# A Review of Recent Progress in Seismic Waves Propagation Modeling Using Machine Learning Based Methods

Oscar Rincón-Cardeño[1, 2,*]

[1] School of Applied Sciences and Engineering, Universidad EAFIT, Medellín, Colombia

[2] Applied Mechanics Research Group from Universidad EAFIT

[*] Correspondence: orincon@eafit.edu.co

https://github.com/oscar-rincon/Review-Seismic-Waves

**Abstract**

Numerical modeling has been crucial for addressing problems across various scientific and engineering disciplines involving partial differential equations. In particular, wave propagation modeling has seen significant development in scientific computation. Standard numerical modeling methods have demonstrated notable accuracy; however, their computational cost can be substantial. Recently, alternative methods based on machine learning have emerged, offering a promising balance between computational cost and accuracy when applied to wave propagation problems. In this work, we present a review of methods developed and used to model wave propagation, with a special emphasis on computational seismology. We discuss the fundamentals of wave propagation modeling, standard numerical methods, and recent advances in solving differential equations through these approaches. We conduct a systematic review of the literature to identify applications where these methods, either standalone or in hybrid approaches with standard numerical methods, have demonstrated efficiency in terms of computational time. The results of this review provide insights into the potential of machine learning techniques for wave propagation modeling and their impact on computational seismology.

**Keywords:** wave equations, numerical methods, machine learning, partial differential equations, computational seismology.

## Introduction

Wave propagation is a physical phenomenon governed by partial differential equations, which hold significant importance across various applied sciences and engineering fields. However, analytical solutions are not always available in many practical situations and numerical methods are usually required to approximate the exact solutions. Consequently, these methods have been applied to solve the partial differential equations (Seriani and Oliveira, 2020).

In the field of wave propagation, numerous techniques address wave propagation challenges. Classical methods include finite-difference, finite-element and spectral-element methods (Moczo et al.; Virieux et al.; Igel; Komatitsch and Tromp; Chaljub et al., 2007; 2011; 2017; 1999; 2007). In these approaches, the spatial coordinates are discretized. In the context of mathematical modeling, the primary objective is to ensure that the solution methods are computationally efficient without sacrificing accuracy to capture the physical details inherent to the system. However, standard numerical methods often encounter difficulties when addressing complex problems such as irregular geometries, material changes, and mixed boundary conditions. Therefore, the computational demand associated with many common models in computer sciences and engineering has increased the development of innovative strategies.

Research conducted with the use of machine learning has considerably grown in the late 2010s, owing to advancements in hardware, such as graphic processing units and data storage technologies and the growth of available data. Additionally, the discovery of better training practices for neural networks, and the availability of open-source packages like Tensorflow, PyTorch and JAX (Abadi et al.; Paszke et al.; Bradbury et al., 2016; 2019; 2018), as well as the availability of Automatic Differentiation in such packages (Paszke et al.; Baydin et al., 2017; 2017). Particularly, neural networks learning algorithms offer attractive approximation capabilities for any function by mapping the input features to the output targets in a data-driven manner. A version of the Universal Approximation Theorem conclusively demonstrates that neural networks have the capability to accurately approximate a wide variety of nonlinear functions without any dimensionality constraints (Barron, 1993). Therefore computational scientists have explored the potential of machine learning as a numerical tool to model systems governed by partial differential equations (Cuomo et al.; Karniadakis et al., 2022; 2021). From those works, physics-informed neural networks is one of the methods that has gained more attention in the last years, cited by over 10,000 publications (Raissi et al., 2019). Figures 1.A and B illustrate the number of publications related to machine learning, numerical methods, and wave propagation modeling.

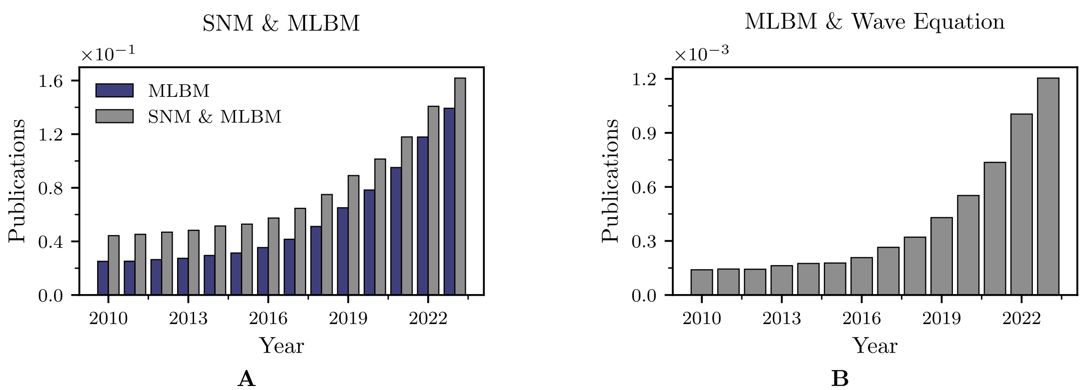


**Figure 1.** The growth of literature related to machine learning and wave propagation modeling is shown. The number of publications was retrieved from Scopus between 2010 and 2023. The relative number of publications is calculated as the number of publications containing the selected terms relative to the total number of publications in Scopus during the same period. The chosen terms were machine learning-based methods (MLBM) and standard numerical methods (SNM) (A), as well as MLBM specifically associated with wave propagation modeling (B).

Remarkable reviews have been conducted to address the increasing use of machine learning algorithms across various engineering and scientific disciplines (Vadyala et al.; Deng et al.; Lino et al., 2022; 2023; 2023). Also emphasis has been placed on the application of neural networks to model seismic waves (JingBo et al., 2023). However, there is uncertainty, given the rapid growth of the field, about in which cases machine learning methods can be an appropriate alternative to standard numerical methods to solve partial differential equations (Grossmann et al.; McGreivy and Hakim, 2023; 2024). Although in principle, machine learning methods have the potential to learn a surrogate model able to approximate the solution of a partial differential equation, some methods can be more efficient than others according to the problem being solved. This is particularly relevant in the context of computational seismology, where the complexity of the domain phenomena can be challenging to model. Therefore the aim of this review is to provide insights into the potential of machine learning methods for wave propagation modeling and their impact on computational seismology.

This work provides a mixed review betweem narative and systematic review of the advancements made in partial differential equations modeling through machine learning and their impact. While this area can be applied to a wide range of problems, our focus will be limited to the propagation of mechanical waves. The work is organized into the following sections: Section 1 describes general aspects about wave propagation modeling. Furthermore, in Sections 2 and 3, we identify existing standard and machine learning methods used to solve the diferential equations and particularly on the wave equation. Then, in Section 4 we systematically review the recent advances in wave propagation modeling achieved through these emerging methods and identify when they can be an alternative to traditional numerical methods or in an hibrid way when they can improve the solver performance in terms of computational time. We aimed to answer the following research question:

**What machine learning techniques have been used as a complement or alternative to traditional numerical methods for modeling the wave equation in computational seismology?**

We define a complementary approach as one in which both machine learning and traditional numerical methods are employed together for modeling, particularly when synthetic data generated by numerical methods is used to train the machine learning model. Conversely, an alternative approach is considered when machine learning techniques are used as the primary solver, relying on experimental data. Furthermore, our scope includes studies where physics-informed neural networks (PINNs) are applied to solve inverse problems. While PINNs have generally been found less accurate and computationally inefficient for forward problems due to the extensive time required for model training, they have shown promise as alternatives to standard numerical methods in inverse problems (Haghighat et al.; Raissi et al.; Hao et al., 2021; 2020; 2023). This is largely attributed to their versatility in handling varying amounts of data and their ability to incorporate physical laws directly into the model.

