

CREATING A UNIVERSAL INFORMATION LANGUAGE

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ABSTRACT. Welkin is a formalized programming language to store information. We introduce its use cases and rigorously define its syntax and semantics. From there, we introduce the bootstrap, making Welkin completely self-contained under a meta-theory based on combinators, equivalent to a provably minimal fragment of arithmetic.

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1. INTRODUCTION

Information Management (IM) is an open area of research as a result of the depth and breadth of disciplines. In terms of depth, many areas are often specialized, requiring an immense understanding of the broader concepts involved and nomenclature used. This specialization is evident in the sciences, as explored in [FG13], [CF14]. Additionally, in terms of breadth, creating common representations shared across sub-disciplines can be difficult. For example, mathematics has extremely diverse disciplines, and connecting these areas is an open problem in scalability [Car+21]. Moreover, creating a standardized form across communities is challenging. In other subjects, like the social sciences, there are no standard terms [Arc+06], and in the humanities, representing certain artifacts as data is involved [Har+20]. More broadly, IM *itself* is divided from distinct approaches that lack interoperability [AJ23]. Certain frameworks equate IM to Knowledge Management (KM) and assert that information must be true [Edw22]. These problems, in both faithfully and broadly storing information, demonstrate the enormous task of effective IM.

In response to these challenges, several solutions have been proposed, but none have been fully successful. In the sciences, a group of researchers created the Findable Accessible Interoperable Resuable (FAIR) guidelines [Wil+16]. Instead of providing a concrete specification or implementation, FAIR provides best practices for storing scientific information. However, multiple papers have outlined problems with these overarching principles, including missing checks on data quality [Gui+25], missing expressiveness for ethics frameworks [Car+21], and severe ambiguities that affect implementations [Jac+20]. Along with the sciences, there are several projects for storing mathematical information (see [CF09] for more details). Older proposals, including the QED Manifesto [KR16] and the Module system for Mathematical Theories (MMT), aimed to be more general and have seen limited success. More centralized systems, like `mathlib` in the Lean proof assistant [Com20], have seen adoption but do not give equal coverage nor are interoperable with other systems. Beyond more “hard” fields, IM in the humanities has few models, including an adaption of FAIR [Har+20] and discipline specific, linked databases in the PARNTHEOS project [Hed+19]. Each of these proposals, even within specific fields, fail to accommodate for all of the mentioned challenges.

In addition to domain specific proposals, there are approaches for general IM which still fail to resolve all issues. One prominent example is Burgin’s theory of information [Bur09] that comprehensively includes many separate areas for IM, including the complexity-based Algorithmic Information Theory (AIT), through a free parameter called an “infological system”, which encompasses domain specific terminology and concepts. In contrast to other approaches, Burgin’s generalized theory is flexible and enables greater coverage of different kinds of information [Mik23]. Despite this coverage Burgin does not closely tie the free parameter with his formal analysis of AIT, making it unclear how to use this in a practical implementation. Broad frameworks for IM, along with the specific proposals, have severe shortcomings, highlighting major obstacles for IM.

This thesis introduces a language to resolve these issues. I call this language **Welkin**, based on an old German word meaning cloud [Dic25]. The core result of this thesis is proving that Welkin satisfies three goals: is **universal**, **scalable**, and **standardized**. For details, see Table 1. The core idea is to generalize Burgin’s free parameter and enable arbitrary representations in the theory, controlled by a computable system. The notion of representation builds on Peirce’s semiotics, or the study of the relationship between a symbol, the object it represents, and the interpreter or interpretation that provides it that meaning [Atk23]. Moreover, to address queries on the validity of truth, we use a relative notion that includes a context managed by a formal system. Truth can then be determined on an individual basis, providing flexibility to any discipline. The focus then shifts to the usefulness of representations based on a topological notion of how “foldable” a structure is, which we call **coherency**. This approach is inspired by coherentism, a philosophical position that states truth is determined in comparison to other truths. [Bra23]. We incorporate ideas from coherentism to identify which representations identify their corresponding objects, and we define information as an invariant under these coherent representations. We include definitions on a *working* basis as what is most practical, not an epistemological stance that can be further clarified in truth systems. Additionally, we keep the theory as simple as possible to make scalability and standardization straight-forward.

