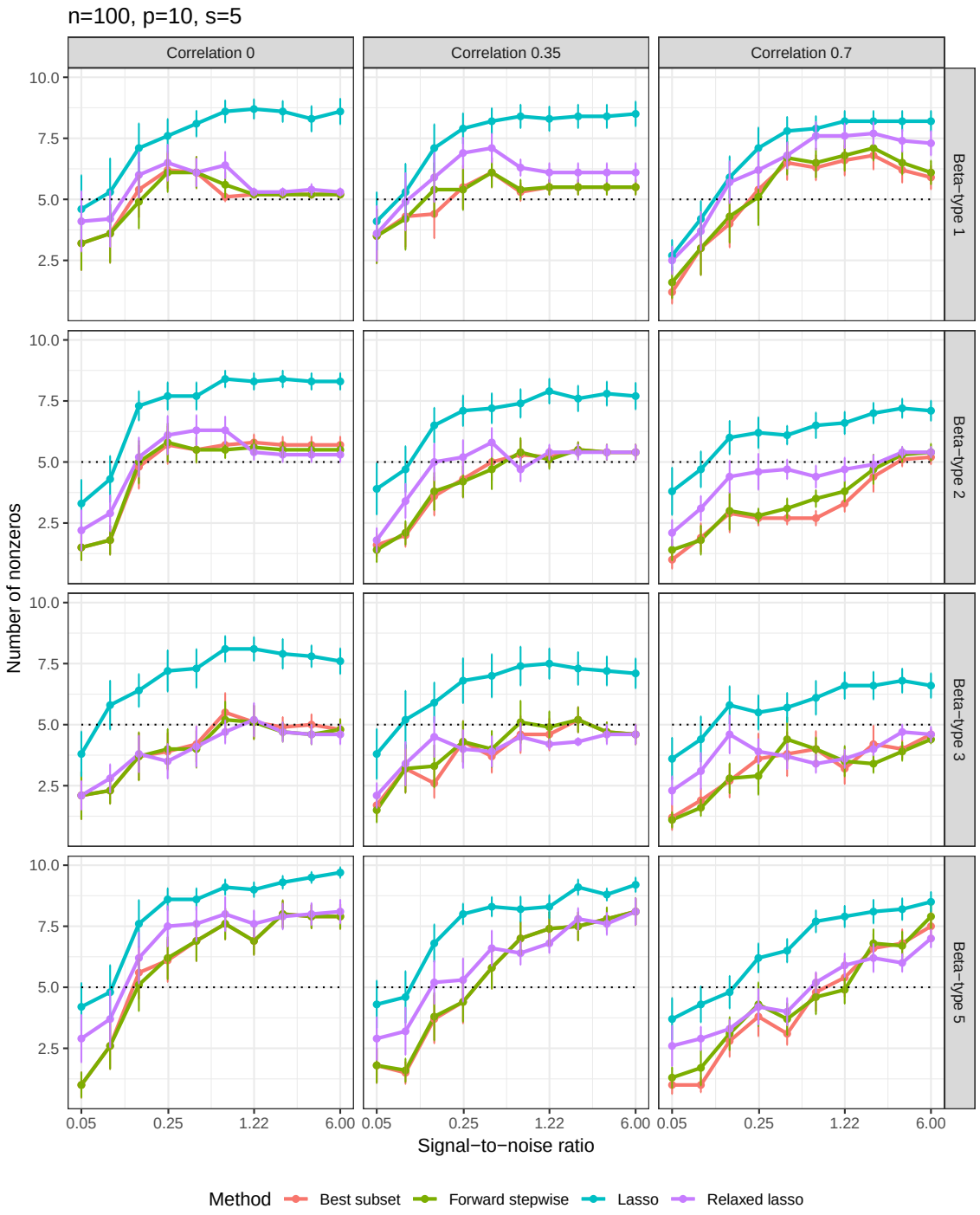
A.1.1 Relative test error (to Bayes)



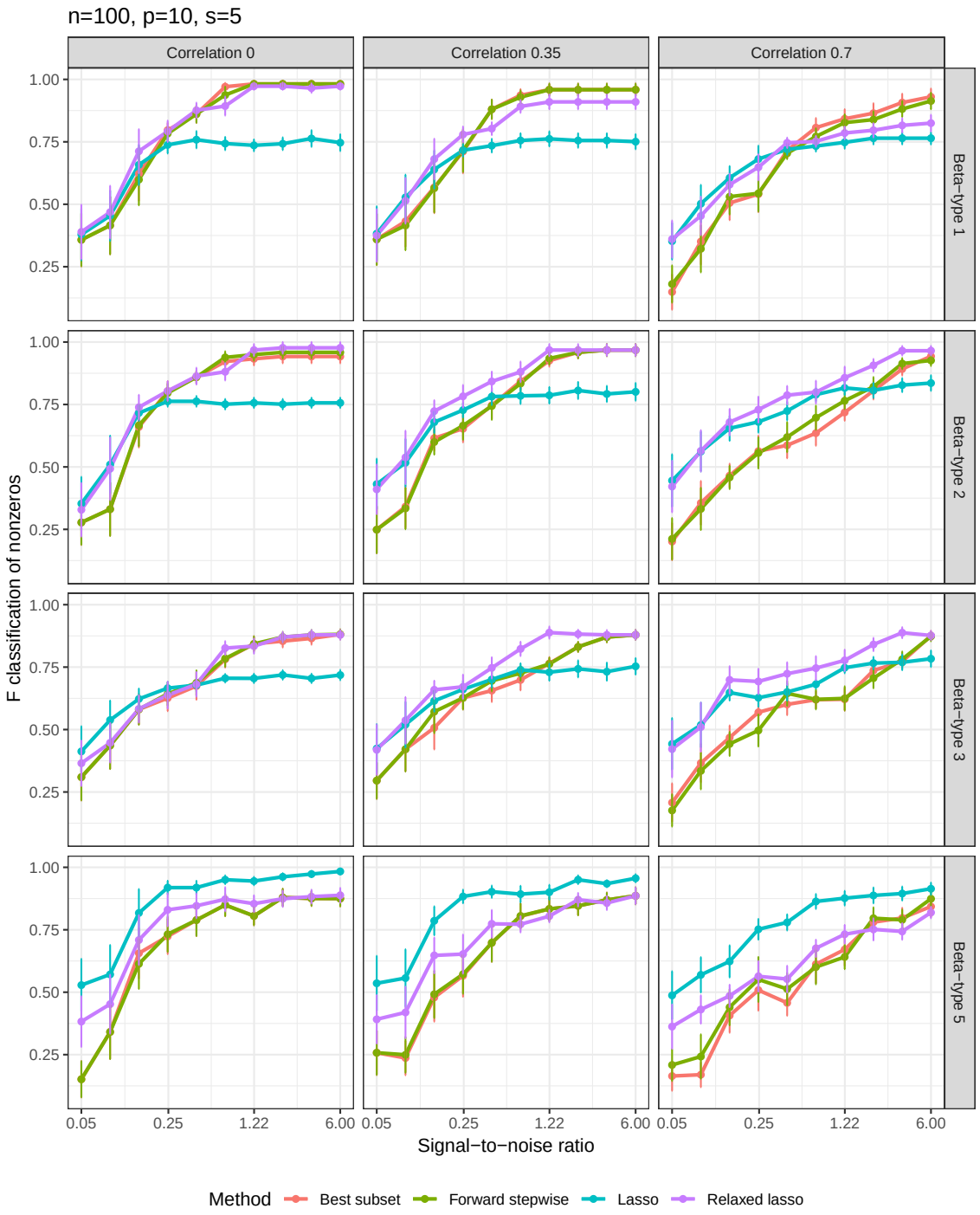
A.1.2 Proportion of variance explained



A.1.3 Number of nonzero coefficients

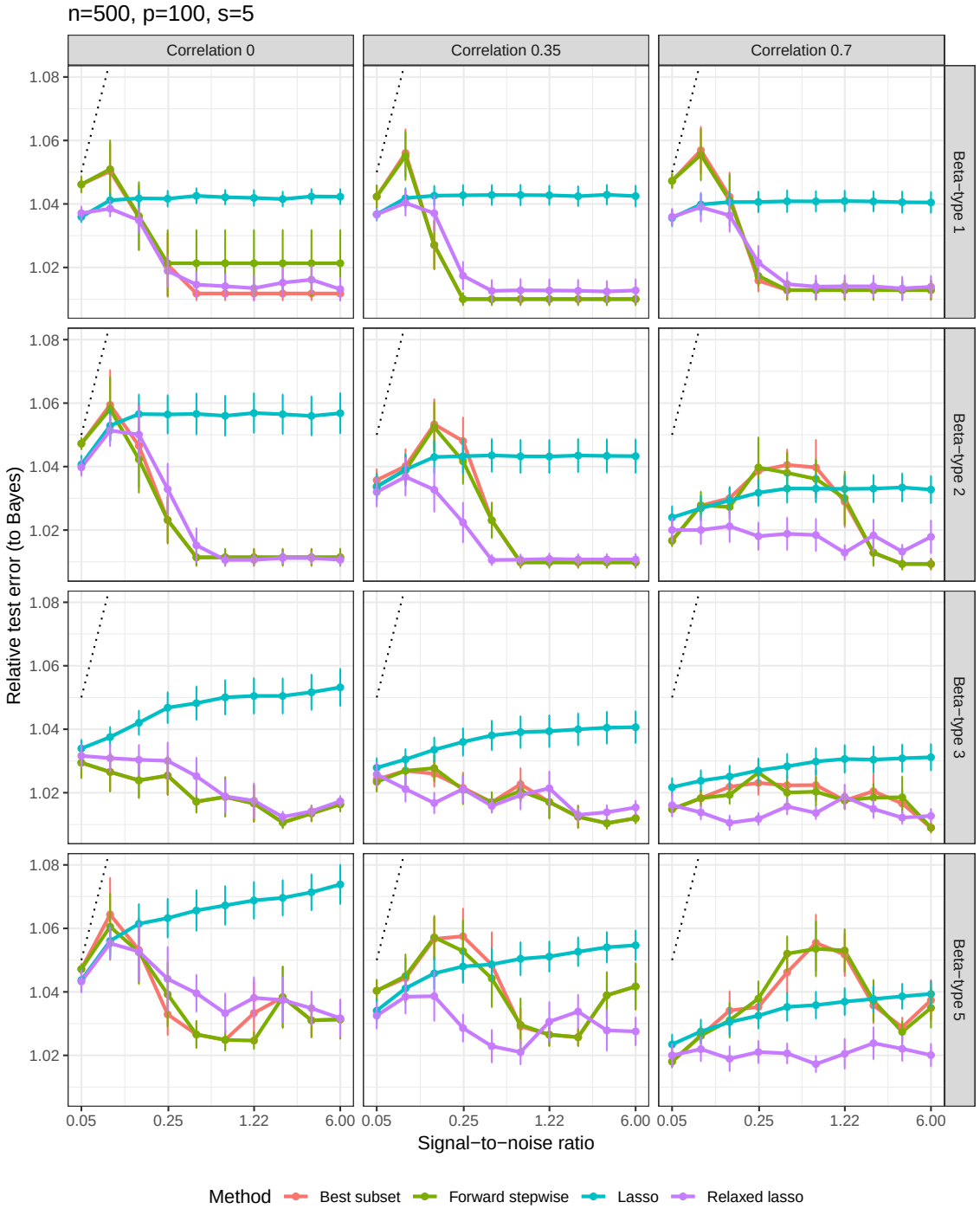


A.1.4 F-score

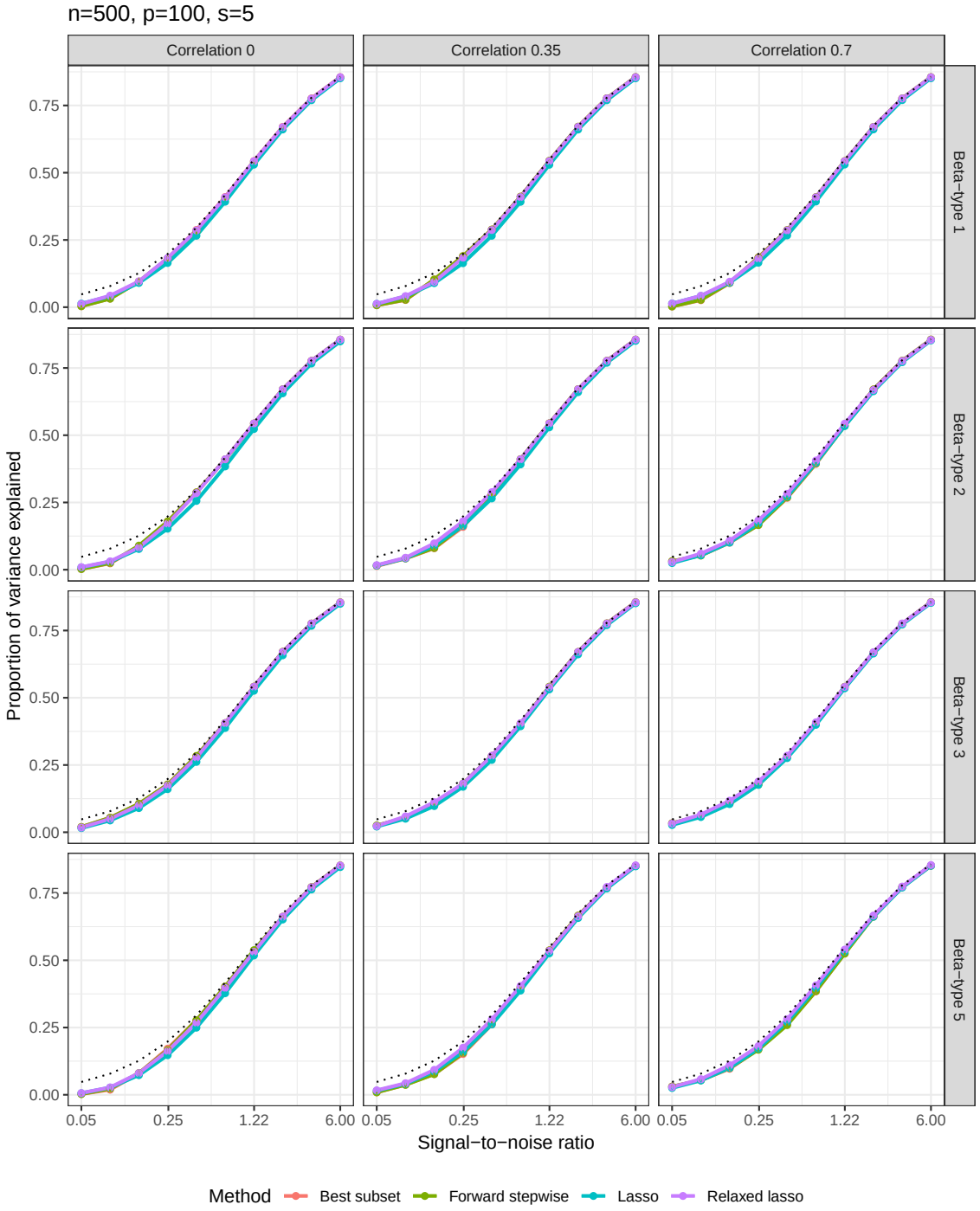

n=100, p=10, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

A.2 Medium setting: $n = 500$, $p = 100$, $s = 5$

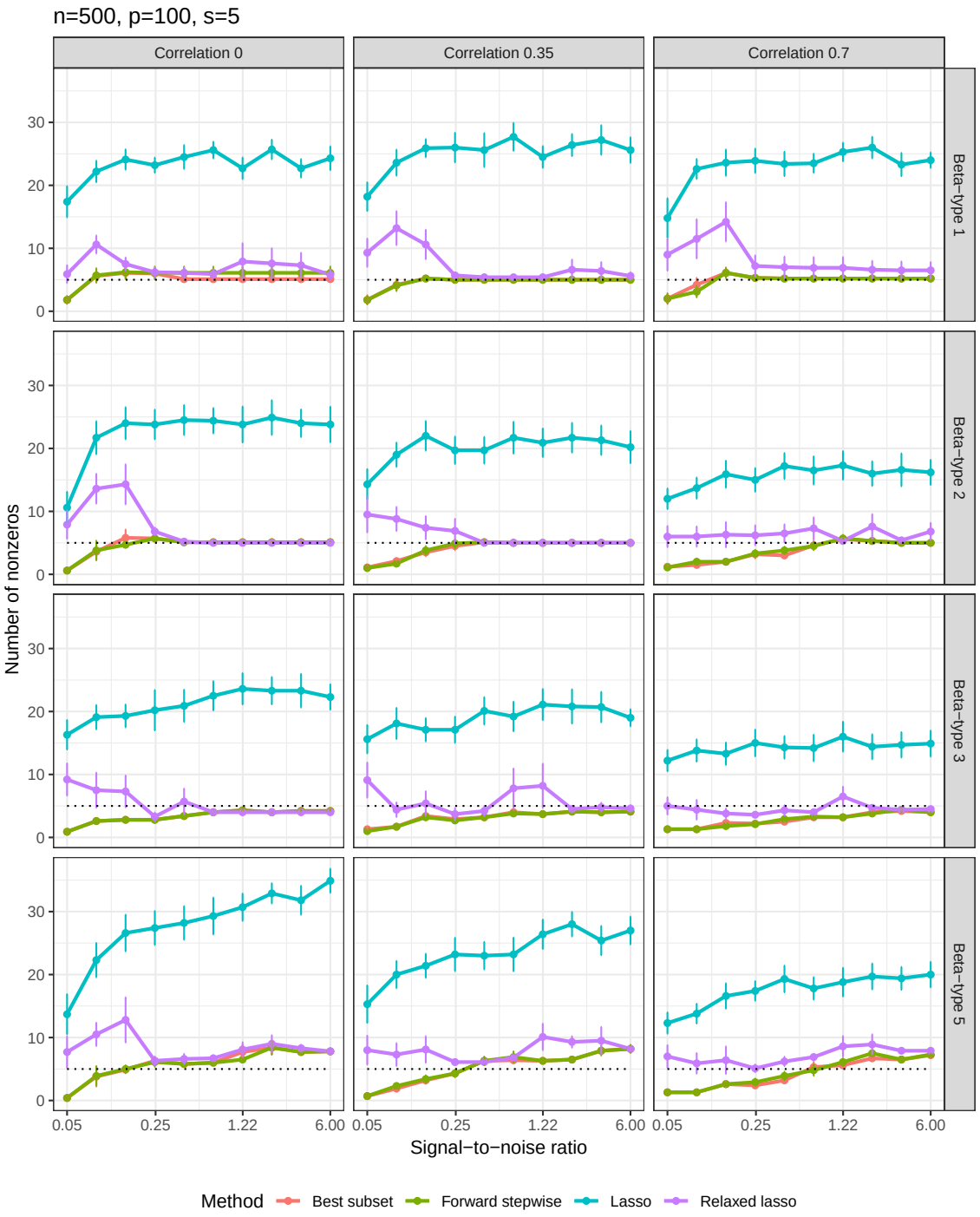
A.2.1 Relative test error (to Bayes)



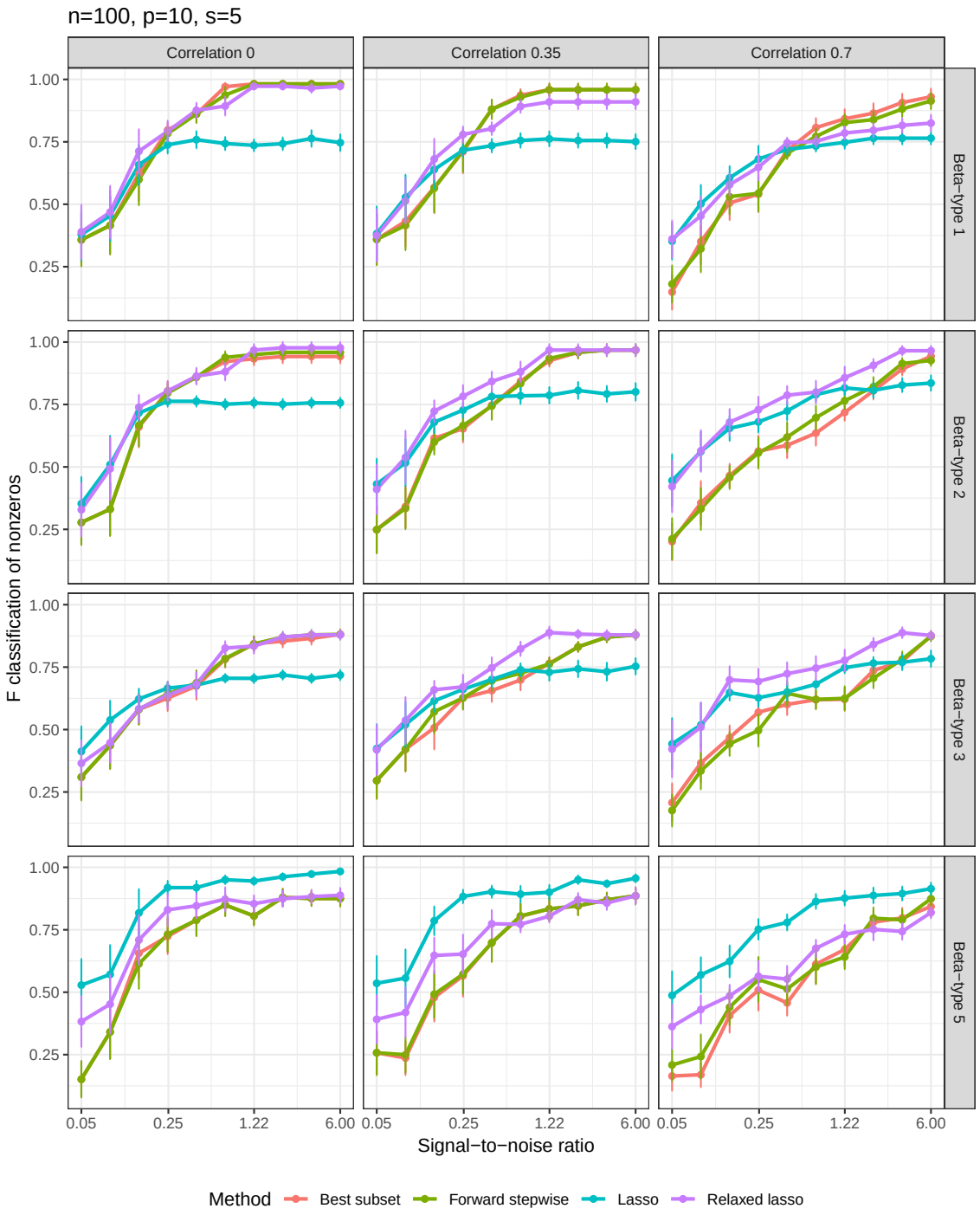
A.2.2 Proportion of variance explained



A.2.3 Number of nonzero coefficients



A.2.4 F-score


n=100, p=10, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
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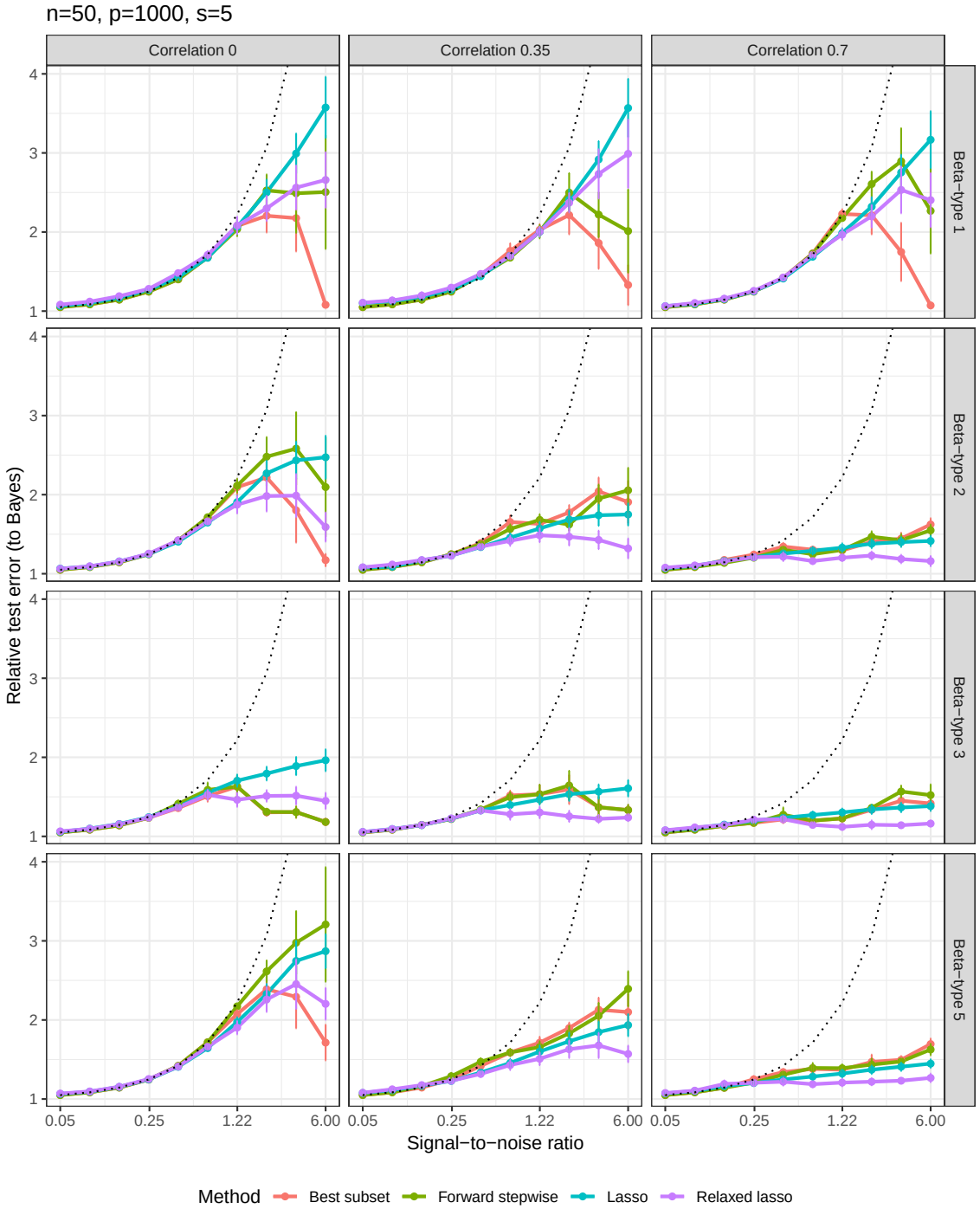
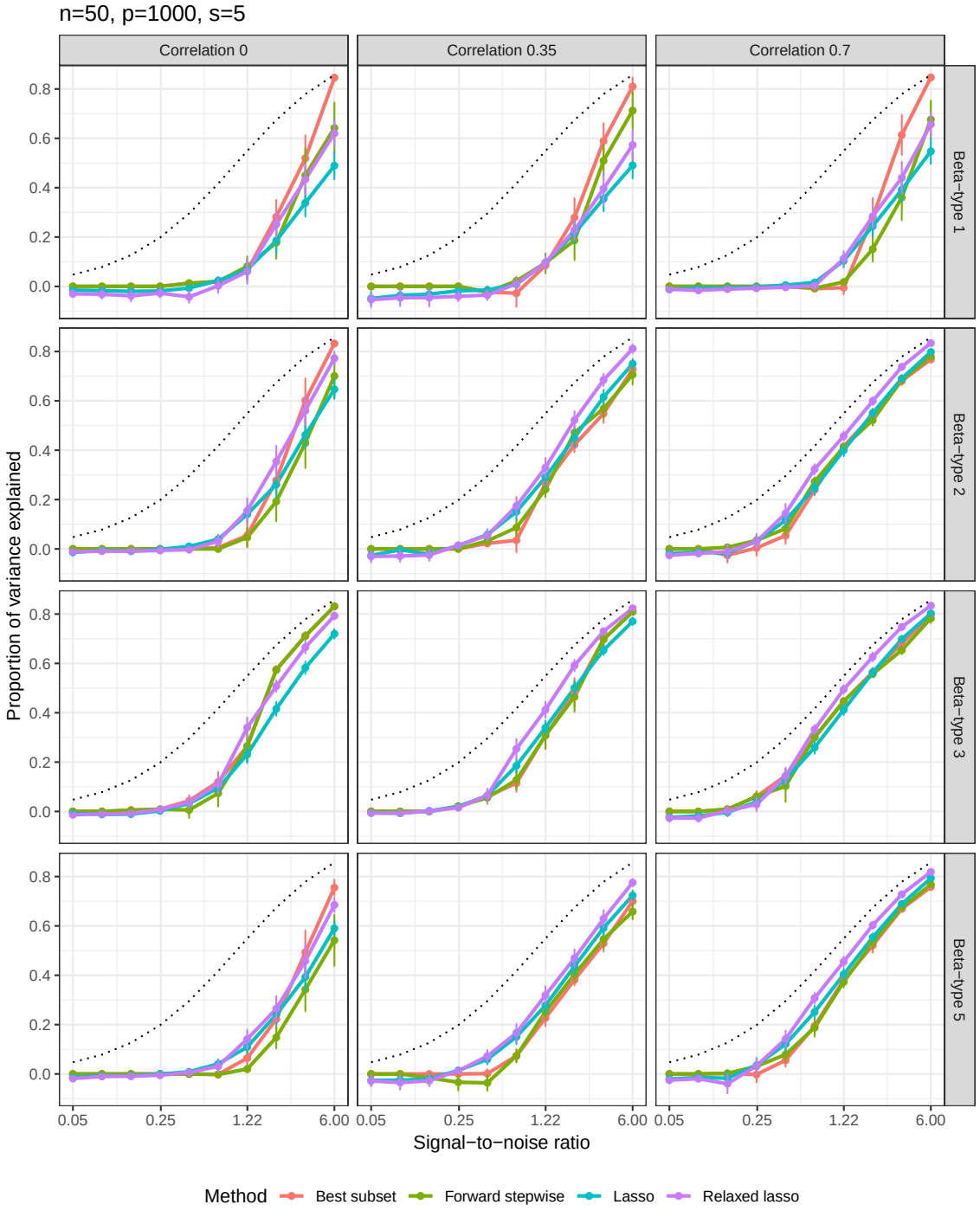
0.50
0.25
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

