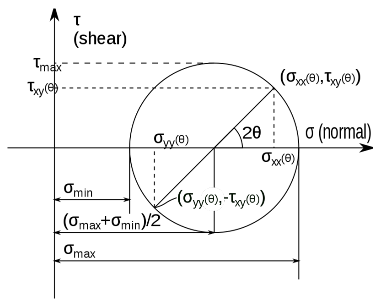
Análisis de deformaciones

- $\vec{r}$: vector de posición original.
- $\vec{\rho}$: vector de posición final.
- A: matriz de transformación.
- D: matriz de transformación de desplazamientos.
- Para $D = \varepsilon + \omega$
- ε-tamaño y
- ω-orientación
- $\vec{\Delta}$: vector de desplazamientos.
- J: Tensor gradiente de desplazamientos.
- Para $J = \varepsilon + \omega$
- ε-Componente simétrica y
- ω-Componente anti-simétrica.

Circulo de Mohr

$$C = \left(\frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}, 0\right)$$

$$r = \sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right)^2 + \bar{\gamma}^2}$$



$$R = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

Transformaciones lineales

$$\vec{\Delta} = D \cdot \vec{r}$$

Caso 2D

$$\vec{\Delta} = \begin{Bmatrix} \Delta_x \\ \Delta_y \end{Bmatrix} \qquad D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{bmatrix} \qquad \vec{r} = \begin{Bmatrix} r_x \\ r_y \end{Bmatrix}$$

$$D = \varepsilon + \omega = pI + S + C + \omega$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \bar{\gamma} \\ \bar{\gamma} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} & 0 \\ 0 & \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} \end{bmatrix} + \begin{bmatrix} \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2} & 0 \\ 0 & \frac{\varepsilon_{yy}-\varepsilon_{xx}}{2} \end{bmatrix} + \begin{bmatrix} 0 & \bar{\gamma} \\ \bar{\gamma} & 0 \end{bmatrix} \qquad \omega = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix}$$

Tensor de deformaciones unitarias

$$J = \varepsilon + \omega$$

$$[J] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$[\varepsilon] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix} \qquad [\omega] = \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & 0 & \frac{1}{2}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) & 0 \end{bmatrix}$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{bmatrix}$$