Goal 1	Universality	The language must enable any user created parameters, whose symbolic representation is accepted a computable function. Every computable function must be definable in the language.
Goal 2	Scalability	The database must appropriately scale to broad representations of information. Local queries must be efficient. Certificates must be available to prove cases where optimal representations have been achieved.
Goal 3	Standardization	The language needs a rigorous and formal specification. Moreover, the bootstrap must be formalized, as well as an abstract machine model. The grammar and bootstrap must be fixed to ensure complete forwards and backwards compatibility.

TABLE 1. Goals for the Welkin language.

This thesis is organized according to Table 2.

Section 2	Motivating Example	Introduces a high-level example, with geographic maps, to explain the core concept in Welkin.
Section 3	Foundations	Explains the meta-theory.
Section 4	Syntax	Provides the grammar and proof that it is unambiguous.
Section 5	Semantics	Explains how ASTs are validated and processed. Develops representations and coherency, and connects these to a working definition of information.
Section 6	Information Organization	Develops a Greedy algorithm to locally optimize information. Creates a certificate that demonstrates when a representation is optimal relative to the current information database.
Section 7	Bootstrap	Bootstraps the language.
Section 8	Conclusion	Concludes with possible applications, particularly in programming languages and broader academic knowledge management.

TABLE 2. Organization for the thesis.

2. MOTIVATING EXAMPLE

3. FOUNDATIONS

3.1. Words.

To keep this thesis self-contained, all recursive definitions are included. For simplicity, we will use the notation a_0, \dots, a_n for a finite list of items. We will revisit the notion “finite” more rigorously in Section 7.

Definition 3.1.0. The **alphabet of binary words** is $\mathcal{A}_{\text{bword}} ::= \text{bit} \mid \cdot \mid w$, where $\text{bit} ::= 0 \mid 1$. A **binary word** is defined recursively: the symbols 0 or 1 are strings, or if w is a string, then so are $w.0$ and $w.1$. We set concatenation to be right-associative, i.e., $(w.w').w'' = w.(w'.w'')$, and safely abbreviate $w.w'$ as ww' . We write $w \in \text{bword}$ to denote that w is a binary word.

For simplicity, we extend the alphabet to include two common bases: decimal and hexadecimal.

Definition 3.1.1. The **alphabet of words** $\mathcal{A}_{\text{word}} ::= \mathcal{A}_{\text{bword}} + \mathcal{A}_{\text{dec}} + \mathcal{A}_{\text{hex}}$, where:

- $\mathcal{A}_{\text{dec}} ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$
- $\mathcal{A}_{\text{hex}} ::= A \mid B \mid C \mid D \mid E \mid F$

A **word** is a concatenation of either only binary digits, only decimal digits, or hexadecimal digits. Each of these are denoted with different prefixes: decimal has none, binary uses $0b$, and hexadecimal uses $0x$.

Using binary words simplifies the bootstrap, so while these digits are included, they are *defined* in terms of binary words, see Section 7.

3.2. Arithmetic.

4. SYNTAX

Now, the base encoding for Welkin is in US-ASCII, formally defined below.

4.1. Encoding.

Definition 4.1.0. US-ASCII consists of 256 symbols, listed in Figure 1.

FIGURE 1. US-ASCII codes and glyphs.

4.2. Strings.

We reserve the term **string** when a word is explicitly enclosed in delimiters, namely single or double quotes. The precise definition is involved, due to including quotes within a string, which are called “escaped quotes”. This is one reason why the base encoding is fixed to US-ASCII encoding: the negation operation cannot be expressed as a natural regular expression over *any* set of terminals, but can if this set is fixed.

Definition 4.2.1. A **single-quoted string** is defined recursively.

The definition of double-quoted string is analogous.

4.3. Grammar.

Definition 4.3.2. BNF consists of productions. Writing $r := a_1 \mid \dots \mid a_n$ is shorthand for the rules $r := a_1, \dots, r := a_n$. A **derivation** is a sequence of steps, recursively defined by starting with the empty derivation, and if d is a derivation and s is a step, then $d.s$ is a derivation.

Now, we formalize an unambiguous form of EBNF for our use case.

Definition 4.3.3. EBNF is a superset of BNF with the abbreviations:

Welkin’s grammar is displayed in Listing 1, inspired by a minimal, C-style syntax. Note that the smallest string accepted by Welkin is $\{\}$ and not the empty string.

TODO: Ensure this is actually LL(1)! Probably need to massage some productions.

```

start ::= (term ",")* term
term  ::= arc | graph
arc   ::= unit "-" unit "->" unit
        | unit "<-" unit "-" unit
        | unit "-" unit "-" unit
graph ::= unit? { term* }
base  ::= unit | string
unit  ::= int

```

LISTING 1. The grammar for Welkin, shown in EBNF notation (see Definition 4.3.6).

