Learning Feature Representations

Module 1 Homework

Oscar Carlsson
Jimmy Aronsson

November 28, 2020

In this document, we summarize our work on the first Homework.

Exercise 1

In the first exercise, we fit a multivariate Gaussian to MNIST patches'

NCE

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

$$J(\theta) \propto \mathbb{E}_{x\sim p_d}\left[\log \frac{p_\theta(x)}{p_\theta(x)+\nu p_n(x)}\right] + \nu \mathbb{E}_{x\sim p_n}\left[\log \frac{\nu p_n(x)}{p_\theta(x)+\nu p_n(x)}\right]. \tag{1}$$

We approximate the right-hand side of equation (1) using the empirical estimate

$$\frac{1}{N}\sum_{i=1}^{N} \log \frac{p_\theta(x_i)}{p_\theta(x_i)+\nu p_n(x_i)} + \frac{\nu}{M}\sum_{j=1}^{M} \log \frac{\nu p_n(x'_j)}{p_\theta(x'_j)+\nu p_n(x'_j)},$$

where $x_i \sim p_d$ are training examples and $x'_j \sim p_n$ are drawn from the noise distribution $\mathcal{N}(\mathbf{0}, \Sigma_n)$. We have further chosen the model distribution $\mathcal{N}(\mathbf{0}, \Sigma_\theta)$, and the above expression can thus be simplified quite a bit. Start by rewriting both terms in the following way:

$$\begin{aligned}\log \frac{p_\theta(x)}{p_\theta(x)+\nu p_n(x)} &= -\log\left(1+\nu\frac{p_n(x)}{p_\theta(x)}\right),\\ \log \frac{\nu p_n(x)}{p_\theta(x)+\nu p_n(x)} &= -\log\left(\frac{p_\theta(x)}{p_n(x)}+\nu\right),\end{aligned}$$

and then insert the relative probability

$$w(x) = \frac{p_n(x)}{p_\theta(x)} = \sqrt{\frac{|\Lambda_n|}{|\Lambda_\theta|}}\exp\left(-\frac{1}{2}x^T\left(\Lambda_n-\Lambda_\theta\right)x\right),$$

to obtain the relatively simple expression

$$\boxed{J(\theta) \mathrel{\underset{\sim}{\propto}} -\frac{1}{N}\sum_{i=1}^{N}\log\left(\nu w(x_i)+1\right) - \frac{\nu}{M}\sum_{j=1}^{M}\log\left(w(x'_j)^{-1}+\nu\right)}$$

Insert figures of samples, loss, etc

Score Matching

$$\begin{aligned}\nabla_x \log p_\theta(x) &= -\Lambda(x-\mu)\\ \Delta \log p_\theta(x) &= -\operatorname{tr}(\Lambda)\end{aligned}$$

Exercise 2

The first step was to extract 50,000 image patches of resolution 28×28. We solved this problem by running the following loop: In each iteration, an image from the `Flickr30k` dataset is loaded, converted to grayscale, and split into multiple patches using the method `tf.image.extract_patches`. Two such patches are selected at random and saved, before moving on to the next iteration, and the program terminates after saving 50,000 patches. See `create_image_patches.py` for details.

Next, we computed a constrained Gaussian representing the above data Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.