Homework Assignment 10: Applied Probabilistic Models

Monte Carlo Simulations

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1 Introduction

In this work, Monte Carlo simulations are employed to estimate expected value in the exercises. To perform simulations, sampling distributions are considered to compare the expected values with the theoretical values obtained in the previous assignment.

The expected value of a random variable can be considered as the long-run average value of its outcomes when the number of repeated trials is large [2], and Monte Carlo simulations play an important role for this kind of experiments. An example of Monte Carlo simulation is repeating an experiment a large enough number of times to make the results practically equivalent to doing it over and over forever [1].

For the analysis, the R software is used in its version 4.0.2 [5], and the code used is available on the GitHub repository [4]. This work is run on a MacBook Air with an Intel Core i5 CPU @ 1.8 GHz and 8 GB RAM.

2 Exercises

For this work exercises that have been simulated are provided in the book Grinstead and Snell [3].

2.1 Exercise 1 page 247

For this exercise, a deck is given which consists of cards between 2 through 10. The player will win a dollar if the number of the card is odd and loses one dollar otherwise. The calculated expected value $E(X) = -\frac{1}{9}$. Then it is executed by simulations (10 000 repetitions), which yields very close values with respect to the previously calculated. In turn, other repetitions of this experiment are executed, represented in Figure 1, which gives a threshold of all the expected values around the theoretical one.

2.2 Exercise 6 page 247

In this situation, let X denote the sum of two numbers that turn up when a die is rolled twice, and Y the difference of the numbers. This exercise requires demonstrate that $E(XY) = E(X)E(Y)$. As states before, simulations are executed to validate conclusions. The experiments are executed 10 000 times and then means are calculated the same amount of times also, Figure 2 shows very similar results.

