# Equivariance versus Augmentation for Spherical Images

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## Abstract

We analyze the role of rotational equivariance in CNNs applied to spherical images. We compare the performance of the group equivariant networks known as S2CNNs and standard non-equivariant CNNs trained with an increasing amount of data augmentation. Our models are trained and evaluated on single or multiple items from the MNIST or FashionMNIST dataset projected onto the sphere. For the *invariant* task of image classification, we find that by considerably increasing the amount of data augmentation and the size of the networks, it is possible for the standard CNNs to reach at least the same performance as the equivariant network. In contrast, for the *equivariant* task of semantic segmentation, the non-equivariant networks are consistently outperformed by the equivariant networks with significantly fewer parameters. We also analyze and compare the inference latency and training times of the different networks, enabling detailed tradeoff considerations between equivariant architectures and data augmentation for practical problems. Our code is publicly available.

## Equivariance

- Many machine learning problems have an inherent symmetry
- Equivariant neural networks build the symmetries of the problem into the network architecture
- A network $\mathcal{N}$ is equivariant with respect to a symmetry group $G$ iff

$$\mathcal{N}(T_g\, x) = T_g'\, \mathcal{N}(x) \qquad \forall g \in G$$

- Widely used in applications such as quantum chemistry, medical imaging, . . .
- Equivariant networks require a specialized architecture
- Equivariance is guaranteed to hold exactly

## Data augmentation

- Enlarge training set by adding transformed data points
- Increases training time
- No need for specialized architecture, thus easy to implement
- No guarantee for exact equivariance

## Dataset

- MNIST digits projected onto the sphere with classification labels and segmentation masks
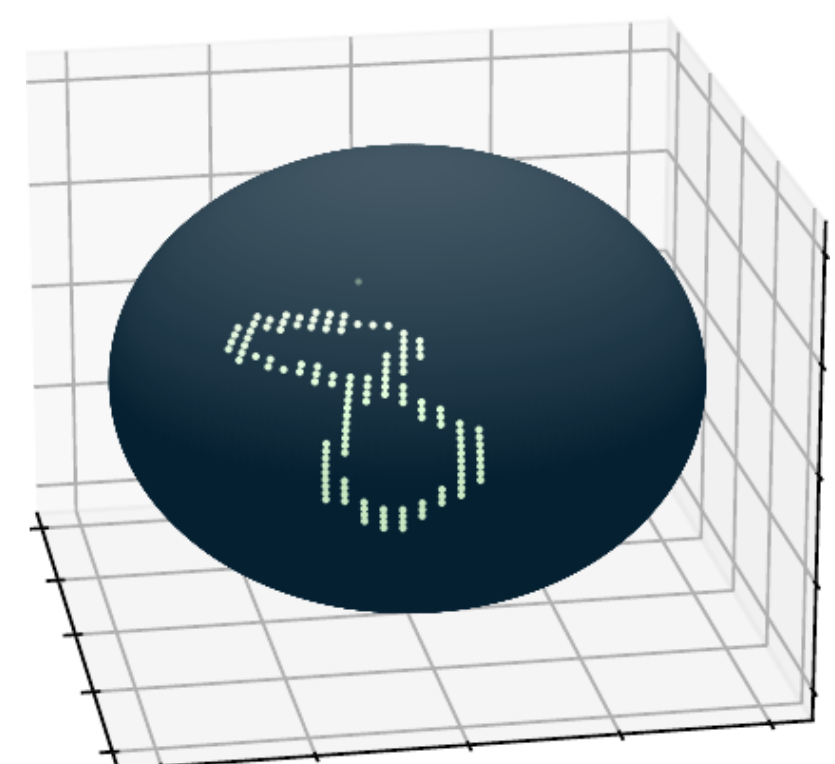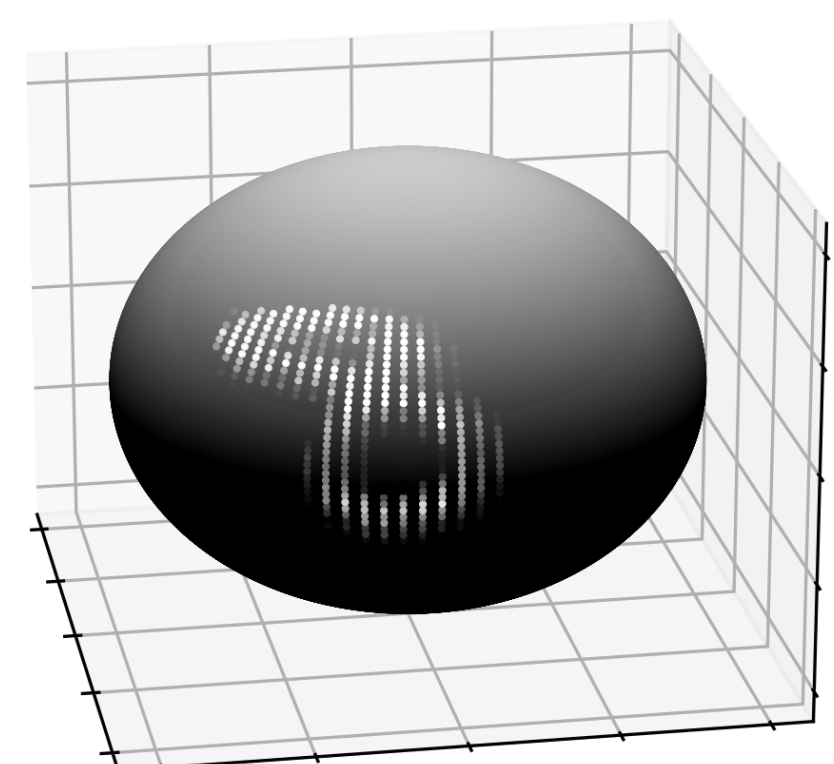