A.3 High-5 setting: $n = 50$, $p = 1000$, $s = 5$

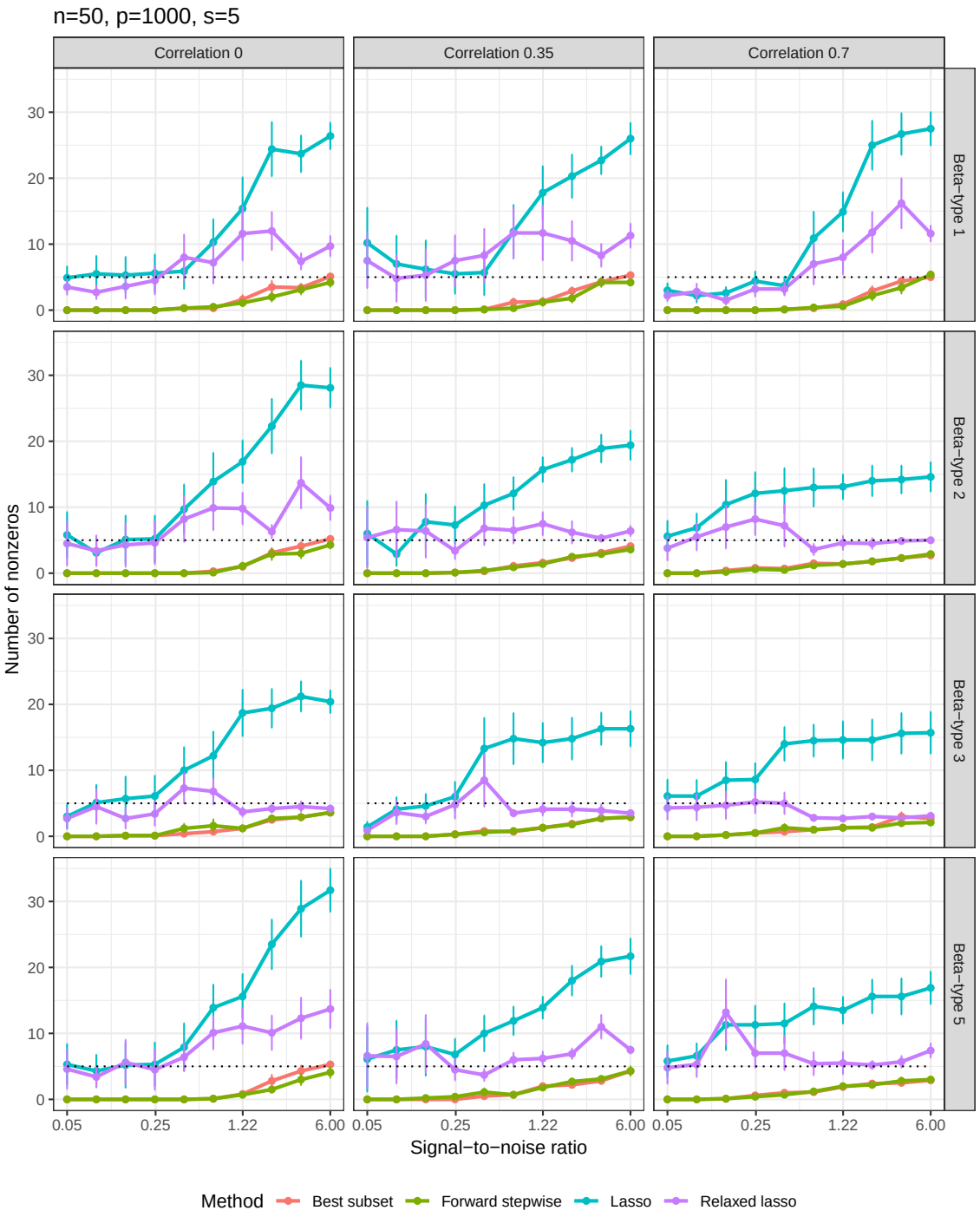
A.3.1 Relative test error (to Bayes)



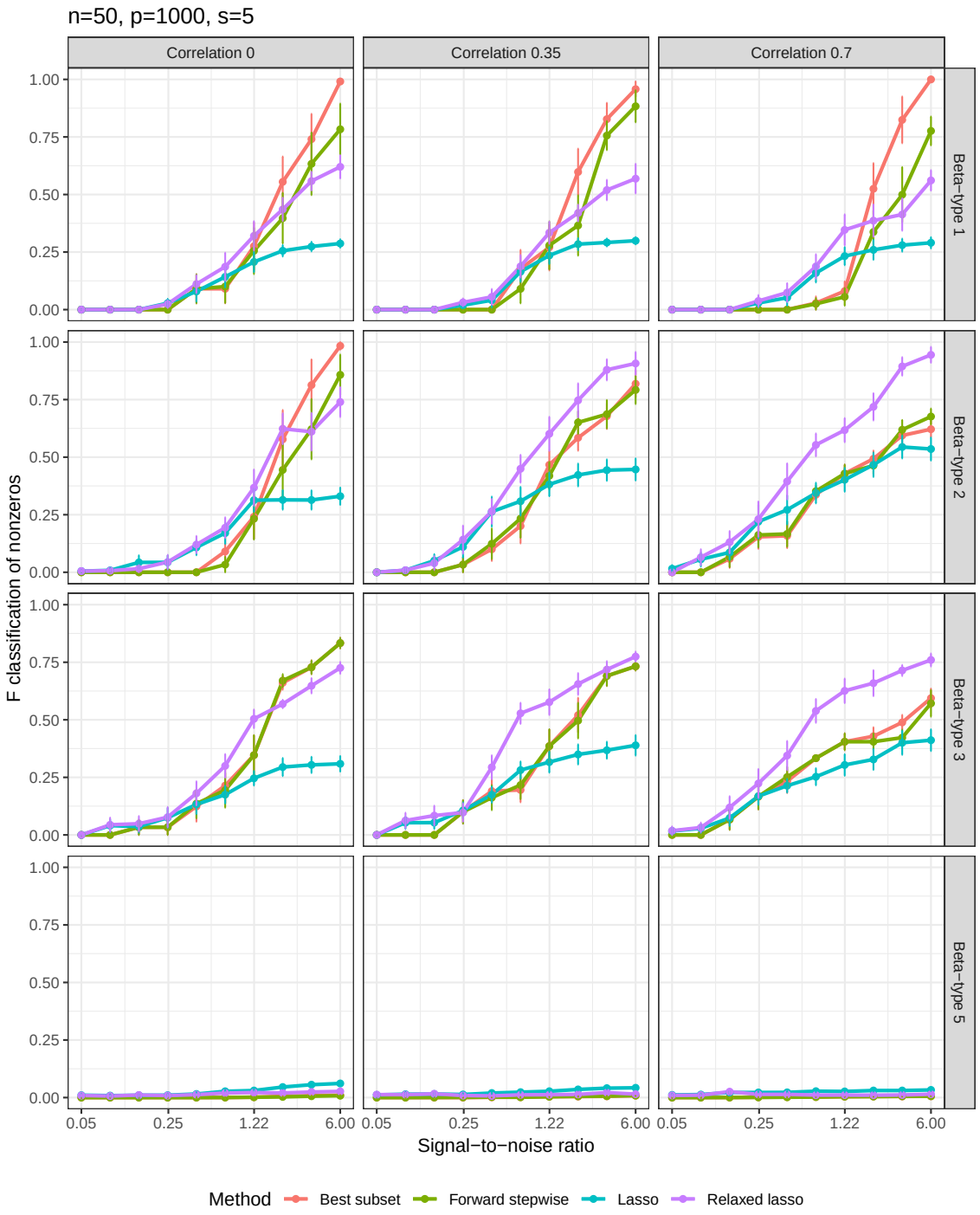
A.3.2 Proportion of variance explained



A.3.3 Number of nonzero coefficients

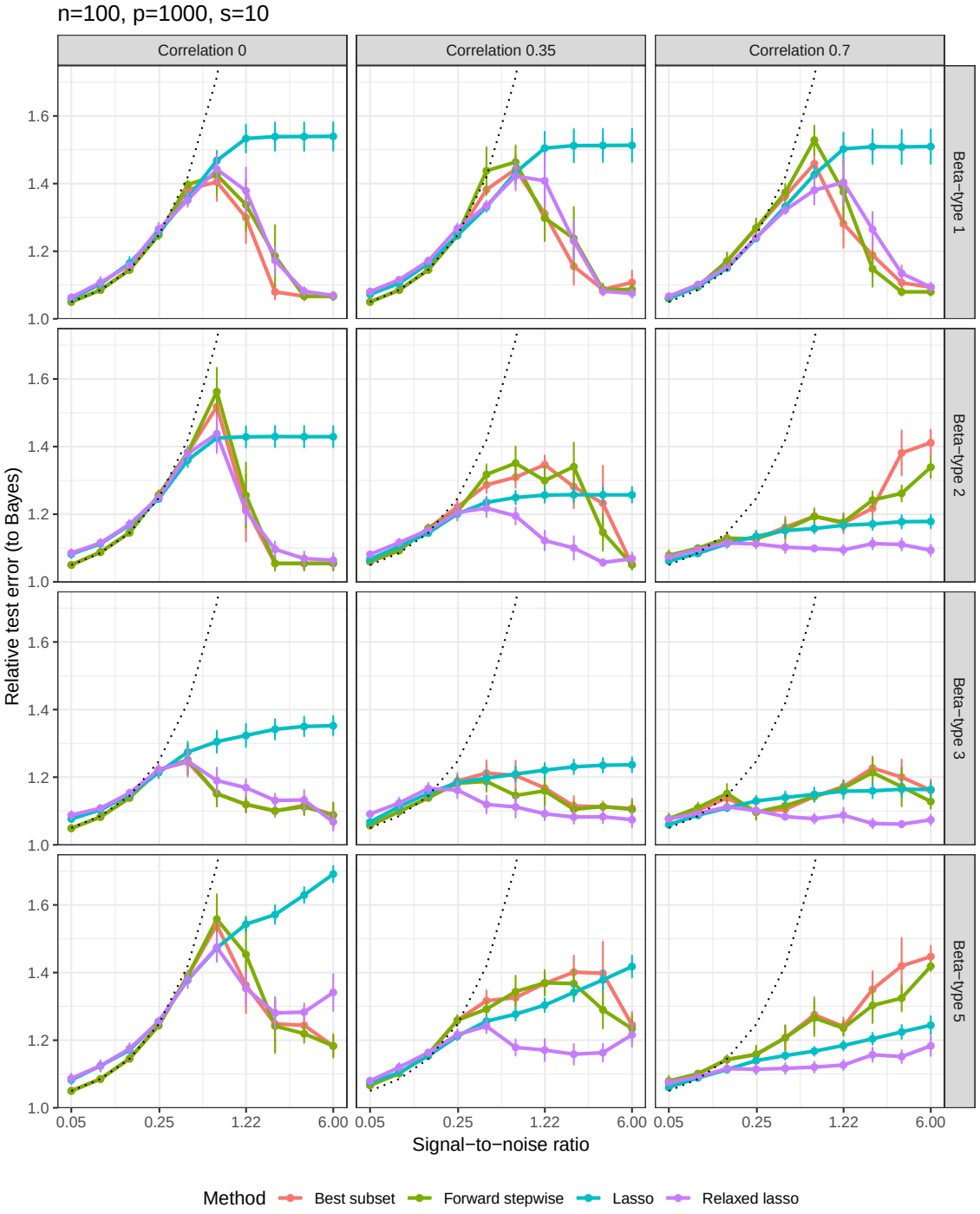


A.3.4 F-score

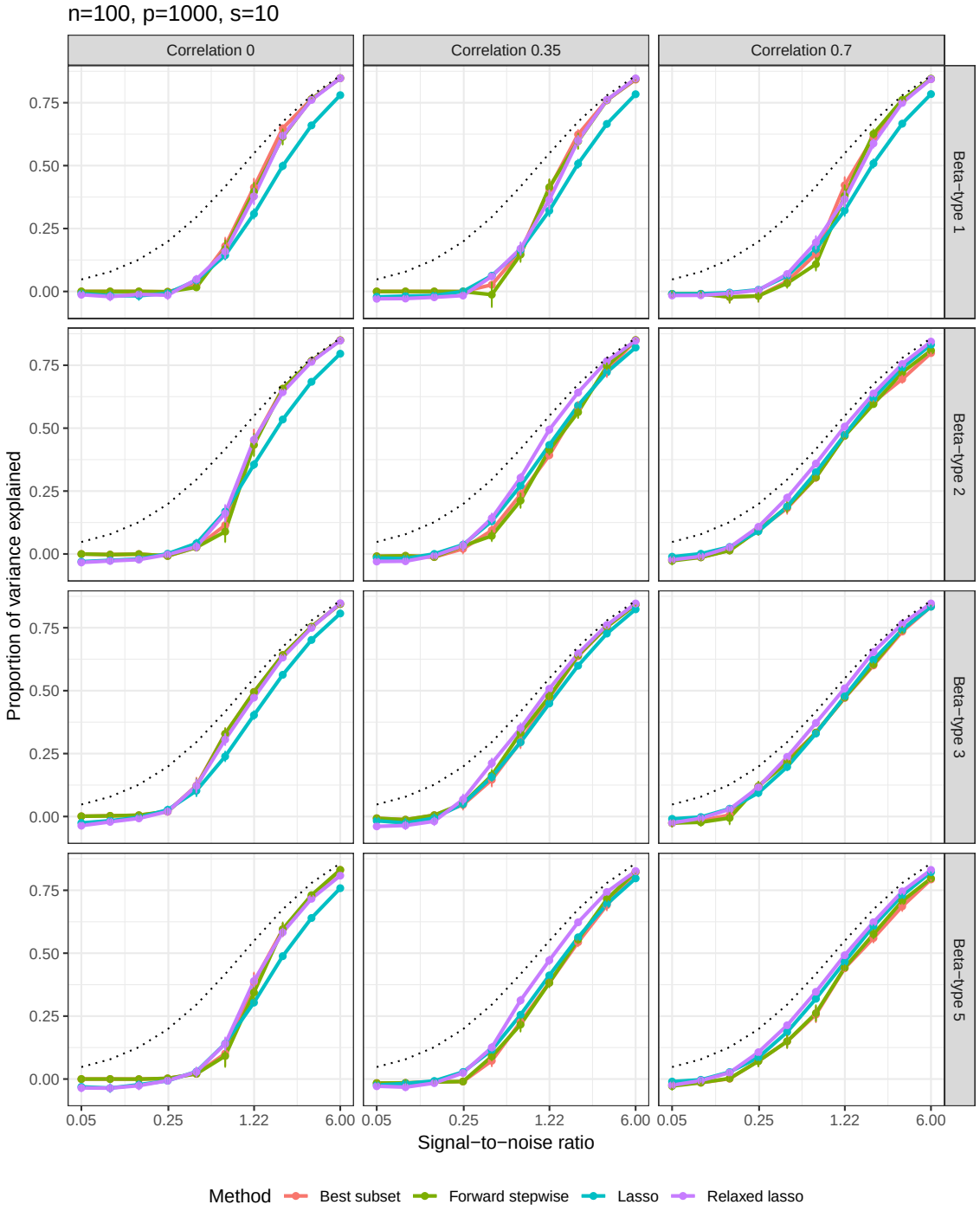

n=50, p=1000, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

A.4 High-10 setting: $n = 100$, $p = 1000$, $s = 10$

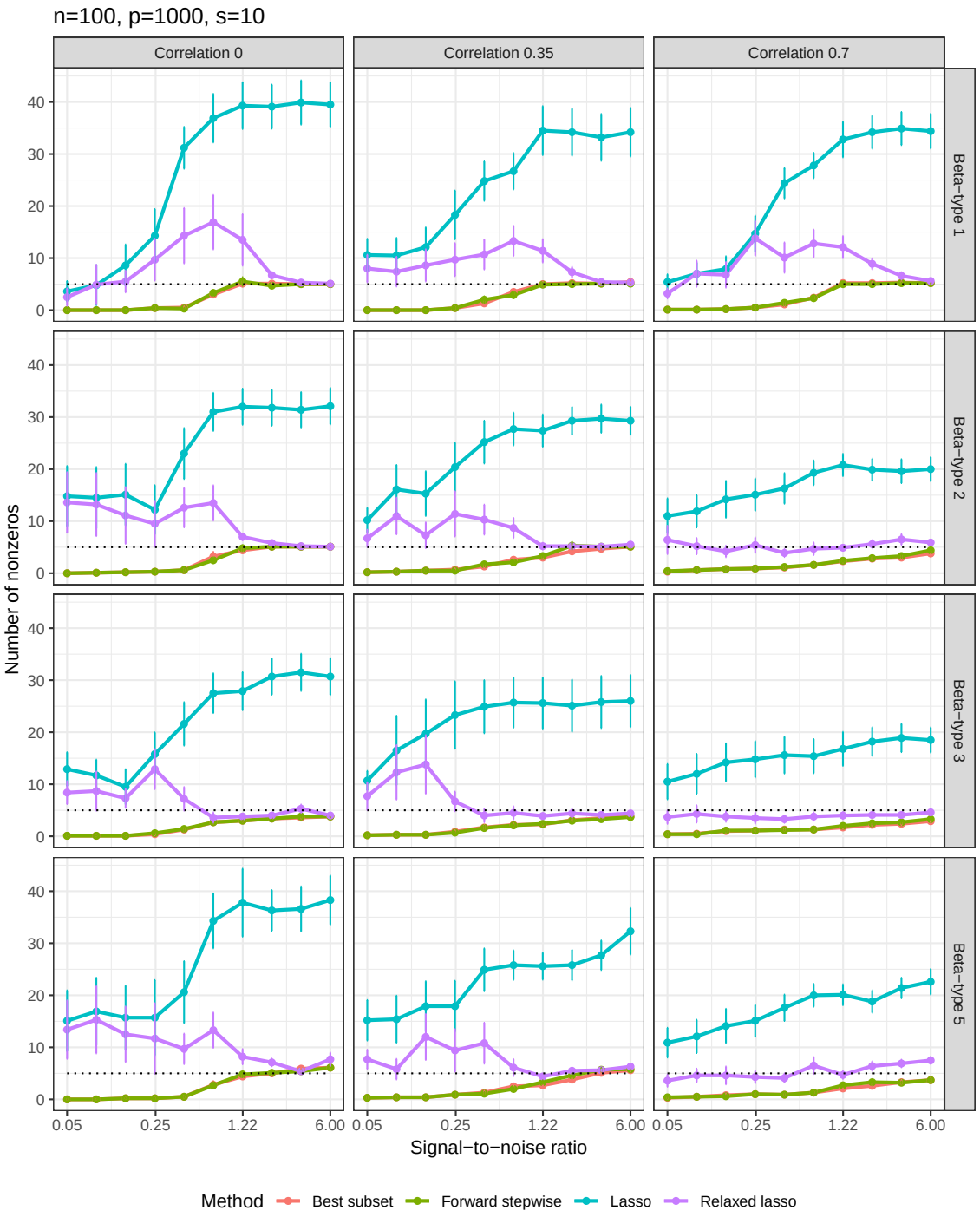
A.4.1 Relative test error (to Bayes)