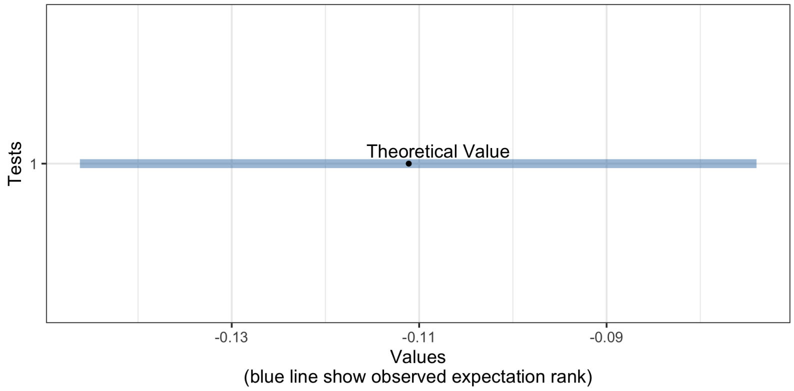

Figure 1: Expected value threshold in experiments in Exercise 1 page 247

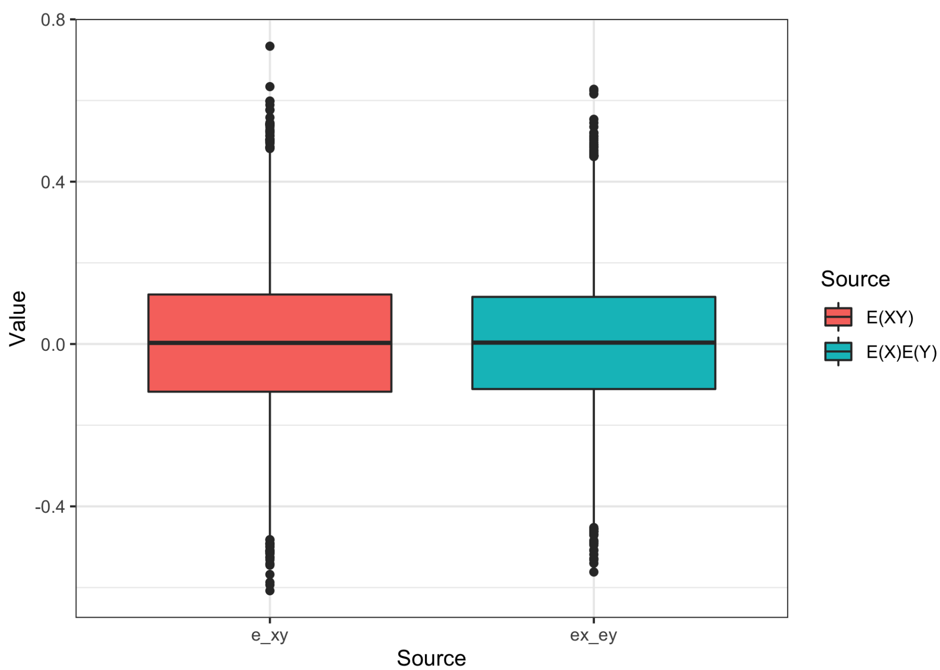

Figure 2: Boxplot of the expected values for both situations in Exercise 6 page 247

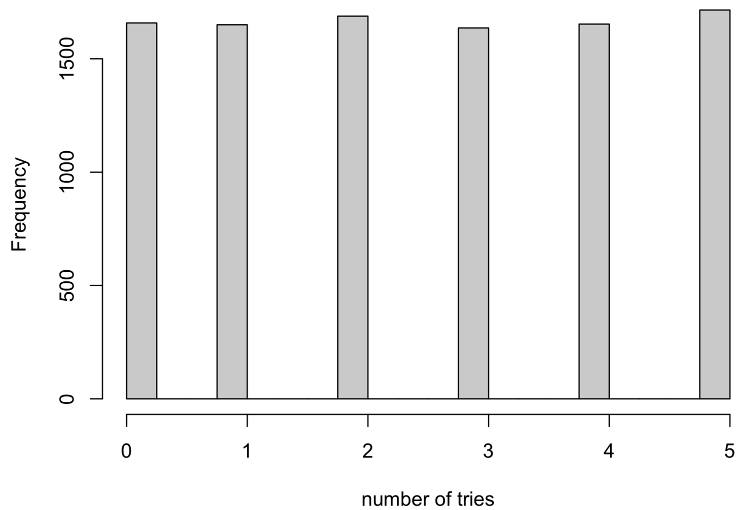

Figure 3: Histogram of the number of tries before opening the door in Exercise 18 page 249

2.3 Exercise 18 page 249

In this exercise, six similar keys are given and let X be the number of tried keys before the success of opening the door. This situation gives an expected value $E(X) = 2.5$. After computing, the sample means of 10 000 tries the experiment gives a value of 2.4818, which seems to be fairly close to the expected value. Figure 3 shows a histogram of the number of tries throughout the 10 000 repetitions of this Monte Carlo simulation. In addition, Figure 4 represents a plot that shows after the value of 1 000 repetitions the expected value begins to stabilize, which is an approach to determine how many Monte Carlo experiments are enough.

2.4 Exercise 3 page 278

For this exercise, the density function of the variable lifetime (in hours) of the ACME super light bulb is given. The calculated expected value $E(T) = 40$. The plot of the density function is generated and shown in Figure 5, which shows the highest area of the curve around the calculated expected value of 40.

2.5 Exercise 12 page 280

The variables X and Y are independent, and both are uniformly distributed on $[0, 1]$. The calculated expected value $E(X^Y) = \ln 2$; this value correspond approximately to 0.6931. Simulations are shown in the histogram of Figure 6 and Figure 7, the latter shows how the expected value begin to stabilize as the number of repetition grows around the previously calculated value.

