CSCI 480, Winter 2017
Math Exercises # 2

Andrew Nguyen

Due date: Wednesday, February 1, midnight.

Turn in both the tex and pdf files (not zipped): `math02.tex` and `math02.pdf`.

Exercises for Chapter 4 Use the method of direct proof to prove the following statements.

16. If two integers have the same parity, then their sum is even.

Proposition: The sum of two integers with the same parity is an even integer.

Proof. Suppose that we have two integers, a and b, that have the same parity.

The set of all even numbers is defined as $\{2n : n \in \mathbb{Z}\}$.

The set of all odd odd numbers is defined as $\{2n + 1 : n \in \mathbb{Z}\}$.

Thus, the set of the sum of a and b is either

$(2n + 2n) = (4n)$, if a and b are even,

or,

$((2n + 1) + (2n + 1)) = (4n + 2)$, if a and b are odd.

For any integer, $\{n \in \mathbb{Z}\}$, whether the summed number falls into $(4n)$ or $(4n + 2)$, the integer will be even.

Therefore, the sum of any two integers with the same parity is an even integer.

Exercises for Chapter 5 Use the method of contrapositive proof to prove the following statements.

12. Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

Proposition: Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

Proof. Suppose a is even.

Since a is even, the set of all a in $\mathbb{Z}$ is defined as: $\{2n : n \in \mathbb{Z}\}$

Accordingly, the set of all a^2 in $\mathbb{Z}$ must be: $\{4n^2 : n \in \mathbb{Z}\}$; this is the set of all values of a^2 where a is an even integer.

Since $4n^2$ is a multiple of 4, all the values of a^2, where a is even, must also be divisible by 4.

Therefore, for all values of a^2 that are not divisible by 4, a must be odd.

Exercises for Chapter 6 Use the method of proof by contradiction to prove the following statements.

18. Suppose $a, b \in \mathbb{Z}$. If $4 \mid (a^2 + b^2)$, then a and b are not both odd.

Proposition: Suppose $a, b \in \mathbb{Z}$. If $4 \mid (a^2 + b^2)$, then a and b are not both odd. (Can be even-even or odd-even but not odd-odd)

Proof. Suppose that there exist values of a and b that are both odd, but still allow $4 \mid (a^2 + b^2)$ to hold true.

By definition of an odd number, both a and b must exist within the set of numbers: $\{2n + 1 : n \in \mathbb{Z}\}$

Further, in being divisible by 4, $(a^2 + b^2)$ must sum to some number c that lies within the set defined as $\{4n : n \in \mathbb{Z}\}$

Exercises for Chapter 7 State clearly which method of proof you are using.

24. If $a \in \mathbb{Z}$, then $4 \nmid (a^2 - 3)$.

Proposition: if $a \in \mathbb{Z}$, then $4 \nmid (a^2 - 3)$

Direct proof. As stated in problem 12 under Exercise 5 above, if a^2 is not divisible by 4, then a must be odd. It follows then, that a must be even if a^2 is divisible by 4.

For all even values of a, $4 \nmid (a^2 - 3)$ must hold true. An even value of a would produce an even valued a^2 as justified by: $(2n) * (2n) = (4n^2)$, which, for all values of n still produces an even number.

Exercises for Chapter 8

20. Prove that $\{9^n : n \in \mathbb{Q}\} = \{3^n : n \in \mathbb{Q}\}$.

Exercises for Chapter 9 Each of the following statements is either true or false. If a statement is true, prove it. If a statement is false, disprove it.

18. If $a, b, c \in \mathbb{N}$, then at least one of $a - b$, $a + c$, and $b - c$ is even.

Exercises for Chapter 10

2. For every integer $n \in \mathbb{N}$, it follows that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

6. For every natural number n, it follows that

$$\sum_{i=1}^{n} (8i - 5) = 4n^2 - n$$

10. For every integer $n \geq 0$, it follows that $3 \mid (5^{2n} - 1)$.

14. Suppose $a \in \mathbb{Z}$. Prove that $5 \mid 2^n a$ implies $5 \mid a$ for any $n \in \mathbb{N}$.