Compact Answers to Temporal Regular Path Queries (Supplementary Material)

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1 INTRODUCTION

This document provides detailed definitions and proofs for the article *Compact answers to Temporal Regular Path Queries*, submitted at CIKM 2023.

As opposed to the structure adopted in the article, the result here are grouped by topic (inductive representation, finiteness, complexity, etc) rather than representation ($\mathcal{U}^{[t]}$, $\mathcal{U}^{[d]}$, etc.). This allows us to emphasize which proofs differ from one representation to the other.

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2 INDUCTIVE REPRESENTATION

2.1 $(\!|q|\!)_G^{[t,d],b,e}$

2.1.1 *Definition.*

2.1.2 *Correctness.*

Operator $\mathsf{path}_1/\mathsf{path}_2$**.**
Let $\mathbf{u}_1 = \langle o_1, o_2, \tau_1, \delta_1\rangle$ and $\mathbf{u}_2 = \langle o_3, o_4, \tau_2, \delta_2\rangle$, with $o_2 = o_3$.
Let also $\mathbf{u}_1 \overline{\bowtie} \mathbf{u}_2 = \langle o_1, o_4, \tau_1', \delta_1 + \delta_2, b, e\rangle$, with

$$\tau = (((\tau_1 + \delta_1) \cap \tau_2) \ominus \delta_1) \cap \tau_1$$
$$b = \max(b_1, b_2 - b_{\delta_1})$$
$$e = \min(e_1, e_2 - e_{\delta_1})$$

For $i \in \{1, 2\}$ and $t \in \tau_i$, we use $\delta_i(t)$ for the interval

$$_{\delta_i}\lfloor\, b_{\delta_i} + \max(0, b_i - t)\ ,\ e_{\delta_i} - \max(0, t - e_i)\,\rfloor_{\delta_i}$$

And similarly to what we did for $\mathcal{U}^{[t,d]}$, we use R_i for be the binary relation over $\mathcal{T}$ specified by the time points and distances in $\mathbf{u}_i$, i.e. $R_i = \{(t, t + d) \mid t \in \tau_i, d \in \delta_i(t)\}$.
Then the intervals in the set $\mathbf{u}_1 \overline{\bowtie} \mathbf{u}_2$ should intuitively represent this relation $R_1 \bowtie R_2$, i.e. we need to prove that *(i)* $\tau = \mathsf{dom}(R_1 \bowtie R_2)$ and that *(ii)* for each $t \in \tau$,

$$t + \delta(t) = \{t' \mid (t, t') \in R_1 \bowtie R_2\}$$

For *(ii)* , let $t \in \tau$. We use

- a for the least value s.t. $(t, a) \in \mathsf{range}(R_1) \cap \mathsf{dom}(R_2)$, and
- a' for the least value s.t. $(a, a') \in R_2$

Then a' is also the least value s.t. $(t, a') \in R_1 \bowtie R_2$.
Analogously, we use z for the greatest value s.t. $(t, z) \in \mathsf{range}(R_1) \cap \mathsf{dom}(R_2)$, and z' for the greatest value s.t. $(z, z') \in R_2$. Then z' is also the greatest value s.t. $(t, z') \in R_1 \bowtie R_2$.
From Lemma **??**:

- $\{t\} \times (t + [a, z]) \subseteq R_1$, and
- $[a, b] \times [a', z'] \subseteq R_2$

Therefore $[a', z'] = \{c \mid (t, c) \in R_1 \bowtie R_2\}$.

To conclude the proof, we show that $t + \delta_t = [a, z]$.
We only prove that $t + b_{\delta_t} = a$ (the proof that $t + e_{\delta_t} = z$ is symmetric).
Following the definition of b, we consider 2 cases:

(1) $b_1 < b_2 - b_{\delta_1}$
(2) $b_1 \geq b_2 - b_{\delta_1}$

For Case (1), we get

$$b_1 < b_2 - b_{\delta_1} \quad (1)$$
$$\max(b_1, b_2 - b_{\delta_1}) = b_2 - b_{\delta_1} \quad (2)$$
$$b = b_2 - b_{\delta_1} \quad \text{from the definition of } b \quad (3)$$

And in Case (1) still, we get:

$$b_1 < b_2 - b_{\delta_1} \quad (4)$$
$$0 < b_2 - b_{\delta_1} - b_1 \quad (5)$$
$$\max(0, b_2 - b_{\delta_1} - b_1) = b_2 - b_{\delta_1} - b_1 \quad (6)$$

Next, we consider two subcases:

(i) $t < b_2 - b_{\delta_1}$
(ii) $t \geq b_2 - b_{\delta_1}$

In Case (i), we get

$$
\begin{aligned}
t <& \quad b_2 - b_{\delta_1} && (7)\\
0 <& \quad b_2 - b_{\delta_1} - t && (8)\\
\max(0, b_2 - b_{\delta_1} - t) =& \quad b_2 - b_{\delta_1} - t && (9)
\end{aligned}
$$

