Assignment 4

Q1. 9 functions, 6 injections, 0 surjections, 0 bijections, and 3 with none of the properties.

Q2.

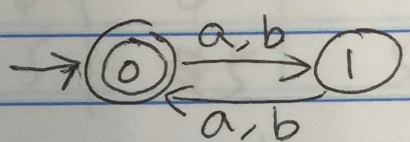
$L = \{\langle\rangle, \langle a, a\rangle, \langle a, a, a, a\rangle, \ldots\}$

The mapping from $\mathbb{N}$ to L maps each n to a list of length $2n$ is a bijection. So, L is countable.

Q3. $((a+b)(a+b)(a+b))^*$ or

$(aba + abb + aab + aaa + bba + bbb + bab + baa)^*$

Q4.



Q5. a.

		a	b	Λ
S	0	$\{0, 2\}$	$\emptyset$	$\{1\}$
	1	$\emptyset$	$\{1, 2\}$	$\emptyset$
	2	$\{3\}$	$\emptyset$	$\emptyset$
F	3	$\emptyset$	$\emptyset$	$\cancel{\emptyset}\ \{2\}$

b. $\lambda(0) = \{0, 1\}$

$\lambda(1) = \{1\}$

$\lambda(2) = \{2\}$

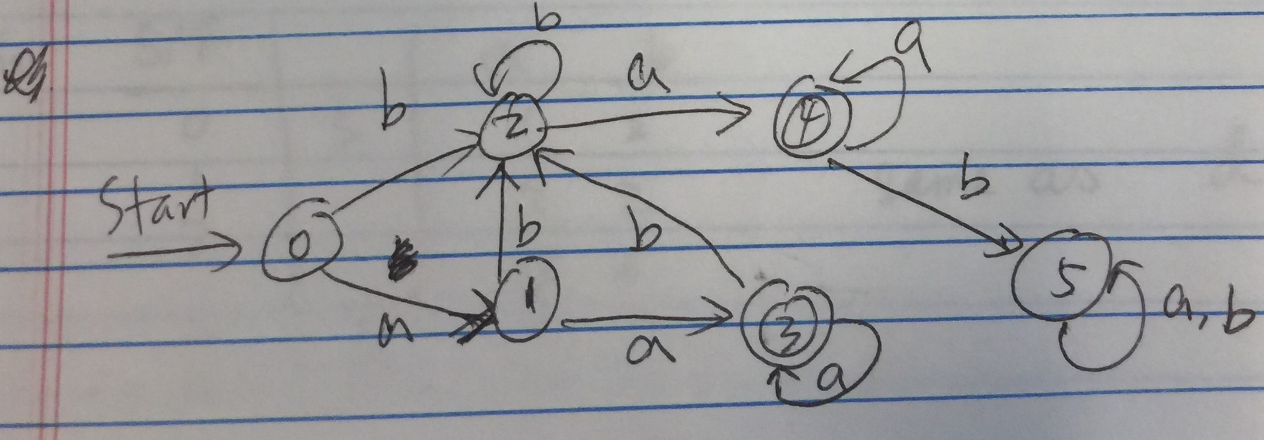
$\lambda(3) = \{2, 3\}$

	T_D	a	b
c. S	{0,1}	{0,1,2}	{1,2}
	{0,1,2}	{0,1,2,3}	{1,2}
	{1,2}	{2,3}	{1,2}
F	{0,1,2,3}	{0,1,2,3}	{1,2}
F	{2,3}	{2,3}	ϕ
	ϕ	ϕ	ϕ

d.

S	0	1	2
	1	3	2
	2	4	2
F	3	3	2
F	4	4	5
	5	5	5

← Renamed the states



(see the word file)

e. $E_0 = \{\{0,1\},\{0,2\},\{0,5\},\{1,2\},\{1,5\},\{2,5\},\{3,4\}\}$

$E_1 = \{\{3,4\},\{0,5\},\{1,2\}\}$

$E_2 = \{\{1,2\}\}$

$E_3 = \{\ \}$

$E_4 = \{\ \}$

f. $[0] = \{0\}$

$[1] = \{1\}$

$[2] = \{2\}$

$[3] = \{3\}$

$[4] = \{4\}$

$[5] = \{5\}$

g.

S/F		a	b
[0]	S	[1]	[2]
[1]		[3]	[2]
[2]		[4]	[2]
[3]	F	[3]	[2]
[4]	F	[4]	[5]
[5]		[5]	[5]

h.

S/F		a	b
0	S	1	2
1		3	2
2		4	2
3	F	3	2
4	F	4	5
5		5	5

same as d.

(By contradiction)

Proof: Let $L = \{w \mid w = \{a,b\}^*$ and w is a palindrome of even length$\}$ and suppose L is regular.

Using the pumping lemma we'll choose $s = a^m bba^m$. Then s can be rewritten as $s = a^m bba^m = xyz$, where $y \neq \Lambda$ and $|xy| \le m$.

It follows that $y = a^i$ for some $i > 0$. Then $\underline{xz = a^{m-i} bba^m}$, which is not in L.

(pumping up or pumping down)

This contradicts the pumping lemma result that $xy^kz \in L$ for all $k \geq 0$. Thus L is not regular.