# 1 Modeling of Wave Propagation

A dynamic model, such as wave propagation in a medium, aim to describe through a function how a system changes over time. These models typically rely on differential equations to characterize the system's evolution. A general formulation of the governing equation for a physical problem can be expressed as:

$$D(u(x,t);\lambda) = f(x,t), \quad x \in \Omega, \quad t \in [0,T].$$

Here, $D$ represents the differential operator acting on the solution $u(x,t)$ to the differential equation, which is parameterized by $\lambda$, and $f(x,t)$ is a source term. The symbols $\Omega$ and $\partial\Omega$ denote the spatial domain and its boundary, respectively. Equation 1 can be applied to model various systems. The corresponding boundary and initial conditions are given by:

$$B(u(x,t)) = g(x,t), \quad x \in \partial\Omega, \quad t \in [0,T]$$

and

$$u(x,0) = h(x,0), \quad x \in \Omega.$$

The prescribed initial and boundary conditions are characterized by $h(x)$ and $g(x, t)$, respectively. In mathematical modeling, two general approaches are commonly used: the forward and inverse problems. The inverse problem involves determining the causes of a set of observations (Röth and Tarantola; Tarantola, 1994; 2005), such as inferring the properties of a medium based on its response to wave propagation. This approach is the opposite of the forward problem, which calculates the effects based on known causes. Since the inverse problem starts with the effects and seeks to determine the causes, it typically requires iterative forward modeling, making it computationally complex. A schematic representation of forward and inverse modeling using numerical methods is shown in Figure 2.

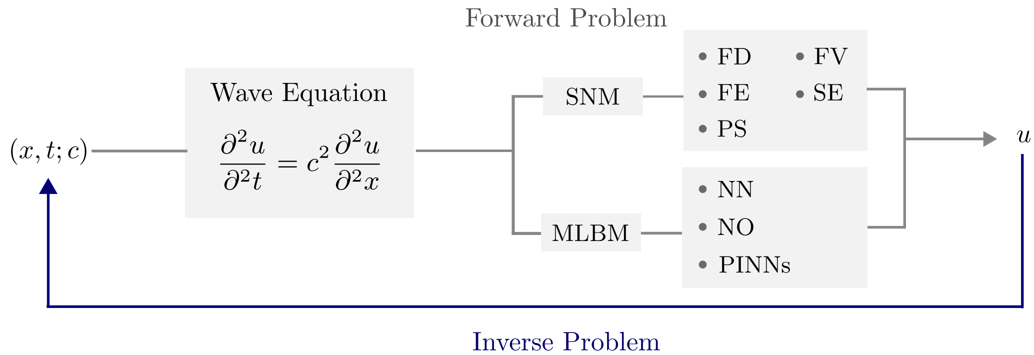


**Figure 2.** Scheme of the forward and inverse problems encountered in solving partial differential equations. In the forward scenario, the inputs $(x, t; c)$ are employed to characterize a model across PDEs. Subsequently, the PDEs are resolved through either standard numerical methods (SNM) or neural networks based methods (MLBM) to derive a solution $u$. Standard numerical methods such as: finite differences (FD), finite elements (FE), pseudo-spectral (PS), finite volumes (FV), and spectral elements (SE). Also, deep learning techniques include, for example, Physics Informed Neural Networks (PINNs), Neural Operator (NO), and Neural Networks (NN). In the case of the inverse problem, the objective is to determine the parameters, for example, the wave speed $c$ starting from the solution $u$.

Inverse problems are closely tied to simulation, and solving them is crucial for many real-world tasks. Moreover, some complex physical problems require determining the properties of a physical system governed by partial differential equations from observational data, rather than solving them directly to obtain a function that satisfies them (Galiounas et al.; Ren et al.; McCann et al., 2022; 2024; 2017). The objective is to estimate a set of latent or unobserved parameters of a system based on real-world observations. Within the framework described by Equation 1, the task involves estimating $\lambda$ given $u$. Inversion can be exceedingly challenging since often requires numerous forward simulations to align the predictions of the physical model with the set of observations.

Despite being the most elementary among mechanical wave equations, the scalar (acoustic) wave equation is widely used to study seismic waves and in medical applications (Moseley; Alkhadhr and Almekkawy, 2022; 2023). The second-order linear wave equation in a homogeneous medium can be expressed as (Carcione, 2002):

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \nabla^2 u(x,t) = f(x,t) \ ,$$

where $\nabla^2 = \sum_{i=1}^{d} = \frac{\partial}{\partial^2 x_i}$, $u(x, t)$ describes the pressure of the generated waves, and $f(x, t)$ is a source term that describes the strength and duration of the source.

Another common expression used to describe the propagation of seismic waves, for the case of a heterogeneous isotropic medium, is the elastic wave equation (Moseley et al.; Lehmann et al., 2018; 2023). This equation can be expressed as:

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla(\lambda(\nabla \cdot u)) + \nabla\mu \left[\nabla u + (\nabla u)^T\right] + (\lambda + 2\mu)\nabla(\nabla \cdot u) - \mu\nabla \times (\nabla \times u) \ ,$$

where $\rho$ is the material density, $u$ is the displacement vector, and $\lambda$, $\mu$ are the Lamé parameters characterizing the material. These equations are fundamental for modeling the propagation of seismic waves in elastic media. The acoustic wave equation is a simplification that assumes the waves are longitudinal and the medium is homogeneous and isotropic. In contrast, the elastic wave equation accounts for the heterogeneous and anisotropic properties of the medium, allowing for the modeling of both longitudinal and transverse waves.

Besides the acoustic and elastic equations, there are other important variants of the wave equation used in different contexts of computational seismology. Viscoelastic Wave Equation is a variant tha incorporates damping effects due to the viscosity of the medium. It is useful for modeling wave attenuation in real geological media that exhibit viscoelastic behavior.

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla(\lambda(\nabla \cdot u)) + \nabla\mu \left[\nabla u + (\nabla u)^T\right] + (\lambda + 2\mu)\nabla(\nabla \cdot u) - \mu\nabla \times (\nabla \times u) - \eta\frac{\partial u}{\partial t} \ ,$$

where $\eta$ is the viscosity coefficient. Anisotropic Wave Equation describe propagation in anisotropic media, the elastic properties vary with direction. The wave equation is modified to include additional terms representing this anisotropy.