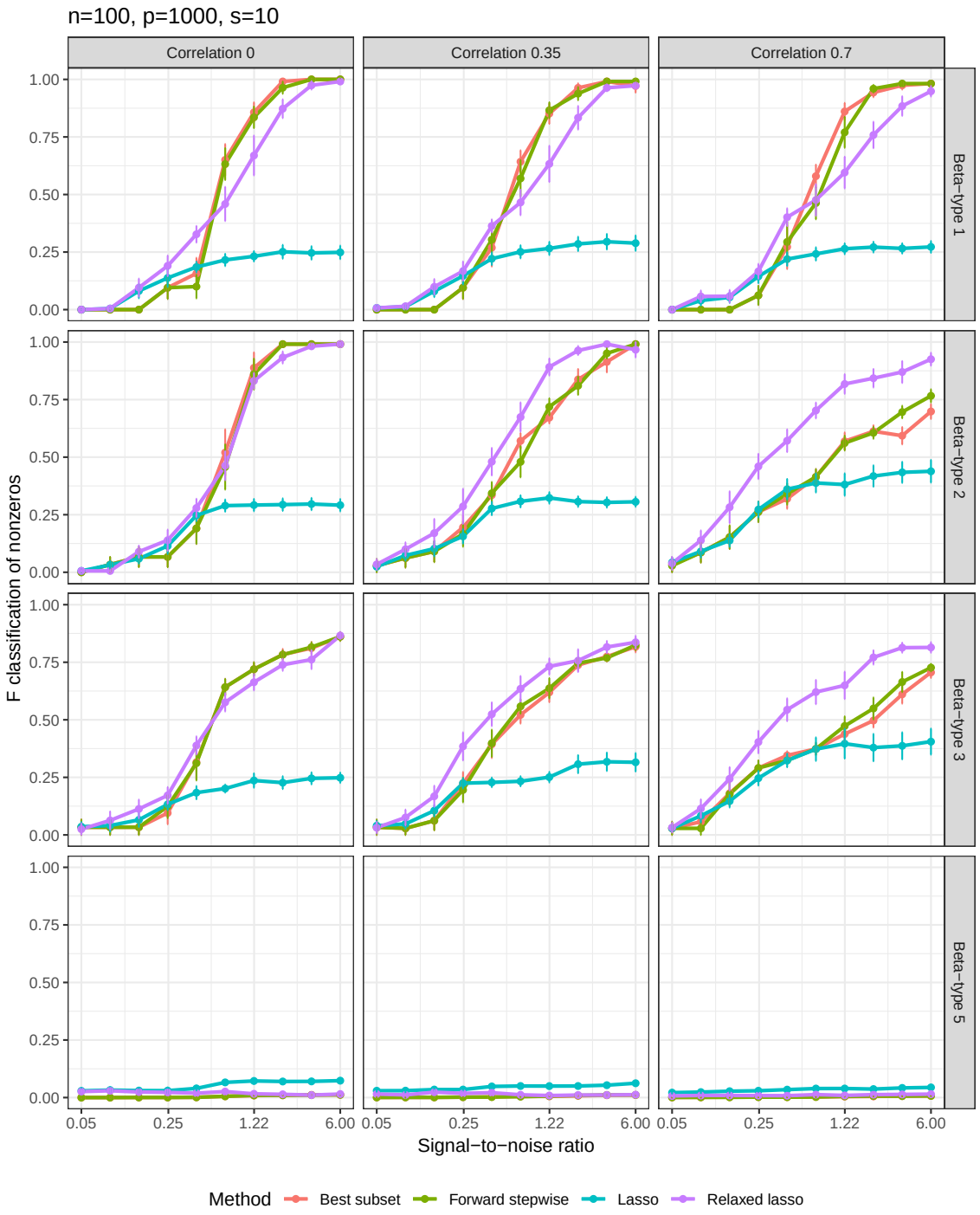
A.4.2 Proportion of variance explained



A.4.3 Number of nonzero coefficients



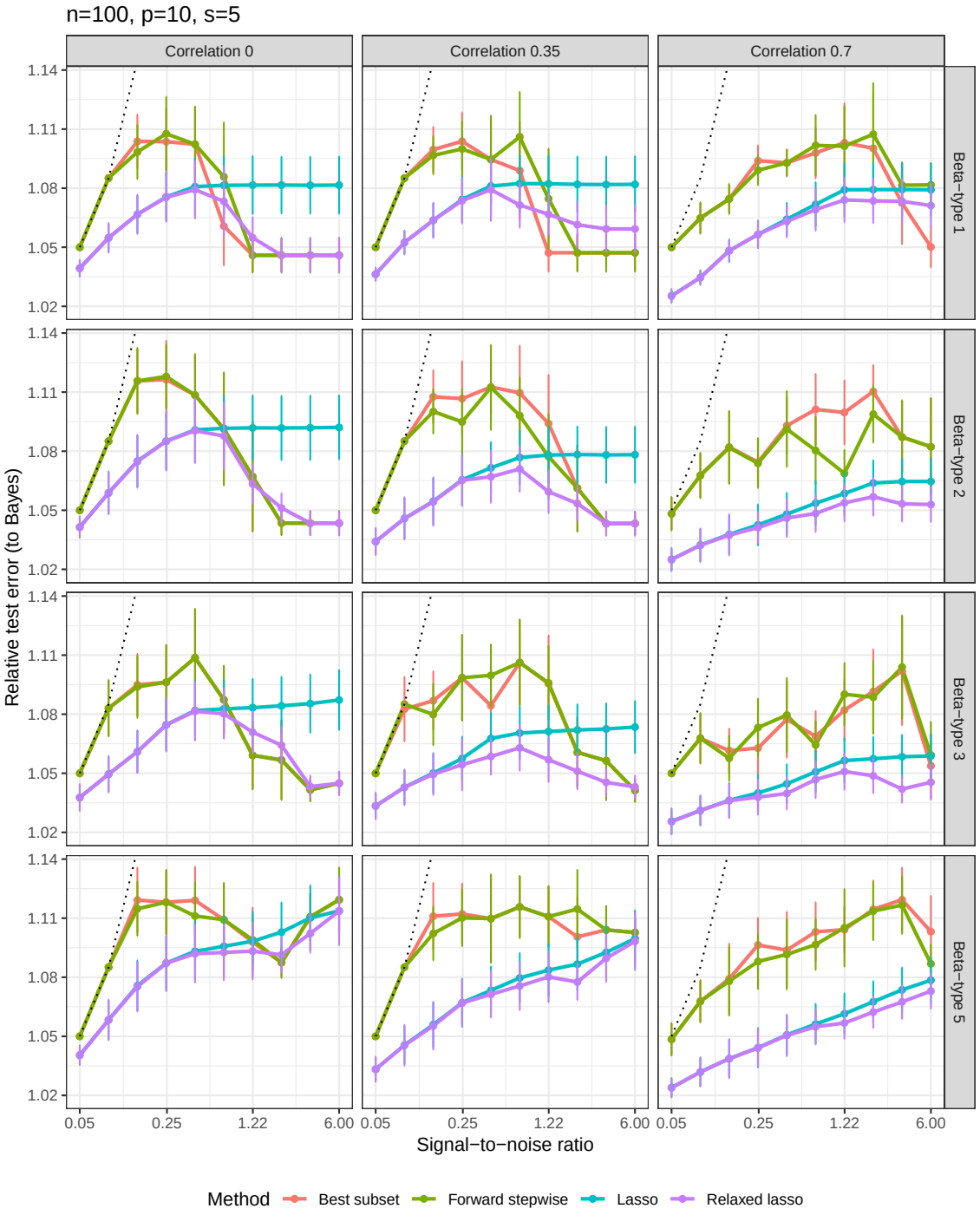
A.4.4 F-score


n=100, p=1000, s=10
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

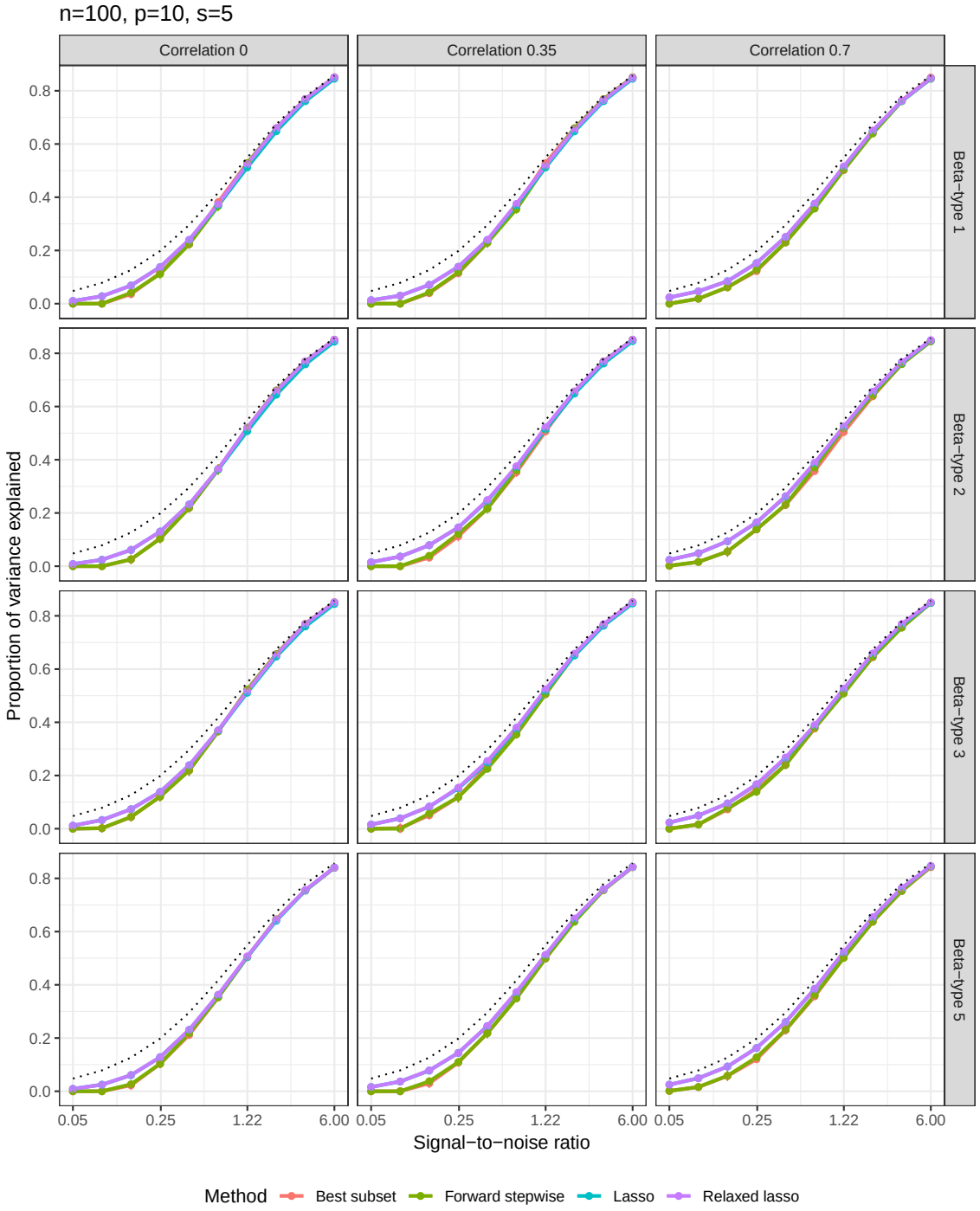
B Oracle tuning

B.1 Low setting: $n = 100$, $p = 10$, $s = 5$

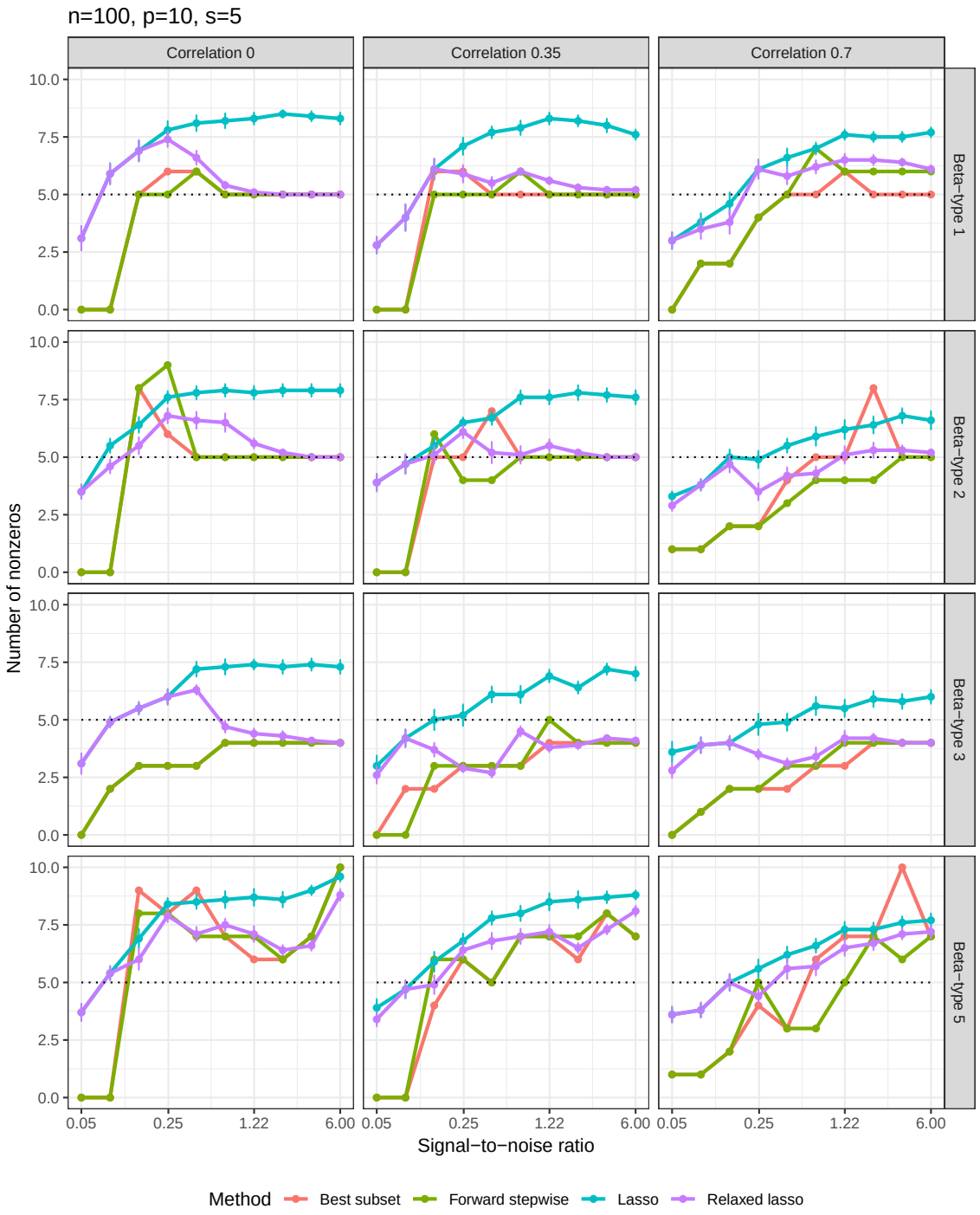
B.1.1 Relative test error (to Bayes)



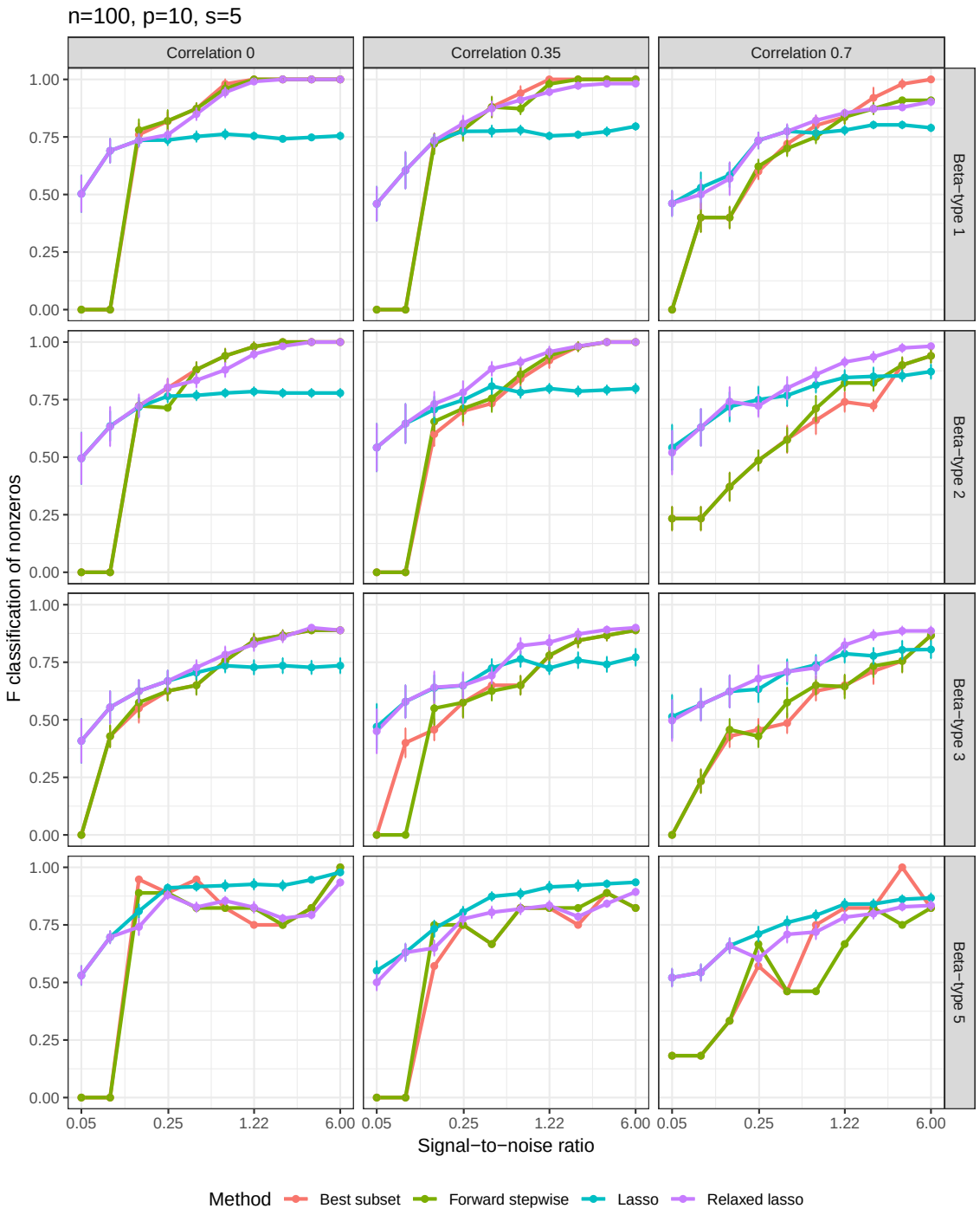
B.1.2 Proportion of variance explained



B.1.3 Number of nonzero coefficients

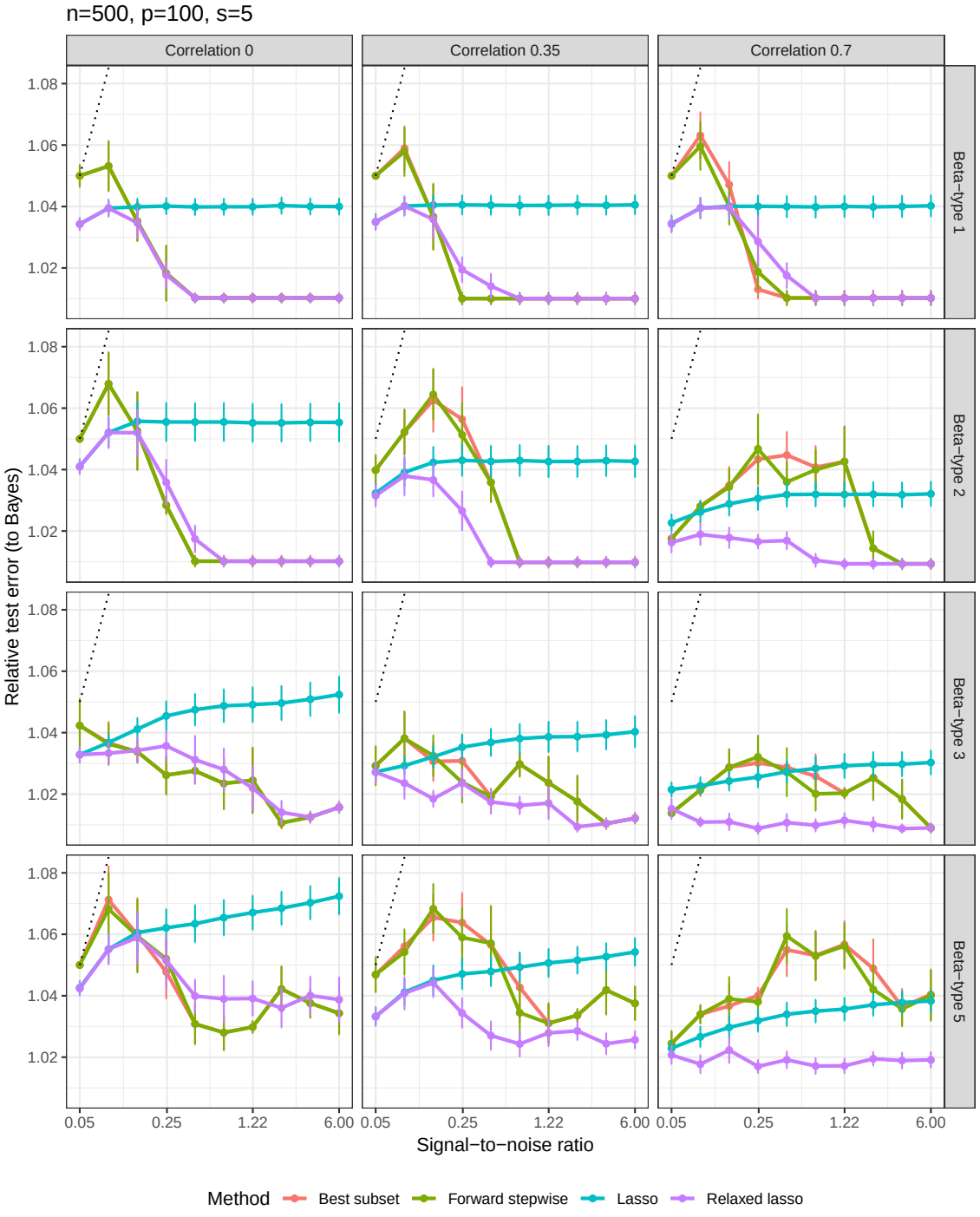


B.1.4 F-score

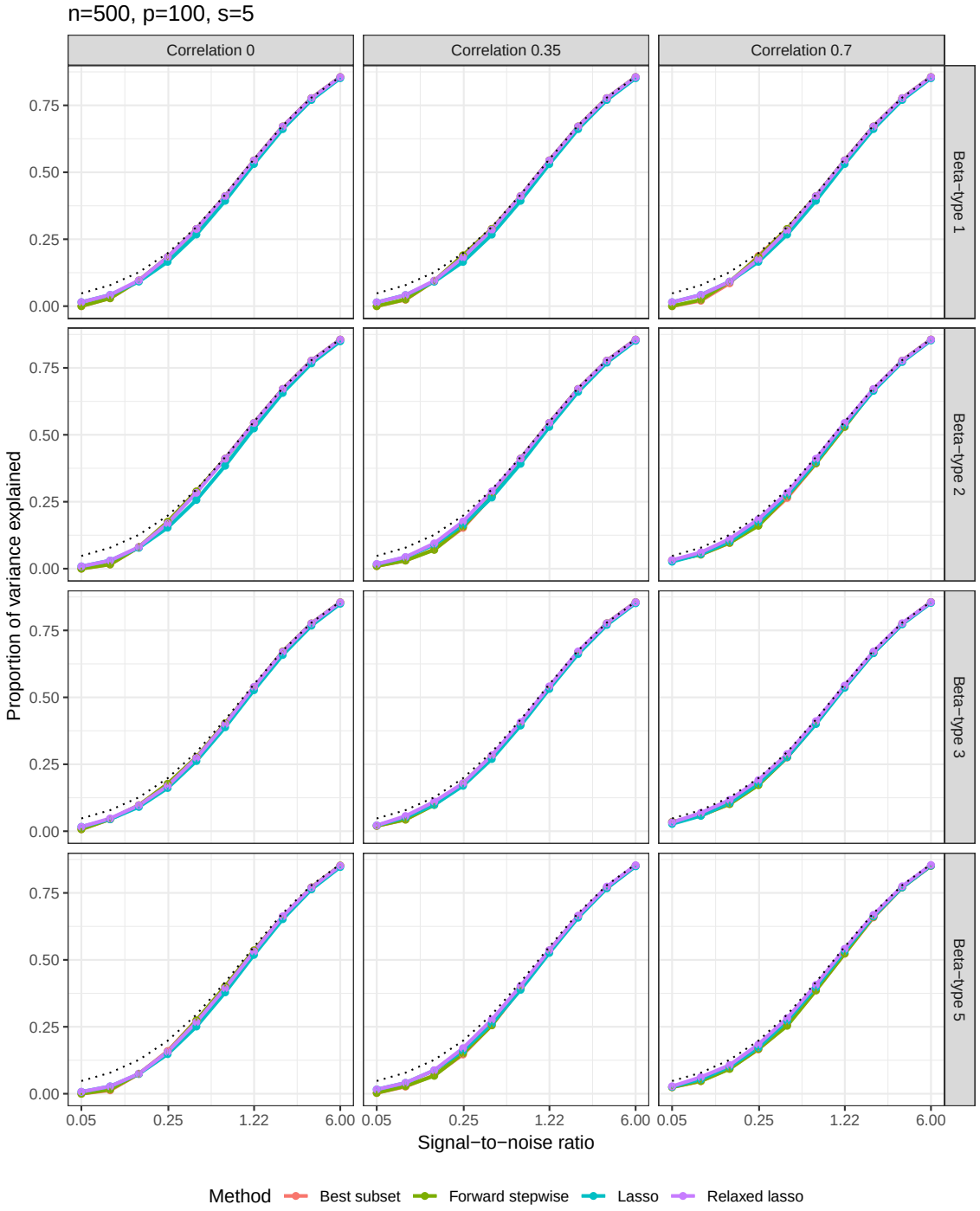

n=100, p=10, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

B.2 Medium setting: $n = 500$, $p = 100$, $s = 5$

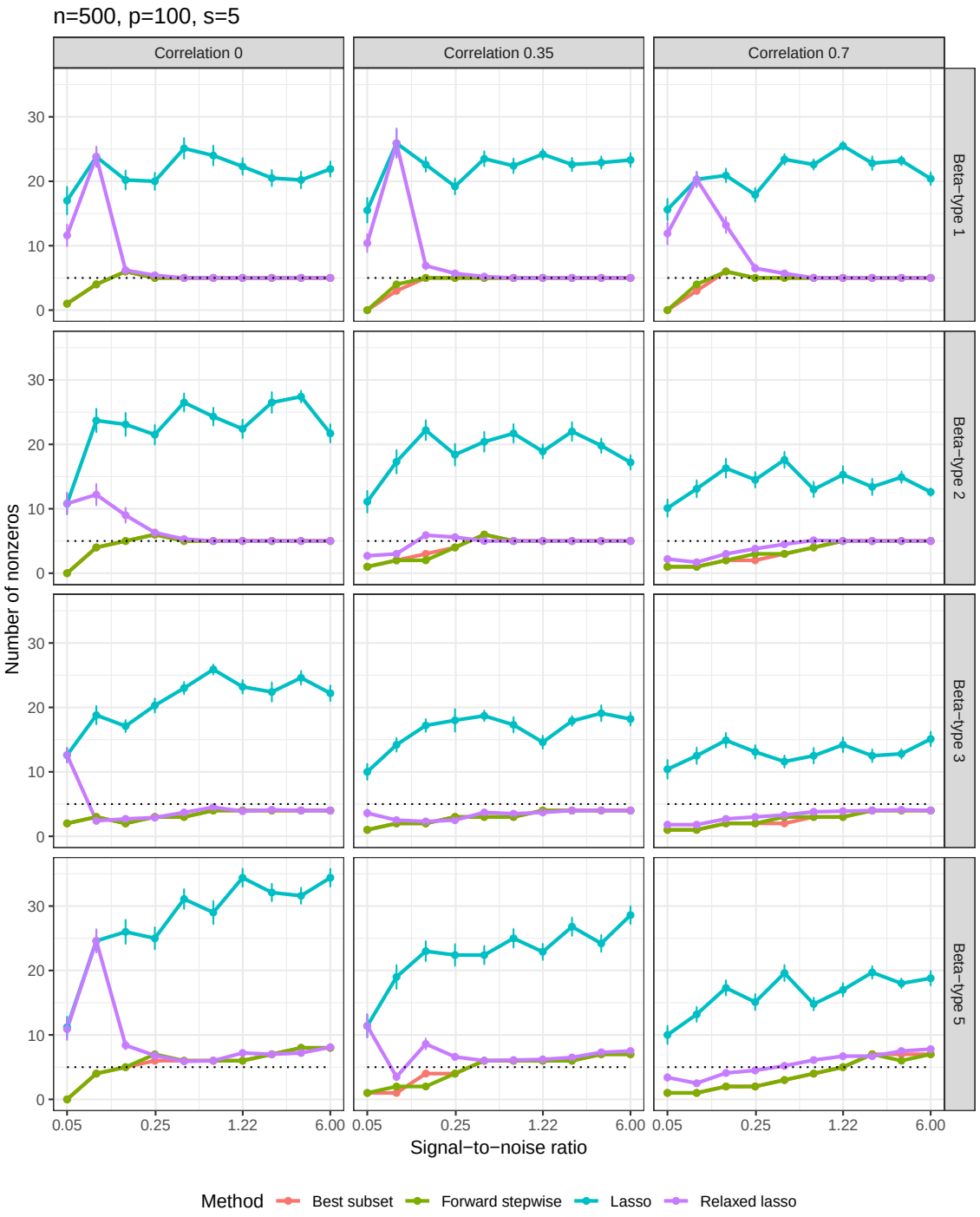
B.2.1 Relative test error (to Bayes)



