CS 301 Summer 2017 Quiz 2 6/29 (25pts)

1. (6pts) Decide whether or not the following pairs of statements are logically equivalent (Yes/No).
   a. $P \wedge Q$ and $\sim(\sim P \vee \sim Q)$ ___Y___
   b. $P \Rightarrow Q$ and $\sim Q \Rightarrow \sim P$ ___Y___
   c. $P \Rightarrow Q$ and $\sim P \vee Q$ ___Y___
   d. $\sim(P \Rightarrow Q)$ and $P \wedge \sim Q$ ___Y___
   e. $P \wedge (Q \vee \sim Q)$ and $(\sim P) \Rightarrow (Q \wedge \sim Q)$ ___Y___
   f. $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$ ___N___

   P   P V F   T F F

2. (4pts) Write the following as an English sentence. Say whether it is true or false.

   $\forall n \in Z, \exists m \in Z, m = n+5.$

   True

   for any integer n, there exists an integer m s.t. $m = n+5$.

3. (5pts) Direct Proof: Suppose a,b and d are integers. If $d|(a + b)$ and $d|a$, then $d|b$.

Proof. Suppose a, b and d are integers, and $d|(a+b)$ and $d|a$.
According to the definition of divisibility $(a+b) = dm$ for some $m \in \mathbb{Z}$
and $a = dn$ for some integer n.

Thus $b = dm - dn = d(m-n)$.

Since $m-n \in \mathbb{Z}$ Therefore $d|b$.

4. (5pts) Contrapositive proof: Suppose $x \in R$. If $x^2 + 5x < 0$ then $x < 0$.

Proof. (Contrapositive) Suppose $x \in \mathbb{R}$, and $x \geq 0$,

then $x^2 \geq 0$, and $5x \geq 0$

therefore $x^2 + 5x \geq 0$.

D.

5. (5pts) Proof by Contradiction: If 3n+2 is odd, then n is odd.

Proof (Proof by contradition)

Suppose 3n+2 is odd, and n is ~~is~~ not odd. ~~or~~ that is,

~~then~~ n is even.

Thus $n = 2m$ for some integer m.

Then $3n+2 = 3 \cdot 2m + 2 = 2(3m+1)$ is even.

Contradiction!

CS 301 Summer 2017 Quiz 3 7/6 (25pts)

1. (5pts) Suppose A, B and C are sets. If $B \subseteq C$, then $A \times B \subseteq A \times C$.

Proof: Case 1. when $A \times B = \emptyset$.

If $A \times B = \emptyset$, then $A \times B \subseteq A \times C$.

Case 2. when $A \times B \neq \emptyset$.

Suppose $(a, b) \in A \times B$. By def. of Cartesian Product, we have $a \in A$ and $b \in B$. Since $B \subseteq C$, ~~then~~ we have $b \subseteq C$. Since $a \in A$ and $b \in C$, we have $(a, b) \in A \times C$.

Thus $A \times B \subseteq A \times C$.

2. (5pts) Suppose A, B, and C are sets, and $C \neq \emptyset$. Prove that if $A \times C = B \times C$, then $A \subseteq B$.

Proof. Case 1. when $A = \emptyset$.

If $A = \emptyset$, then $A \subseteq B$.

Case 2. when $A \neq \emptyset$.

Suppose $a \in A$, since $C \neq \emptyset$, suppose $c \in C$, then $(a, c) \in A \times B$, since $A \times C = B \times C$, we have $(a, c) \in B \times C$.

thus $a \in B$.

therefore $A \subseteq B$.

3. (5pts) Suppose A, B, and C are sets, and $C \neq \emptyset$. Prove that if $A \times C = B \times C$, then $B \subseteq A$.

Proof. Case 1. when $B = \emptyset$,

If $B = \emptyset$, then $B \subseteq A$.

Case 2. when $B \neq \emptyset$,

Suppose $b \in B$, since $C \neq \emptyset$, suppose $c \in C$, then $(b, c) \in B \times C$. Since $B \times C = A \times C$, we have $(b, c) \in A \times C$.

thus $b \in A$.

therefore $B \subseteq A$.

4. (1pt) Suppose A, B, and C are sets, and $C \neq \emptyset$. Prove that if $A \times C = B \times C$, then $B = A$.

Proof. Since we've proved both $A \subseteq B$ and $B \subseteq A$, we have $B = A$.

Each of the following statements is either true or false. If a statement is true, prove it. If a statement is false, disprove it.

5. (3pts) For all sets A and B, if $A - B = \emptyset$, then $B \neq \emptyset$.

False.

Disproof: It is false.

One counter example: $A = \emptyset$ and $B = \emptyset$.

6. (3pts) Suppose A, B and C are sets. If $A = B - C$, then $B = A \cup C$.

False.

Disproof. One counter example:

$A = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

$C = \{4\}$.

7. (3pts) There exists a set X for which $\mathbb{R} \subseteq X$ and $\emptyset \in X$.

True.

Proof. Let $X = \mathbb{R} \cup \{\emptyset\}$.

CS 301 Summer 2017 Quiz 4 7/13 (25pts)

1. (4pts) Prove that $3 \mid 4^n - 1$ for any integer $n \geq 1$.

Proof (Induction).

Basis: $n=1$, $4^n - 1 = 4^1 - 1 = 3$. the statement holds.

Induction: Assume $3 \mid 4^k - 1$ for $n = k$.

Show $3 \mid 4^{k+1} - 1$ for $n = k+1$.

Since $3 \mid 4^k - 1$, $4^k - 1 = 3a$ (*) for some integer a.