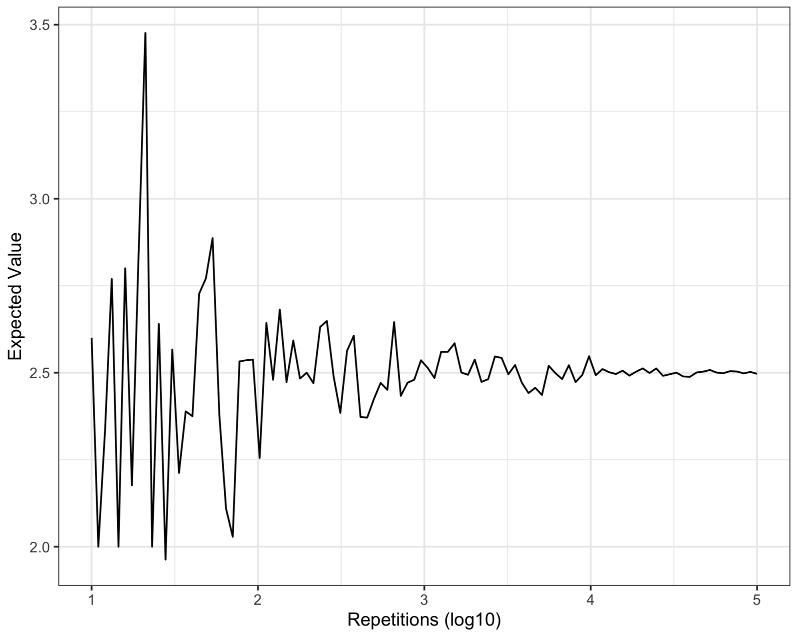

Figure 4: Stabilization of the expected value in the number of tries before opening the door in Exercise 18 page 249

