CSCI 301, Winter 2017
Math Exercises # 3

YOUR NAME HERE

Due date: Friday, February 10, midnight.

Exercises for Section 11.1

12. Prove that the relation $|$ (divides) on the set $\mathbb{Z}$ is reflexive and transitive.

Exercises for Section 11.2

8. Define a relation R on $\mathbb{Z}$ as xRy if and only if $x^2 + y^2$ is even. Prove R is an equivalence relation (prove from the definitions).

Exercises for Section 11.4

4. Write the addition and multiplication tables for $\mathbb{Z}_6$

Exercises for Section 12.2

8. A function $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ is defined as $f(m, n) = (m + n, 2m + n)$. Verify whether this function is injective and whether it is surjective.

Exercises for Section 12.3

2. Prove that if a is a natural number, then there exist two unequal natural numbers k and ℓ for which $a^k - a^\ell$ is divisible by 10.

Exercises for Section 12.5

6. The function $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by the formula $f(m, n) = (5m + 4n, 4m + 3n)$ is bijective. Find its inverse.

Exercises for Section 12.6

6. Given a function $f : A \to B$ and a subset $Y \subseteq B$, is $f(f^{-1}(Y)) = Y$ always true? Prove or give a counterexample.

Exercises for Section 13.1

A. Show that the two given sets have equal cardinality by describing a bijection from one to the other. Describe your bijection with a formula (not as a table).

10. $\{0, 1\} \times \mathbb{N}$ and $\mathbb{Z}$

Exercises for Section 13.2

14. Suppose $A = \{(m, n) \in \mathbb{N} \times \mathbb{R} : n = \pi m\}$. Is it true that $|\mathbb{N}| = |A|$? Prove or disprove it.

Exercises for Section 13.3

8. Prove or disprove: The set $\{(a_1, a_2, a_3, \ldots) : a_i \in \mathbb{Z}\}$ of infinite sequences of integers is countably infinite.