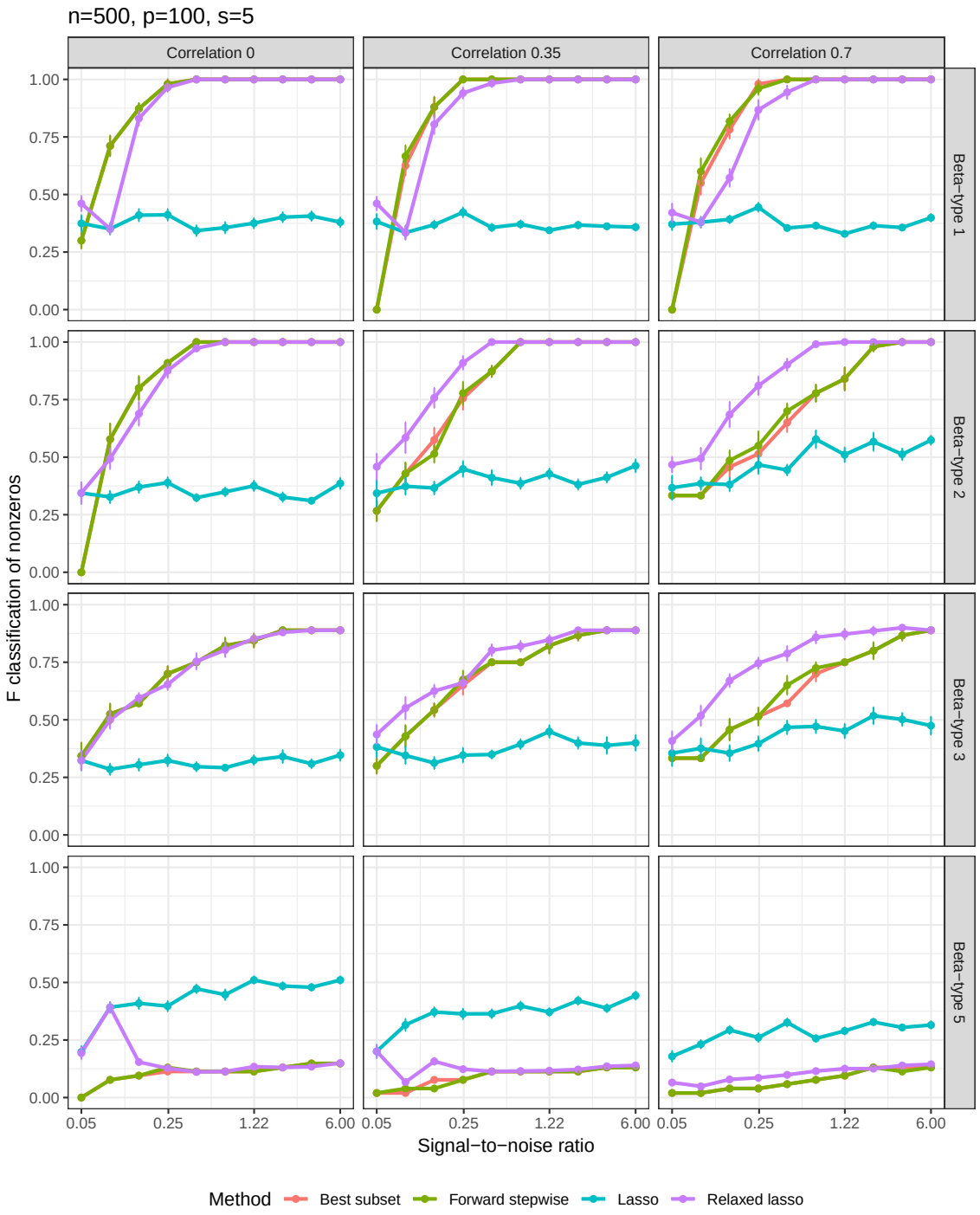
B.2.2 Proportion of variance explained



B.2.3 Number of nonzero coefficients

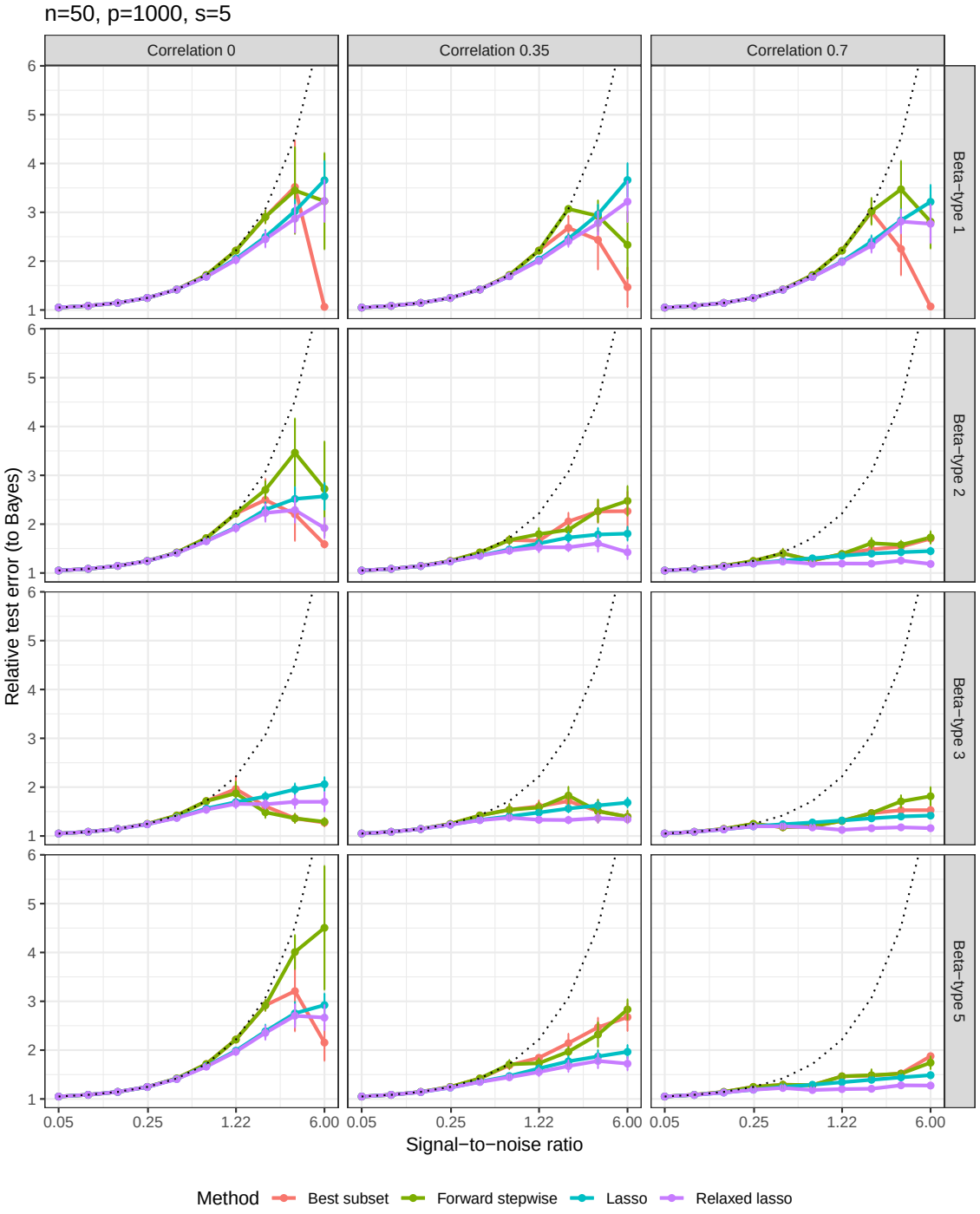


B.2.4 F-score

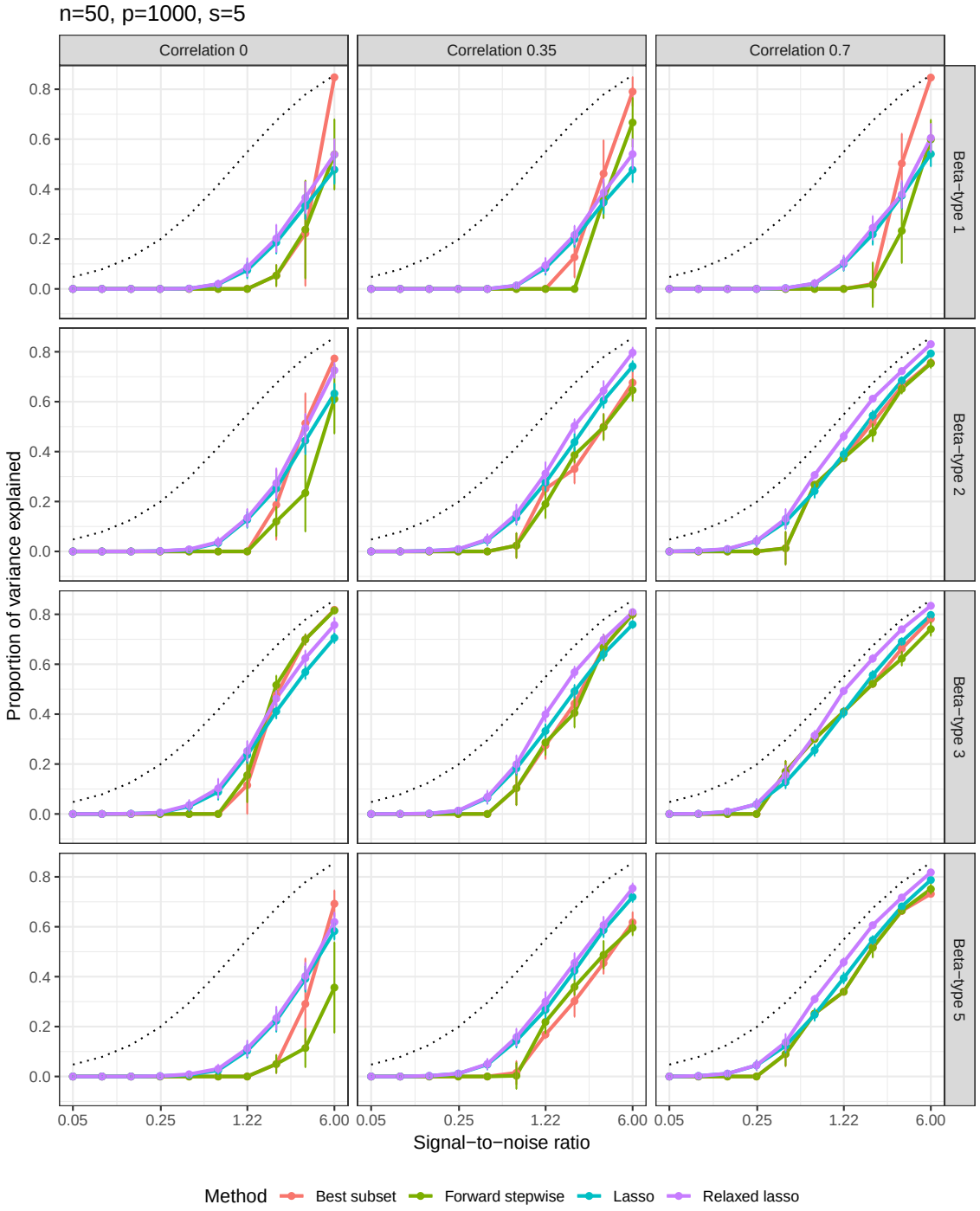

n=500, p=100, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

B.3 High-5 setting: $n = 50$, $p = 1000$, $s = 5$

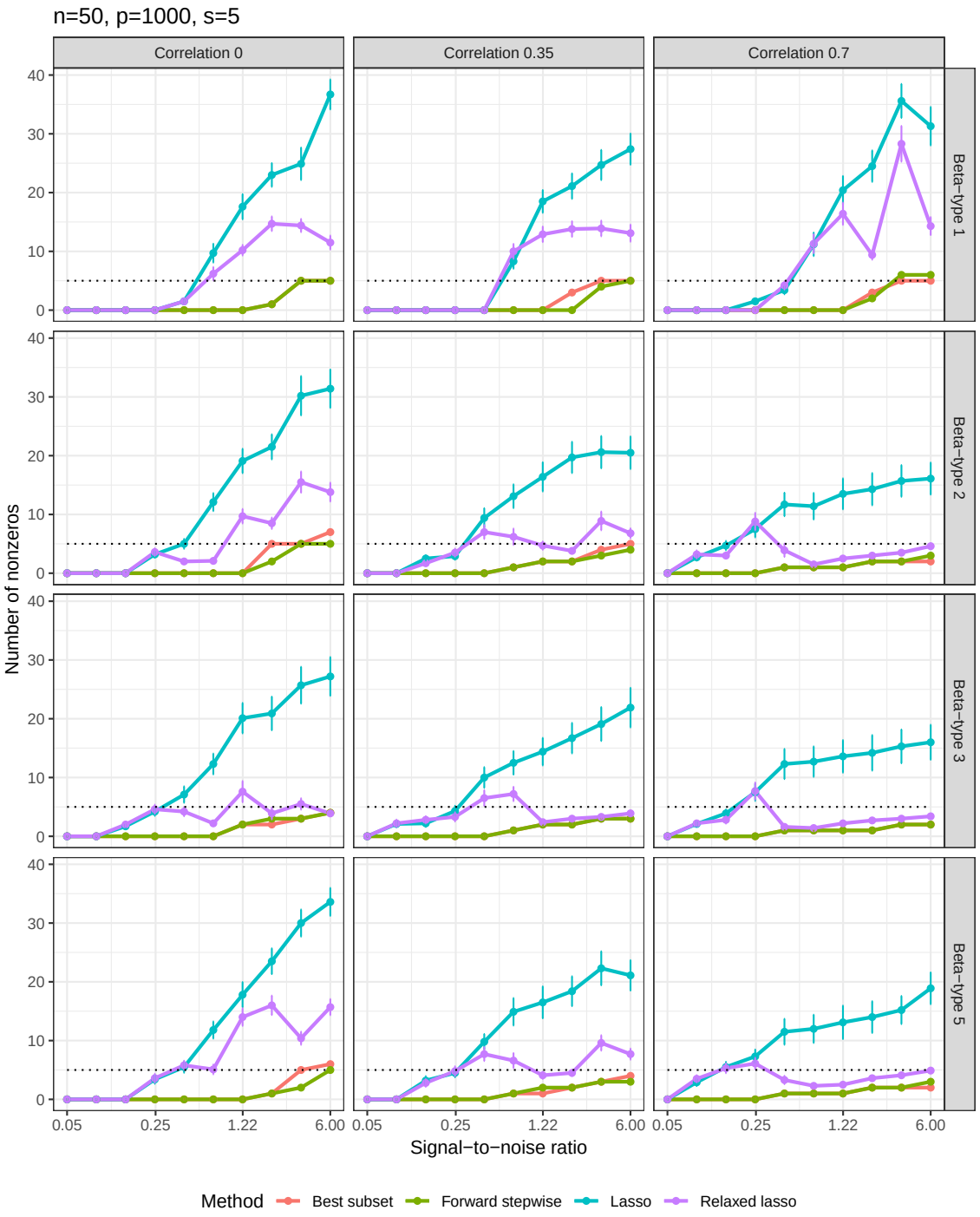
B.3.1 Relative test error (to Bayes)



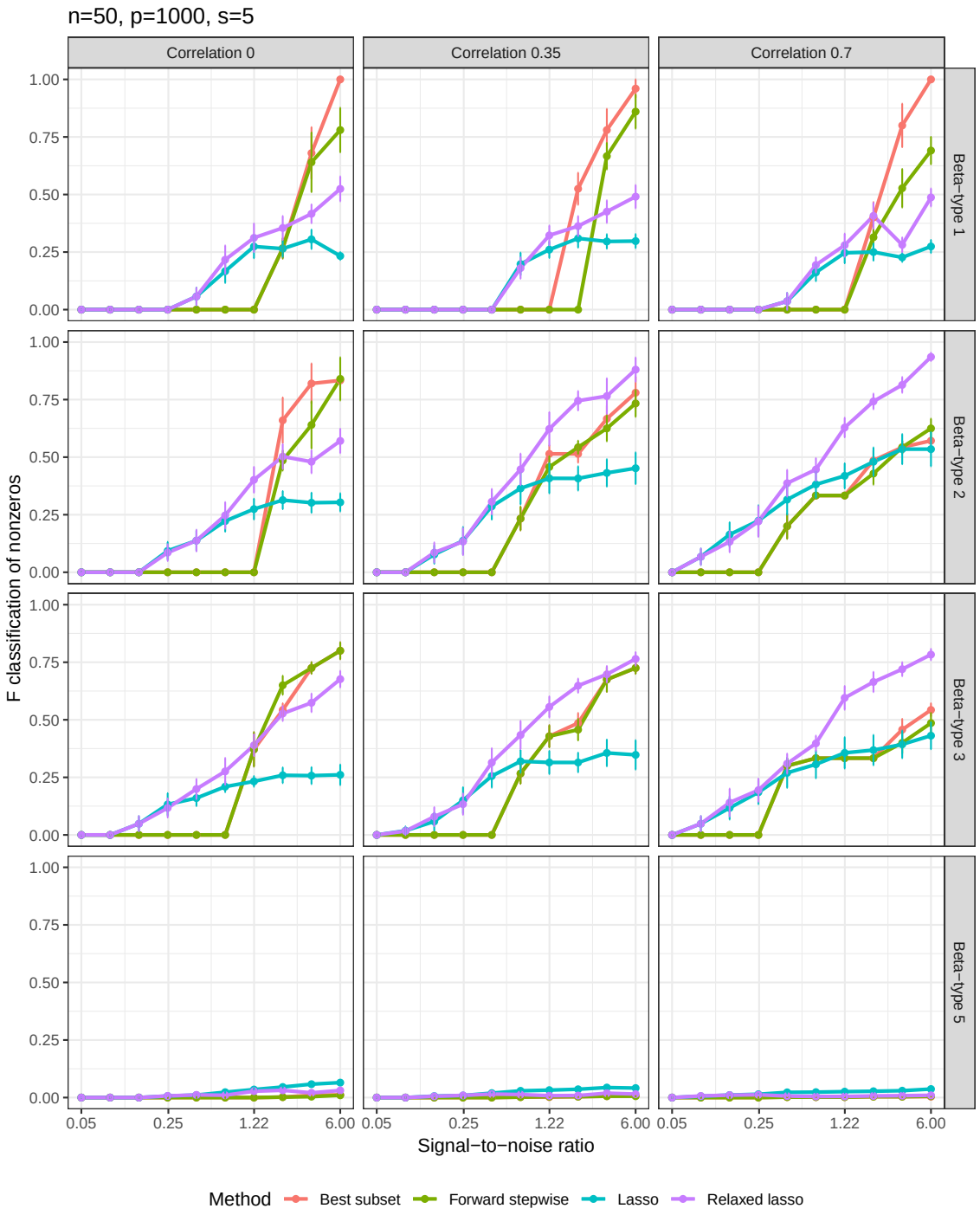
B.3.2 Proportion of variance explained



B.3.3 Number of nonzero coefficients

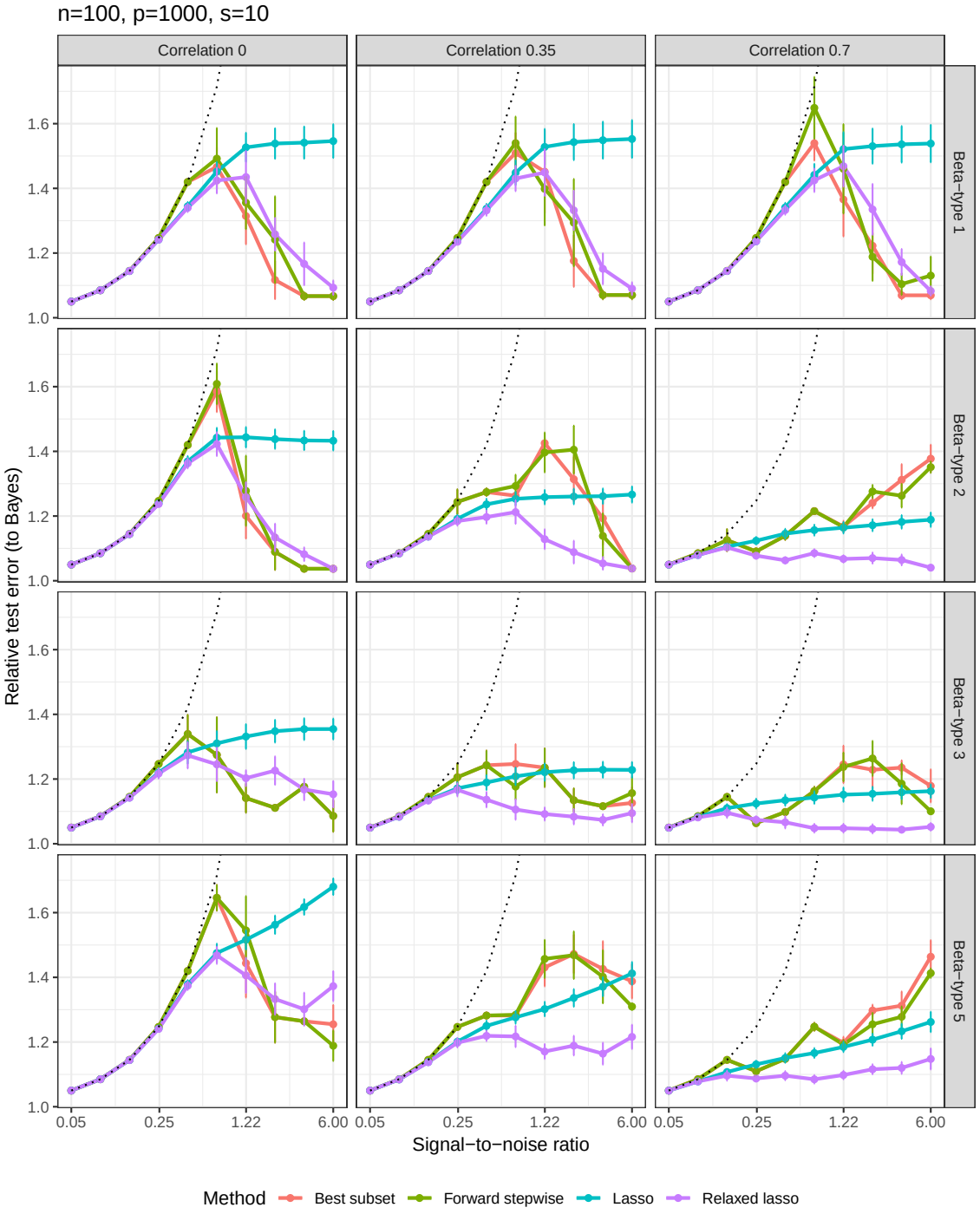


B.3.4 F-score

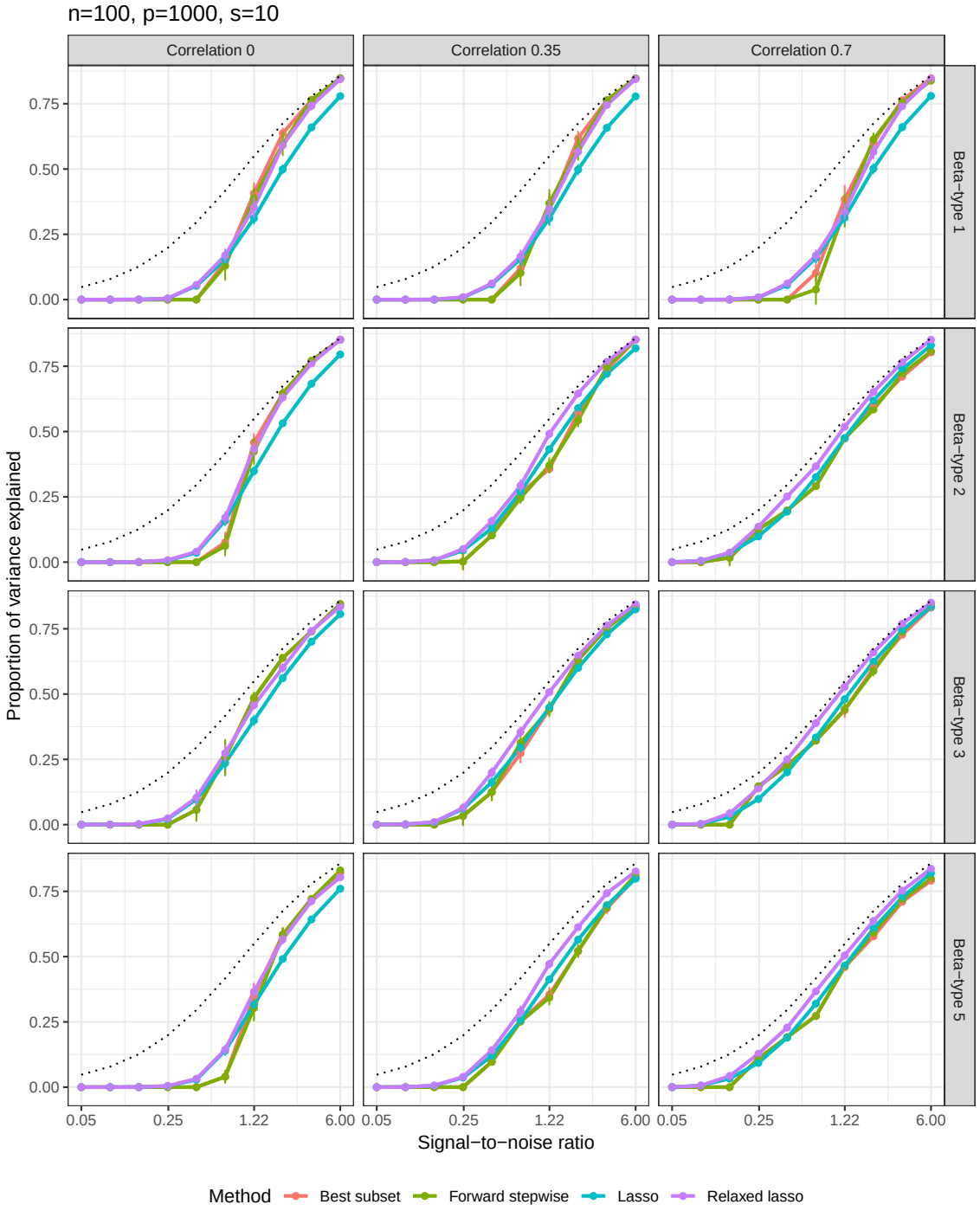

n=50, p=1000, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

B.4 High-10 setting: $n = 100$, $p = 1000$, $s = 10$

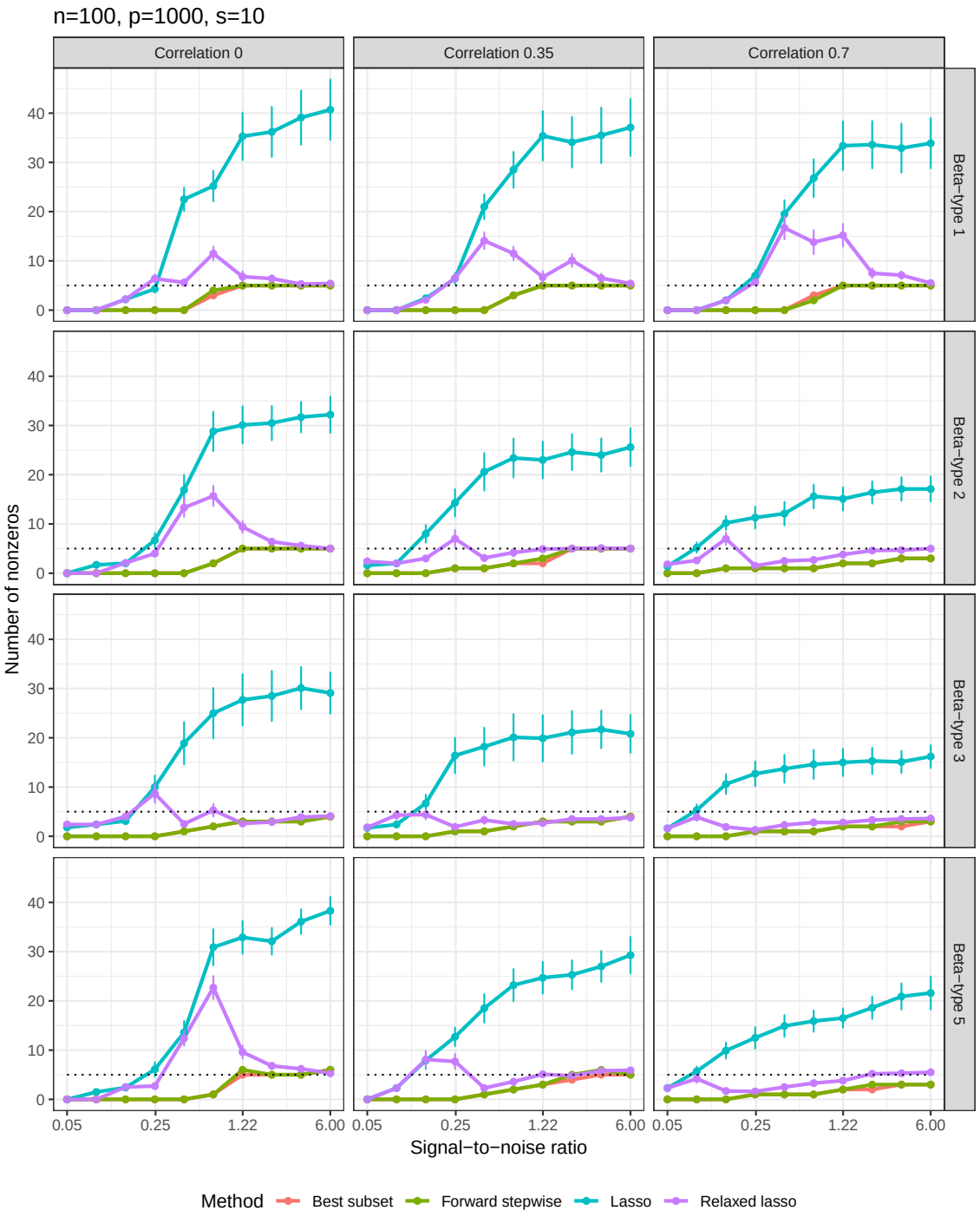
B.4.1 Relative test error (to Bayes)



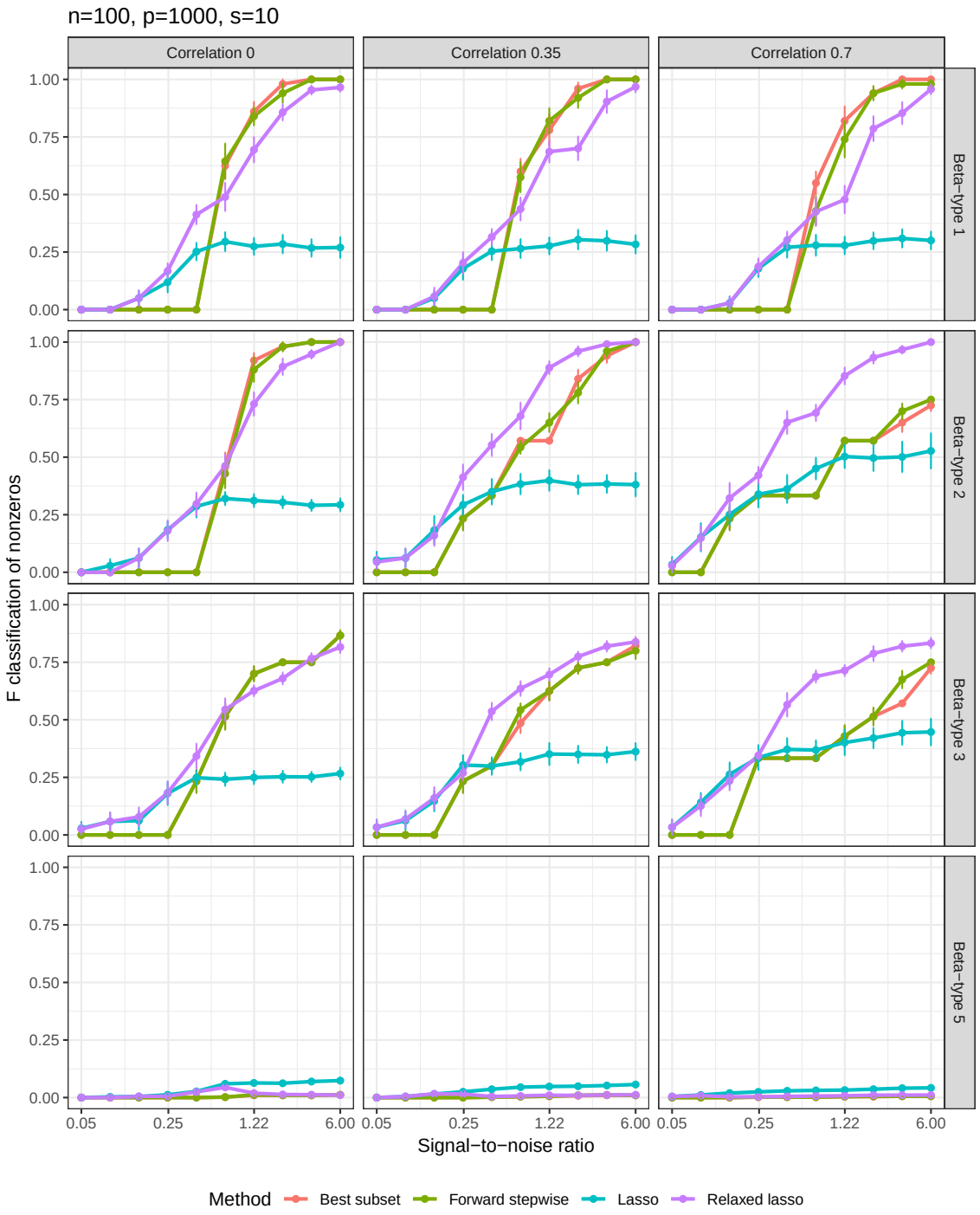
B.4.2 Proportion of variance explained



B.4.3 Number of nonzero coefficients



B.4.4 F-score


n=100, p=1000, s=10
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
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Beta−type 5
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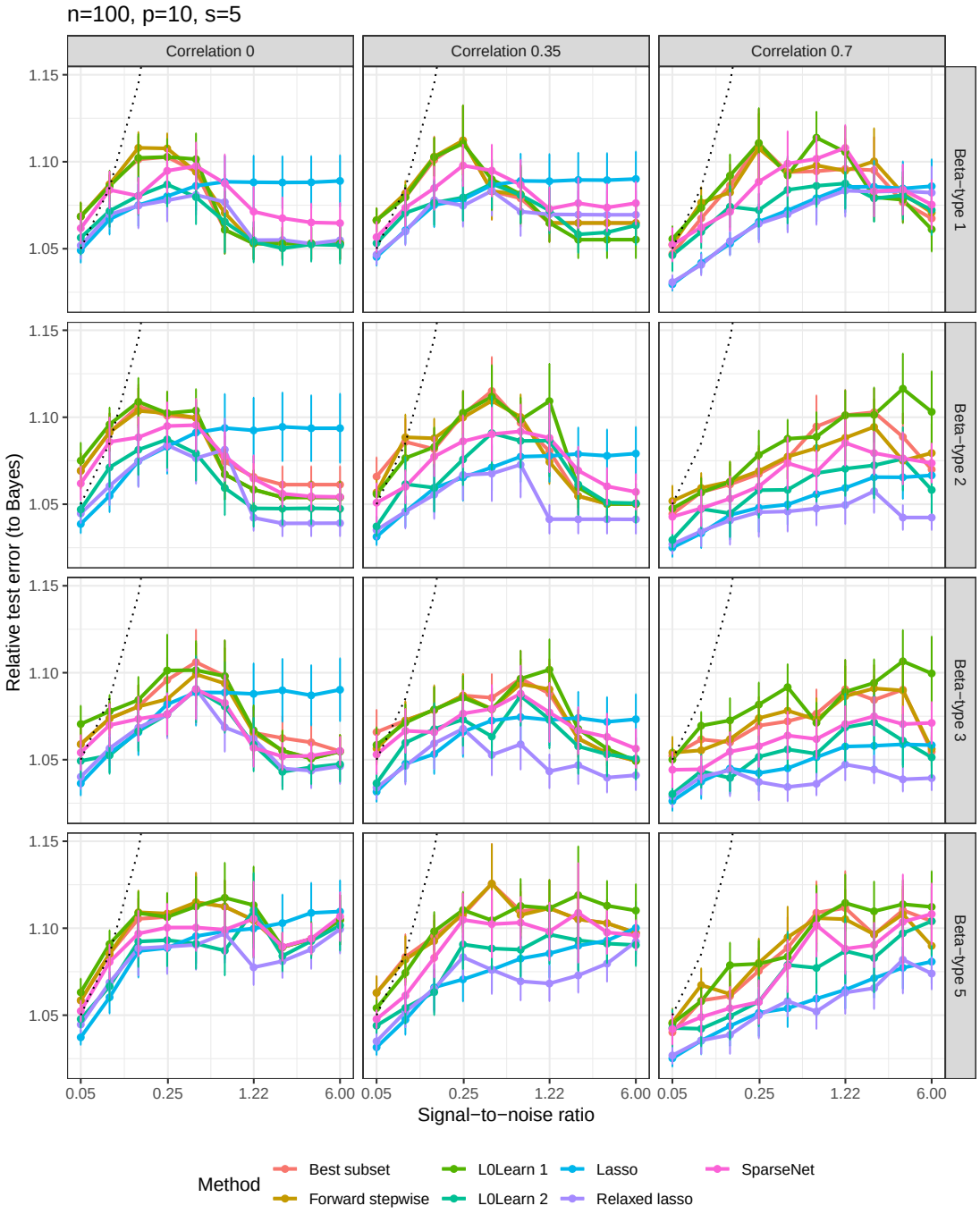
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
Lasso
Relaxed lasso

