

CREATING A UNIVERSAL INFORMATION LANGUAGE

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ABSTRACT. Welkin is a formalized programming language to store information. We introduce its use cases and rigorously define its syntax and semantics. From there, we introduce the bootstrap, making Welkin completely self-contained under the meta-theory of Goedel’s System T (equi-consistent to Peano Arithmetic).

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1. INTRODUCTION

Undergraduates are taught many things in lecture, books, and papers, but there is one unspoken truth: researchers have extremely *diverse* communities. Each community has their own approaches, their own conferences, and their own formatting. They even have a distinct preference to chalk, dry erase marker, or neither. For a long time, I didn’t notice this aspect about research. I started my mathematics journey in secondary education, independently reading papers from set theory, logic, and more. I was fortunate to have vast archives available on the internet, but my ability to *collaborate* in research was limited. This quickly changed in college[TODO: maybe make this a stronger sentence?]. Everywhere I went, I saw the multitude of research groups, from those mentioned in lecture to the experiences I worked in to the faculty panels in the COSMOS math club. I worked in several research groups and found how rich and fulfilling each was. This fundamentally changed my perception of research as a whole. I learned that diversity *drives* research, including in the sciences, liberal arts, and many more subjects I have hardly explored.

Because of how diverse research is, these communities are extremely independent and build separate repositories of knowledge. For example, I wrote a poster for a cryptography class, concerning an MIT researcher who created programs of com-

TODO: Create a stronger connection to the fact that the papers I read were ONLINE. This is crucial!

TODO: clean up transitions with examples

mon cryptographic schemes and mathematically proved their correctness. I met this researcher at a conference and discovered that *neither* knew about the other! They provided their own contributions, but I initially assumed that *everyone* contributing to cryptography work on common projects. As another example, at my time in the Budapest Semesters in Mathematics, I adopted a key tool in program verification to find proofs for a combinatorial problem. According to my advisor, that approach had *never* been considered before in his group. The boundaries between these communities isn't so clear, and it seems to take years to begin to *remotely* find them.

The separation of these communities raises a key question: *can* knowledge across disciplines be bridged together?

In addition to this [FIRST CLASS OF PROBLEMS NAME], another major hurdle is truth management. [DISCUSS Problems with truth + corrections from papers don't propagate!] What can be done is addressing *information*, the storage of the *asserted* facts themselves, regardless of truth. As one example, suppose a scientist claims, "X is true about Y". One could debate the veracity of that claim, but what we can say is, "This scientist claims, 'X is true about Y'". Even if we doubt that, we could do: "This claim can be formulated: 'This scientist claims 'X is true about Y''". By using these justifications, stating that a claim is expressible, the *syntactic expression* of the claim can be separated from its *semantic truth value*.¹ I will make this more rigorous in later sections, but this means we can build knowledge bases on top of information systems using flexible extensions.

Information has been extensively studied through *measurements* in Algorithmic Information Theory (AIT). The founding idea of AIT is the Minimum Description Length (MDL) principle, that the best definition for an object is the smallest description that describes it. To formalize this idea, Kolmogorov defined a description as a *program*, and the Kolmogorov complexity of a string as the length of the *smallest program* that computes that string (see [1]). This program is defined via a Turing-computable programming language, and there is a different constant factor depending on the language, but AIT focuses on the asymptotic complexity. A cornerstone of this framework is providing the reason underlying cause of Gödel's incompleteness theorems, Turing's halting problem, Tarski's undefinability of truth, and more: *not all information can be compressed into a finite description*. This view was articulated by Chaitin on information compression. He defined Ω as the probability that a random Turing machine will halt and proved that it cannot be compressed computably. This result is a major theme in AIT, to address the limits of computation.

In addition to addressing these limits, Chaitin's results have profound consequences for the nature of mathematics. He explains:

Mathematics...has infinite complexity, whereas any individual theory would have only finite complexity and could not capture all the richness of the full world of mathematical truth. [2]

¹One might be worried about a paradox, such as "This claim is expressible: this claim is not expressible." We will avoid this using a clear separation of the overarching metatheory and object theory, with the former being syntactical in nature. To express this separation, we write quotes around the claim itself.

TODO: cite main claims about journals/research communities/etc as needed

TODO: REWORD as needed + merge with discussions on FAIR.

Chaitin's claim extends beyond mathematics; the extent of research areas are so vast that the idea of a *single* theory would fail to faithfully reproduce these disciplines. The study of the areas *themselves* is needed to faithfully represent them. This has been explored in Béziau's field of Universal Logic [3], where the aim is to study *logics* and not a *single* logic. In short, Chaitin's result, and the works in Universal Logic and others, demonstrate that research must be represented *flexibly* as well as faithfully.