## Research question

Analyze trade-off between data augmentation and equivariance for invariant and equivariant tasks.
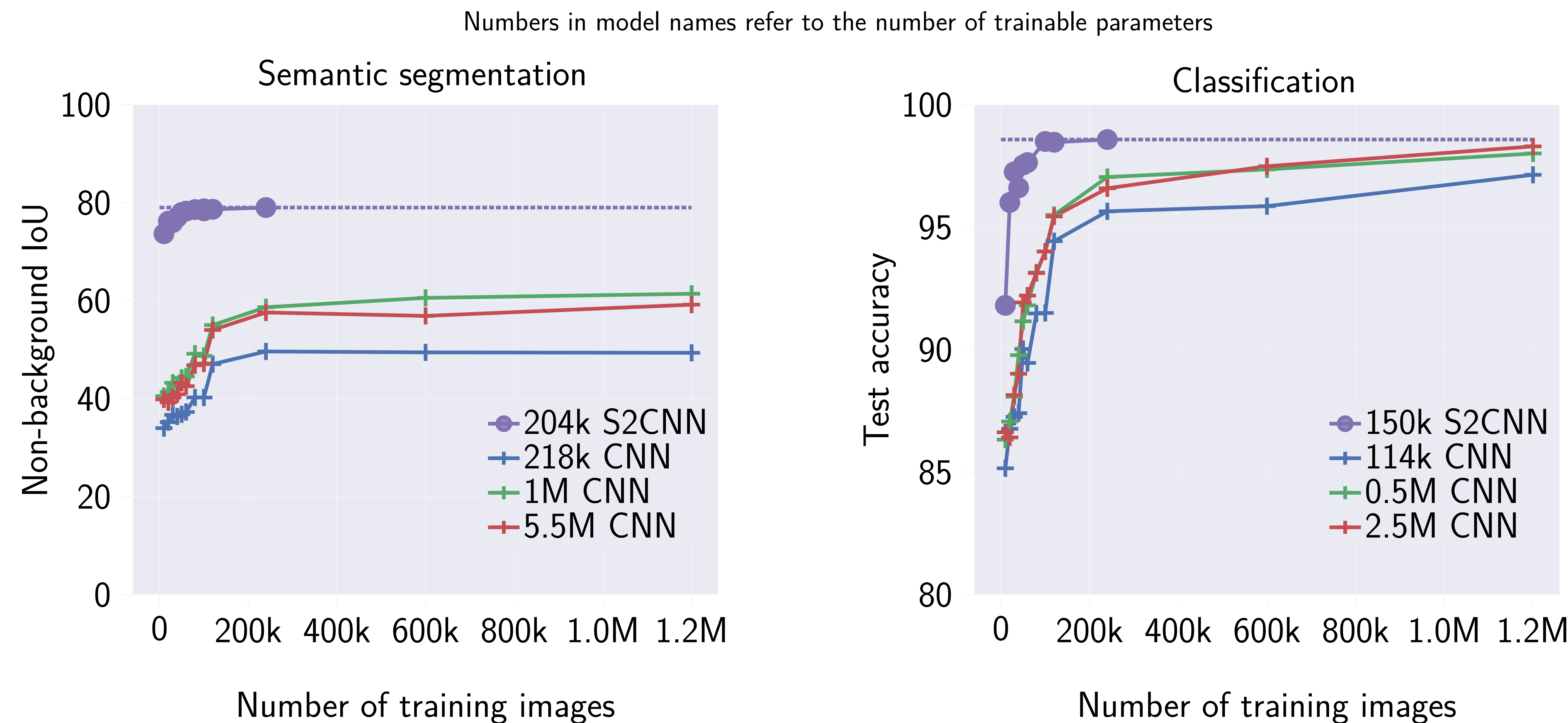
## Key takeaway