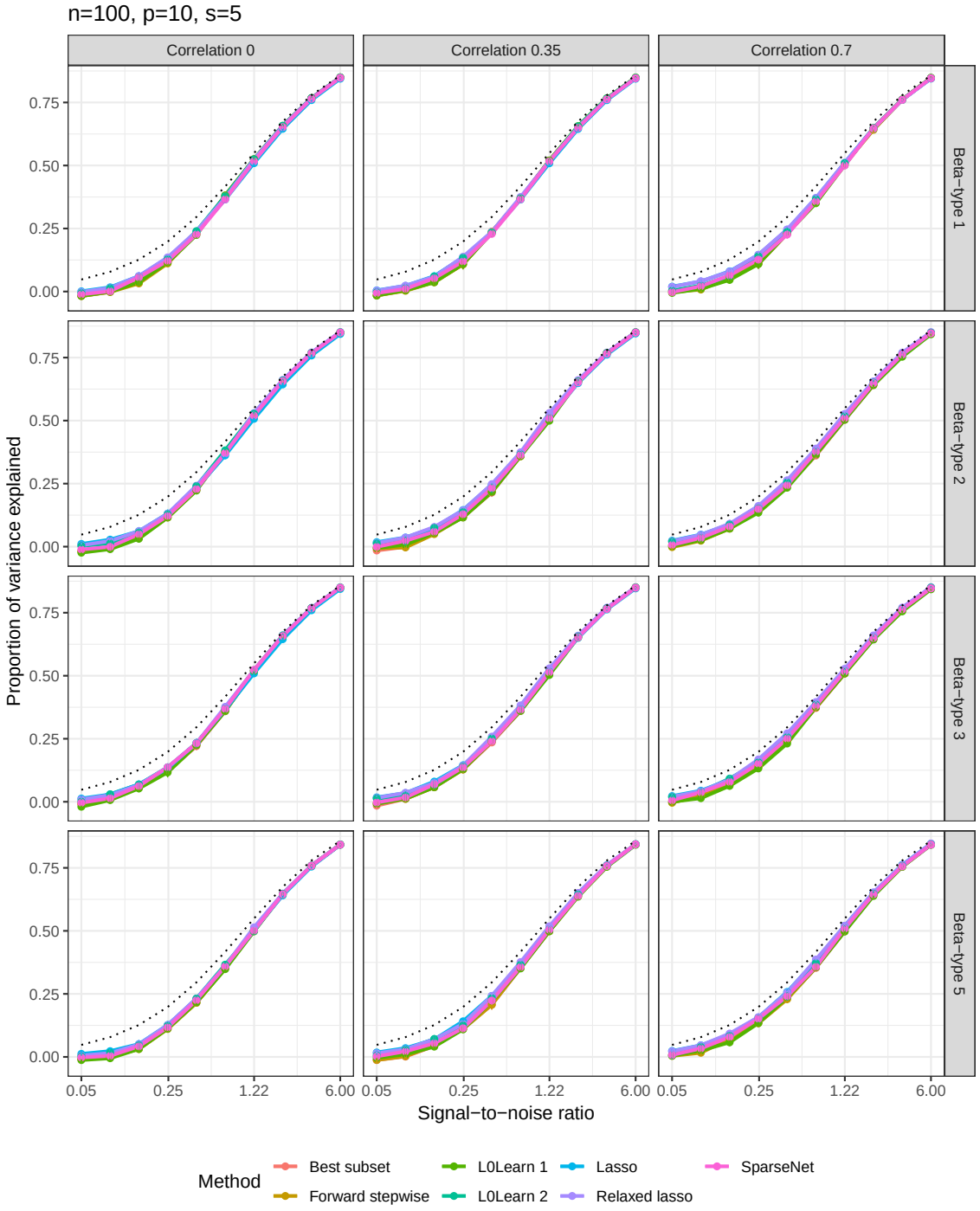
C Validation tuning: now including L0Learn-1, L0Learn-2 and SparseNet

C.1 Low setting: $n = 100$, $p = 10$, $s = 5$

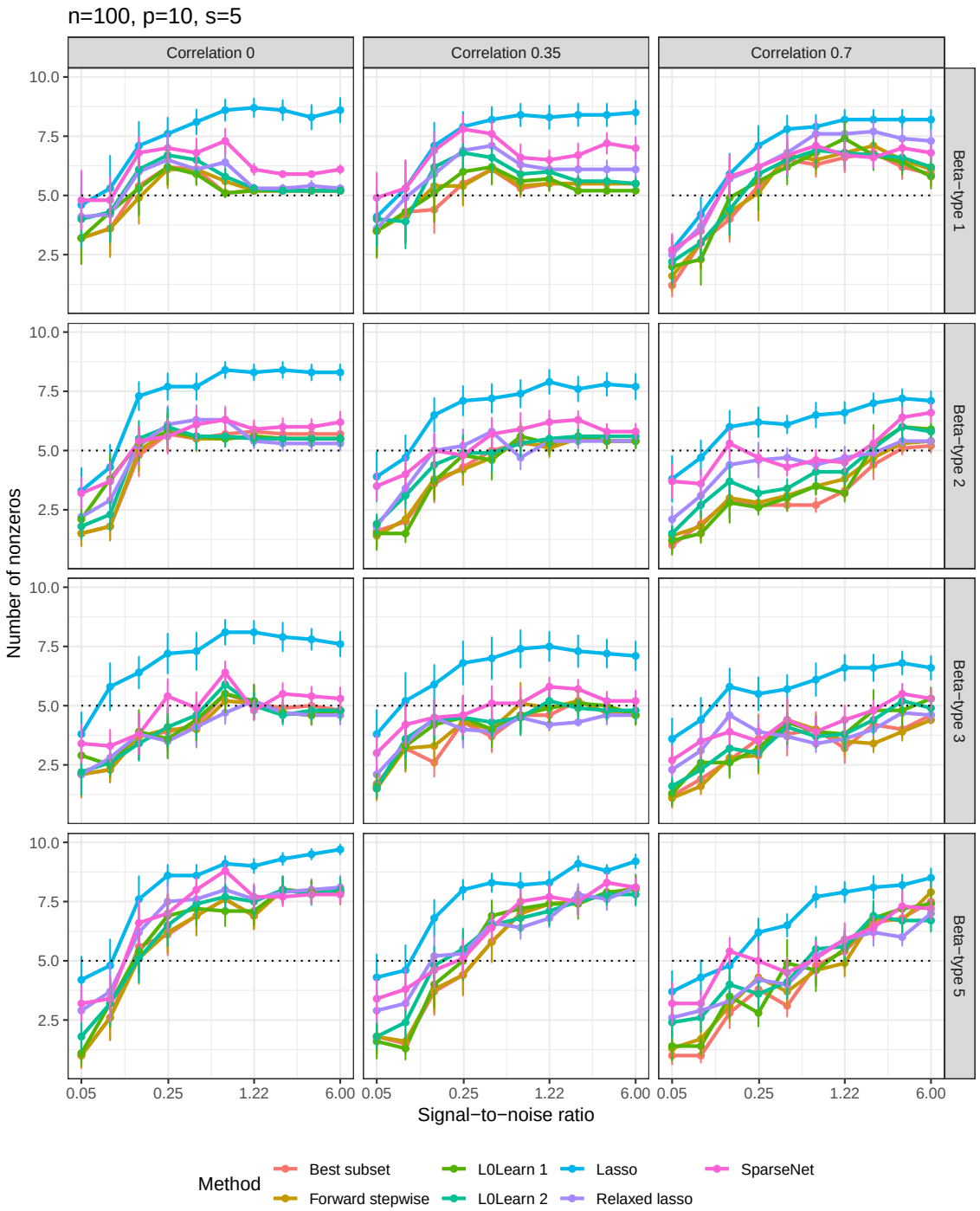
C.1.1 Relative test error (to Bayes)



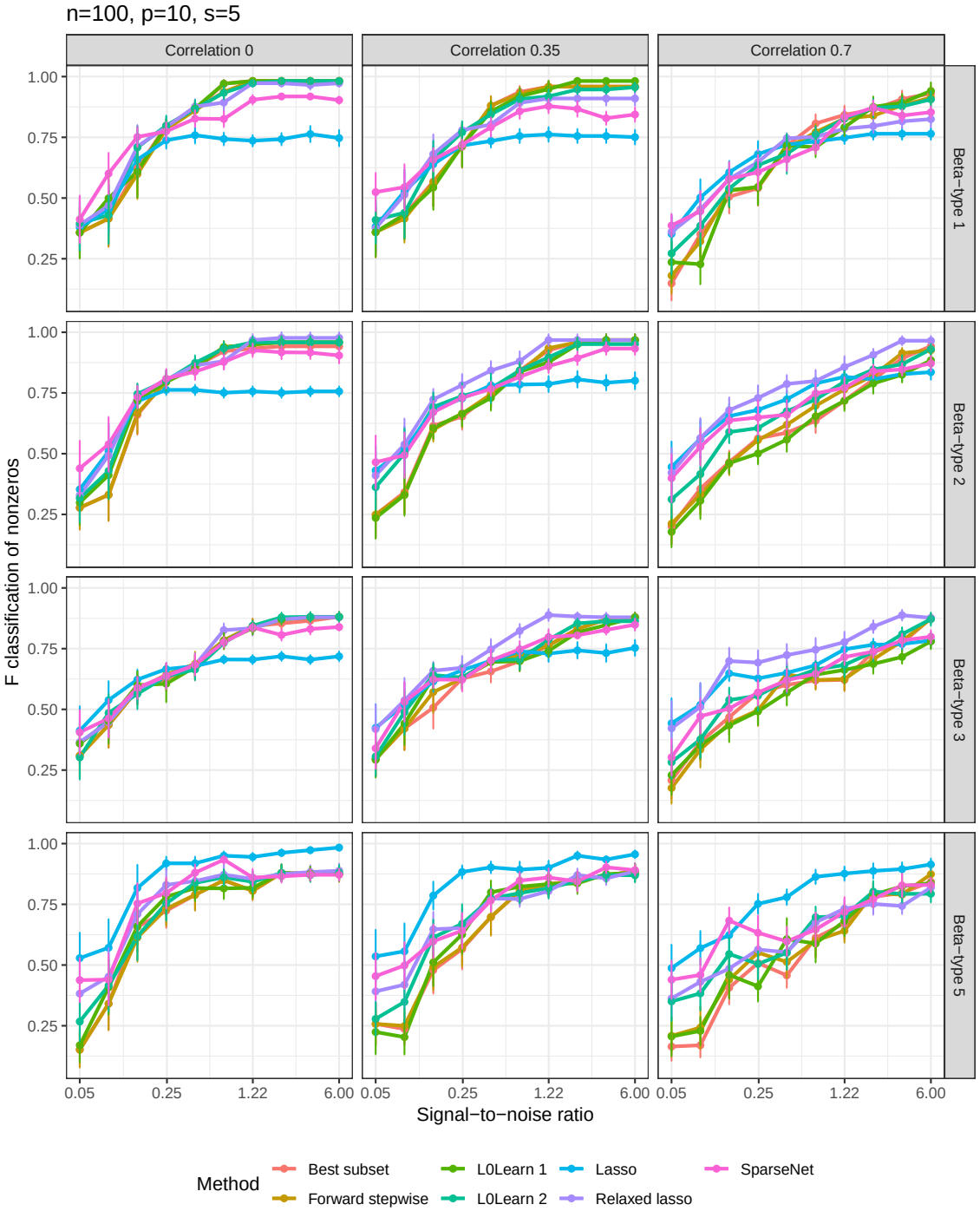
C.1.2 Proportion of variance explained



C.1.3 Number of nonzero coefficients



C.1.4 F-score


n=100, p=10, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
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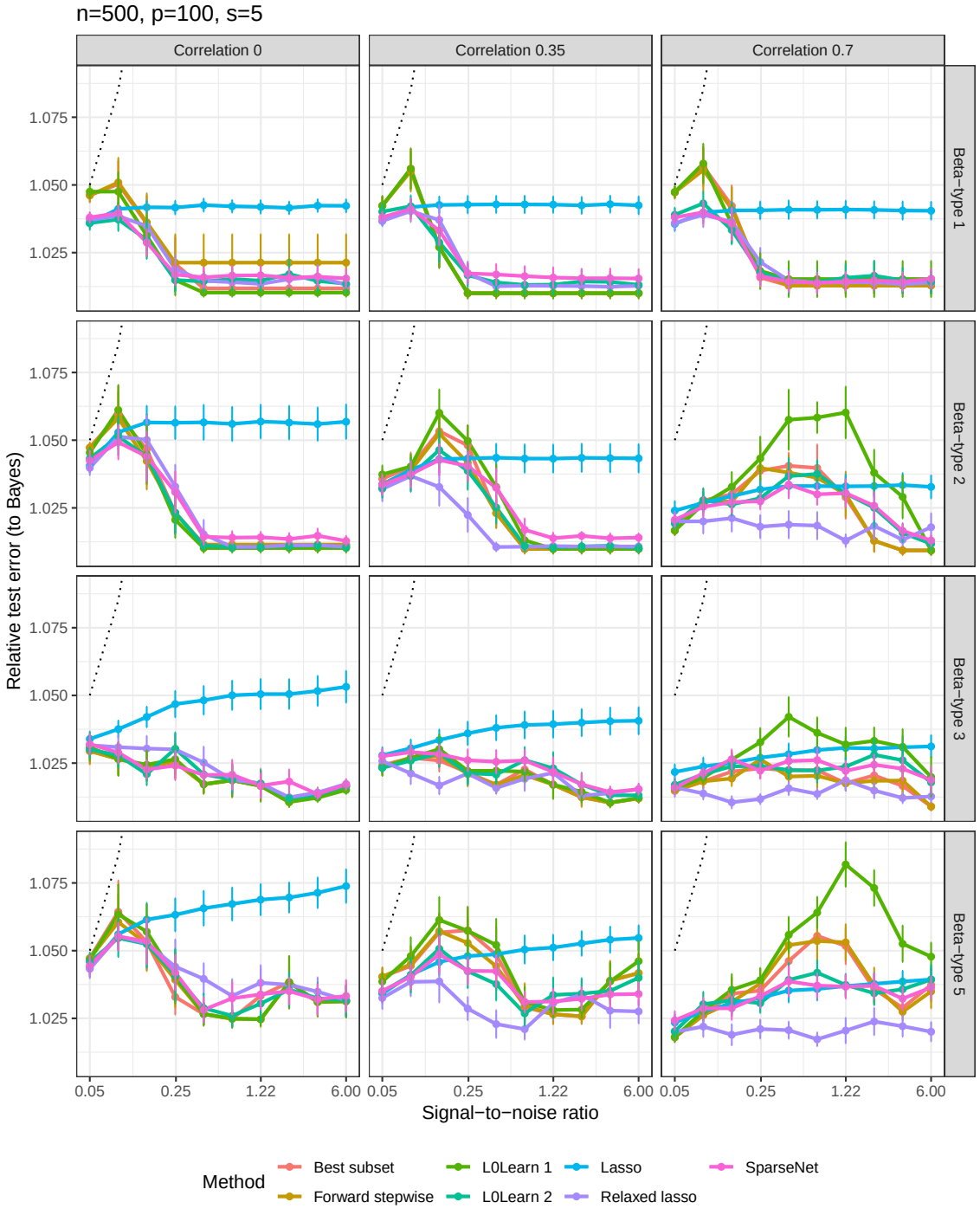
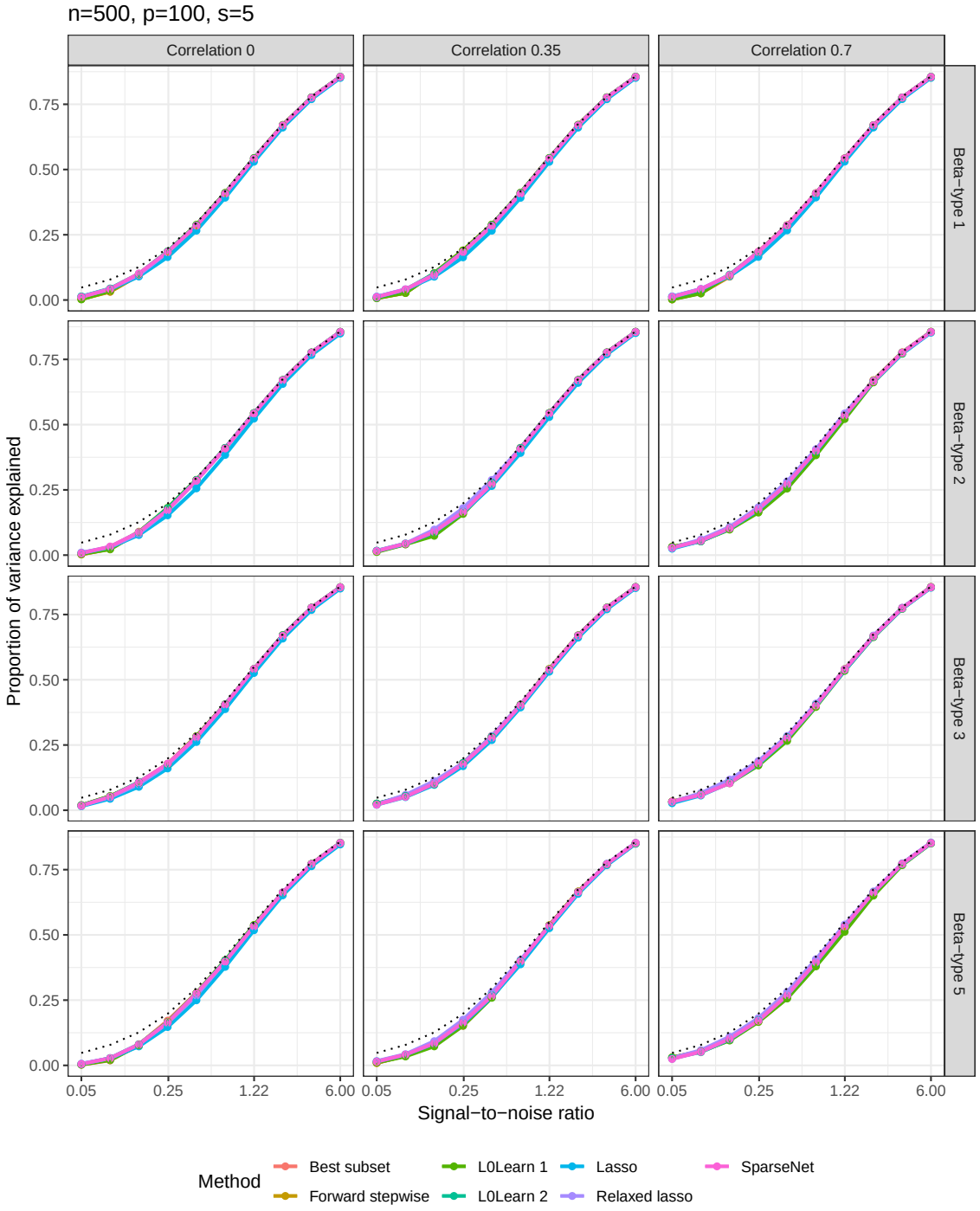
0.75
0.50
0.25
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

C.2 Medium setting: $n = 500$, $p = 100$, $s = 5$

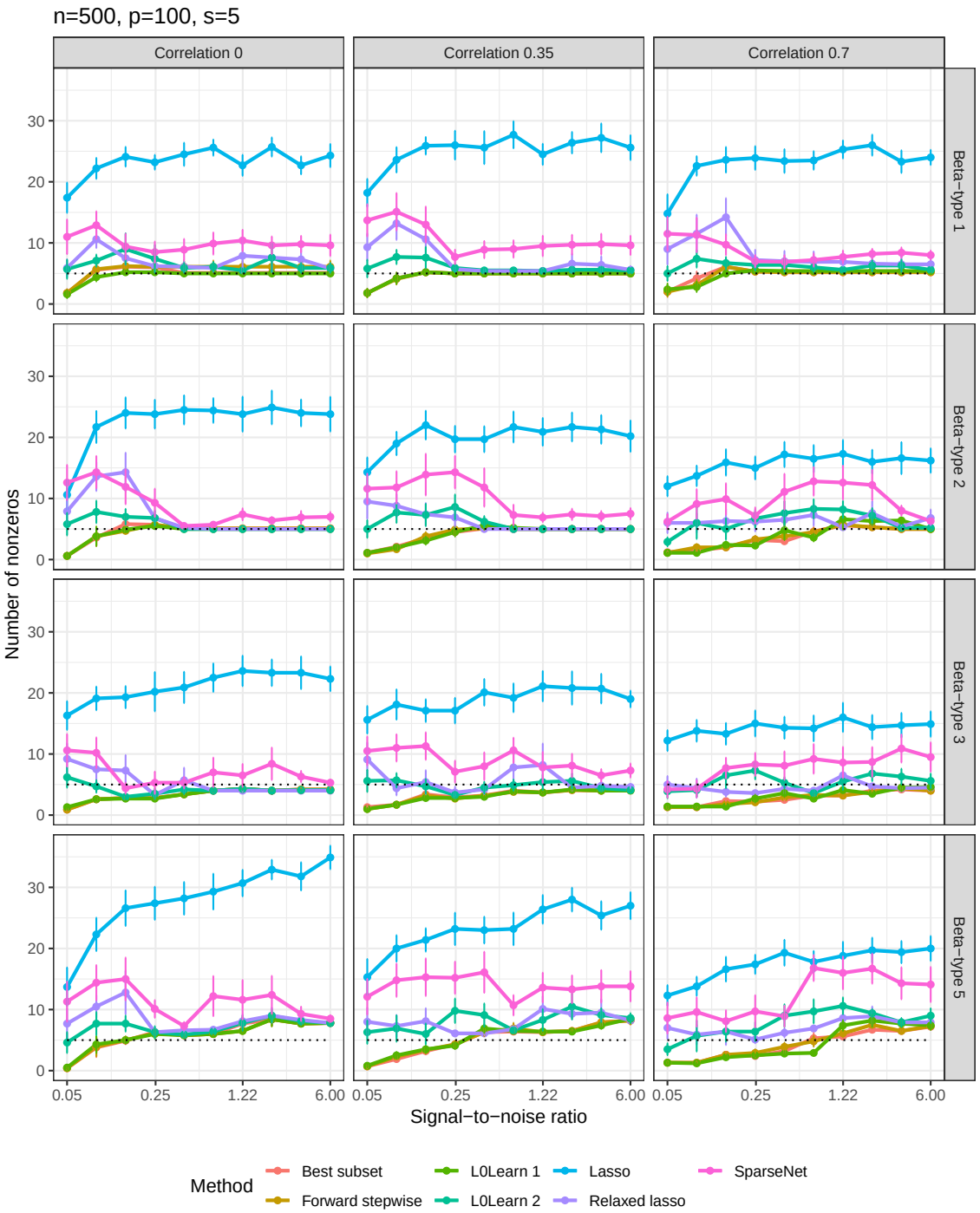
C.2.1 Relative test error (to Bayes)



