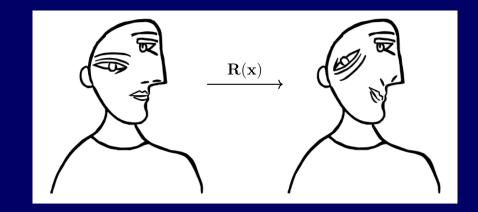
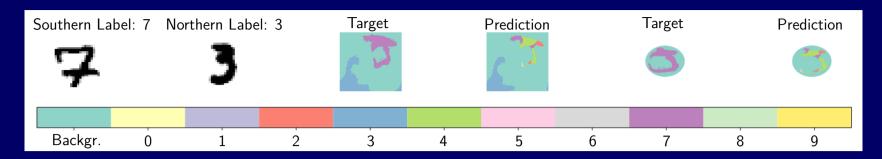
# Local rotations don't matter with appropriate layers.





# Gauge equivariant convolutional neural networks

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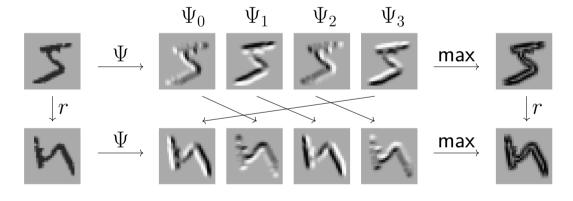
# Intro

How does one deal with rotations? Options are:

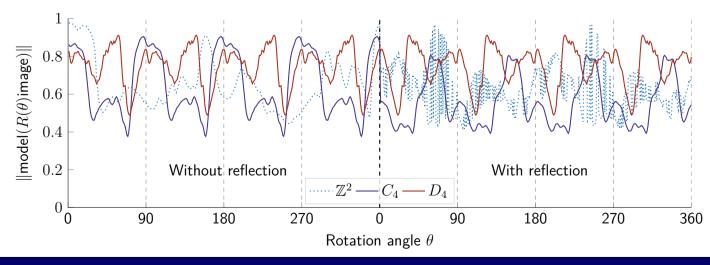
- You don't, you know all your data will have your prefered orientation. (Dangerous: real life throws curveballs at your models)
- Make sure that all your data has the right orientation. (A lot of work, either manually or making an algorithm to unrotate data)
- Augement training data so that everything is represented. (Every orientation becomes a **lot** of data to deal with)
- Modify your architecture and layers to deal with the rotation automatically. (The easy way.)

(Bullet image source: "rotation" by Adrien Coquet from the Noun Project)

## One way: transform kernels [Cohen and Welling 2016]



# Magnitude of classification invariance for four fold rotation symmetry applied to single MNIST digit



### Some mathematics

A map  $\Phi$  is equivariant with respect to a transformation T of the data if it doesn't matter if one transforms the data before or after one applies the map  $\Phi$ :

$$\Phi \circ T = T \circ \Phi. \tag{1}$$

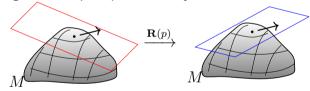
An example is that normal convolutions are equivariant to translation. One can extend this to a larger equivariance if we allow convolution kernels and data to be functions on a group:

$$[\Psi \star f](g) = \int_G \Psi(g^{-1}g')f(g') \ dg'.$$
 (2)

This is equivariant if the kernel transforms in a special way under the group action:

$$\Psi(hgh') = \rho_2(h)\Psi(g)\rho_1(h'). \tag{3}$$

[Cohen and Welling 2016] discretise this to allow for easy computation, see figure on the left. Generalise this to local transformations by taking a viewpoint of local coordinates changes, see figure on top of poster and just below:



Equivariant convolutions in this general context was introduced by [Cheng et al. 2019] without much mathematical details. These details are expanded on in our upcoming article and in [Carlsson 2020].

# **Current work**

We're currently examining the limits of the more general framework, how it compares to different classical approaches, in both classification and semantic segmentation tasks.

# References

Carlsson, Oscar (2020). "Gauge Equivariant Convolutional Neural Networks". In.

Cheng, Miranda C. N. et al. (June 6, 2019). Covariance in Physics and Convolutional Neural Networks. arXiv: 1906.02481 [hep-th, stat].

Cohen, Taco S. and Max Welling (June 3, 2016). Group Equivariant Convolutional Networks. arXiv: 1602.07576 [cs, stat].



