Learning Feature Representations

Module 1 Homework

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In this document, we summarize our work on the first Homework.

Exercise 1

In the first exercise, we fit a multivariate Gaussian to MNIST patches'

Noise distribution $\mathcal{N}(\mathbf{0}, \Sigma_n)$ Model distribution $\mathcal{N}(\mathbf{0}, \Sigma_\theta)$

NCE

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$$J(\theta) \propto \mathbb{E}_{x\sim p_d}\left[\log \frac{p_\theta(x)}{p_\theta(x) + \nu p_n(x)}\right] + \nu \mathbb{E}_{x\sim p_n}\left[\log \frac{\nu p_n(x)}{p_\theta(x) + \nu p_n(x)}\right]. \tag{1}$$

We approximate the right-hand side of equation (1) using the empirical estimate

$$\frac{1}{N}\sum_{i=1}^{N} \log \frac{p_\theta(x_i)}{p_\theta(x_i) + \nu p_n(x_i)} + \frac{\nu}{M}\sum_{j=1}^{M} \log \frac{\nu p_n(x'_j)}{p_\theta(x'_j) + \nu p_n(x'_j)},$$

where $x_i \sim p_d$ are training examples and $x'_j \sim p_n$ are drawn from the noise distribution. Next, we simplify the above expression above by rewriting both terms in the following way:

$$\log \frac{p_\theta(x)}{p_\theta(x) + \nu p_n(x)} = -\log\left(1 + \nu \frac{p_n(x)}{p_\theta(x)}\right),$$
$$\log \frac{\nu p_n(x)}{p_\theta(x) + \nu p_n(x)} = -\log\left(1 + \frac{1}{\nu}\frac{p_\theta(x)}{p_n(x)}\right),$$

and then insert the relative probability

$$w(x) = \frac{p_n(x)}{p_\theta(x)} = \sqrt{\frac{|\Lambda_n|}{|\Lambda_\theta|}} \exp\left(-\frac{1}{2}x^T\left(\Lambda_n - \Lambda_\theta\right)x\right),$$

to obtain the relatively simple expression

$$J(\theta) \underset{\sim}{\propto} -\frac{1}{N}\sum_{i=1}^{N}\log\left(\nu w(x_i)+1\right) - \frac{\nu}{M}\sum_{j=1}^{M}\log\left(\frac{1}{\nu w(x_j')}+1\right). \tag{2}$$

We found that $w(x)$ is typically very small in practice, hence the sum $(\nu w)^{-1}+1$ is dominated by the first term. Its logarithm can thus be approximated by the numerically more stable expression

$$\log\left(\frac{1}{\nu w(x)}+1\right) \approx -\log \nu w(x) = \frac{1}{2}x^T(\Lambda_n - \Lambda_\theta)x - \frac{1}{2}\log\left(\nu^2\frac{|\Lambda_n|}{|\Lambda_\theta|}\right).$$

We could also completely remove the first sum in equation (2), as $\log(\nu w(x)+1) \approx \log 1$. Nevertheless, we decided to keep the first sum since it didn't cause computational problems and we didn't want our estimate to be independent of the real training data. Thus, our final estimate is

$$\boxed{J(\theta) \underset{\sim}{\propto} -\frac{\nu}{2}\log\left(\nu^2\frac{|\Lambda_n|}{|\Lambda_\theta|}\right) - \frac{1}{N}\sum_{i=1}^{N}\log\left(\nu w(x_i)+1\right) + \frac{\nu}{2M}\sum_{j=1}^{M}{x_j'}^T(\Lambda_n-\Lambda_\theta)x_j'}$$

Insert figures of samples, loss, etc

Score Matching

$$\nabla_x \log p_\theta(x) = -\Lambda_\theta(x-\mu)$$
$$\Delta \log p_\theta(x) = -\operatorname{tr}(\Lambda)$$

Exercise 2

The first step was to extract 50,000 image patches of resolution 28×28. We solved this problem by running the following loop: In each iteration, an image from the `Flickr30k` dataset is loaded, converted to grayscale, and split into multiple patches using the method `tf.image.extract_patches`. Two such patches are selected at random and saved, before moving on to the next iteration, and the program terminates after saving 50,000 patches. See `create_image_patches.py` for details.

Next, we computed a constrained Gaussian representing the above data Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.