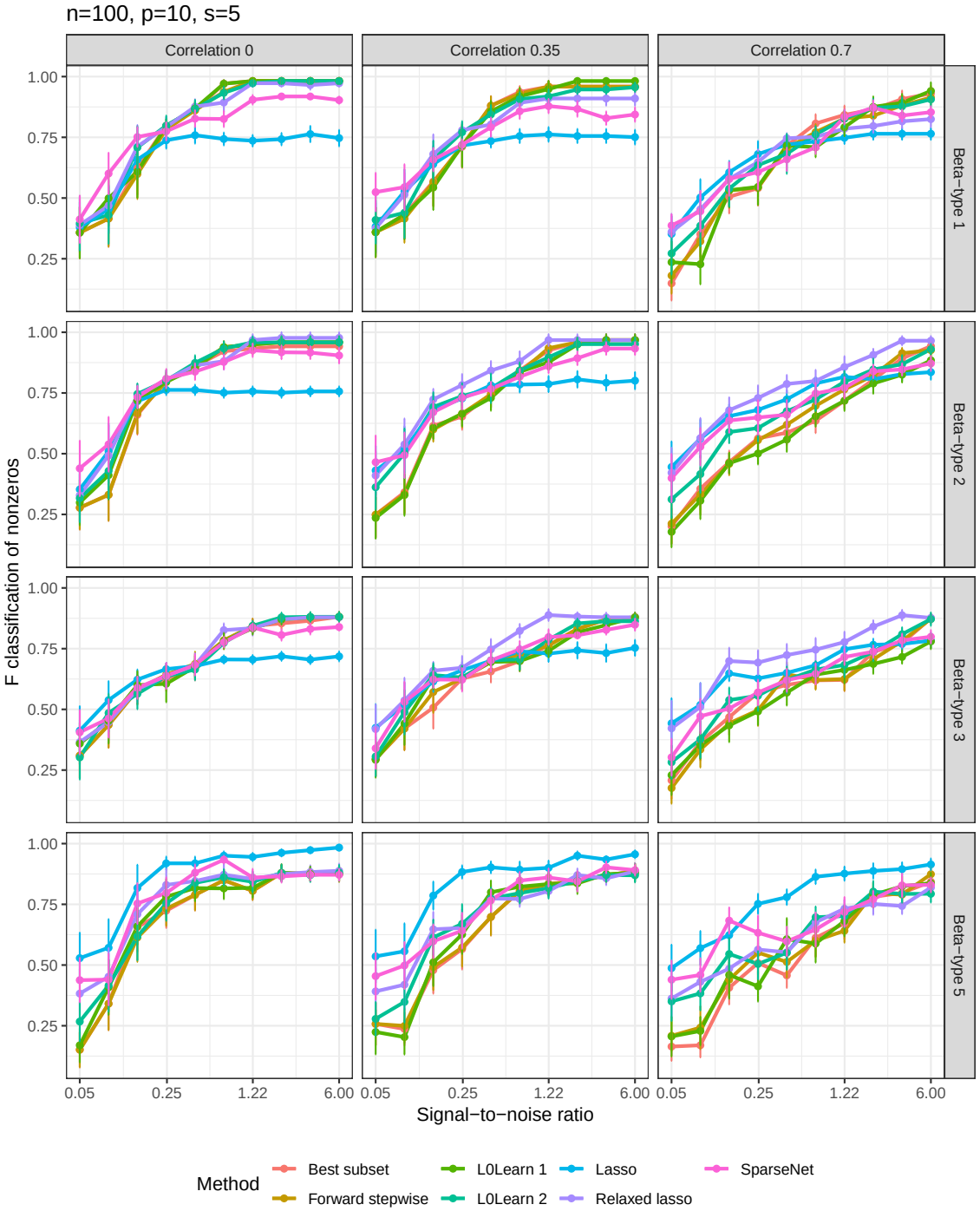
C.2.2 Proportion of variance explained



C.2.3 Number of nonzero coefficients

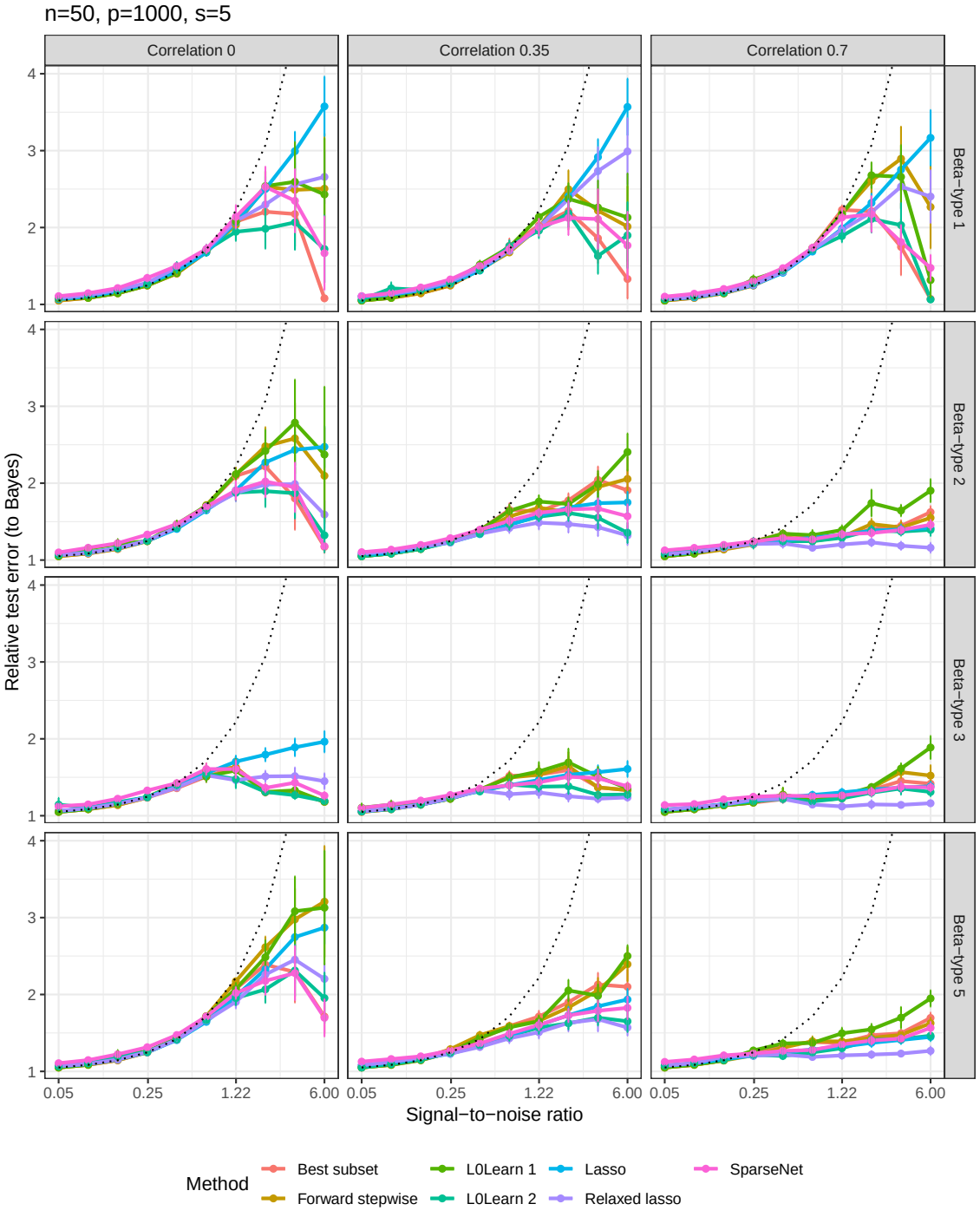


C.2.4 F-score

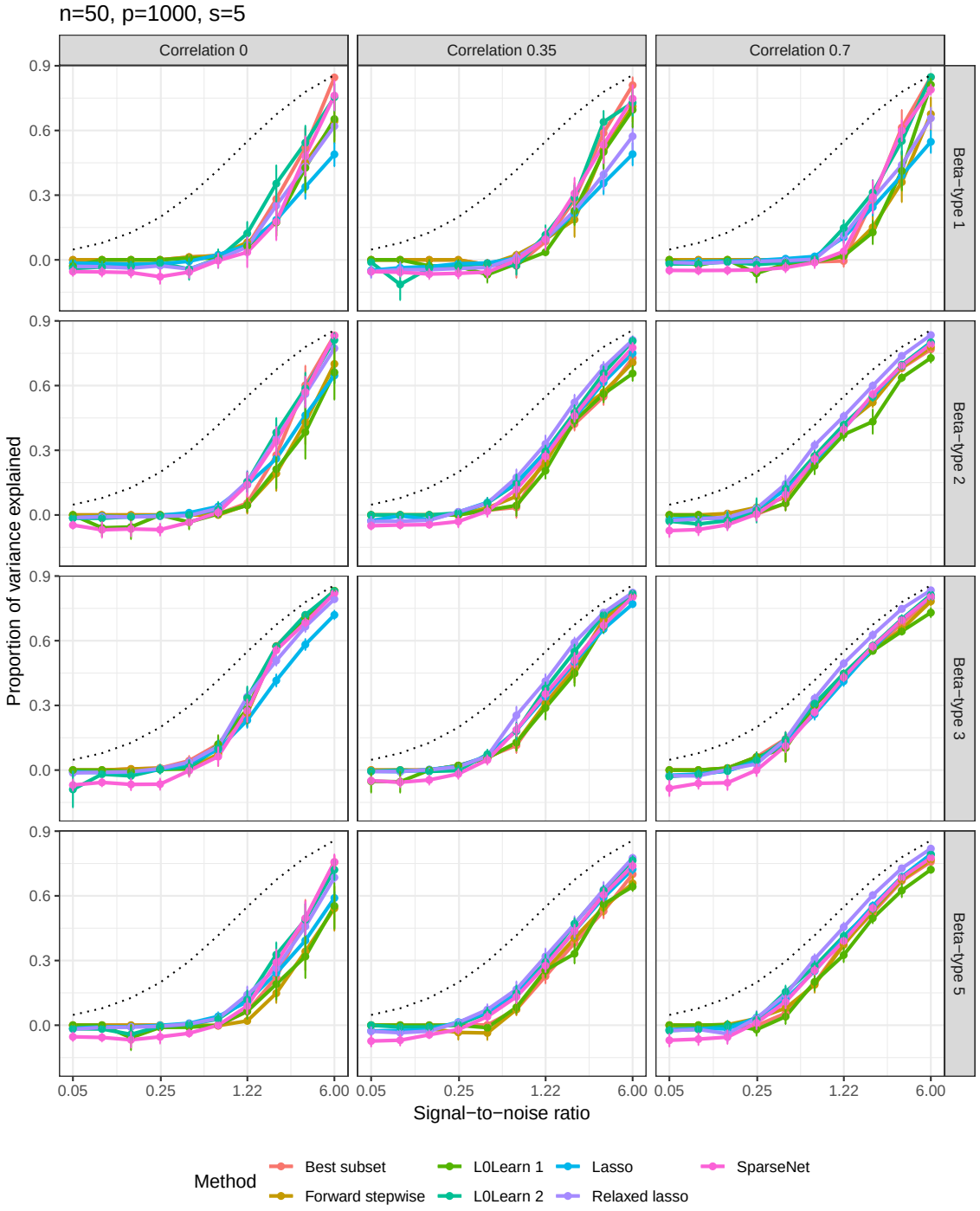

n=100, p=10, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

C.3 High-5 setting: $n = 50$, $p = 1000$, $s = 5$

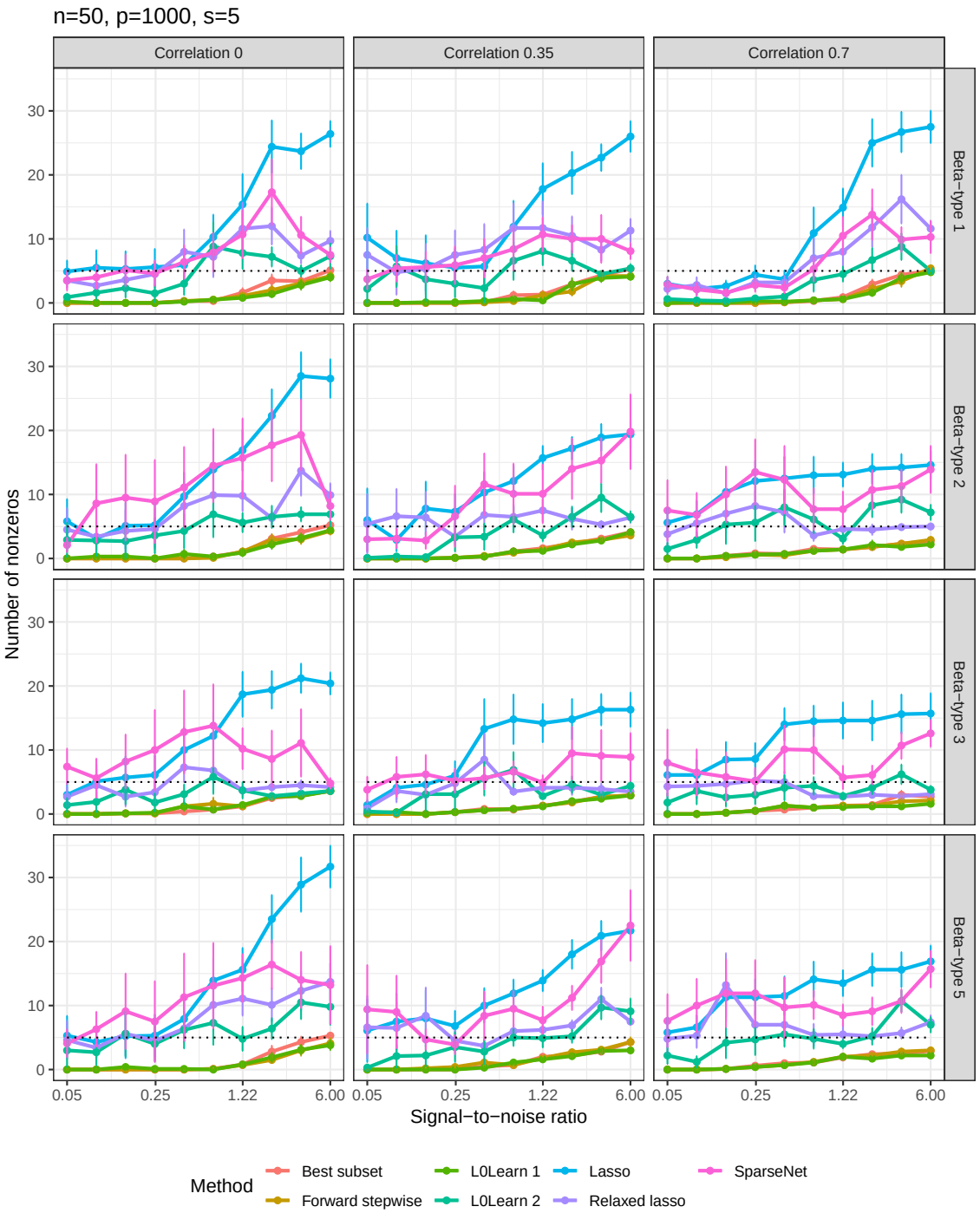
C.3.1 Relative test error (to Bayes)



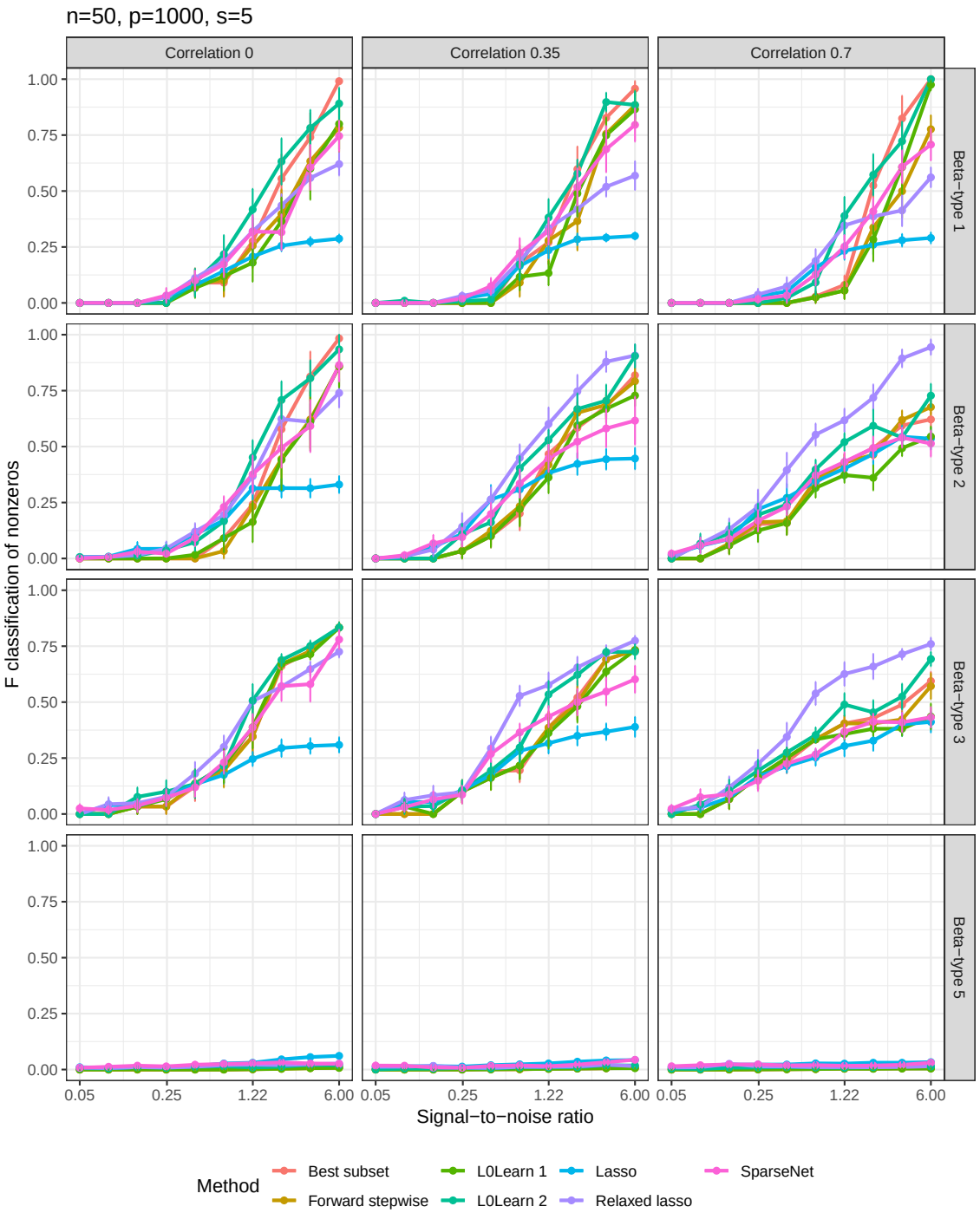
C.3.2 Proportion of variance explained



C.3.3 Number of nonzero coefficients

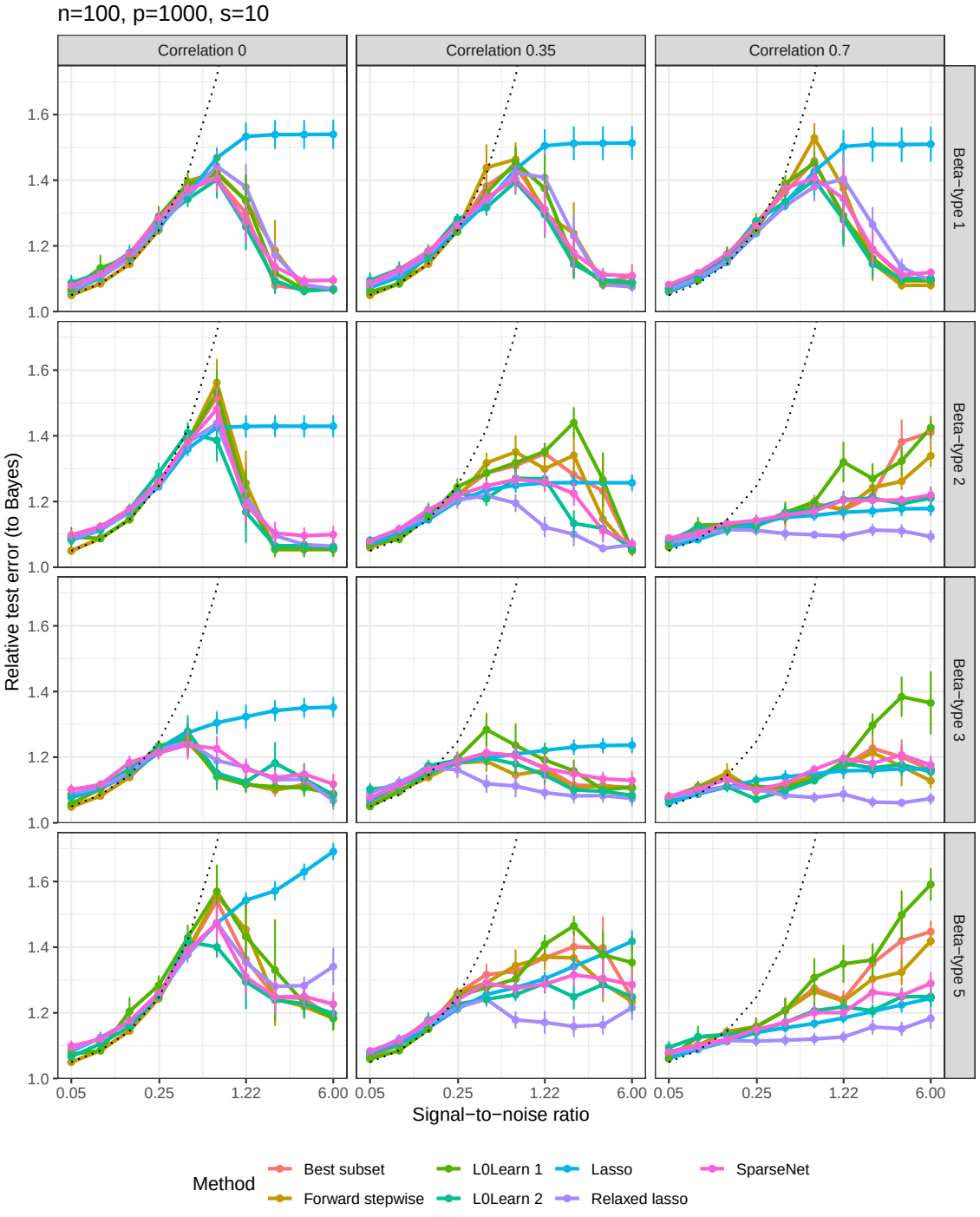


C.3.4 F-score

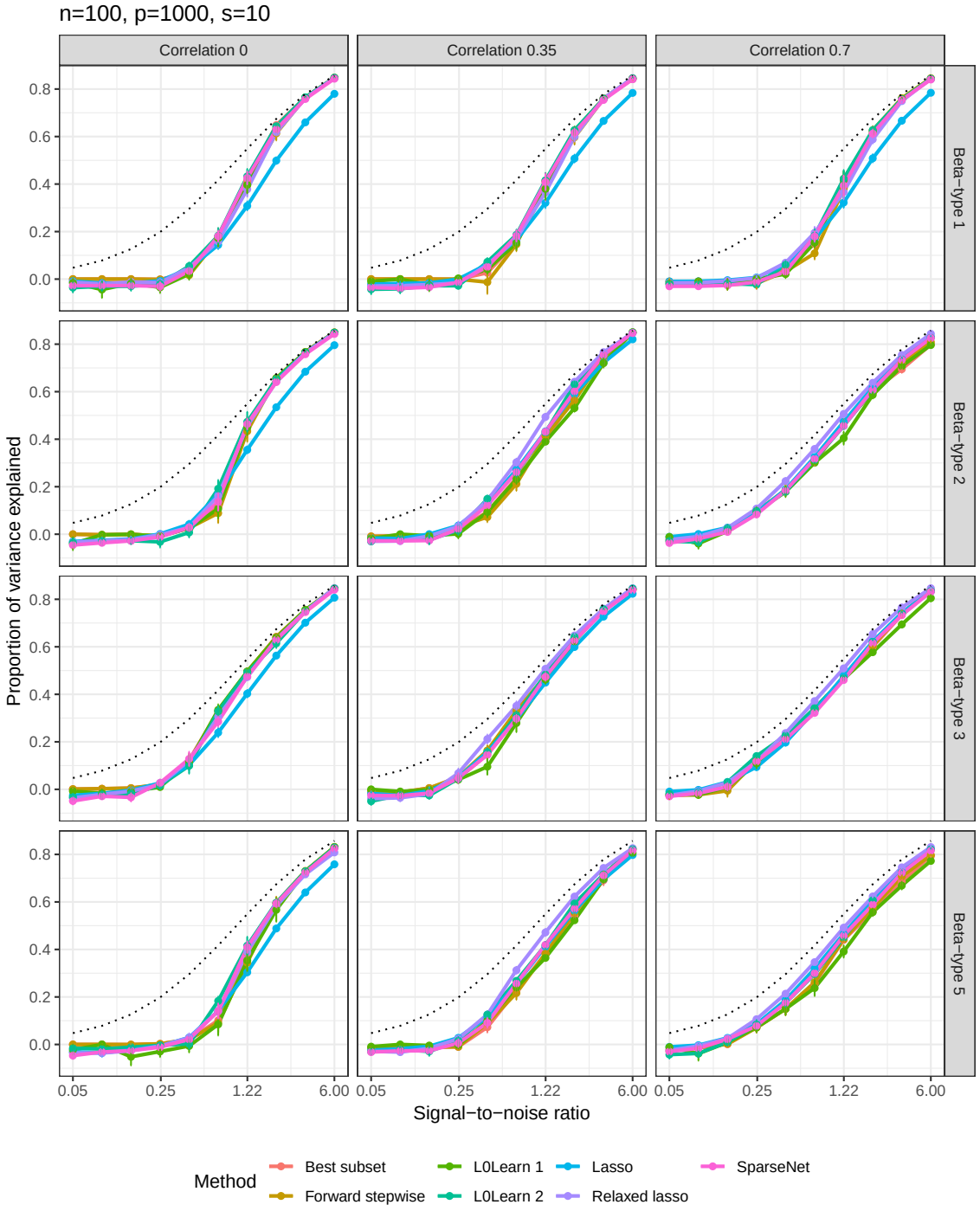

n=50, p=1000, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

C.4 High-10 setting: $n = 100$, $p = 1000$, $s = 10$

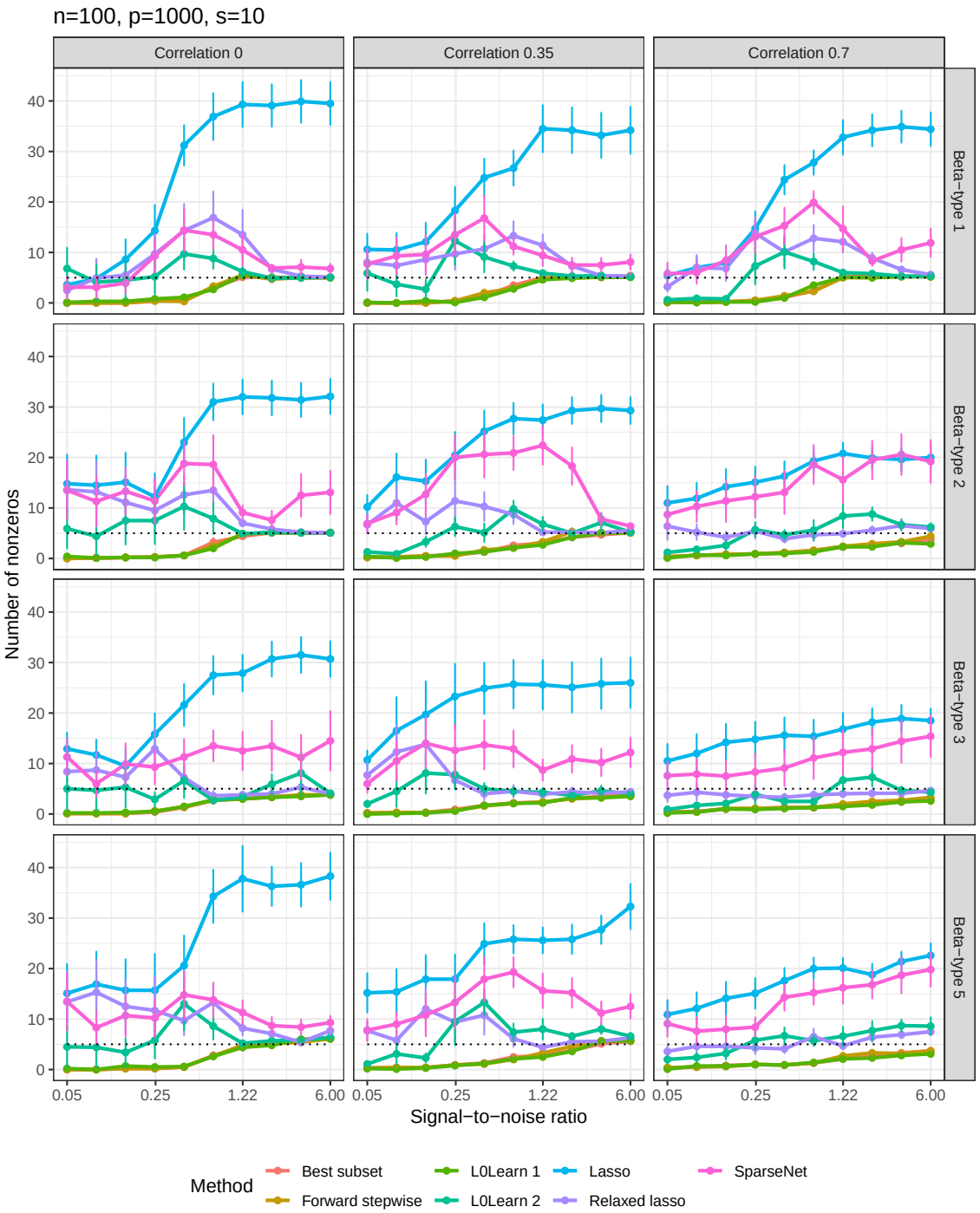
C.4.1 Relative test error (to Bayes)