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + f \ ,$$

where $\sigma$ is the anisotropic stress tensor and $f$ is a source term. Nonlinear Wave Equation are considered in situations where wave amplitudes are very large, linear approximations are insufficient, and nonlinear terms must be considered in the wave equation.

$$\frac{\partial^2 u}{\partial t^2} - c^2\nabla^2 u + \beta\frac{\partial u^2}{\partial x^2} = f(x_i, t) \ ,$$

where $\beta$ is a nonlinearity coefficient. These variants allow for more precise and realistic modeling of seismic wave propagation in different types of media and under various conditions. The choice of the appropriate wave equation depends on the characteristics of the medium and the seismic phenomenon being studied. Tradicionally, the wave equation and its applications to computational seismology have been solved using standard numerical methods (Igel, 2017).

## 2 Standard Numerical Methods

In the past decades various numerical methods have been proposed to solve physics systems by partial differential equations such as the wave equation. The finite-difference method is among the most popular to solve partial differential equations, and particularly the wave equation. This possibly associated with its straightforward concept and easy implementation. A complete review of the finite-differences method applied to wave propagation can be found the Moczo et al. (2014). Partial derivatives are approximated by discrete operators involving differences between adjacent grid points. The finite difference method suits for tackling issues related to simple geometric structures. In contrast, other methods such as the finite element offers more grid flexibility, facilitating the handling of intricate geometric boundaries.

In wave propagation simulations, the partial differential equations are typically discretized on a staggered grid (Madariaga; Virieux, 1976; 1986). This approach facilitates the resolution of the rupture propagation problem. Particularly an approach was proposed in the work of Zhou et al. (2021) a finite-difference method with variable-length temporal and spatial operators was proposed to increase the stability and efficiency of the standard method. Also, Liu et al. (2023) combined a standard staggered-grid, finite-difference approach and the perfectly matched layer absorbing boundary to solve 3D first-order velocity-stress equations of acoustoelasticity to simulate wave propagating.

Finite-element methods are suitable for dealing with intricate shapes and diverse materials because they can use irregular grids. They permit flexibility in size, shape, and approximation order. Nevertheless, a drawback is their high demand for computing power. This methodology involves the transformation of the problem at hand into a system of linear equations utilizing the weak formulation of the pertinent differential equation. This transformation is facilitated by employing an interpolation basis comprised of polynomials defined over disjoint domains, commonly referred to as elements.

Open-source software is available for applying numerical methods to solve the wave equation. For example, FEniCS and DUNE (Langtangen and Logg; Sander, 2016; 2020), offer computing frameworks designed for solving partial differential equations using the finite element method. SPECFEM, which specializes in seismic wave propagation, is widely used in simulations implemented in Fortran (Komatitsch et al.; Komatitsch et al., 2023; 2024). Similarly, SEISMIC_CPML (Komatitsch and Martin, 2007) uses finite differences for modeling. These implementations of standard methods have enabled effective simulations of the wave equation.

A significant difficulty in using standard methods for wave propagation simulations is their computational cost. Their accuracy is achieved at the expense of the number of points in the grid. Modeling a complex domain can entail a huge amount of grid points, with the wavefield requiring iterative updates across the entire grid at each time step. Associated with the required discretization is the challenge when dealing with high-dimensional systems. The curse of dimensionality can lead to a rapid increase in computational cost as the number of dimensions grows. Additionally, model evaluation and storage could be significantly costly (Saloma, 1993), and their limited capacity to incorporate measured data into their predictions makes them less ideal for use in inverse problems. There is considerable scientific interest in employing machine learning techniques to address these challenges.

## 3 Machine learning Methods

The field of machine learning has recently shown significant promise in approximating predictions of physical phenomena. These methods are capable of capturing highly nonlinear physics and provide substantially faster inference times compared to traditional simulations. Consequently, machine learning has been employed as an alternative to conventional methods, leveraging its capability as a universal function approximator (Hornik, 1991). For example, support vector machines have been used to solve ordinary and partial differential equations. Although this method was originally designed for classification tasks, an extention to the method that apply least square to the objetive function has been proposed to solve differential equations (Mehrkanoon et al.; Mehrkanoon and Suykens, 2012; 2015).

Neural network based methods are a subset of machine learning, whose models are composed of an artificial neural network with a single or multiple processing layers (Figure 3.A). It have shown potential in overcoming the limitations of multiple approaches in various fields such as computer vision, natural language processing, and genomics (LeCun et al.; Goodfellow et al., 2015; 2016). The fundamental architecture of a neural network architecture is conformed by an input layer, an output layer, and an arbitrary number of hidden layers. Particularly, in a fully connected neural network, neurons in adjacent layers are connected with each other but neurons within a single layer share no connection (Figure 3.B). Furthermore, neural networks methods have emerged as an attractive tool to augment and complement conventional numerical solvers of partial differential equations, thereby enabling the tackling of challenges across multiple dimensions, scales, and parameterization with the promise of efficiency and precision.

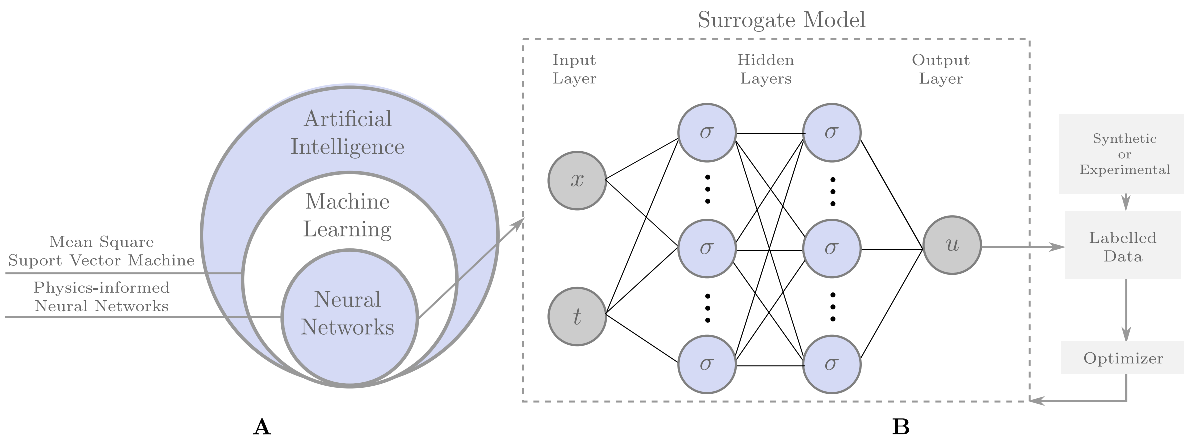


**Figure 3.** Artificial Intelligence subsets and artifical neural networks. (A) Deep learning as a subset of machine learning and artificial intelligence and (B) basic architecture of artificial neural networks.

They essentially model the partial differential equation solution by a deep neural network and train the network's parameters to approximate the solution. Data-driven neural networks methods are capable of directly learning the trajectory of a system of partial differential equations from available data (Li et al.; Li et al., 2020; 2021). Alternatively, syntetic data generated by standard numerical methods can be used to train the neural network. Therefore, a surrogate model can be used to predict the solution of the partial differential equation at a reduced computational cost. While keeping an acceptable level of accuracy. For example, one of the most popular types of deep neural networks is known as convolutional neural networks. A convolutional neural network convolves learned features with input data, and uses 2D convolutional layers, making this architecture well suited to processing 2D data, such as images.