4.4. Unambiguity.

Definition 4.4.4. A grammar is LL(1) if ...

Theorem 4.4.5. *Welkin's grammar is LL(1). Hence, this grammar is unambiguous, i.e., every string accepted by the language has exactly one derivation.*

Proof. Consider the corresponding LL(1) table. □

5. SEMANTICS

5.1. ASTs.

- Semantics on ASTs
 - Terms: graphs
 - For ease of use, include a null node that is the root of the tree. This represents the module itself.
- For information organization: integrate with previous section
 - Emphasize how this is a useful tool and can ensure **new** information content is being created (at least, that can be distinguished from the current module). If already existing, but that doesn't match the user's expectations, they need to refine it! OR, maybe it **does** match similarly with something else! (e.g., hidden connections between math and music)
- Emphasize pragmatics as well, via units

5.2. Representations.

Now we develop the formal framework to discuss information in terms of units, enabling a complete mechanization of Welkin's meta-theory. To keep this section self-contained, we explicitly provide all recursive definitions.

Definition 5.2.0. The **alphabet of units** is $\mathcal{A}_{\text{unit}} = u \mid \mathcal{A}_{\text{word}}$. A **unit ID** is combination of symbols u_w , where $w \in \text{word}$.

Definition 5.2.1. A **free parameter** is a parameter given an associated ID. No further restrictions are imposed.

We now define representations recursively, using unit IDs and free parameters as the base case.

Definition 5.2.2. Units are recursively defined:

- **Base case:** IDs and free parameters are units.
- **Recursive step:**
 - **Parts::** if u_1, \dots, u_n are finitely many units, then so is their combination $\{u_1, \dots, u_n\}$. A combination defined without a provided ID is called an **anonymous unit**.
 - **Representations:** If u, w, v are units, so is $v \rightarrow u$. We say v **represents** u . or conversely, u is **represented by** v .

Key equalities:

- $u.\{\} = u$. Acts as a sort of * operator from other languages.
 - To use one level up: $.u$
- $\left(u \xrightarrow{v} w\right) \in x \Leftrightarrow x(u) \xrightarrow{x(v)} x(w)$, where $x(u)$ is $x.u$ if $u \in x$ or u otherwise.

Example 5.2.3. Consider a house with a dog, a cat, and a person. We can represent the house as unit house, the dog as unit dog, the cat by unit cat, and the person by unit person. In our Welkin file, we add, house { dog, cat, person}. The person has an internal concept of pet and uses it to represent both the dog and cat, which we write as person { animal --> dog, animal --> cat}, under the scope of house.

Parts of units are denoted as $u.u'$. Scoping is included to provide namespaces. Moreover, parts enable **interpretations**. We write $u \xrightarrow{v} u'$ in case $u, v, (u \rightarrow v) \in u'$, so u represents v **via** u' . In this case, we say u' is a **context** to $u \rightarrow v$. Note that unlabeled representations can have multiple contexts.

Example 5.2.4. Consider the recursive definition of a binary tree: either it is a null (leaf) node, or it contains two nodes, left and right. We can model this as follows:

- First, create units for each of the notions: tree {null, left, right}.
- Next, we write, tree { null --> .tree, .tree --> left, .tree --> right, { left, .right } --> .tree}. Notice that we refer to the *namespace*, thereby enabling recursion. By our scoping rules, writing tree would be a *new* unit.
- To impose that the left subtree is *distinct* from the right one, we can use symbols.

An important idea in this example is that the abstraction could be defined *first*, or a concrete model could. For this reason, the choice of how entities are represented is flexible.

Definition 5.2.5. A unit u is **non-trivial** if it is non-empty and does not contain all relations. A unit u is **coherent relative to a context** u' if $u + u'$, the union of these units, is non-trivial.

Remark 5.2.6. This definition is a natural generalization of consistency in first-order logic. We will frequently rely on this result throughout the thesis.

Theorem 5.2.7. A representation is preserves information modulo \equiv iff the representation modulo \equiv is coherent.

Remark 5.2.8. This theorem enables truth management via specific contexts, specified as units. The task of finding core truths is then free, left open to flexibility accommodate for any truth management.

5.3. Universal Systems.

Inspired by [Mes12], we prove that scoping is strictly more expressive than without.