The problem with a flexible representation system is precisely *how* to accomplish this. AIT provides asymptotic results on information *measurement*, but does not provide a guide on the fixed representation to use. Chaitin created a LISP variant, designed specifically for the ease of implementation and analysis [4], but this does not address the faithful representations of other languages. Additionally, Universal Logic provides a single definition of a logic, one which can be tedious in exotic logics. Each of these issues underly the importance of *organization* itself, which emerge in the proliferation of general-purpose programming languages.

As a concrete example of organizational challenges, consider a Python program in Listing 1.

```
int main() {
    return 0;
}

def main():
    pass

if __init__ == "__main__":
    pass
```

LISTING 1. Example programs in C (left) and Python (right).

In light of the persistence of organizational issues, another step is required beyond Chaitin's reasoning: we must consider *universal building blocks* themselves. Sticking to Turing machines is sufficient in AIT and studying asymptotics, but the *actual* storage must take place. This is the motivation and driving force of this thesis.

1.1. Goals.

The aim of this thesis is to create a universal information language to standardize *all* formal representations. I call this language **Welkin**, an old German word meaning cloud [5]. This aim will be made more precise in the later sections, where we will formally define a verifier for a programming language.

- **Goal 1:** universality. This language applies to ANY checker. Needs to be extremely flexible towards this goal, so we can **ONLY** assume, at most, we are getting a TM as an input, or it's somehow programmed.
- **Goal 2:** standardized. Needs to be rigorously and formally specified.
- **Goal 3:** optimal reuse. With respect to some criterion, enable *as much* reuse on information as possible.
- **Goal 4:** efficiency. Checking if we have enough information *given* a database much be efficient!

1.2. Organization.

- Section 2. Foundations: define the meta-theory used + verifiers. Also outline the Trusted Computing Base.
- Section 3. Information Systems: explore information in the context of verifiers. Then synthesize a definition to satisfy Goal 1.

TODO: Explain all claims concretely but concisely, as well as cite other possibilities (e.g., HoTT for logics)! And add anything else besides AIT.

TODO: Finish transpilation example! Just show we care about storing the underlying semantics.

- Section 4. Information Reuse. Develops the optimal informal system (w.r.t to a metric defined in this system) to satisfy Goal 3.
- Section 5. Syntax: Go over the simple LL(1) grammar, which is similar to JSON and uses python syntax for modules.
- Section 6. Semantics: Defines information graphs and their correspondence with the optimal informal system in Section 3.
- Section 7. Bootstrap. Fulfill Goal 2 with both the Standard AND the complete bootstrap.
- Section 7. Prototype. Time permitting, develop a prototype to showcase the language, implemented in python with a GUI frontend (Qt) and possibly a hand-made LL(1) parser.
- Section 8. Conclusion. Reviews the work done in the previous sections. Then outlines several possible applications.

2. FOUNDATIONS

2.1. Abstract Objects/Slates.

- Brief philosophical discussion
 - Discuss Kit Fine's arbitrary objects.
 - * Address symbol grounding problem + circularity: Fine uses FOL to define a notion **used** to construct FOL
 - *
 - Emphasize **pragmatism**, echoing intro. It matters how we can **use it** for this language, **not** epistemological statements or certainty (what "is" or "isn't")
 - Establish notion of a slate
 - Bring up notion of "tabula rasa"
 - Want a "clean slate" that can be "assigned an interpretation" arbitrarily
 - * Make main defense as to why this is universal; need to allow **any** extensions, so need to be arbitrarily imbued by interpreters/oracles.
 - * Note: no guarantees on what interpreters there "are" or limitations, e.g., humans have finite lifespans.
 - * Main point: to ensure arbitrary interpretation, need clean slates! Argue this is pragmatic (i.e., useful practically)
 - This is completely informal and depends on the interpreter.
 - Idea for formalization: treat **handles/ids** slates as the discrete objects
 - * Analogy: sorting through an inventory of dishes. Will connect back to organization!
 - * THEN can use formal systems/computable functions around those IDs
- to make formal claims
- * Why formal systems? Because we want to assert claims! Important pragmatically! Just like keeping track of inventory or specific points! Or being a historian!

- * Use to define information! Expand on how this improves notion of infons
- * Powerful aspect: can shape AROUND new slates! Provide examples in Slate Logic

2.2. Slate Logic.

- Definition
 - Define binary strings. Assign these to slates.
 - * Have **slate variables**. This is our entry point into arbitrary interpreters.
 - * Can change meaning based on interpretation/context! Emphasize how there can be a many to one relationship, and we need to increase formal systems available to distinguish between them!
 - * Emphasize need for a function that can enumerate these slate variables. So NOT just one slate variable. Maybe provide lemma on impossibility of doing more (within a formal system?)
 - Combinators
 - * Want a **simple** presentation to define theory. We do require substitution (variables are important here!), but want to present theory with combinators.
 - * Emphasize that this is a bootstrap/easier way to start. Just like starting somewhere on a map and then relocating (make this clearer!)
 - Justifications: what we can assert about **formal objects**
 - * Inspired by Artemov's logic of proofs. Will connect back in next two sections with serial consistency!
- Examples
 - Sorting dishes (analogy from before)
 - Map analogy, with places as IDs AND paths
 - * Emphasize that new objects can be given IDs arbitrarily; that is why we need (countably) infinitely many IDs!