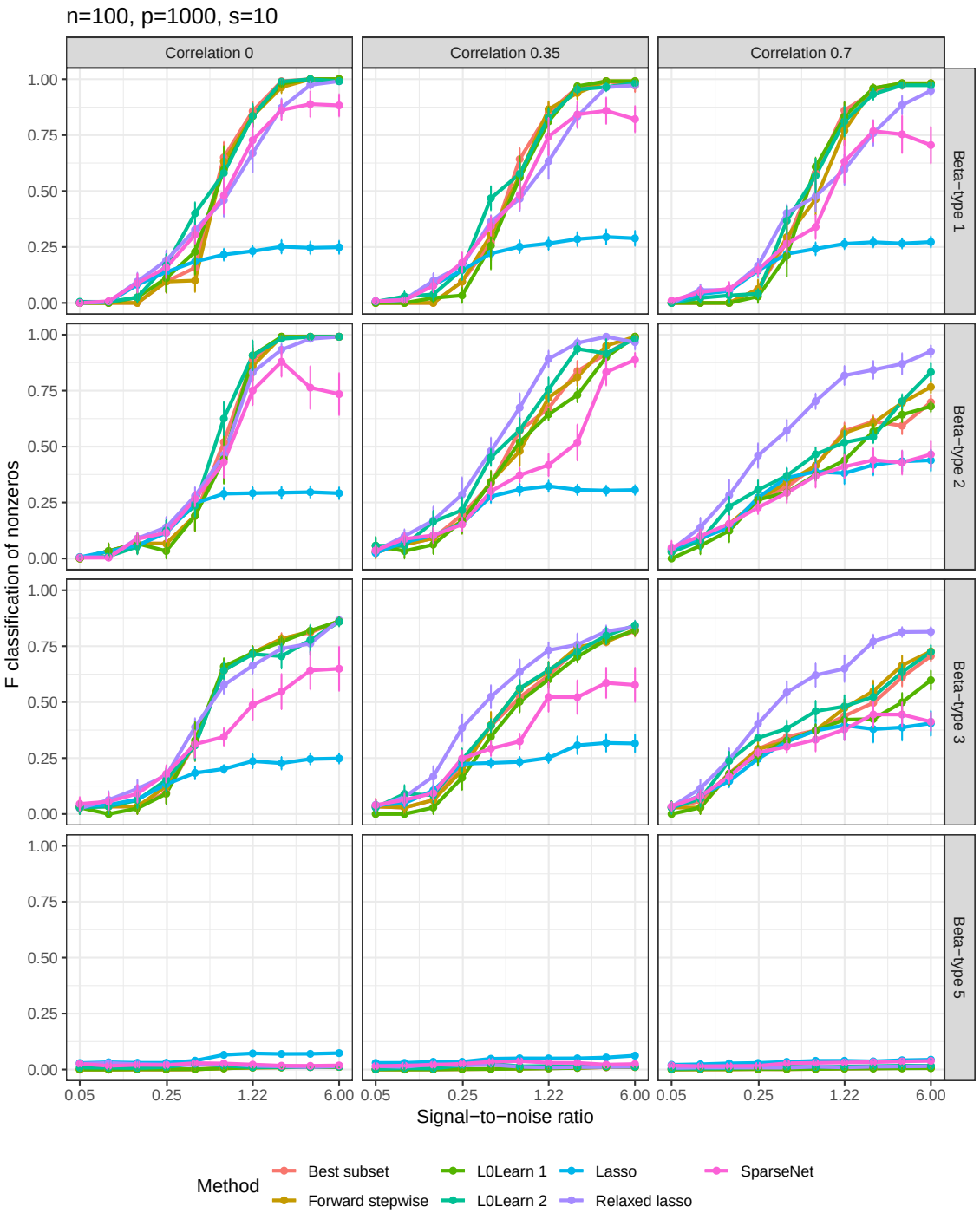
C.4.2 Proportion of variance explained



C.4.3 Number of nonzero coefficients



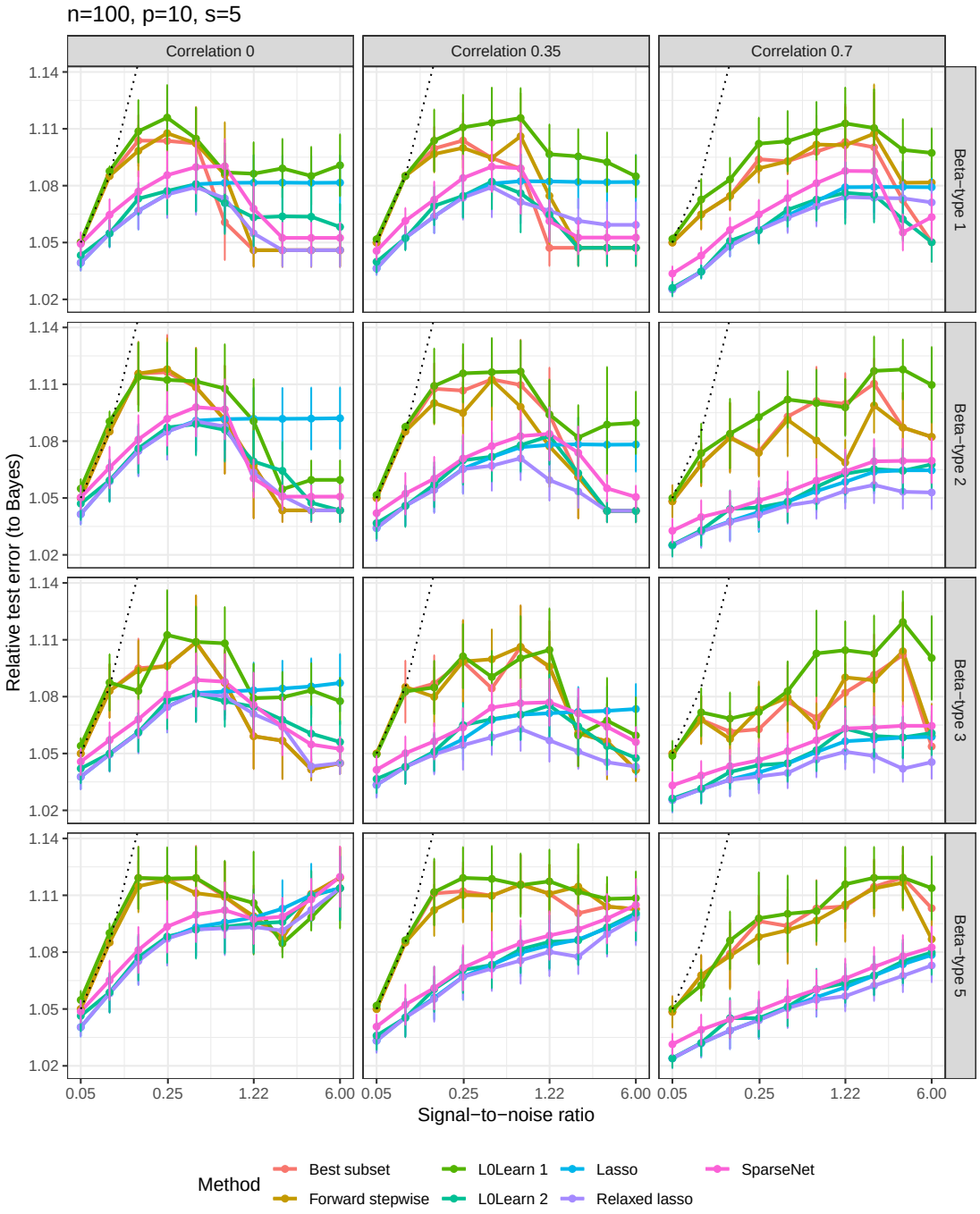
C.4.4 F-score


n=100, p=1000, s=10
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

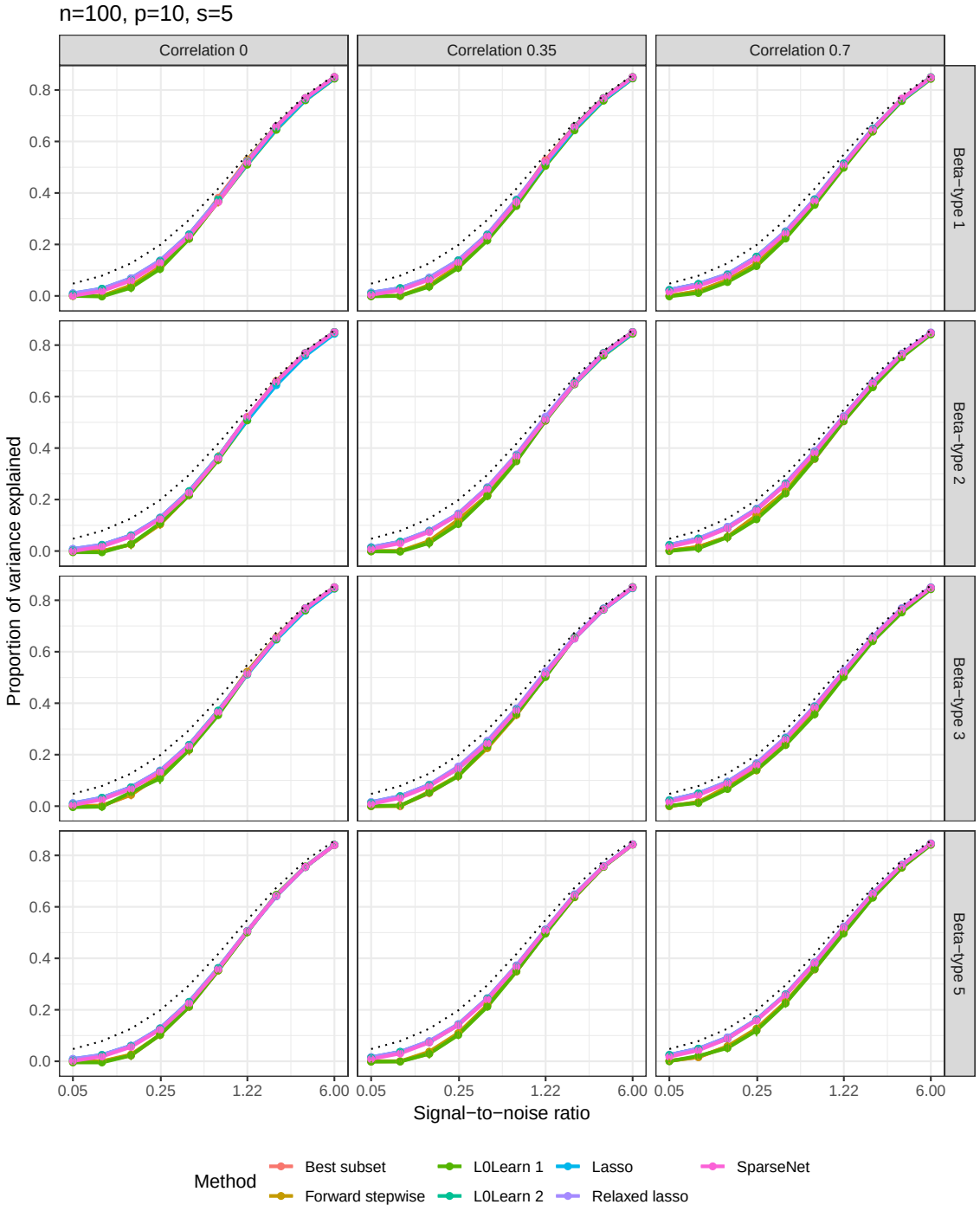
D Oracle tuning: now including L0Learn-1, L0Learn-2 and SparseNet

D.1 Low setting: $n = 100$, $p = 10$, $s = 5$

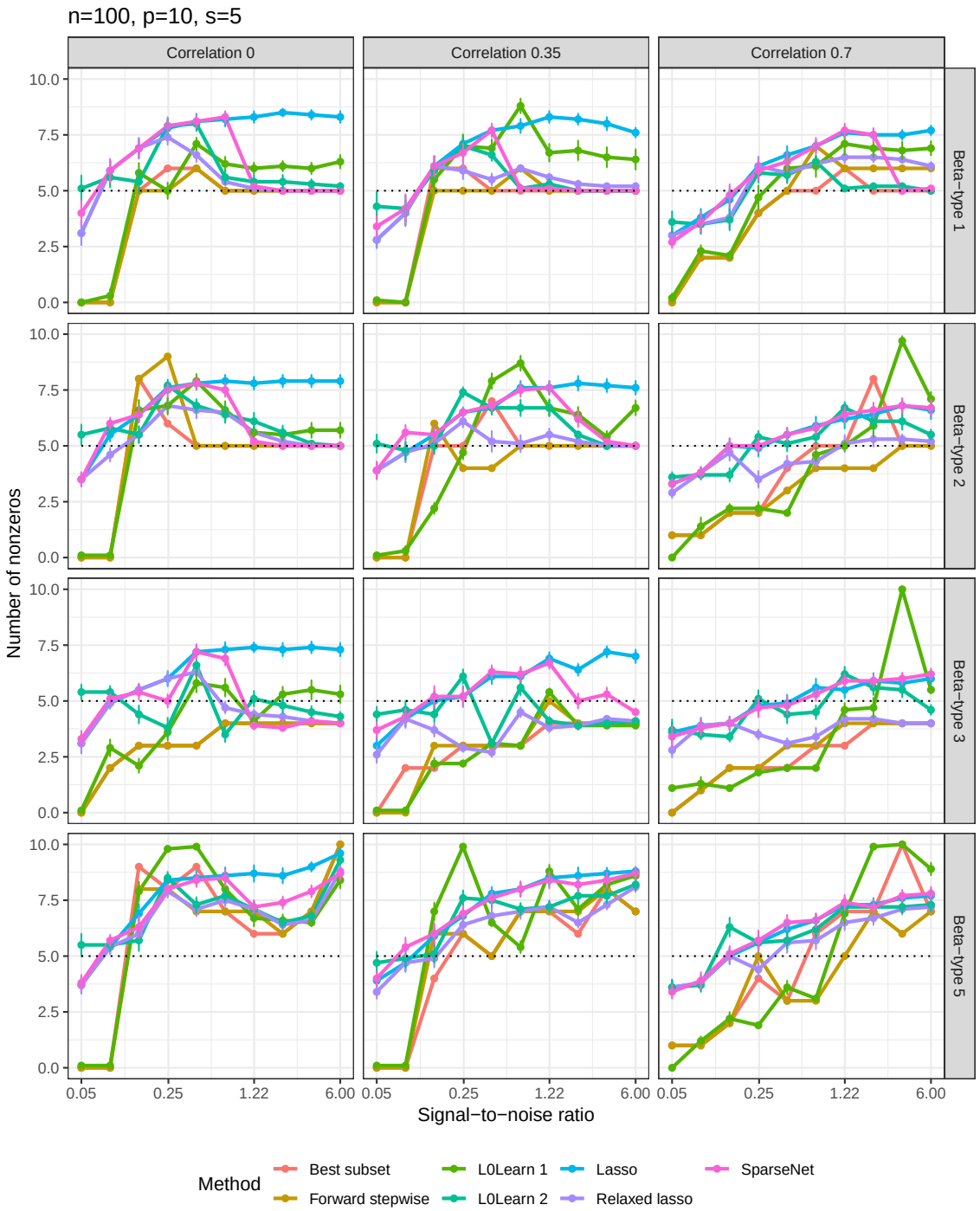
D.1.1 Relative test error (to Bayes)



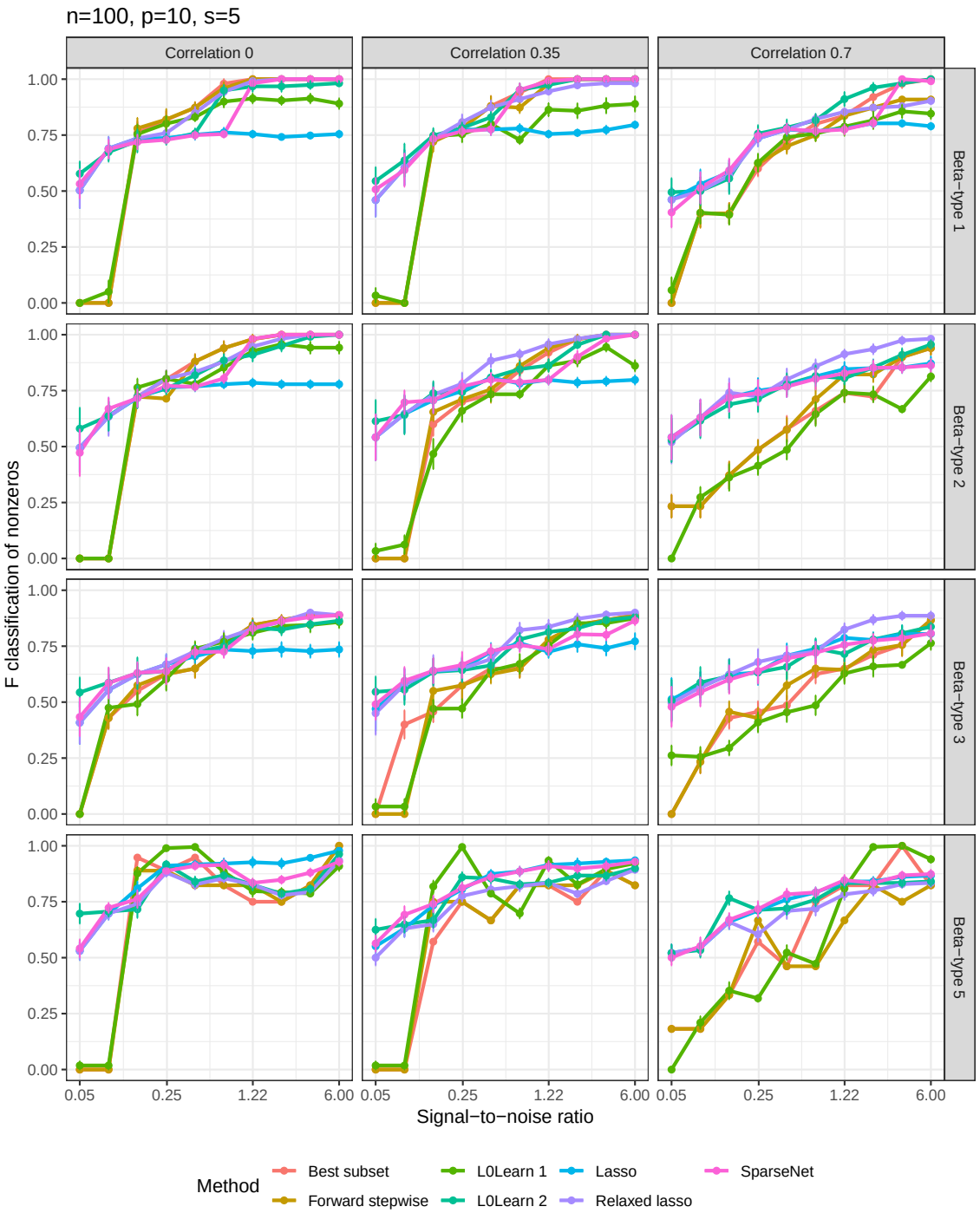
D.1.2 Proportion of variance explained



D.1.3 Number of nonzero coefficients

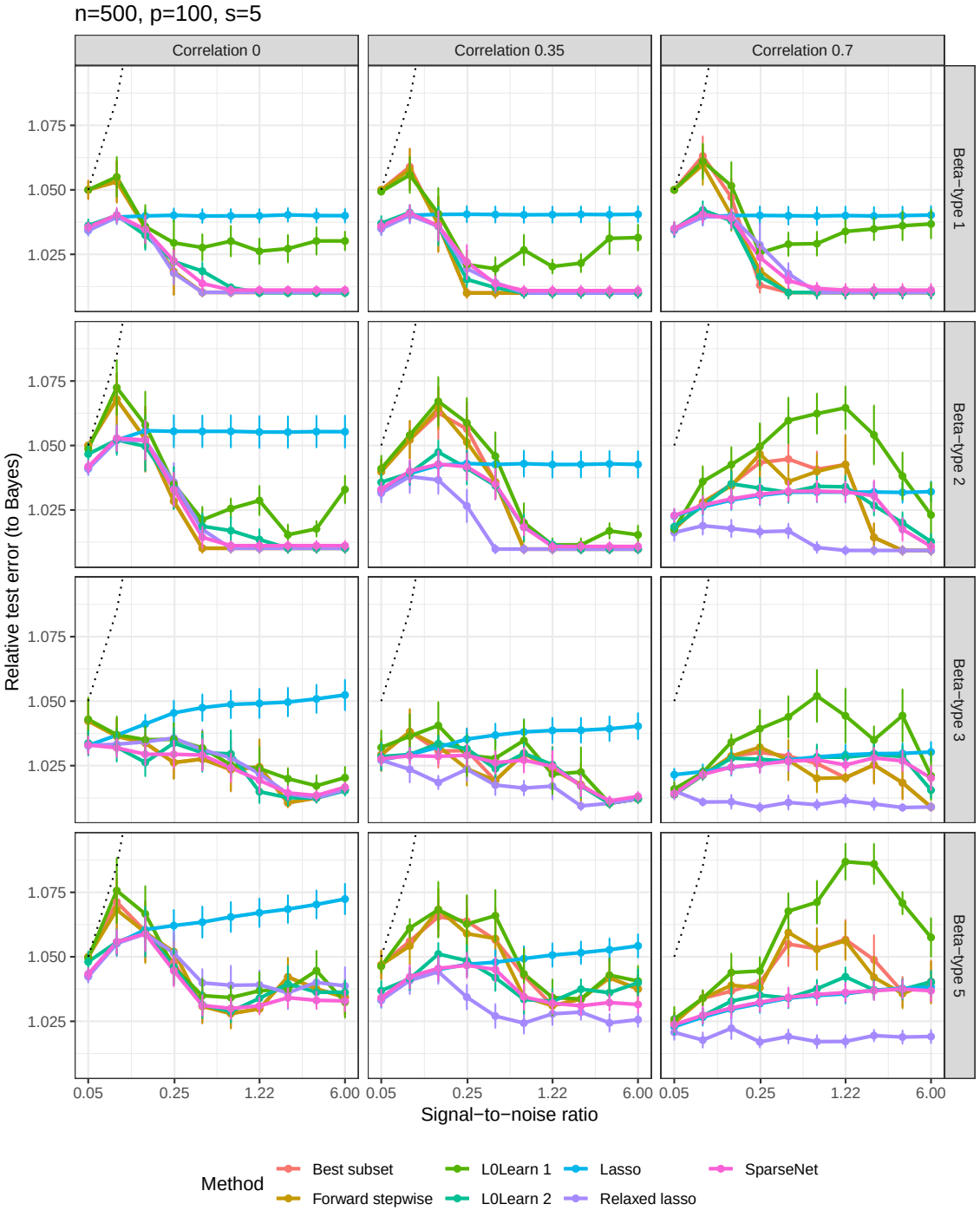


D.1.4 F-score

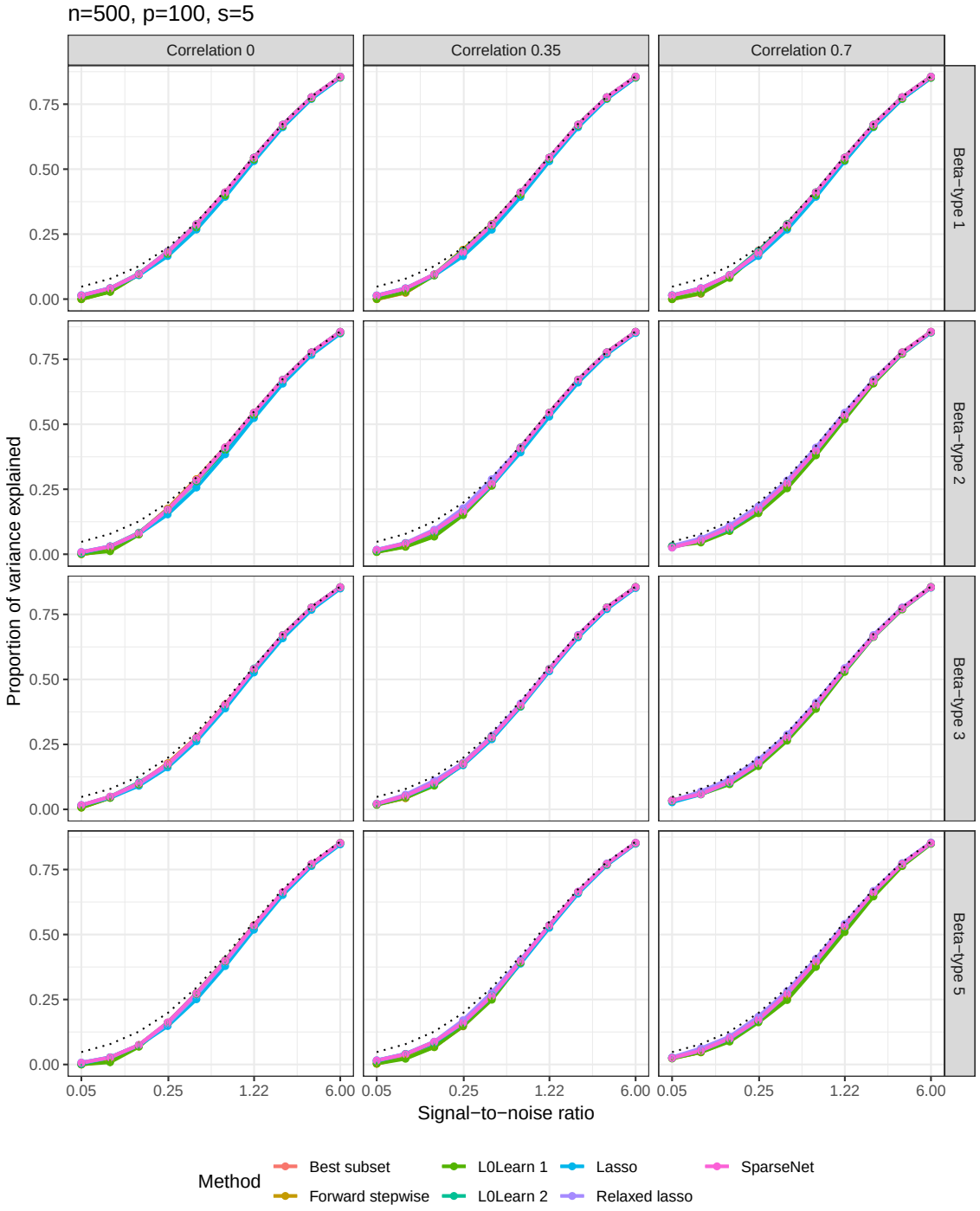

n=100, p=10, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

D.2 Medium setting: $n = 500$, $p = 100$, $s = 5$

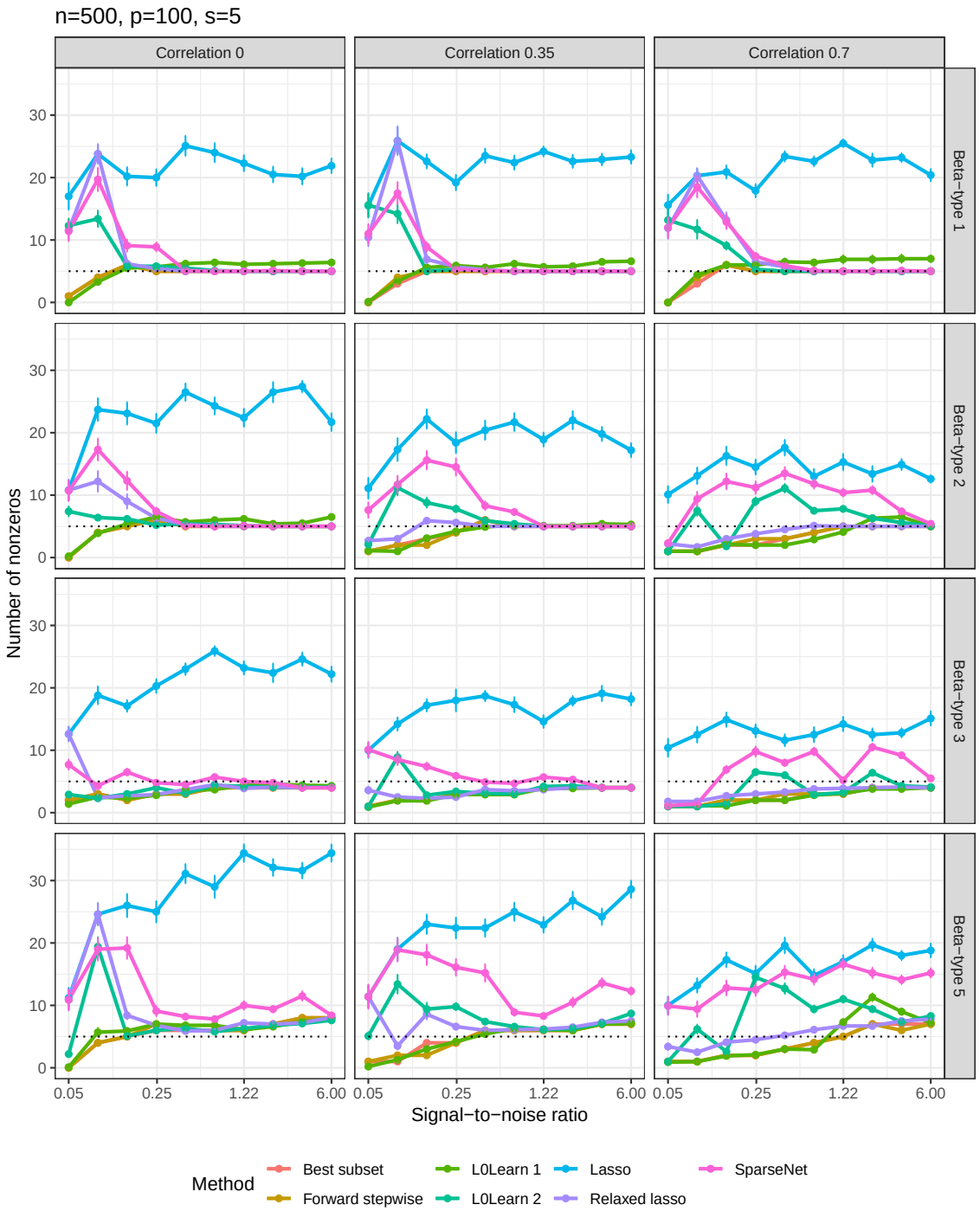
D.2.1 Relative test error (to Bayes)