For equivariant tasks, the performance of non-equivariant networks trained with data augmentation saturates well below the performance of much smaller equivariant models.

For invariant tasks, the performance of larger non-equivariant networks trained with data augmentation can reach the performance of equivariant networks, although the non-equivariant networks take longer to train.

## Main results

### Performance

Numbers in model names refer to the number of trainable parameters



Semantic segmentation

- 204k S2CNN
- 218k CNN
- 1M CNN
- 5.5M CNN

Classification

- 150k S2CNN
- 114k CNN
- 0.5M CNN
- 2.5M CNN

### Throughput and training times

Due to the specialized architecture the equivariant model has much higher latency at similar parameter counts than the non-equivariant models.

| Model | Latency (ms) | Throughput (N/s) |
| --- | --- | --- |
| 204k S2CNN | 111.0(6) | 9.00(4) |
| 200k CNN | 5.93(24) | 169.0(58) |

At matched accuracy the total training time for the non-equivariant model trained with data augmentation is much higher than the training time for the equivariant model trained without data augmentation.

| Model | Accuracy | Training time |
| --- | --- | --- |
| 150k S2CNN | 97.64% | 15h |
| 5M CNN | 97.49% | 26h |

## Further results

### Rotated vs non-rotated test images

When training is performed only on unrotated images, the non-equivariant models outperform the equivariant models on unrotated test data. On rotated test data, the non-equivariant performance deteriorates whereas the equivariant performance is unaffected.
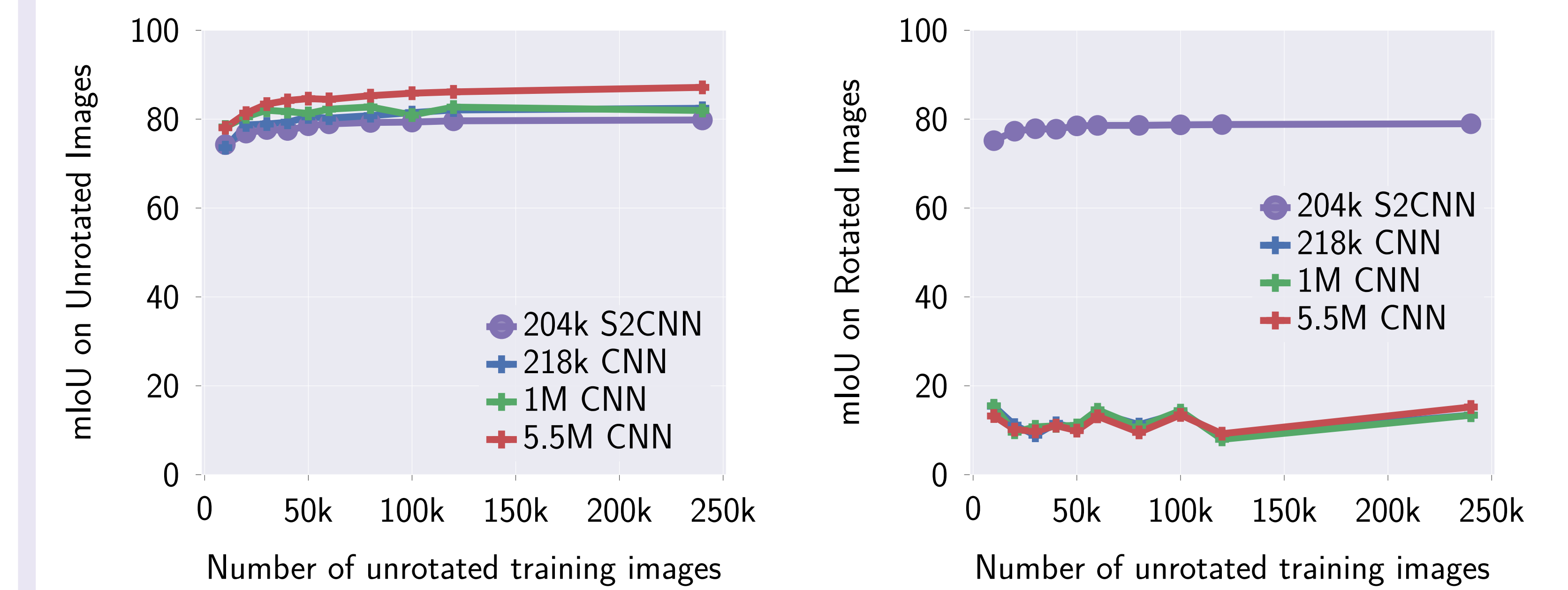


