

CSCI 301, Winter 2017

Math Exercises # 5

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Due date:

Construct a context-free grammar for each of the languages in questions 1 to 7.

1. $\{0^{2n}1^n : n \geq 0\}$

$$S \rightarrow \epsilon \mid 00S1$$

2. $\{w : w \text{ contains at least three 1s}\}$

$$S \rightarrow A1A1A1A$$

$$A \rightarrow \epsilon \mid 0A \mid 1A$$

3. $\{w : \text{the length of } w \text{ is odd and its middle symbol is } 0\}$

$$S \rightarrow 0S0 \mid 1S1 \mid 0S1 \mid 1S0 \mid 0$$

4. $\{w : w \text{ is a palindrome}\}$

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

5. $\{w : w \text{ starts and ends with the same symbol}\}$

$$S \rightarrow 0A0 \mid 1A1$$

$$A \rightarrow \epsilon \mid 0A \mid 1A$$

6. $\{w : w \text{ starts and ends with different symbols}\}$

$$S \rightarrow 0A1 \mid 1A0$$

$$A \rightarrow \epsilon \mid 0A \mid 1A$$

7. $\{a^m b^n : 0 \leq m \leq n \leq 2m\}$

$$S \rightarrow \epsilon \mid ab \mid Sb \mid aaSb$$

Because of the conditions set on n and m : $0 \leq m \leq n \leq 2m$; the number of b's ranges from the number of a's to double the number of a's, and cannot be outside of that range. eg: if there are 5 a's, $aaaaa$, then we must have at least 5 b's but not more than 10 b's.

$aaaaabbbbb$ fulfills the requisites along with $aaaaabbbbbbb$, and $aaaaabbbbbbb$.

So then, for every a we input, we must input at least one b as well.