All these approaches employ machine learning algorithms and others such as support vector machines, random forests, Gaussian processes have been also applied to model physical systems. However, they are implemented mainly as black-box tools. The constructed neural network can be thought of being ignorant of the mathematical description of the physical phenomenon. In order to overcome this limitation physics-informed neural networks architectures have been proposed. Where the activation and the loss functions are designed according to the context of the problem.

**Question:** When is necessary to involve a PDE in the model?

There has been an increasing interest in leveraging physics-informed neural networks to solve forward and inverse problems where full or partial knowledge of the governing equations is known since the published works of Raissi and Karniadakis (2018), Raissi et al. (2018) and Raissi et al. (2019). The core concept of PINNs is to minimize an energy functional that represents the residual of the PDE along with its initial and boundary conditions. Although similar ideas for constraining neural networks using physical laws have been explored in previous studies (Lagaris et al., 1998). The general principle of physics-informed neural networks is to integrate deep neural networks and physical laws to learn the underlying consistent dynamics from small or zero labeled data (Karniadakis et al., 2021). As universal approximators, neural networks have the potential to represent any partial differential equation. They make use of the powerful tool that is automatic differentiation. This capability eliminates the need for the discretization step, thereby avoiding discretization-based physics errors as well. Instead a random sampling of the domain is implemented. Physics-informed neural networks aim to address physical systems governed by the equation

$$u_{tt} - D[u(t, x); \lambda] = 0,$$

where $x \in \mathbb{R}^D$ and $t \in \mathbb{R}$. The expression $N[u(t, x); \lambda]$ denotes an underlying differential operator that characterizes the physical system, parametrized by $\lambda$. The function $u(t, x)$ represents the system's solution. The loss function is of the general form

$$L := \beta_{\text{pde}} L_{\text{pde}}(\sigma) + \beta_{\text{ic}}(\sigma) L_{\text{ic}} + \beta_{\text{bc}} L_{\text{bc}}(\sigma),$$

where

$$\begin{aligned}
\mathcal{L}_{pde}(\boldsymbol{\sigma}) &= \frac{1}{n_{pde}} \sum_{i=1}^{n_{pde}} \left| u_{tt} - \mathcal{D}\left[\hat{u}\left(t, \boldsymbol{x}_i; \boldsymbol{\sigma}\right)\right] - f\left(t, \boldsymbol{x}_i\right) \right|^2, \\
\mathcal{L}_{bc}(\boldsymbol{\sigma}) &= \frac{1}{n_{bc}} \sum_{i=1}^{n_{bc}} \left| \hat{u}\left(t, \boldsymbol{x}_i; \boldsymbol{\sigma}\right) - g\left(t, \boldsymbol{x}_i\right) \right|^2, \\
\mathcal{L}_{ic}(\boldsymbol{\sigma}) &= \frac{1}{n_{ic}} \sum_{i=1}^{n_{ic}} \left| \hat{u}\left(0, \boldsymbol{x}_i; \boldsymbol{\sigma}\right) - h\left(t, \boldsymbol{x}_i\right) \right|^2,
\end{aligned}$$

and $\mathcal{L}_{\text{pde}}$ represents the residuals of the PDEs, $\mathcal{L}_{\text{ic}}$ represents the error at the collocation points at the initial time point, and $\mathcal{L}_{\text{bc}}$ represents the error at the collocation points on the boundaries. The terms $n_{pde}$, $n_{bc}$, and $n_{ic}$ denote the number of collocation points used for the PDE residuals, boundary conditions, and initial conditions, respectively. The coefficients $\beta_{\text{ic}}$ and $\beta_{\text{bc}}$ are training hyper-parameters. Figure 4 illustrates the application of physics-informed neural networks to the wave equation.

One major drawback of these methods is the difficulty of transferring knowledge between different configurations. For example, when solving the wave equation, CNNs and PINNs are trained with a fixed velocity parameter and cannot predict anything for a different velocity value.

One of the main challenges in numerically modeling mechanical is associated with the dimensionality, given the computational complexity. Tackling complex high-dimensional systems comes with significant challenges. Despite this, machine learning-based algorithms offer promising prospects for solving partial differential equations, as indicated by studies such as the one by Blechschmidt and Ernst (2021). Most of the applications are implemented in one dimensional or two-dimensional domains. In Lehmann et al. (2023) the Fourier Neural Operator method is applied to model sesimic waves.

Emerging machine learning methods for solving partial differential equations can face difficulties in establishing fair comparison points with standard numerical methods. McGreivy and Hakim identified two common pitfalls. First, comparing the runtime of a less accurate machine learning method to a more accurate standard numerical method, whereas a fair approach would be to make the comparison under similar accuracy levels. Second, evaluating the standard numerical

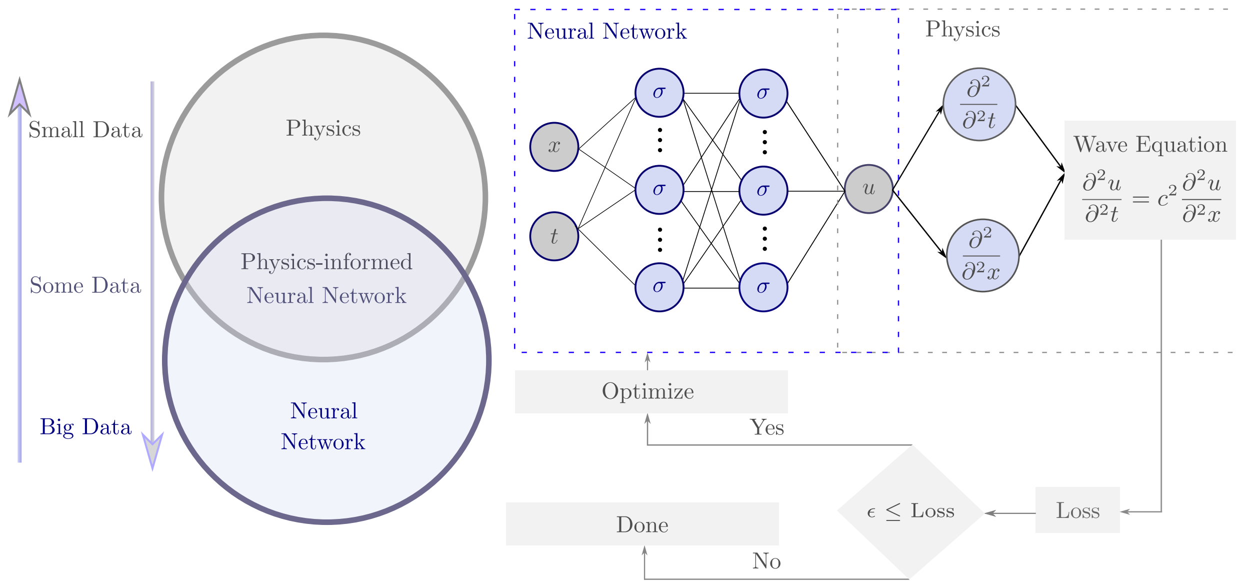


**Figure 4.** Physics-informed neural networks scheme applied to the wave equation.

method that is not suitable for the partial differential equation being solved. These two criteria are essential for properly evaluating performance, but they are not always followed.