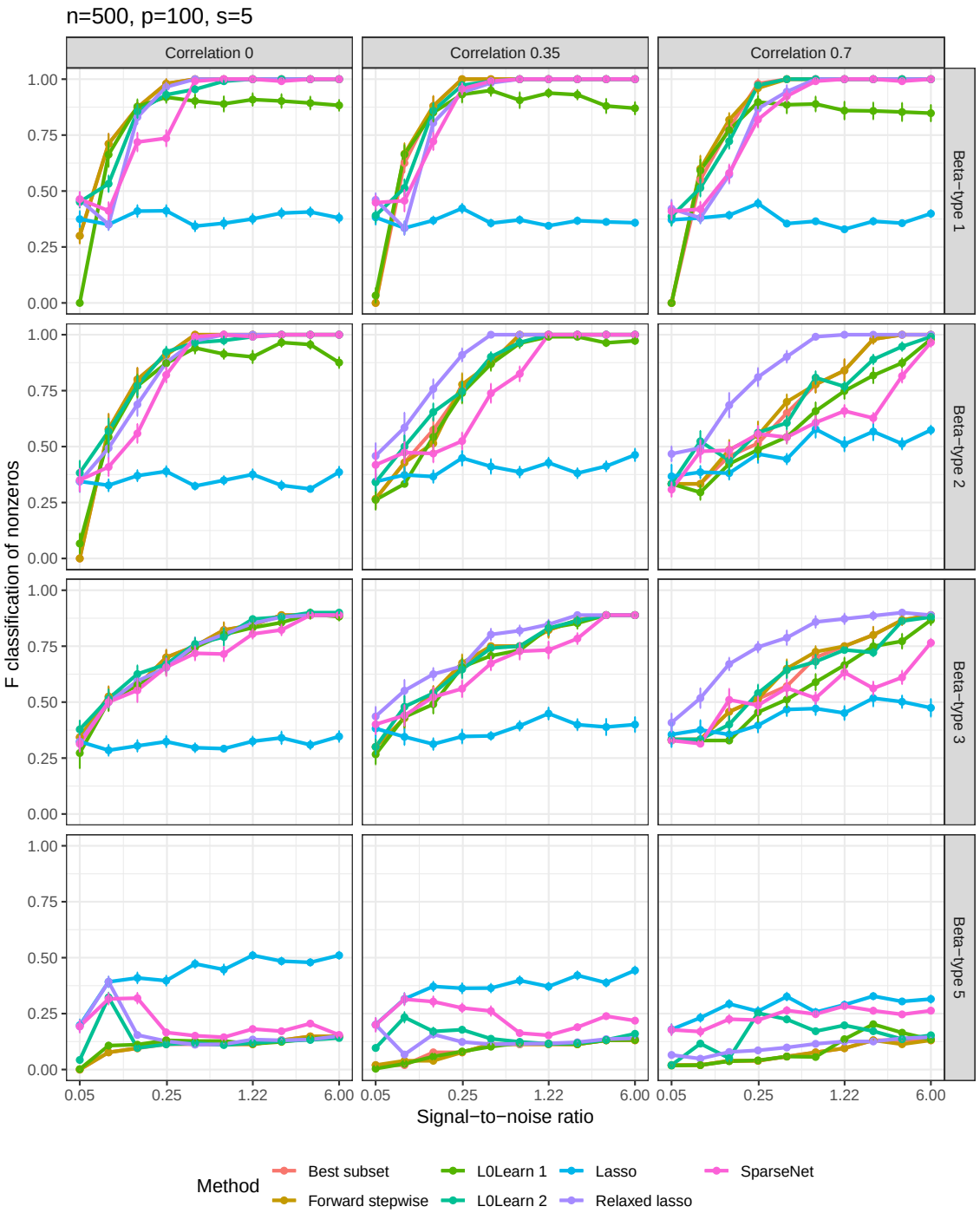
D.2.2 Proportion of variance explained



D.2.3 Number of nonzero coefficients

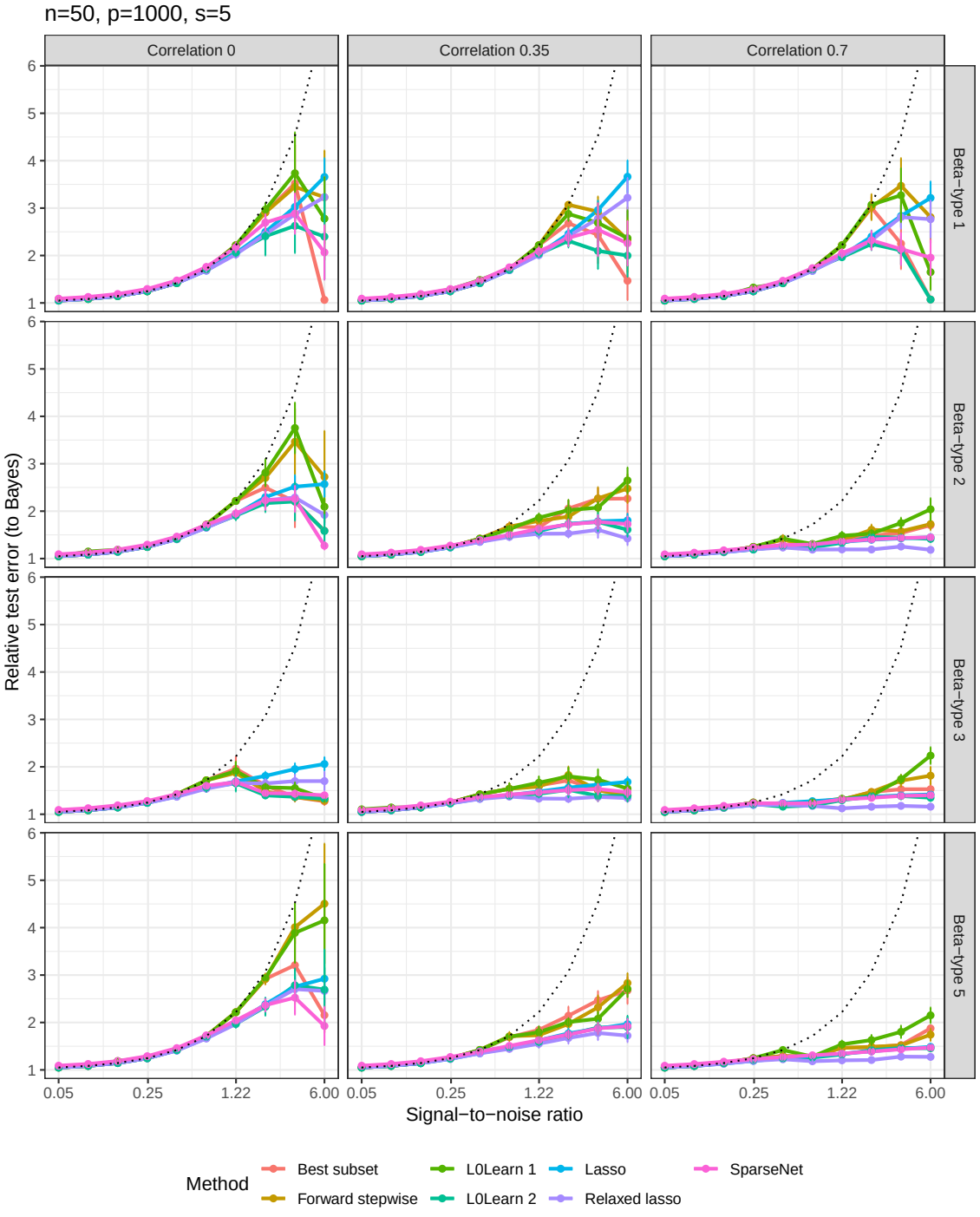


D.2.4 F-score

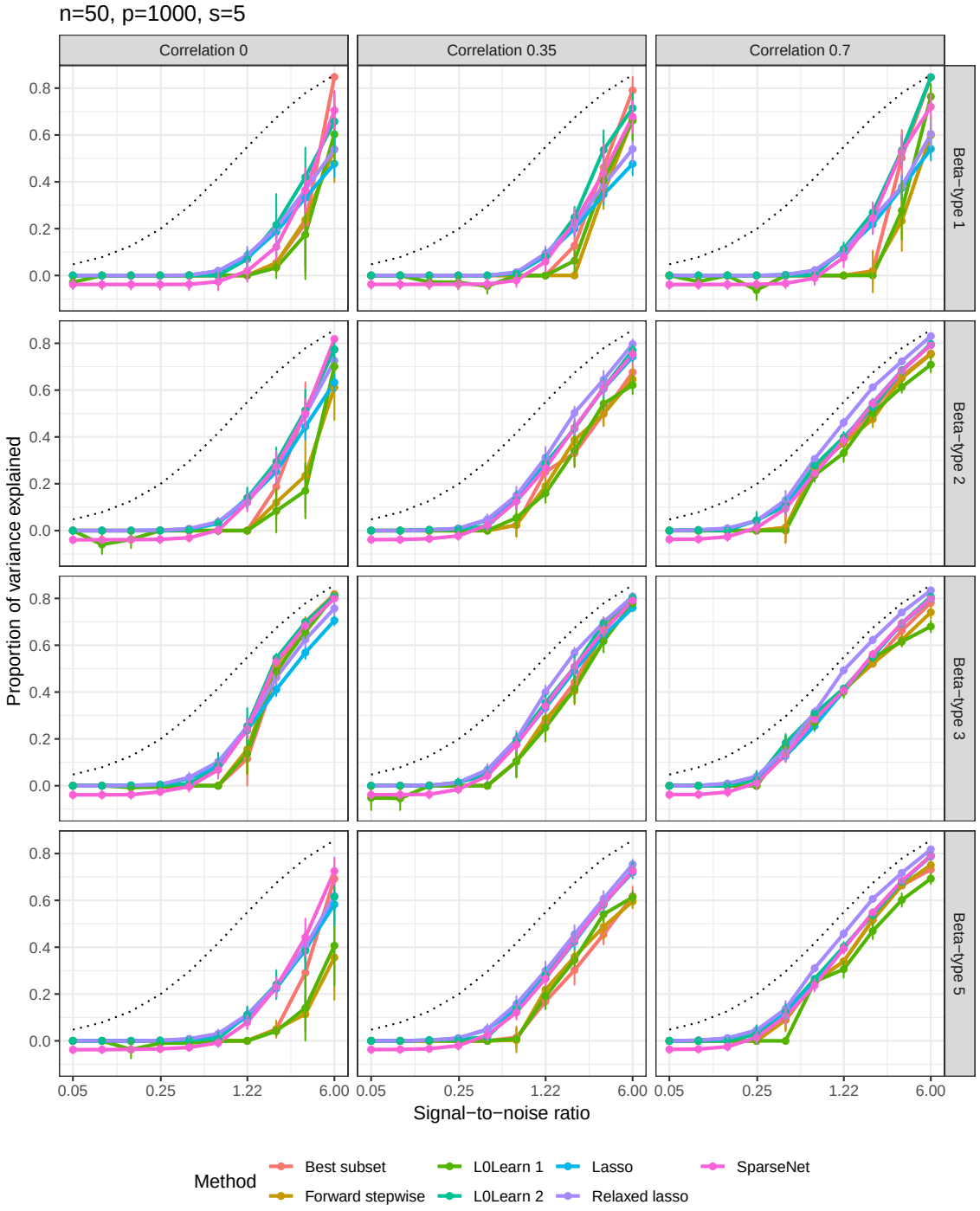

n=500, p=100, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

D.3 High-5 setting: $n = 50$, $p = 1000$, $s = 5$

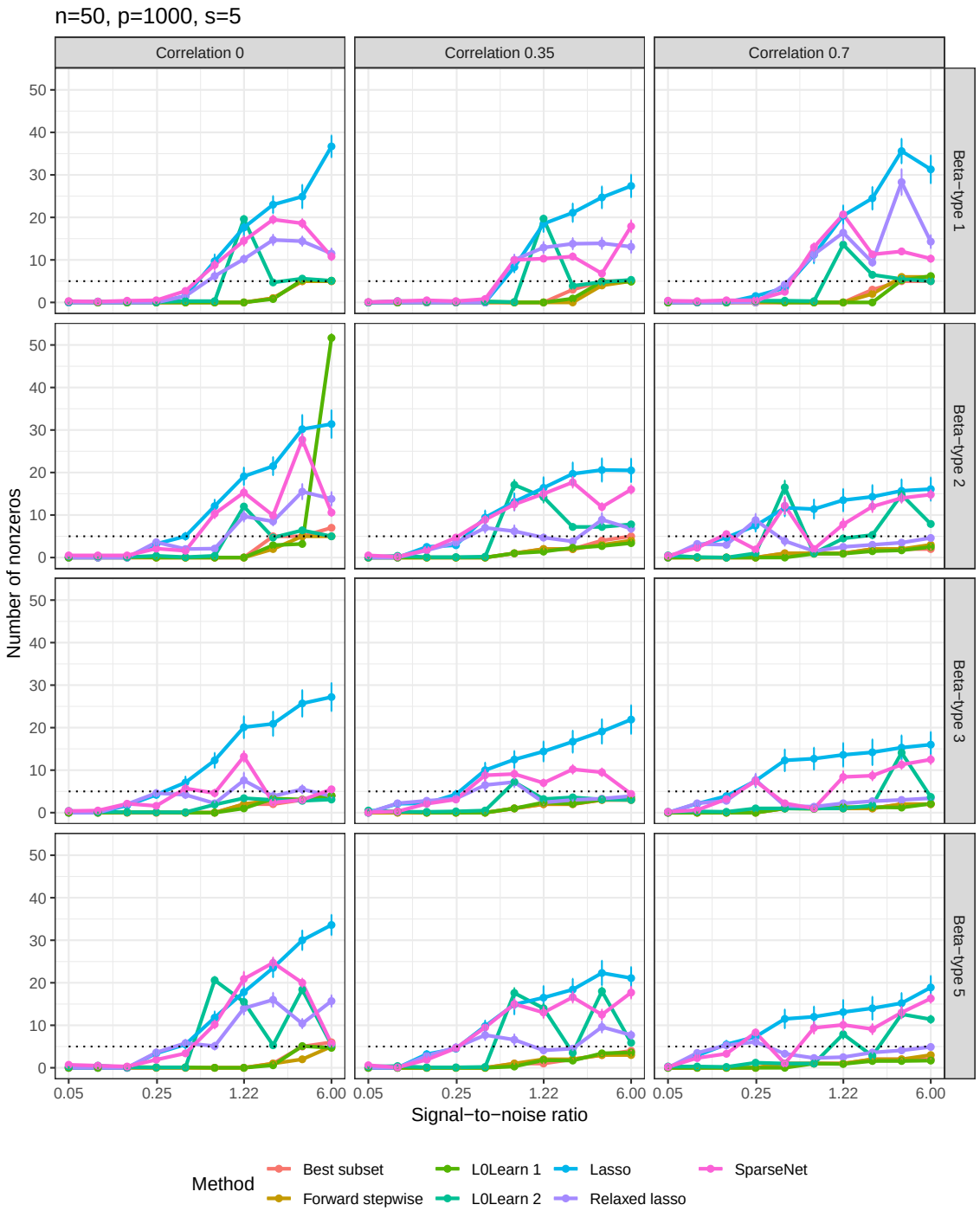
D.3.1 Relative test error (to Bayes)



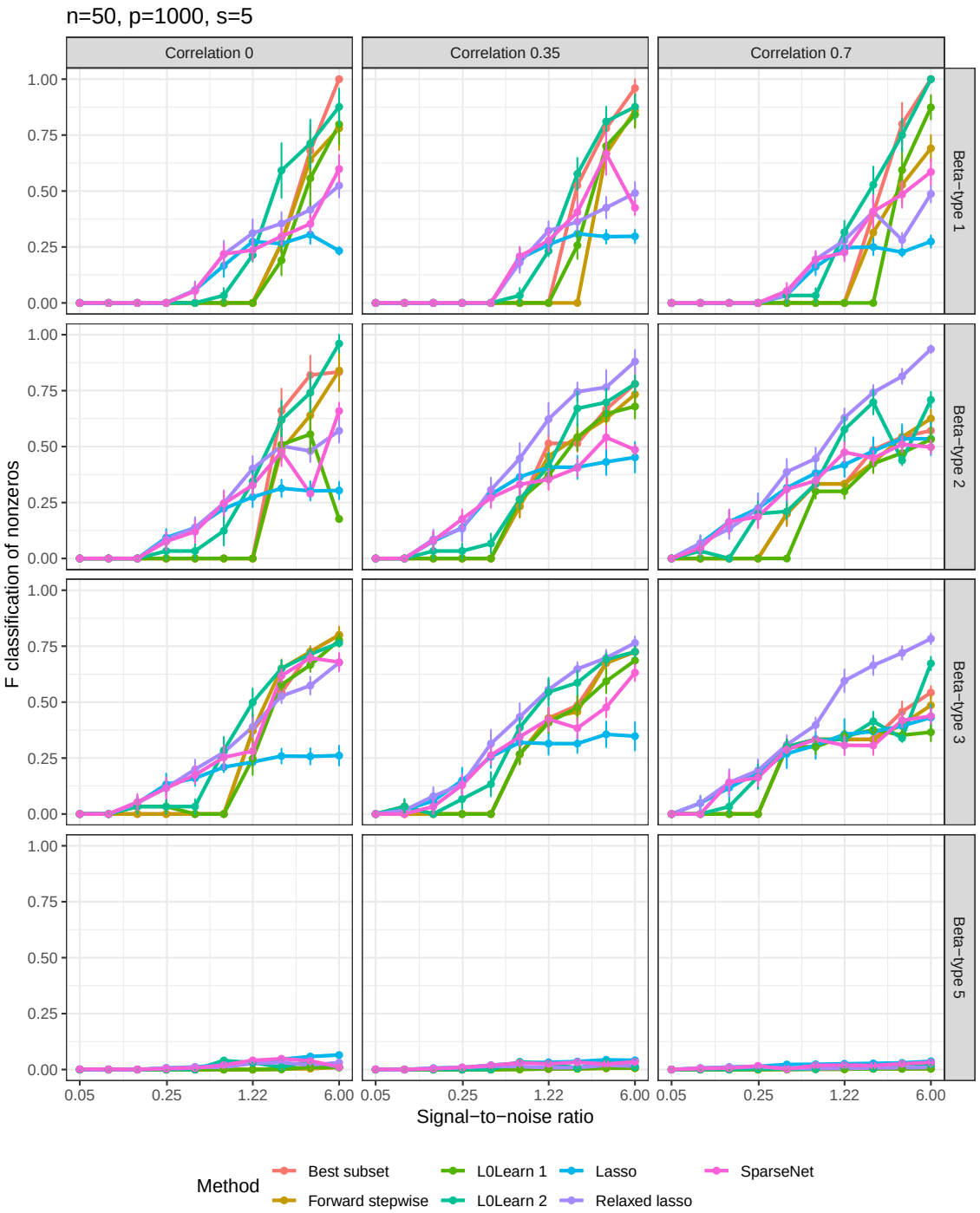
D.3.2 Proportion of variance explained



D.3.3 Number of nonzero coefficients



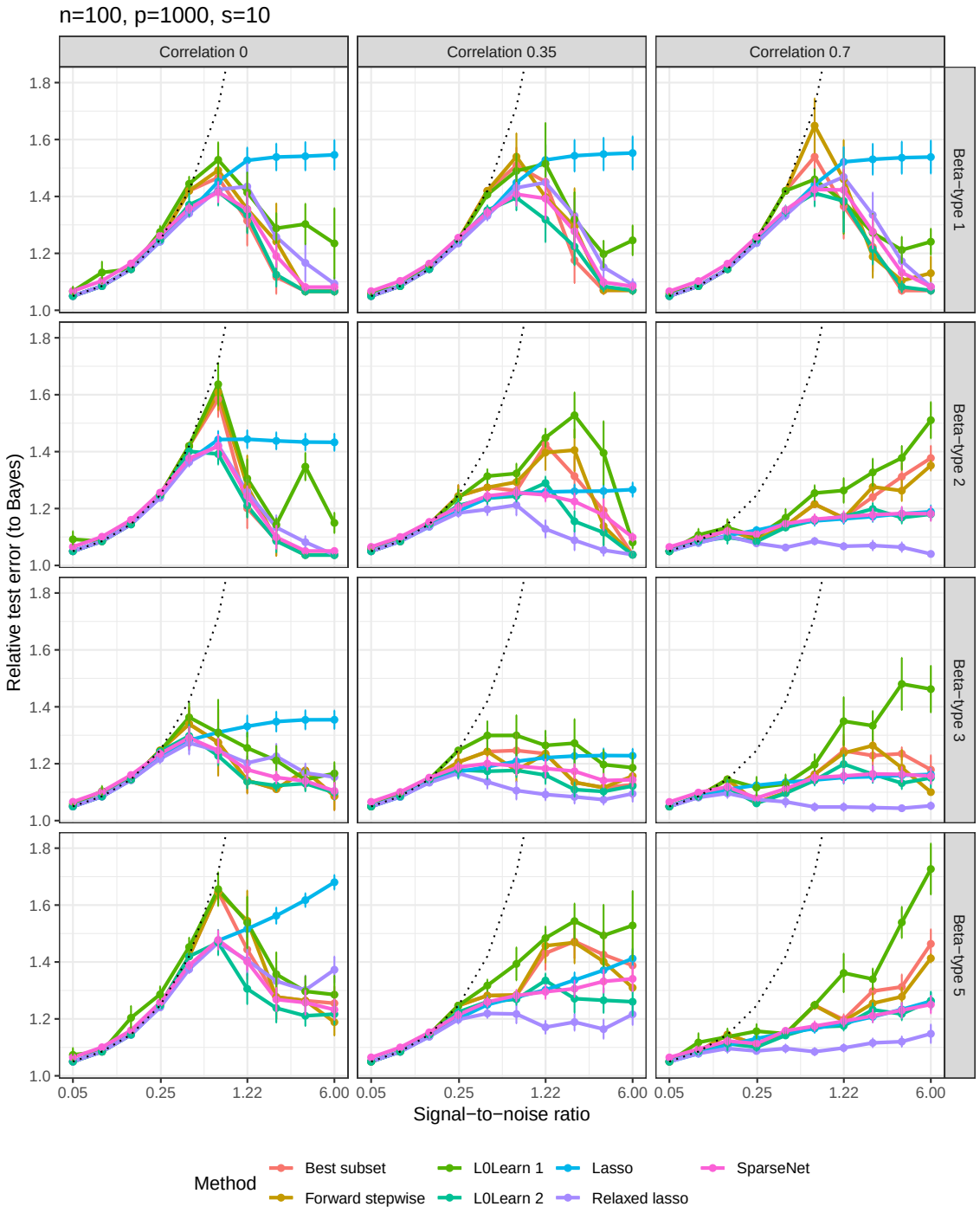
D.3.4 F-score


n=50, p=1000, s=5
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet

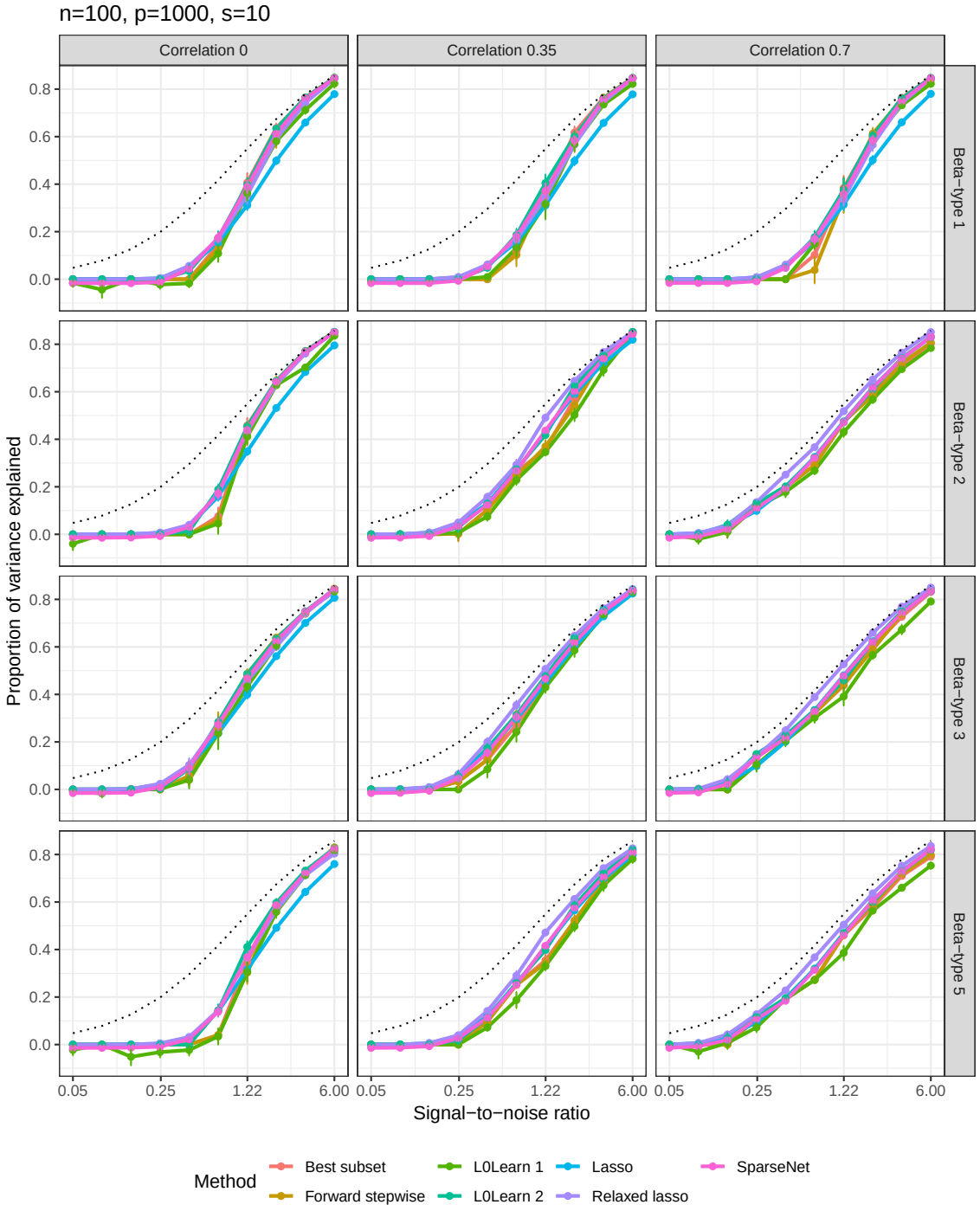
D.4 High-10 setting: $n = 100$, $p = 1000$, $s = 10$

D.4.1 Relative risk (to null model)

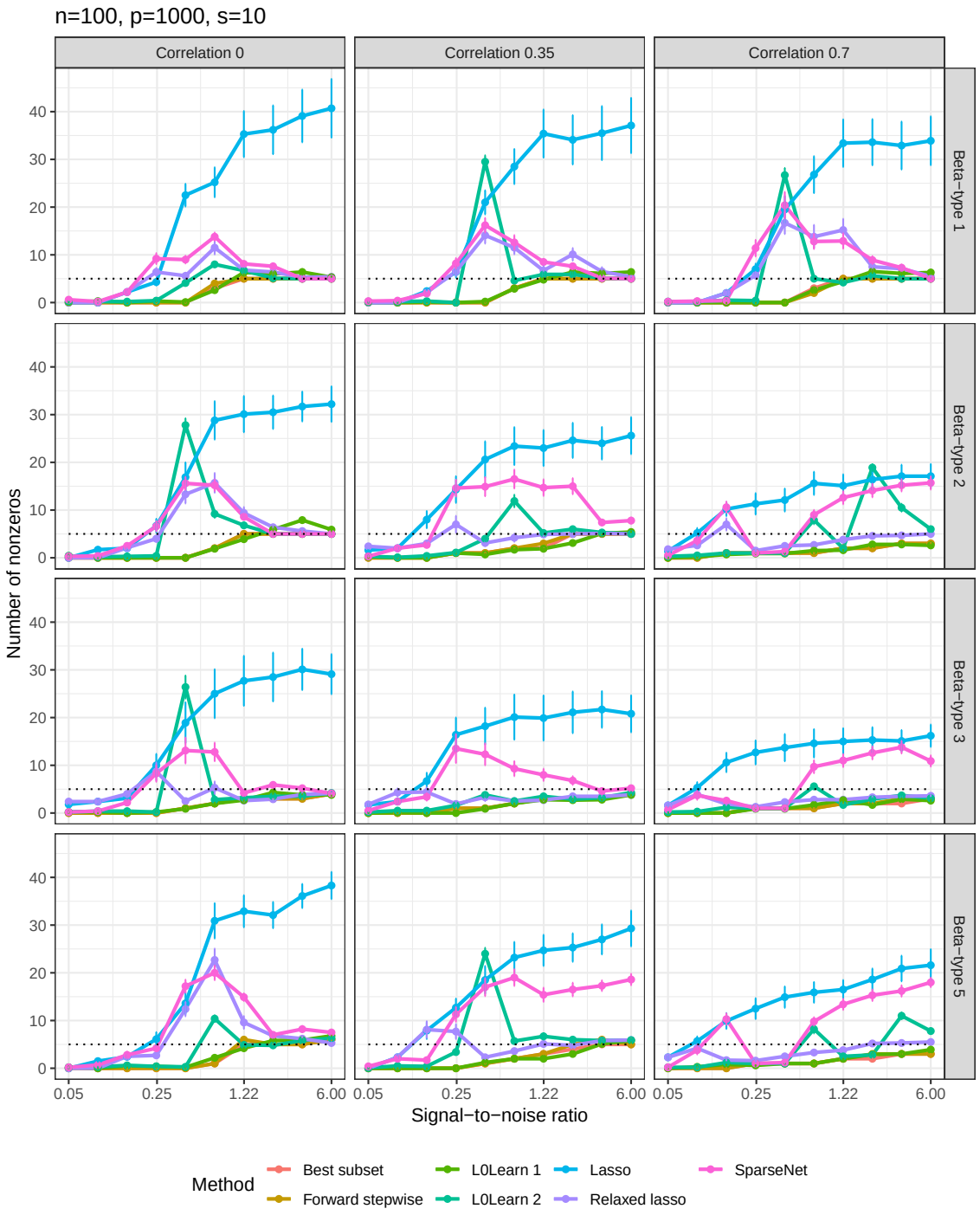
D.4.2 Relative test error (to Bayes)



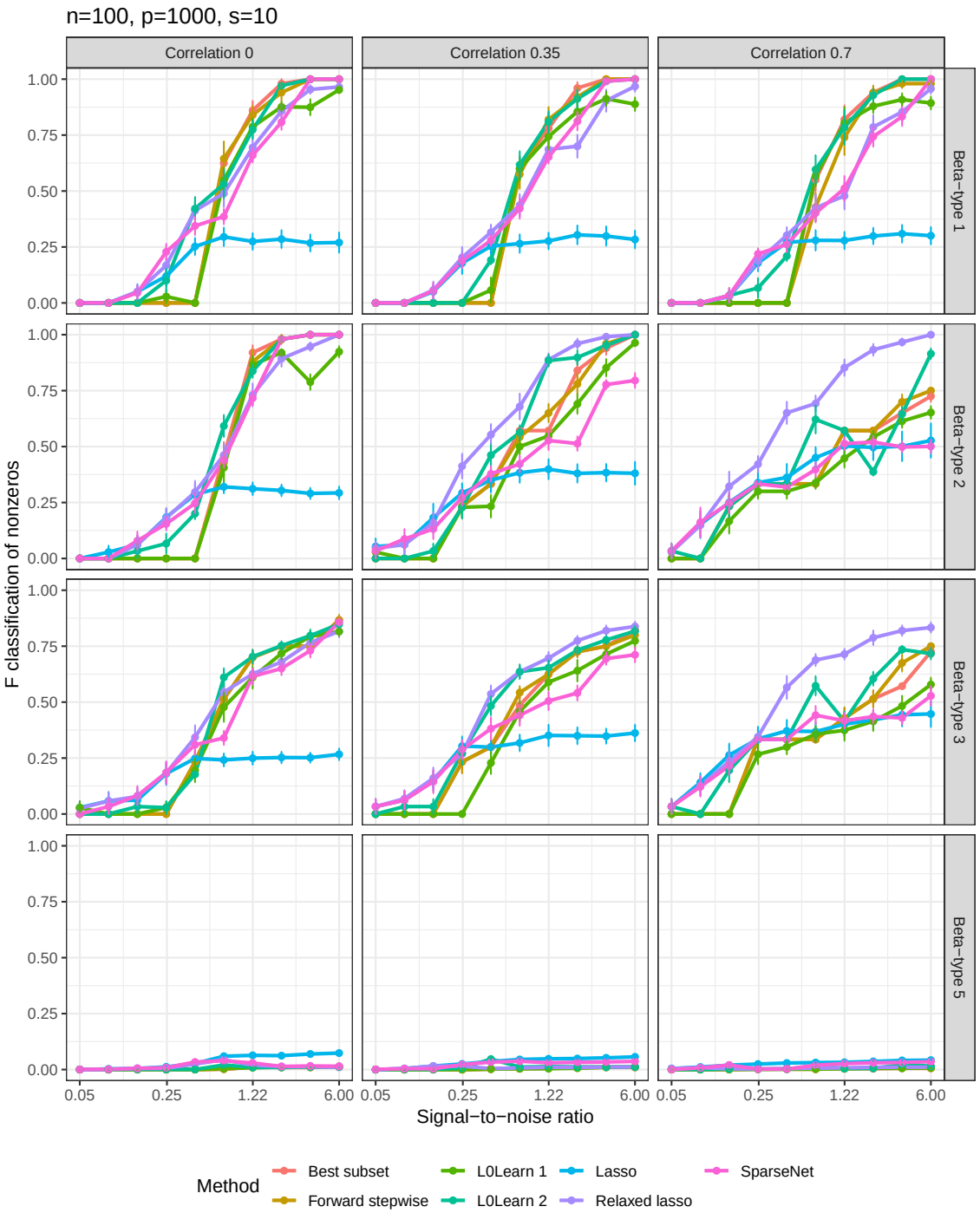
D.4.3 Proportion of variance explained



D.4.4 Number of nonzero coefficients



D.4.5 F-score


n=100, p=1000, s=10
Correlation 0
Correlation 0.35
Correlation 0.7
Beta−type 1
Beta−type 2
Beta−type 3
Beta−type 5
F classification of nonzeros
1.00
0.75
0.50
0.25
0.00
0.05
0.25
1.22
6.00
Signal−to−noise ratio
Method
Best subset
Forward stepwise
L0Learn 1
L0Learn 2
Lasso
Relaxed lasso
SparseNet