2.3. Coherency.

- Definability
 - Show that this generalizes Padoa definability
 - * Classic example where this is used: showing congruence is not definable in terms of betweenness
 - * On the reals, $x \mapsto 2x$ is monotonic but does not preserve congruence.
 - * In HOL, only one direction shows. Determine how to strengthen to claim to ensure equivalence
 - Basic idea: notion A is definable via notion B iff every map that preserves B also preserves A
 - Want to use this basic idea to talk about information preservation
- Coherency of a System
 - What systems are **useful** or can talk about other systems?

- * Don't want: empty system or one with ALL The rules. These are will have low usefulness
- * Want: complex, intricate structures. Problem is, lots of notions for this! Need to determine a general notion, using slates!
- Use coherency as basis for information organization

2.4. Translations Between First Order Logic.

- Want easy access to first order logic
 - Review literature. Notable examples:
 - * SMT solvers in Rocq + Lean (via monomorphization of types)
 - Problem: abstractions are hard to convey! Lots of "bloat"
 - BUT SMT solvers are very well established, particularly with Gödel's completeness theorem.
 - How to get best of both worlds? Solution: slates!
- First step: define extension to first order logic (let's call it, say, FOL(Slate))
 - Add slates as a special sort, but focuses on first order terms.
 - * Emphasize that there are FOL theories **weaker** than combinators.
 - So, with a coherency argument, argue that FOL can be powerful **precisely because** RE is possible, WITH the combination of the completeness theorem. (Not possible in all logics!).
- Second step: show that FOL(Slate) is equivalent to FOL by treating slates as an additional sort.
 - Straightforward, but emphasize rule on slates on making meaningful/useful abstractions!
 - IF time allows, provide experiments, but mostly argue why, based on the argument for slates, this would work.
 - Argue that you could AT LEAST embed the necessary abstractions via slates. And organization will help show this is feasible with a theoretical argument (but it's not exponential time. It is (hopefully) ACTUALLY feasible.)
- Final step: show that there is an equivalent embedding that **preserves** slates.
 - Important part: preservation up to iso!
 - Maybe bring up Jose Mesegeru's "epsilon-representation distance" notion

3. UNIVERSAL SYSTEMS

- Provide previous section (translation to FOL) as a major example
 - Generalize from the case of a formal system from an earlier draft
 - Earlier definition: (D, R) , with D a grammar and R a set of RE rules
 - Universal system: $U = (D_U, R_U)$ is universal if, for each formal system S , there is a term t in D_U such that derivations in S are reflected and preserved via t in D_U . So they are faithfully encoded
 - Earlier proof: a system is universal iff it induces a computable, RE full sub-category of the category of formal systems.
 - Refine these ideas to use slates + coherency from before.
- Can involve more ambitious encodings!

- Also develop reflection!
- Hint at topic of next section, or smooth out transition. Next section is discussing **which** universal system to use or how to effectively translate between them

4. INFORMATION ORGANIZATION

- Main question: **which** universal system to choose? Is this practical?
 - What is a suitable criterion for a base theory?
 - Recall aim: want to mechanically store systems for a database
 - * What if possible performance degradation? Will we get stuck if we start with one architecture? Will we have to adjust later?
 - * Aim is to ensure architecture is completely flexible and can automatically adapt
 - * One key metric: ability to store as many systems coherently as possible, i.e., store as much information as possible
 - Main problem: Blum's speedup theorem
 - * Briefly generalize this for slate logic
 - * Show that no single way to completely organize systems based on a computable metric.

This is part of the need for new search techniques!

 - * Want to separate search from storage though, but we want to improve stored results **with** new results. This forms the idea behind the database architecture: have a simple way to store results that automatically gets better with new techniques/results.
 - * Need explicit proofs for this! Not sure how to store certificates...

4.1. Impossible Classes.

The reason to restrict our transformations is two-fold. First, we need to ensure we can *verify* them efficiently. Determining whether a morphism between two formal systems exist can be reduced to the Halting problem, and is therefore not practical for defining an optimal formal system. Second, if we include those transformations that we *can* effectively check, no optimal formal system exists.

Theorem 4.1.0. *With respect to the class of all computable transformations that can be computably verified, there is no optimal formal system.*

4.2. Efficient Querying.

Instead of making proofs most efficient as is, we want to support finding optimal representations. But we want to do this from an efficiently queryable system, which is the most optimal.

5. THE WELKIN LANGUAGE

5.1. **Syntax.**

5.2. **Semantics.**

6. BOOTSTRAP

7. CONCLUSION

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