Now from the definition of δ_t,

$$
\begin{aligned}
b_{\delta_t} =& \quad b_{\delta_1} + b_{\delta_2} + \max(0, b - t) && && (10)\\
=& \quad b_{\delta_1} + b_{\delta_2} + \max(0, b_2 - b_{\delta_1} - t) && \text{from (3)} && (11)\\
=& \quad b_{\delta_1} + b_{\delta_2} + b_2 - b_{\delta_1} - t && \text{from (9)} && (12)\\
=& \quad b_{\delta_2} + b_2 - t && && (13)\\
b_{\delta_t} + t =& \quad b_{\delta_2} + b_2 - t + t && && (14)\\
=& \quad b_{\delta_2} + b_2 && && (15)
\end{aligned}
$$

Next, from the definition of a'

$$
\begin{aligned}
a' =& \quad b_{\delta_2(a)} + a && (16)\\
=& \quad b_{\delta_2} + \max(0, b_2 - a) + a && (17)
\end{aligned}
$$

And, from the definition of a

$$
\begin{aligned}
a =& \quad b_{\delta_1(t)} + t && (18)\\
=& \quad b_{\delta_1} + \max(0, b_1 - t) + t && (19)
\end{aligned}
$$

Then we have two further subcases:

(I) $t \geq b_1$, or
(II) $t < b_1$

In case (I):

$$
\begin{aligned}
t \geq& \quad b_1 && && (20)\\
0 \geq& \quad b_1 - t && && (21)\\
\max(0, b_1 - t) =& \quad 0 && && (22)\\
a =& \quad b_{\delta_1} + t && \text{from (19)} && (23)\\
\max(0, b_2 - a) =& \quad \max(0, b_2 - b_{\delta_1} - t) && && (24)\\
=& \quad b_2 - b_{\delta_1} - t && \text{from (9)} && (25)\\
=& \quad b_2 - a && \text{from (23)} && (26)
\end{aligned}
$$

In case (II):

$$
\begin{aligned}
t <& \quad b_1 && && (27)\\
0 <& \quad b_1 - t && && (28)\\
\max(0, b_1 - t) =& \quad b_1 - t && && (29)\\
a =& \quad b_{\delta_1} + b_1 - t + t && \text{from (19)} && (30)\\
=& \quad b_{\delta_1} + b_1 && && (31)\\
\max(0, b_2 - a) =& \quad \max(0, b_2 - b_{\delta_1} - b_1) && && (32)\\
=& \quad b_2 - b_{\delta_1} - b_1 && \text{from (6)} && (33)\\
=& \quad b_2 - a && \text{from (31)} && (34)\\
& && && (35)
\end{aligned}
$$

So in both cases (I) and (II), we get

$$\max(0, b_2 - a) = b_2 - a$$

Thefore from (17)

$$a' = b_{\delta_2} + b_2 - a + a \tag{36}$$
$$= b_{\delta_2} + b_2 \tag{37}$$
$$= t + b_{\delta_t} \quad \text{from (15)} \tag{38}$$

which concludes the proof for Case (1)- (i).

We continue with Case (1)- (ii).
From Case (ii):

$$t \geq b_2 - b_{\delta_1} \tag{39}$$
$$0 \geq b_2 - b_{\delta_1} - t \tag{40}$$
$$\max(0, b_2 - b_{\delta_1} - t) = 0 \tag{41}$$

Now from the definition of δ_t:

$$b_{\delta_t} = b_{\delta_1} + b_{\delta_2} + \max(0, b - t) \tag{42}$$
$$= b_{\delta_1} + b_{\delta_2} + \max(0, b_2 - b_1 - t) \quad \text{from (3)} \tag{43}$$
$$= b_{\delta_1} + b_{\delta_2} \quad \text{from (41)} \tag{44}$$
$$b_{\delta_t} + t = b_{\delta_1} + b_{\delta_2} + t \tag{45}$$

Next, from Case (1) and Case (ii), by transitivity, we get

$$b_1 \leq t \tag{46}$$
$$\max(0, b_1 - t) = 0 \tag{47}$$

And from the definition of a

$$a = b_{\delta_1(t)} + t \tag{48}$$
$$= b_{\delta_1} + \max(0, b_1 - t) + t \tag{49}$$
$$= b_{\delta_1} + t \quad \text{from (47)} \tag{50}$$
$$\geq b_{\delta_1} + b_2 - b_{\delta_1} \quad \text{from Case (ii)} \tag{51}$$
$$\geq b_2 \tag{52}$$
$$0 \geq b_2 - a \tag{53}$$
$$\max(0, b_2 - a) = 0 \tag{54}$$

Therefore from (17) and (54)

$$a' = b_{\delta_2} + a \tag{55}$$
$$= b_{\delta_2} + b_{\delta_1} + t \quad \text{from (50)} \tag{56}$$
$$= b_{\delta_t} + t \quad \text{from (15)} \tag{57}$$

which concludes the proof for Case (1)- (ii).