8. Let G be the grammar:

$$S \rightarrow aB \mid bA$$

$$A \rightarrow a \mid aS \mid bAA$$

$$B \rightarrow b \mid bS \mid aBB$$

For the string $aaabbabbba$, find a

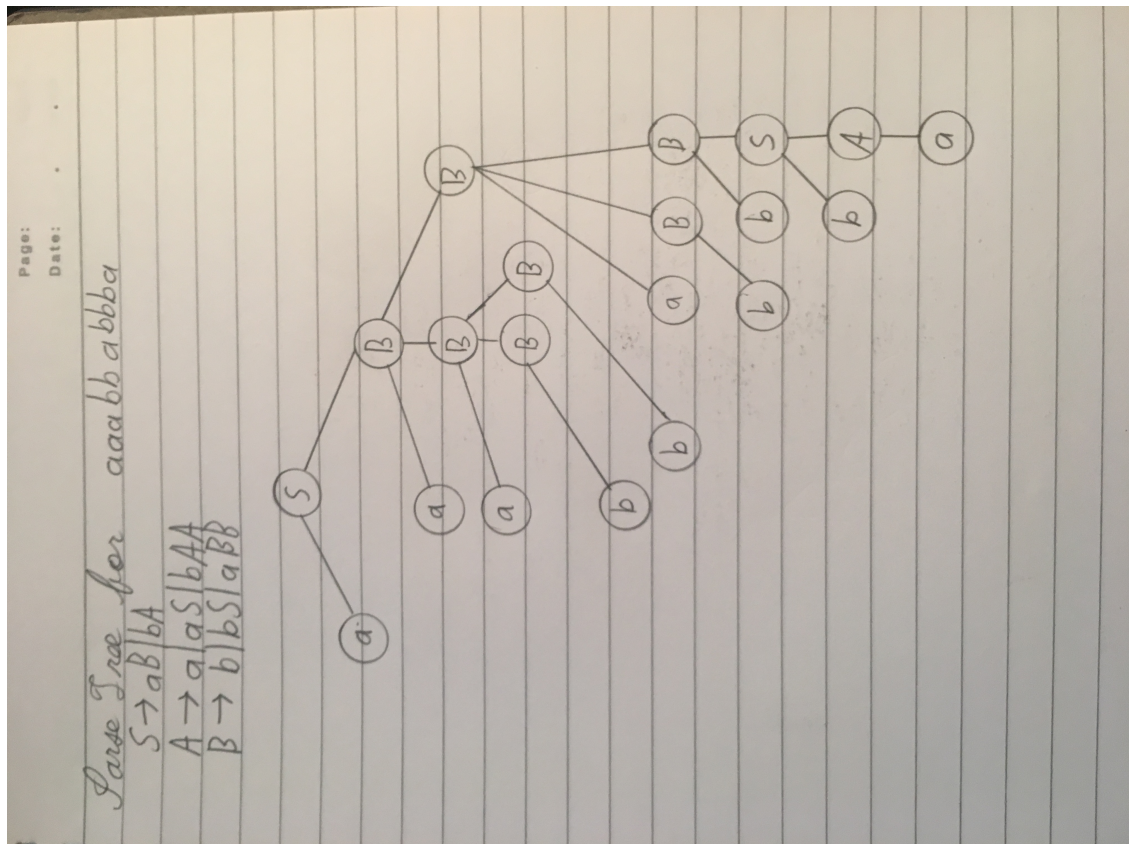
(a) leftmost derivation,

$S \rightarrow aB$
 $aB \rightarrow aaBB$
 $aaBB \rightarrow aaaBBB$
 $aaaBBB \rightarrow aaabBB$
 $aaabBB \rightarrow aaabbB$
 $aaabbB \rightarrow aaabbaBB$
 $aaabbaBB \rightarrow aaabbabB$
 $aaabbabB \rightarrow aaabbabbS$
 $aaabbabbS \rightarrow aaabbabbA$
 $aaabbabbA \rightarrow aaabbabbba$

(b) rightmost derivation,

$S \rightarrow aB$
 $aB \rightarrow aaBB$
 $aaBB \rightarrow aaBaBB$
 $aaBaBB \rightarrow aaBaBbS$
 $aaBaBbS \rightarrow aaBaBbbA$
 $aaBaBbbA \rightarrow aaBaBbba$
 $aaBaBbba \rightarrow aaBabbba$
 $aaBabbba \rightarrow aaaBBabbba$
 $aaaBBabbba \rightarrow aaaBbabbba$
 $aaaBbabbba \rightarrow aaabbabbba$

(c) parse tree.



(d)

9. Convert the following grammar to Chomsky normal form:

$$\begin{aligned} S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid aS \mid a \\ B &\rightarrow aBB \mid bS \mid b \end{aligned}$$

Follow the steps documented in my notes and the text, and show the resulting grammar after each step.

Step 1 Eliminate the start variable from the right-hand side of rules.

Step 2 Eliminate ϵ -rules.

Step 3 Eliminate unit-rules.

Step 4 Eliminate all rules having more than two symbols on the right-hand side.

Step 5 Eliminate all rules of the form $A \rightarrow u_1 u_2$ where u_1 and u_2 are not both variables.

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid aS \mid a \\ B &\rightarrow aBB \mid bS \mid b \end{aligned}$$

$$\begin{aligned} S_0 &\rightarrow bA \mid aB \\ S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid abA \mid aaB \mid a \\ B &\rightarrow aBB \mid bbA \mid bAB \mid b \end{aligned}$$

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid abA \mid aaB \mid a \\ B &\rightarrow aBB \mid bbA \mid bAB \mid b \end{aligned}$$

Introduce new vars

$$\begin{aligned} C &\rightarrow AA \\ D &\rightarrow BB \\ E &\rightarrow a \\ F &\rightarrow b \\ G &\rightarrow ab \\ H &\rightarrow ba \\ I &\rightarrow aa \\ J &\rightarrow bb \end{aligned}$$

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow FA \mid EB \\ A &\rightarrow FC \mid GA \mid IB \mid a \\ B &\rightarrow ED \mid JA \mid HB \mid b \\ C &\rightarrow AA \\ D &\rightarrow BB \\ E &\rightarrow a \\ F &\rightarrow b \\ G &\rightarrow ab \\ H &\rightarrow ba \\ I &\rightarrow aa \\ J &\rightarrow bb \end{aligned}$$