Different extentions of the classical work where PINNs was originally proposed have emerged. Kharazmi et al. (2019) proposed variational physics-informed neural networks which instead trained physics-informed neural networks using the variational form of the underlying differential equations. A neural network is still used to approximate the solution of the differential equation, but it is combined with a set of analytical test functions to compute the residual of the variational form of the equation in its physics loss term. Furthermore, they used quadrature points to estimate the corresponding integrals in the variational loss, rather than random collocation points. They found that the variational physics-informed neural networks was able to solve differential equations including Poisson's equation with similar or better accuracy to a physics-informed neural networks trained using the strong form, whilst requiring less collocation points to train. However, most of these extensions have not yet been applied to wave propagation modeling.

Various open-source frameworks are available for solving partial differential equations using emerging machine learning methods. Python packages such as NeuroDiffEq (Chen et al., 2020) and DeepXDE (Lu et al., 2021) facilitate the solving of both ordinary and partial differential equations using neural networks as function approximators. A similar implementation in the Julia programming language is NeuralPDE (Zubov et al., 2021). Additionally, PINNs-Torch (Bafghi and Raissi, 2023) enables the application of Physics-Informed Neural Networks using PyTorch, offering improved performance compared to the original model.

## 4 Applications

This section presents a systematic review of the literature on the application of machine learning methods to model wave propagation. Our goal is to assist researchers interested in applying these emerging techniques to wave propagation modeling. Given the broad scope of machine learning applications, we focus on evaluating the potential of these methods within computational seismology. A search strategy was employed to identify relevant publications that address the research question outlined in the introduction. Figure 5 displays the flowchart and the total number of studies meeting these criteria.

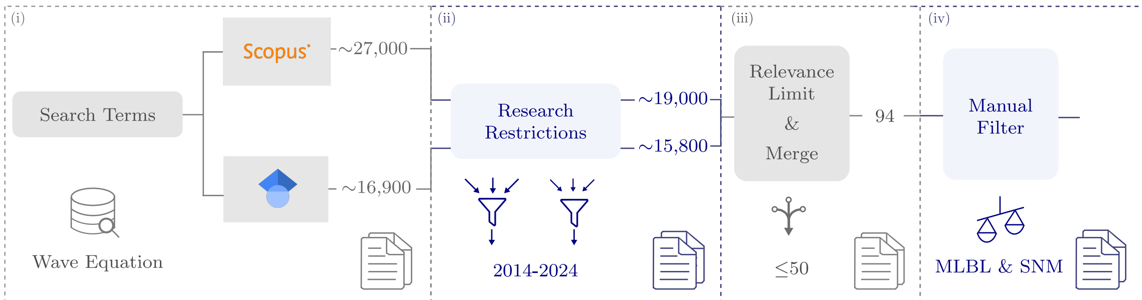


**Figure 5.** Search flowchart and number of publications after each step. During the systematic review process, Scopus and Google Scholar were utilized with the relevant search terms (i), and the research was restricted to works in English and within the time frame of 2014-2024 (ii). The resulting lists were then sorted by relevance and limited to a maximum of 50 entries, with duplicates removed (iii). Finally (iv), a manual filter was applied by reading the titles and abstracts to ensure the publications were pertinent to our chosen field.

Initially, our search focused on how machine learning has been used for modeling wave propagation. The initial search was conducted using the following query on the Scopus and Google Scholar websites as an initial filter:

```
("machine learning" OR "deep learning" OR "neural networks") AND
("wave propagation" OR "wave equation") AND (modeling OR modelling
OR model OR simulation)
```

Google Scholar provided references from both indexed journals and preprint platforms like arXiv, which is useful for identifying recent literature in this rapidly evolving field.

The search was limited to articles published between 2014 and 2024. Non-English documents were also excluded. The resulting list was then sorted by relevance, with the analysis limited to the first 50 results from both databases, which were then merged, and duplicate works removed. The final list was manually filtered by reading the titles and abstracts, with relevance to the chosen field, which was computational seismology. Also, works that applied physics-informed neural networks works that were not used to solve inverse problems were excluded. Cited references were also reviewed to identify additional relevant articles.

The number of articles that successfully passed the manual filtering process was _. From these, we extracted key information to address the research question. The articles were summarized based on the following criteria: the application domain within computational seismology, the machine learning method employed. If synthetic data was used, standard numerical method used was mentioned. Table 1 provides a summary of the main findings from the reviewed articles. The list was sorted by date of publication, with the most recent articles appearing first.

| Article | Wave Equation Type | MLBM | Application | Dataset |
|---|---|---|---|---|
| Ren et al. (2024) | 2D-acoustic | PINNs | FWIs | FEM |
| Song and Wang (2022) | 2D-frequency | Method 2 | Application 2 | Efficiency 2 |
| Example Article 3 | Type 3 | Method 3 | Application 3 | Efficiency 3 |
| Example Article 4 | Type 4 | Method 4 | Application 4 | Efficiency 4 |

Table 1: Summary of the reviewed articles and their main findings.

# 5 Conclusions

In this review, we have discussed the advancements in wave propagation modeling achieved through machine learning methods, with a focus on computational seismology. We have provided an overview of the fundamentals of wave propagation modeling, standard numerical methods, and the recent advances in machine learning methods to solve differential equations. A systematic review of the literature was conducted to identify the applications where machine learning methods have demonstrated to improve the computational performance to standard numerical methods in terms of computational time and accuracy. It is important to recognize that deep learning methods should complement, rather than replace, standard numerical techniques for solving partial differential equations. Traditional methods have been refined over decades to meet robustness and computational efficiency criteria in real-world applications. While this review focuses on computational seismology applications, the discussed methods can be applied to other fields where the wave equation is relevant. Future research should aim to integrate the strengths of both machine learning and traditional numerical methods, exploring hybrid approaches that can leverage the advantages of each technique.

# References

[1] G. Seriani and S. P. Oliveira. Numerical modeling of mechanical wave propagation, 9 2020. ISSN 18269850.

[2] Peter Moczo, Johan O.A. Robertsson, and Leo Eisner. The finite-difference time-domain method for modeling of seismic wave propagation. In Ru-Shan Wu, Valerie Maupin, and Renata Dmowska, editors, *Advances in Wave Propagation in Heterogenous Earth*, volume 48 of *Advances in Geophysics*, pages 421–516. Elsevier, 2007. doi: https://doi.org/10.1016/S0065-2687(06)48008-0. URL `https://www.sciencedirect.com/science/article/pii/S0065268706480080`.

[3] Jean Virieux, Henri Calandra, and René-Edouard Plessix. A review of the spectral, pseudo-spectral, finite-difference and finite-element modelling techniques for geophysical imaging. *Geophysical Prospecting*, 59(5):794–813, 2011. doi: https://doi.org/10.1111/j.1365-2478.2011.00967.x. URL `https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1365-2478.2011.00967.x`.

[4] Heiner Igel. *Computational seismology: a practical introduction*. Oxford University Press, 2017.