We continute with Case (2).
In this case, we get

$$b_1 \geq b_2 - b_{\delta_1} \tag{58}$$
$$\max(b_1, b_2 - b_{\delta_1}) = b_1 \tag{59}$$
$$b = b_1 \quad \text{from the definition of } b \tag{60}$$

And from Case (2) still, we derive

$$b_1 \geq b_2 - b_{\delta_1} \tag{61}$$
$$0 \geq b_2 - b_{\delta_1} - b_1 \tag{62}$$
$$\max(0, -b_{\delta_1} - b_1) = 0 \tag{63}$$

As well as

$$b_1 \geq b_2 - b_{\delta_1} \tag{64}$$
$$b_1 + b_{\delta_1} \geq b_2 \tag{65}$$

Next, we distinguish two subcases, namely

(a) $t < b_1$ and
(b) $t \geq b_1$

We start with Case (a).
In this case,

$$\begin{aligned} t <& \ b_1 && (66) \\ 0 <& \ b_1 - t && (67) \\ \max(0, b_1 - t) =& \ b_1 - t && (68) \end{aligned}$$

And from the definition of δ_t:

$$\begin{aligned} b_{\delta_t} =& \ b_{\delta_1} + b_{\delta_2} + \max(0, b - t) && && (69) \\ =& \ b_{\delta_1} + b_{\delta_2} + \max(0, b_1 - t) && \text{from (60)} && (70) \\ =& \ b_{\delta_1} + b_{\delta_2} + b_1 - t && \text{from (68)} && (71) \\ b_{\delta_t} + t =& \ b_{\delta_1} + b_{\delta_2} + b_1 - t + t && && (72) \\ =& \ b_{\delta_1} + b_{\delta_2} + b_1 && && (73) \end{aligned}$$

Next, from the definition of a

$$\begin{aligned} a =& \ b_{\delta_1(t)} + t && && (74) \\ =& \ \max(0, b_1 - t) + b_{\delta_1} + t && && (75) \\ =& \ b_1 - t + b_{\delta_1} + t && \text{from (68)} && (76) \\ =& \ b_1 + b_{\delta_1} && && (77) \end{aligned}$$

So from (65) (78)

$$\begin{aligned} a \geq& \ b_2 && && (79) \\ 0 \geq& \ b_2 - a && && (80) \\ \max(0, b_2 - a) =& \ 0 && && (81) \\ b_{\delta_2} + \max(0, b_2 - a) =& \ b_{\delta_2} && && (82) \\ b_{\delta_2(a)} =& \ b_{\delta_2} && && (83) \\ b_{\delta_2(a)} + a =& \ b_{\delta_2} + a && && (84) \\ a' =& \ b_{\delta_2} + a && \text{from the defintiion of } a' && (85) \\ a' =& \ b_{\delta_2} + b_1 + b_{\delta_1} && \text{from (77)} && (86) \\ a' =& \ b_{\delta_t} + t && \text{from (73)} && (87) \end{aligned}$$

which concludes the proof for Case (2)- (a).

We end with Case (2)- (b). In this case,

$$\begin{aligned} t \geq& \ b_1 && (88) \\ 0 \geq& \ b_1 - t && (89) \\ \max(0, b_1 - t) =& \ 0 && (90) \end{aligned}$$

And from the definition of δ_t:

$$\begin{aligned} b_{\delta_t} =& \ b_{\delta_1} + b_{\delta_2} + \max(0, b - t) && && (91) \\ =& \ b_{\delta_1} + b_{\delta_2} + \max(0, b_1 - t) && \text{from (60)} && (92) \\ =& \ b_{\delta_1} + b_{\delta_2} && \text{from (90)} && (93) \\ b_{\delta_t} + t =& \ b_{\delta_1} + b_{\delta_2} + t && && (94) \end{aligned}$$

Next, from the definition of a

$$\begin{aligned} a =& \ b_{\delta_1(t)} + t && && (95) \\ =& \ \max(0, b_1 - t) + b_{\delta_1} + t && && (96) \\ =& \ b_{\delta_1} + t && \text{from (90)} && (97) \end{aligned}$$

Now from Case (b)

$$
\begin{aligned}
b_1 + \leq \;& t && && (98)\\
b_1 + b_{\delta_1} \leq \;& t + b_{\delta_1} && && (99)\\
b_1 + b_{\delta_1} \leq \;& a && \text{from (97)} && (100)\\
b_2 \leq \;& a && \text{from (65), by transitivity} && (101)\\
b_2 - a \leq \;& 0 && && (102)\\
\max(0, b_2 - a) = \;& 0 && && (103)\\
b_{\delta_2} + \max(0, b_2 - a) = \;& b_{\delta_2} && && (104)\\
b_{\delta_2(a)} = \;& b_{\delta_2} && && (105)\\
b_{\delta_2(a)} + a = \;& b_{\delta_2} + a && && (106)\\
a' = \;& b_{\delta_2} + a && \text{from the defintiion of } a' && (107)\\
= \;& b_{\delta_2} + b_{\delta_1} + t && \text{from (97)} && (108)\\
= \;& b_{\delta_t} + t && \text{from (94)} && (109)
\end{aligned}
$$

which concludes the proof for Case (2)- (b). □

3 FINITENESS

4 COMPLEXITY OF QUERY ANSWERING

5 MINIMIZATION

6 SIZE OF COMPACT ANSWERS