### Multiple digits

Similar results hold for semantic segmentation with four digits projected onto the sphere.
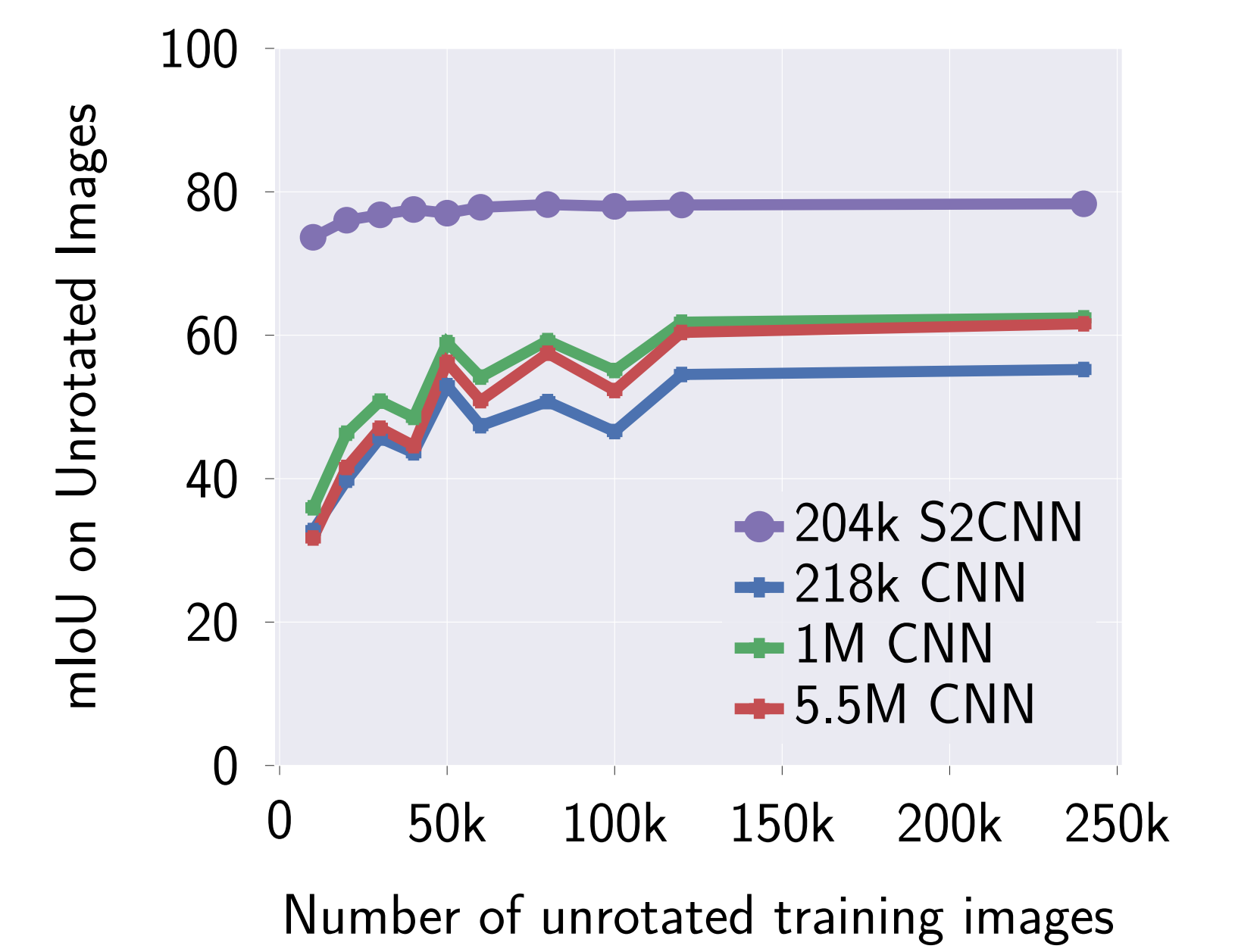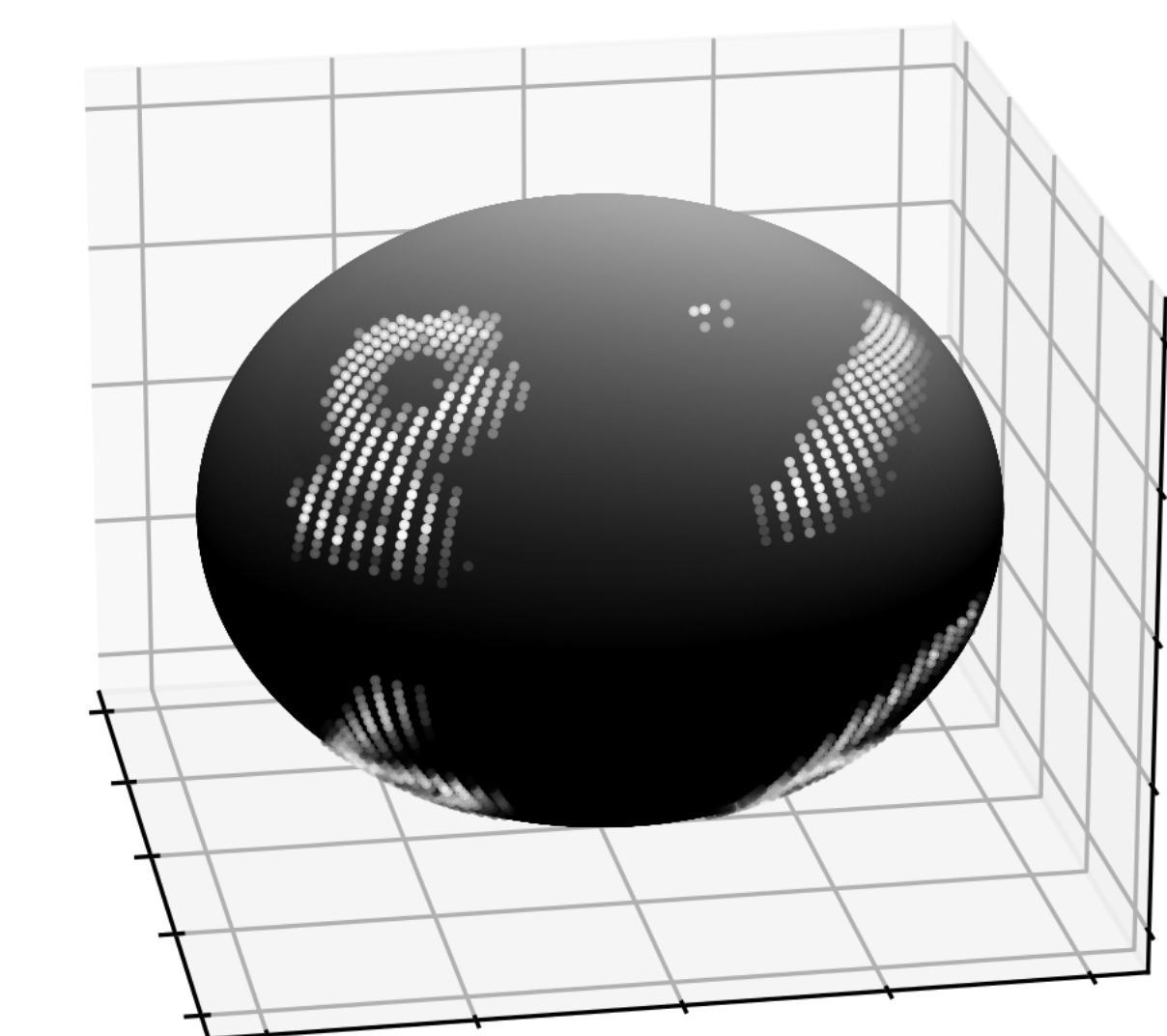


### New equivariant S2CNN layer for semantic segmentation

We added a layer to the S2CNN architecture (Cohen et al., 2018) reducing feature maps on SO(3) to feature maps on $S^2$ for semantic segmentation.

Cohen, T. S., Geiger, M., Köhler, J., and Welling, M. Spherical CNNs. In *6th International Conference on Learning Representations, ICLR*, 2018.