[5] Dimitri Komatitsch and Jeroen Tromp. Introduction to the spectral element method for three-dimensional seismic wave propagation. *Geophysical Journal International*, 139(3):806–

822, December 1999. ISSN 0956-540X. doi: 10.1046/j.1365-246x.1999.00967.x. URL https://doi.org/10.1046/j.1365-246x.1999.00967.x.

[6] Emmanuel Chaljub, Dimitri Komatitsch, Jean-Pierre Vilotte, Yann Capdeville, Bernard Valette, and Gaetano Festa. Spectral-element analysis in seismology. In Ru-Shan Wu, Valerie Maupin, and Renata Dmowska, editors, *Advances in Geophysics*, volume 48 of *Advances in Wave Propagation in Heterogenous Earth*, pages 365–419. Elsevier, January 2007. doi: 10.1016/S0065-2687(06)48007-9. URL https://www.sciencedirect.com/science/article/pii/S0065268706480079.

[7] Martin Abadi, Paul Barham, Jianmin Chen, Zhifeng Chen, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Geoffrey Irving, Michael Isard, Manjunath Kudlur, Josh Levenberg, Rajat Monga, Sherry Moore, Derek G. Murray, Benoit Steiner, Paul Tucker, Vijay Vasudevan, Pete Warden, Martin Wicke, Yuan Yu, and Xiaoqiang Zheng. {TensorFlow}: A System for {Large-Scale} Machine Learning. pages 265–283, 2016. ISBN 978-1-931971-33-1. URL https://www.usenix.org/conference/osdi16/technical-sessions/presentation/abadi.

[8] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Kopf, Edward Yang, Zachary DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. PyTorch: An Imperative Style, High-Performance Deep Learning Library. In *Advances in Neural Information Processing Systems*, volume 32. Curran Associates, Inc., 2019. URL https://proceedings.neurips.cc/paper_files/paper/2019/hash/bdbca288fee7f92f2bfa9f7012727740-Abstract.html.

[9] James Bradbury, Roy Frostig, Peter Hawkins, Matthew James Johnson, Chris Leary, Dougal Maclaurin, George Necula, Adam Paszke, Jake VanderPlas, Skye Wanderman-Milne, and Qiao Zhang. JAX: composable transformations of Python+NumPy programs, 2018. URL http://github.com/google/jax.

[10] Adam Paszke, Sam Gross, Soumith Chintala, Gregory Chanan, Edward Yang, Zachary DeVito, Zeming Lin, Alban Desmaison, Luca Antiga, and Adam Lerer. Automatic differentiation in PyTorch. 2017.

[11] Atılım Günes Baydin, Barak A. Pearlmutter, Alexey Andreyevich Radul, and Jeffrey Mark Siskind. Automatic differentiation in machine learning: a survey. *The Journal of Machine Learning Research*, 18(1):5595–5637, January 2017. ISSN 1532-4435.

[12] A.R. Barron. Universal approximation bounds for superpositions of a sigmoidal function. *IEEE Transactions on Information Theory*, 39(3):930–945, May 1993. ISSN 1557-9654. doi: 10.1109/18.256500. URL https://ieeexplore.ieee.org/abstract/document/256500. Conference Name: IEEE Transactions on Information Theory.

[13] Salvatore Cuomo, Vincenzo Schiano Di Cola, Fabio Giampaolo, Gianluigi Rozza, Maziar Raissi, and Francesco Piccialli. Scientific Machine Learning Through Physics–Informed Neural Networks: Where we are and What's Next. *Journal of Scientific Computing*, 92(3):88, July 2022. ISSN 1573-7691. doi: 10.1007/s10915-022-01939-z. URL https://doi.org/10.1007/s10915-022-01939-z.

[14] George Em Karniadakis, Ioannis G. Kevrekidis, Lu Lu, Paris Perdikaris, Sifan Wang, and Liu Yang. Physics-informed machine learning. *Nature Reviews Physics*, 3(6):422–440, June 2021. ISSN 2522-5820. doi: 10.1038/s42254-021-00314-5. URL https://www.nature.com/articles/s42254-021-00314-5. Number: 6 Publisher: Nature Publishing Group.

[15] M. Raissi, P. Perdikaris, and G. E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378:686–707, 2 2019. ISSN 0021-9991. doi: 10.1016/J.JCP.2018.10.045.

[16] Shashank Reddy Vadyala, Sai Nethra Betgeri, John C. Matthews, and Elizabeth Matthews. A review of physics-based machine learning in civil engineering. *Results in Engineering*, 13: 100316, March 2022. ISSN 2590-1230. doi: 10.1016/j.rineng.2021.100316. URL https://www.sciencedirect.com/science/article/pii/S2590123021001171.

[17] Weikun Deng, Khanh T. P. Nguyen, Kamal Medjaher, Christian Gogu, and Jérôme Morio. Physics-informed machine learning in prognostics and health management: State of the art and challenges. *Applied Mathematical Modelling*, 124:325–352, December 2023. ISSN 0307-904X. doi: 10.1016/j.apm.2023.07.011. URL https://www.sciencedirect.com/science/article/pii/S0307904X23003086.

[18] Mario Lino, Stathi Fotiadis, Anil A. Bharath, and Chris D. Cantwell. Current and emerging deep-learning methods for the simulation of fluid dynamics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 479(2275):20230058, July 2023. doi: 10.1098/rspa.2023.0058. URL https://royalsocietypublishing.org/doi/full/10.1098/rspa.2023.0058. Publisher: Royal Society.

[19] Z. O. U. JingBo, L. I. U. Cai, and Zhao PengFei. Research progress of physics-informed neural network in seismic wave modeling. *Progress in Geophysics*, 38(1):430–448, February 2023. ISSN 1004-2903. doi: 10.6038/pg2023GG0142. URL http://www.progeophys.cn//en/article/doi/10.6038/pg2023GG0142. Publisher: Progress in Geophysics.

[20] Tamara G. Grossmann, Urszula Julia Komorowska, Jonas Latz, and Carola-Bibiane Schönlieb. Can Physics-Informed Neural Networks beat the Finite Element Method?, February 2023. URL http://arxiv.org/abs/2302.04107. arXiv:2302.04107 [cs, math].

[21] Nick McGreivy and Ammar Hakim. Weak baselines and reporting biases lead to overoptimism in machine learning for fluid-related partial differential equations, July 2024. URL http://arxiv.org/abs/2407.07218. arXiv:2407.07218 [physics].

[22] Ehsan Haghighat, Maziar Raissi, Adrian Moure, Hector Gomez, and Ruben Juanes. A physics-informed deep learning framework for inversion and surrogate modeling in solid mechanics. *Computer Methods in Applied Mechanics and Engineering*, 379:113741, June 2021. ISSN 0045-7825. doi: 10.1016/j.cma.2021.113741. URL https://www.sciencedirect.com/science/article/pii/S0045782521000773.

[23] Maziar Raissi, Alireza Yazdani, and George Em Karniadakis. Hidden fluid mechanics: Learning velocity and pressure fields from flow visualizations. *Science*, 367(6481):1026–1030, February 2020. doi: 10.1126/science.aaw4741. URL https://www.science.org/doi/full/10.1126/science.aaw4741. Publisher: American Association for the Advancement of Science.