Lemma 5.3.9. *Representations with interpretations are undefinable in terms of unlabeled representations.*

Proof. It suffices to note that representing partial computable functions requires combinations. But every transformation under unlabeled representations does not preserve these conditions, hence, representations with interpretations are not definable. \square

Theorem 5.3.10. *Any computable function and its trace under a given string can be represented by units. (TODO: make more precise.)*

Proof. We prove there is a natural embedding from the lambda calculus into our meta-theory, using two parts:

- $\lambda x.f$ is represented as $\{x \rightarrow f\}$.
- $f.g$ is represented as the combination of units, or $\{f, g\}$.

Thus, any λ -term can be represented in the meta-theory, completing the proof. \square

Note that there are multiple ways to prove Theorem 5.3.4, infinitely in fact. This motivates the following definition.

Definition 5.3.11. A universal representation system (URS) is a unit that can represent any representation.

Theorem 5.3.12. *A unit is a universal representation system if and only if it can represent any partial computable function. Moreover, any universal representation system can represent any universal representation system. In particular, representing itself is called **reflection**.*

The term *universal* is specifically for expressing *representations* symbolically. The free parameter still needs to be included and is an additional feature on top of partial computable functions. However, the *management* of these symbols is done entirely with partial computable functions.

The next section discusses the issue of *managing* the infinitely many choices for URSs.

TODO: TODO: define the generalization to Padoa's Method clearer.

TODO: Make this more precise and complete proof.

6. INFORMATION ORGANIZATION

- Main question: **which** universal system to choose? Is this practical?
 - What is a suitable criterion for a base theory?
 - Recall aim: want to mechanically store systems for a database
 - * What if possible performance degradation? Will we get stuck if we start with one architecture? Will we have to adjust later?
 - * Aim is to ensure architecture is completely flexible and can automatically adapt
 - * One key metric: ability to store as many systems coherently as possible, i.e., store as much information as possible
 - Main problem: Blum's speedup theorem
 - * Briefly generalize this for slate logic

- * Show that no single way to completely organize systems based on a computable metric.

This is part of the need for new search techniques!

- * Want to separate search from storage though, but we want to improve

stored results **with** new results. This forms the idea behind the database architecture: have a simple way to store results that automatically gets better with new techniques/results.

- * Need explicit proofs for this! Not sure how to store certificates...

6.1. Impossible Classes.

The reason to restrict our transformations is two-fold. First, we need to ensure we can *verify* them efficiently. Determining whether a morphism between two formal systems exist can be reduced to the Halting problem, and is therefore not practical for defining an optimal formal system. Second, if we include those transformations that we *can* effectively check, no optimal formal system exists.

Theorem 6.1.0. *With respect to the class of all computable transformations that can be computably verified, there is no optimal formal system.*

6.2. Efficient Querying.

Instead of making proofs most efficient as is, we want to support finding optimal representations. But we want to do this from an efficiently queryable system, which is the most optimal.

7. BOOTSTRAP

- Provided in bootstrap.welkin file or similar. Main module is welkin, which needs to:
 - Provide slates, so inductive definition of binary strings + variables
 - Explain combinators
 - Explain the basics of universal systems. This is very important!
 - * Can elide proofs IF there is enough information content.
 - * TODO: figure out how proofs might work in this setting
 - Provides syntax and semantics of Welkin itself
- This thesis: will prove that the AST generated from this file

is correct AND that it does, with a suitable interpretation bootstrap itself.

- Explain how slates expand the envelope for implementations, BUT ensures that the final product, the syntax + semantics checkers and the information organization, can be externally seen!

8. CONCLUSION

- Review of thesis
 - Developed slate logic + bi-translation with FOL
 - Developed locally optimal organizational technique that can improve based on annotations/certificates
 - Introduced the language, with a straightforward graph syntax and semantics
 - Builds upon the last section

- Bootstrapped standard + used coherency condition
- Significance
 - Backwards AND forwards compatible standard that bootstraps itself. Easy for implementations!
 - Applications to any human subject
 - * Sciences
 - * Liberal arts
 - * Economics
 - * Etc.
- Future work
 - Programming language semantics + synthesis
 - * Incorporate broader aspects + intent of users! ESSENTIAL for new programming languages to be able to discuss pragmatics in some way!
 - * Also reproducible AND executable specifications, though creating an engine to execute these is far beyond the scope of the thesis
 - Organizing large corpuses of human text
 - Numerous applications to AI and improving results
 - * Emphasize role of symbol grounding problem in AI

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