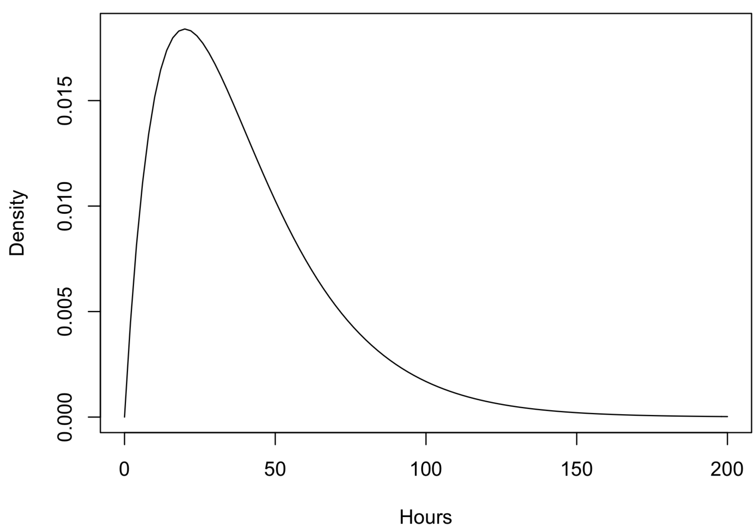

Figure 5: Density curve of the lifetime of the light bulb in Exercise 3 page 278

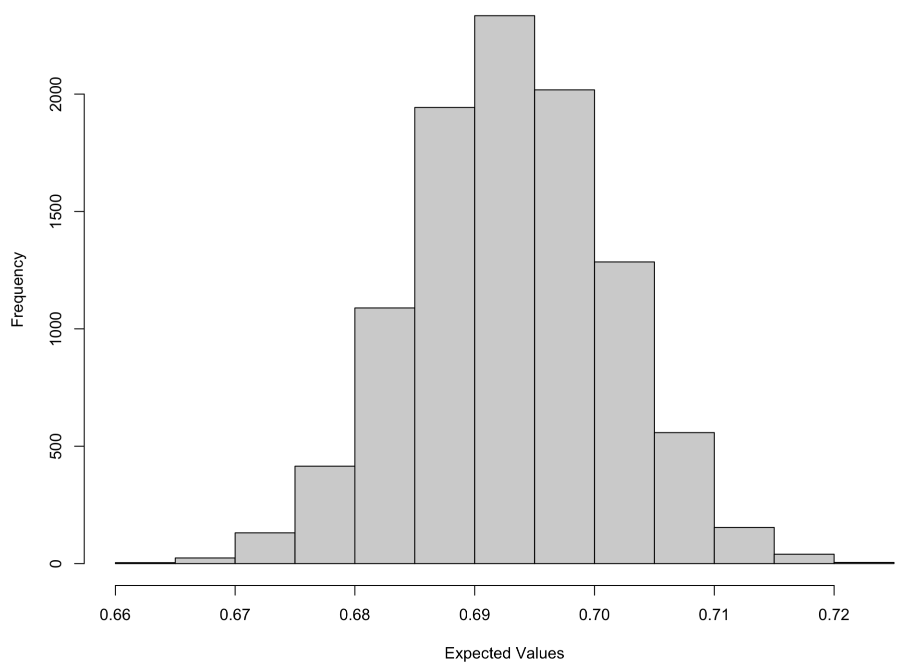

Figure 6: Histogram of the simulated expected values in Exercise 12 page 280

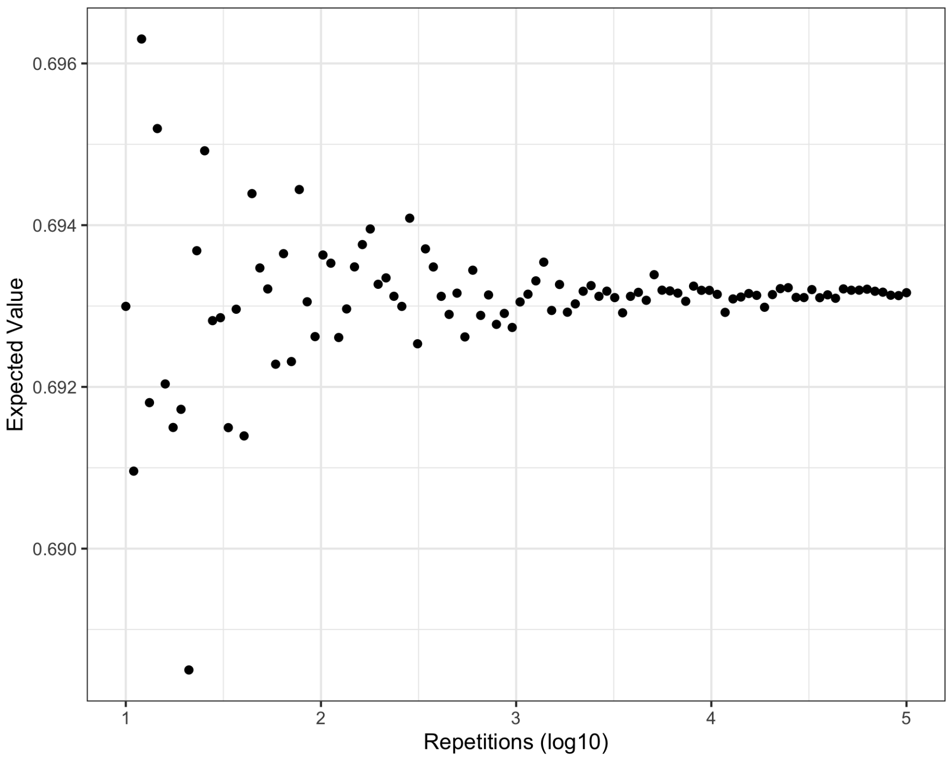

Figure 7: Stabilization of expected values in Exercise 12 page 280

References

[1] Faisal Akbar. Data science: Probability, 2019. https://rstudio-pubs-static.s3.amazonaws.com/487679_4087cbd1e9f74f8b8a4ddcd7eb0de0e6.html, Last accessed on 2020-11-8.

[2] Alexander Gerber Christoph Hanck, Martin Arnold and Martin Schmelzer. Introduction to econometrics with r, 2020. https://www.econometrics-with-r.org/index.html, Last accessed on 2020-11-5.

[3] Charles Miller Grinstead and James Laurie Snell. *Introduction to probability*. American Mathematical Soc., 2012.

[4] Oscar Alejandro Hernandez Lopez. Probability in R. https://github.com/oscaralejandro1907/probability-in-R/blob/master/assignment1/t1.R, 2020.

[5] The R Foundation. The R Project for Statistical Computing. https://www.r-project.org/, 2020.