[24] Zhongkai Hao, Songming Liu, Yichi Zhang, Chengyang Ying, Yao Feng, Hang Su, and Jun Zhu. Physics-Informed Machine Learning: A Survey on Problems, Methods and Applications, March 2023. URL http://arxiv.org/abs/2211.08064. arXiv:2211.08064 [cs, math].

[25] Gunter Röth and Albert Tarantola. Neural networks and inversion of seismic data. *Journal of Geophysical Research: Solid Earth*, 99(B4):6753–6768, 1994. ISSN 2156-2202. doi: 10.1029/93JB01563. URL https://onlinelibrary.wiley.com/doi/abs/10.1029/93JB01563. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1029/93JB01563.

[26] Albert Tarantola. *Inverse Problem Theory and Methods for Model Parameter Estimation*. Society for Industrial and Applied Mathematics, 2005. doi: 10.1137/1.9780898717921. URL https://epubs.siam.org/doi/abs/10.1137/1.9780898717921.

[27] Elias Galiounas, Tom G. Tranter, Rhodri E. Owen, James B. Robinson, Paul R. Shearing, and Dan J. L. Brett. Battery state-of-charge estimation using machine learning analysis of ultrasonic signatures. *Energy and AI*, 10:100188, November 2022. ISSN 2666-5468. doi: 10.1016/j.egyai.2022.100188. URL https://www.sciencedirect.com/science/article/pii/S2666546822000374.

[28] Pu Ren, Chengping Rao, Su Chen, Jian-Xun Wang, Hao Sun, and Yang Liu. SeismicNet: Physics-informed neural networks for seismic wave modeling in semi-infinite domain. *Computer Physics Communications*, 295:109010, February 2024. ISSN 0010-4655. doi: 10.1016/j.cpc.2023.109010. URL https://www.sciencedirect.com/science/article/pii/S0010465523003557.

[29] Michael T. McCann, Kyong Hwan Jin, and Michael Unser. Convolutional Neural Networks for Inverse Problems in Imaging: A Review. *IEEE Signal Processing Magazine*, 34(6):85–95, November 2017. ISSN 1558-0792. doi: 10.1109/MSP.2017.2739299. URL https://ieeexplore.ieee.org/document/8103129. Conference Name: IEEE Signal Processing Magazine.

[30] B. Moseley. *Physics-informed machine learning: from concepts to real-world applications*. PhD thesis, University of Oxford, 2022. URL https://ora.ox.ac.uk/objects/uuid:b790477c-771f-4926-99c6-d2f9d248cb23.

[31] S. Alkhadhr and M. Almekkawy. Wave Equation Modeling via Physics-Informed Neural Networks: Models of Soft and Hard Constraints for Initial and Boundary Conditions. *Sensors*, 23(5), 2023. ISSN 1424-8220. doi: 10.3390/s23052792.

[32] José Carcione. *Wave Propagation in Anisotropic, Anelastic, Porous and Electromagnetic Media*. 2002.

[33] Benjamin Moseley, Andrew Markham, and Tarje Nissen-Meyer. Fast approximate simulation of seismic waves with deep learning. July 2018. URL http://arxiv.org/abs/1807.06873.

[34] Fanny Lehmann, Filippo Gatti, Michaël Bertin, and Didier Clouteau. Fourier Neural Operator Surrogate Model to Predict 3D Seismic Waves Propagation. pages 297–310, 2023. doi: 10.7712/120223.10339.20362. URL http://arxiv.org/abs/2304.10242. arXiv:2304.10242 [physics].

[35] Peter Moczo, Jozef Kristek, and Martin Gális. *The Finite-Difference Modelling of Earthquake Motions : Waves and Ruptures*. 2014. ISBN 9781107028814.

[36] Raul Madariaga. Dynamics of an expanding circular fault. *Bulletin of the Seismological Society of America*, 66(3):639–666, June 1976. ISSN 0037-1106. doi: 10.1785/BSSA0660030639. URL `https://doi.org/10.1785/BSSA0660030639`.

[37] Jean Virieux. p-sv wave propagation in heterogeneous media: Velocity-stress finite-difference method, 1986. URL `http://library.seg.org/`.

[38] Hongyu Zhou, Yang Liu, and Jing Wang. Acoustic finite-difference modeling beyond conventi-onal courant-friedrichs-lewy stability limit: Approach based on variable-length temporal and spatial operators. *Earthquake Science*, 34:123–236, 2021. doi: 10.29382/eqs-2021.

[39] Siqin Liu, Zhusheng Zhou, and Weizu Zeng. Simulation of Elastic Wave Propagation Based on Meshless Generalized Finite Difference Method with Uniform Random Nodes and Damping Boundary Condition. *Applied Sciences (Switzerland)*, 13(3), February 2023. doi: 10.3390/app13031312. Publisher: MDPI.

[40] Hans Petter Langtangen and Anders Logg. Solving pdes in python the fenics tutorial i, 2016. URL `http://www.springer.com/series/13548`.

[41] Oliver Sander. *DUNE — The Distributed and Unified Numerics Environment*, volume 140 of *Lecture Notes in Computational Science and Engineering*. Springer International Publishing, Cham, 2020. ISBN 978-3-030-59701-6 978-3-030-59702-3. doi: 10.1007/978-3-030-59702-3. URL `http://link.springer.com/10.1007/978-3-030-59702-3`.

[42] Dimitri Komatitsch, Jeroen Tromp, Hom Nath Gharti, Daniel Peter, Eduardo Valero Cano, Etienne Bachmann, Alexis Bottero, Quentin Brissaud, Bryant Chow, Paul Cristini, Congyue Cui, Rene Gassmoeller, Michael Gineste, Felix Halpaap, Eric Heien, Jesus Labarta, Matthieu Lefebvre, Nicolas Le Goff, Pieyre Le Loher, Qiancheng Liu, Qinya Liu, Youshan Liu, Zhaolun Liu, David Luet, Roland Martin, Rene Matzen, Ryan Modrak, Christina Morency, Masaru Nagaso, Eric Rosenkrantz, Herurisa Rusmanugroho, Elliott Sales de Andrade, Carl Tape, Jean-Pierre Vilotte, Zhinan Xie, and Zhendong Zhang. Specfem/specfem2d: Specfem2d v8.1.0, December 2023. URL `https://doi.org/10.5281/zenodo.10415228`.