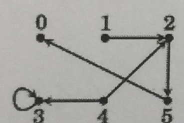
then $4^{k+1} - 4 = 12a$ (multiply 4 on both sides of (*))

then $4^{k+1} - 1 = 12a + 3$

$= 3(4a+1) = 3m$ where $m = 4a+1$

Therefore $3 \mid 4^{k+1} - 1$.

2. (3pts) Here is a diagram for a relation R on a set A. Write the sets A and R.



$A = \{0, 1, 2, 3, 4, 5\}$

$R = \{(1,2), \{2,5\}, \{3,3\}, (4,3), (5,0)\}$ $(4,2)$,

3. (3pts) Consider the subset $R = (\mathbb{R} \times \mathbb{R}) - \{(x,x) : x \in \mathbb{R}\} \subseteq \mathbb{R} \times \mathbb{R}$. What familiar relation on R is this? Explain.

Inequality. ("Not equals to")

All (x,x) where $x \in \mathbb{R}$ have been eliminated from $\mathbb{R} \times \mathbb{R}$.

4. (6pt) Define a relation on Z by declaring xRy if and only if x and y have the same parity. Is R reflexive? Symmetric? Transitive? Explain.

- reflexive. for all $x \in \mathbb{Z}$, $(x,x) \in R$ since x and x have the same parity.

- symmetric. if $(x,y) \in R$ then $(y,x) \in R$. since if x and y have the same parity then y and x also have the same parity.

- Transitive. if $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$.

if x and y have the same parity and y and z have the same parity, then x and z also have the same parity.

5. (4pts) Suppose R is a symmetric and transitive relation on a set A, and there is an element $a \in A$ for which aRx for every $x \in A$. Prove that R is reflexive.

Proof. Since for every $x \in A$, there is an element $a \in A$ $(a, x) \in R$, and R is symmetric, ~~the~~

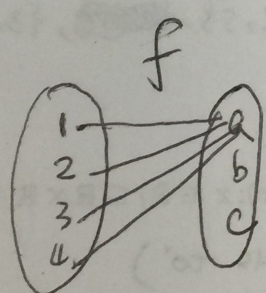
we have $(x, a) \in R$.

Given R is transitive and $(x, a) \in R$ and $(a, x) \in R$ for every $x \in A$.

We have: for every $x \in A$, $(x, x) \in R$.

Therefore, R is reflexive.

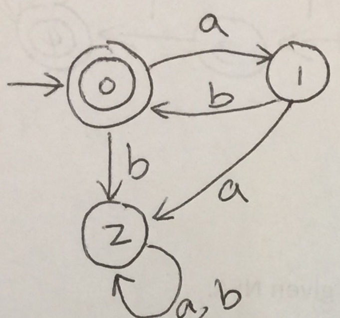
6. (3pts) Let A={1,2,3,4} and B={a,b,c}. Give an example of a function f : A→B that is neither injective nor surjective.



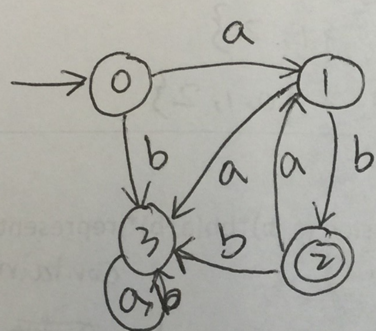
7. (2pts) The function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \pi x - e$ is bijective. Find its inverse.

$$f^{-1}(x) = \frac{x+e}{\pi}$$

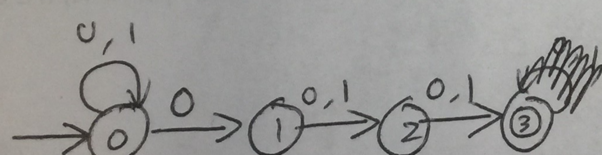
1. (4pts) Construct a DFA for the language $L = \{(ab)^n \mid n \in N, N = \{0, 1, 2, 3, \ldots\}\}$ over the alphabet {a, b}.



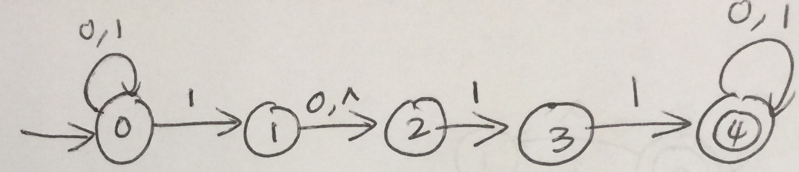
2. (4pts) Construct a DFA for the language $L = \{(ab)^n \mid n \in N, N = \{1, 2, 3, \ldots\}\}$ over the alphabet {a, b}.



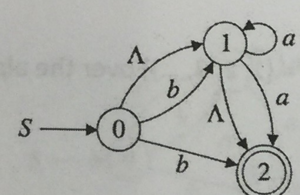
3. (4pts) Construct an NFA that accepts the language L = {w: w is a binary string and w has a 0 in the 3rd position from the right}.



4. (4pts) Construct an NFA that accepts that language L = {w: w is a binary string containing the substring 1011 or 111} over the alphabet {0, 1}.



5. (5pts) Calculate the following λ-closures for the given NFA.

	$\lambda(0) = ?$ {0, 1, 2} $\lambda(1) = ?$ {1, 2} $\lambda(2) = ?$ {2} $\lambda(\{1, 2\}) = ?$ {1, 2} $\lambda(\{0, 1, 2\}) = ?$ {0, 1, 2}

6. (2pts) What language does the regular expression (a+b)*bb(a+b)* represent?

containing the substring

~~at~~ a set of strings over a, b ~~with~~ bb ~~as~~.

~~a substring~~

7. (2pts) Find a regular expression for $\{ab^n \mid n \in N, n \geq 0\} \cup \{ba^n \mid n \in N, n \geq 0\}$.

$ab^* + ba^*$