Project B:

Question 1:

Prove: $-\epsilon y'' + y = 2x + 1$

Given: $0 \leq x \leq 1$, $y(0) = 0$, $y(1) = 0$, $\epsilon = 10^{-3}$,

$$y(x) = 2x + 1 - \frac{sinh(\frac{1-x}{\sqrt{}})+3sinh(\frac{x}{\sqrt{}})}{sinh(\frac{1}{\sqrt{}})}$$

Derivatives:

$$y'(x) = \frac{cosh(\frac{1-x}{\sqrt{}})-3cosh(\frac{x}{\sqrt{}})}{sinh(\frac{1}{\sqrt{}})(\sqrt{})} + 2$$

$$y''(x) = \frac{-sinh(\frac{1-x}{\sqrt{}})-3sinh(\frac{x}{\sqrt{}})}{()\ sinh(\frac{1}{\sqrt{}})}$$

Work:

$$-\epsilon\left(\frac{-sinh(\frac{1-x}{\sqrt{}})-3sinh(\frac{x}{\sqrt{}})}{()\ sinh(\frac{1}{\sqrt{}})} \right) - \left(\frac{sinh(\frac{1-x}{\sqrt{}})+3sinh(\frac{x}{\sqrt{}})}{sinh(\frac{1}{\sqrt{}})} \right) + 2x + 1$$

$$= \frac{sinh(\frac{1-x}{\sqrt{}})+3sinh(\frac{x}{\sqrt{}})}{sinh(\frac{1}{\sqrt{}})} - \left(\frac{sinh(\frac{1-x}{\sqrt{}})+3sinh(\frac{x}{\sqrt{}})}{sinh(\frac{1}{\sqrt{}})} \right) + 2x + 1$$

$$= \frac{3sinh(\frac{x}{\sqrt{}})-3sinh(\frac{x}{\sqrt{}})+sinh(\frac{1-x}{\sqrt{}})-sinh(\frac{1-x}{\sqrt{}})}{sinh(\frac{1}{\sqrt{}})} + 2x + 1$$

$$= \frac{0}{sinh(\frac{1}{\sqrt{}})} + 2x + 1$$

$$= 2x + 1.$$

Therefore $-\epsilon y'' + y = 2x + 1$.