[43] Dimitri Komatitsch, Jeroen Tromp, Rahul Garg, Hom Nath Gharti, Masaru Nagaso, Elif Oral, Daniel Peter, Michael Afanasiev, Rafael Almada, Jean-Paul Ampuero, Etienne Bachmann, Kangchen Bai, Piero Basini, Stephen Beller, Jordan Bishop, Francois Bissey, Celine Blitz, Alexis Bottero, Ebru Bozdag, Emanuele Casarotti, Joseph Charles, Min Chen, Paul Cristini, Clement Durochat, Percy Galvez Barron, Rene Gassmoeller, Dominik Goeddeke, Leopold Grinberg, Aakash Gupta, Eric Heien, Vala Hjoerleifsdottir, Foivos Karakostas, Sue Kientz, Jesus Labarta, Nicolas Le Goff, Pieyre Le Loher, Matthieu Lefebvre, Qinya Liu, Youshan Liu, David Luet, Yang Luo, Alessia Maggi, Federica Magnoni, Roland Martin, Rene Matzen, G. D. McBain, Dennis McRitchie, Matthias Meschede, Peter Messmer, David Michea, David Miller, Ryan Modrak, Vadim Monteiller, Christina Morency, Surendra Nadh Somala, Tarje Nissen-Meyer, Kevin Pouget, Max Rietmann, Elliott Sales de Andrade, Brian Savage, Bernhard Schuberth, Anne Sieminski, James Smith, Leif Strand, Carl Tape, Eduardo Valero Cano, Brice Videau, Jean-Pierre Vilotte, Huihui Weng, Zhinan Xie, Chang-Hua Zhang, and Hejun Zhu. Specfem/specfem3d: Specfem3d v4.1.1, March 2024. URL `https://doi.org/10.5281/zenodo.10823181`.

[44] Dimitri Komatitsch and Roland Martin. An unsplit convolutional perfectly matched layer improved at grazing incidence for the seismic wave equation. *GEOPHYSICS*, 72(5):SM155–

SM167, September 2007. ISSN 0016-8033. doi: 10.1190/1.2757586. URL https://library.seg.org/doi/10.1190/1.2757586. Publisher: Society of Exploration Geophysicists.

[45] Caesar Saloma. Computational complexity and the observation of physical signals. *Journal of Applied Physics*, 74(9):5314–5319, November 1993. ISSN 0021-8979. doi: 10.1063/1.354232. URL https://doi.org/10.1063/1.354232.

[46] Kurt Hornik. Approximation capabilities of multilayer feedforward networks. *Neural Networks*, 4(2):251–257, January 1991. ISSN 0893-6080. doi: 10.1016/0893-6080(91)90009-T. URL https://www.sciencedirect.com/science/article/pii/089360809190009T.

[47] Siamak Mehrkanoon, Tillmann Falck, and Johan A. K. Suykens. Approximate Solutions to Ordinary Differential Equations Using Least Squares Support Vector Machines. *IEEE Transactions on Neural Networks and Learning Systems*, 23(9):1356–1367, September 2012. ISSN 2162-2388. doi: 10.1109/TNNLS.2012.2202126. URL https://ieeexplore.ieee.org/abstract/document/6224185. Conference Name: IEEE Transactions on Neural Networks and Learning Systems.

[48] Siamak Mehrkanoon and Johan A. K. Suykens. Learning solutions to partial differential equations using LS-SVM. *Neurocomputing*, 159:105–116, July 2015. ISSN 0925-2312. doi: 10.1016/j.neucom.2015.02.013. URL https://www.sciencedirect.com/science/article/pii/S0925231215001629.

[49] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *Nature*, 521(7553):436–444, May 2015. ISSN 1476-4687. doi: 10.1038/nature14539. URL https://doi.org/10.1038/nature14539.

[50] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, November 2016. ISBN 978-0-262-33737-3. Google-Books-ID: omivDQAAQBAJ.

[51] Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli, Burigede Liu, Kaushik Bhattacharya, Andrew Stuart, and Anima Anandkumar. Neural Operator: Graph Kernel Network for Partial Differential Equations, March 2020. URL http://arxiv.org/abs/2003.03485. arXiv:2003.03485 [cs, math, stat].

[52] Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli, Burigede Liu, Kaushik Bhattacharya, Andrew Stuart, and Anima Anandkumar. Fourier Neural Operator for Parametric Partial Differential Equations, May 2021. URL http://arxiv.org/abs/2010.08895. arXiv:2010.08895 [cs, math].

[53] Maziar Raissi and George Em Karniadakis. Hidden physics models: Machine learning of nonlinear partial differential equations. *Journal of Computational Physics*, 357:125–141, March 2018. ISSN 0021-9991. doi: 10.1016/j.jcp.2017.11.039. URL https://www.sciencedirect.com/science/article/pii/S0021999117309014.

[54] Maziar Raissi, Paris Perdikaris, and George Em Karniadakis. Numerical Gaussian Processes for Time-Dependent and Nonlinear Partial Differential Equations. *SIAM Journal on Scientific Computing*, 40(1):A172–A198, January 2018. ISSN 1064-8275. doi: 10.1137/17M1120762. URL https://epubs.siam.org/doi/abs/10.1137/17M1120762. Publisher: Society for Industrial and Applied Mathematics.

[55] I.E. Lagaris, A. Likas, and D.I. Fotiadis. Artificial neural networks for solving ordinary and partial differential equations. *IEEE Transactions on Neural Networks*, 9(5):987–1000, September 1998. ISSN 1941-0093. doi: 10.1109/72.712178. URL https://ieeexplore.ieee.org/document/712178. Conference Name: IEEE Transactions on Neural Networks.

[56] Jan Blechschmidt and Oliver G. Ernst. Three ways to solve partial differential equations with neural networks — A review. *GAMM-Mitteilungen*, 44(2): e202100006, 2021. ISSN 1522-2608. doi: 10.1002/gamm.202100006. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/gamm.202100006. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/gamm.202100006.

[57] E. Kharazmi, Z. Zhang, and G. E. Karniadakis. Variational Physics-Informed Neural Networks For Solving Partial Differential Equations, November 2019. URL http://arxiv.org/abs/1912.00873. arXiv:1912.00873 [physics, stat].

[58] Feiyu Chen, David Sondak, Pavlos Protopapas, Marios Mattheakis, Shuheng Liu, Devansh Agarwal, and Marco Di Giovanni. Neurodiffeq: A python package for solving differential equations with neural networks. *Journal of Open Source Software*, 5(46):1931, 2020.

[59] Lu Lu, Xuhui Meng, Zhiping Mao, and George Em Karniadakis. DeepXDE: A deep learning library for solving differential equations. *SIAM Review*, 63(1):208–228, 2021. doi: 10.1137/19M1274067.

[60] Kirill Zubov, Zoe McCarthy, Yingbo Ma, Francesco Calisto, Valerio Pagliarino, Simone Azeglio, Luca Bottero, Emmanuel Luján, Valentin Sulzer, Ashutosh Bharambe, Nand Vinchhi, Kaushik Balakrishnan, Devesh Upadhyay, and Chris Rackauckas. Neuralpde: Automating physics-informed neural networks (pinns) with error approximations, 2021. URL https://arxiv.org/abs/2107.09443.

[61] Reza Akbarian Bafghi and Maziar Raissi. PINNs-Torch: Enhancing Speed and Usability of Physics-Informed Neural Networks with PyTorch. October 2023. URL https://openreview.net/forum?id=nl1ZzdHpab.

[62] Chao Song and Yanghua Wang. High-frequency wavefield extrapolation using the Fourier neural operator. *Journal of Geophysics and Engineering*, 19(2):269–282, April 2022. ISSN 1742-2132. doi: 10.1093/jge/gxac016. URL https://doi.org/10.1093/jge/gxac016.