Signals and Filtering: Overview

- Today: signals and data acquisition
  - Source of deterministic errors and random noises
  - Basics of signal spectrums and Fourier Transform
  - Nyquist theorem, modulation and aliasing
  - Basics of low pass filters

- Signal processing is at the interface of embedded software engineering and ECE.
- Objective:
  - understand the fundamental concepts
  - master issues that a software engineer should do and can do

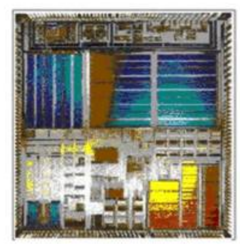
- **A basic tutorial** (written by Hagit Shatkay) on Fourier transform is posted (see lecture2_add_material)

➔ **Do not get confused:**
  - **In the lecture ω = angular velocity = 2π * frequency**
  - **In the tutorial ω = frequency**

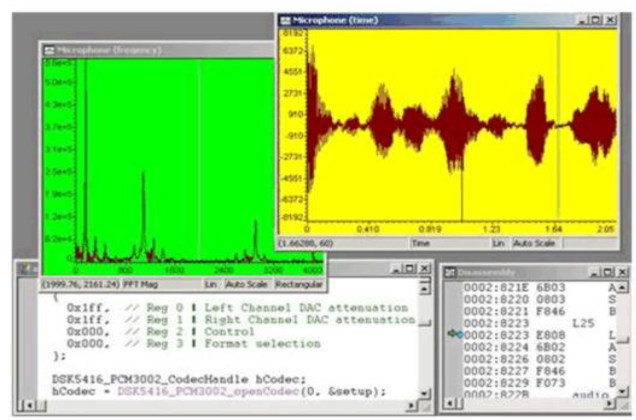
Digital Signal Processing

→ A/D → Digital Signal Processing → D/A →

analog signal
↓
digital signal



digital signal
↓
analog signal



Deterministic Errors

- **Bias** is a measurement error that remains constant in magnitude for all observations. It adds an offset to the true value. Most analog components have this problem over time.
- Solution: calibration by adjusting resistors or by compensating the bias in software. (Did you find bias in Lab 1's A/D – D/A conversion?)

- **Quantization errors**: the resolution is not fine enough for the application needs.
  - Proper configuration of voltage ranges in A/D and D/A (Do you still remember how to do it?)
  - Standard A/D and D/A card gives 12 bit over a given range. However, higher resolution ones are also available.

- **Aliasing**: refers to an effect that causes different signals to become indistinguishable (or *aliases* of one another) when sampled (to be discussed later)

Random Noises

- Noise is the electrical signal that you do not want.

- Internal in the electronics of the A/D - D/A card. It is usually very small.
- Environmental noise :
  - Noise from equipment power transformer. Switching transformers are cheap. The very inexpensive ones are quite noisy. Linear transform is heavy, expensive but quiet.
  - Noise from the power line due to power tools, elevators etc. Buy line voltage conditioner or move to another place to do your work.
  - EMI: electrical shielding and/or using differential mode.
    - Do you still remember what is differential mode inputs?

- Poor grounding is a common source of noise problems. Before trying anything fancy, make sure that your equipment is properly grounded.

Ground

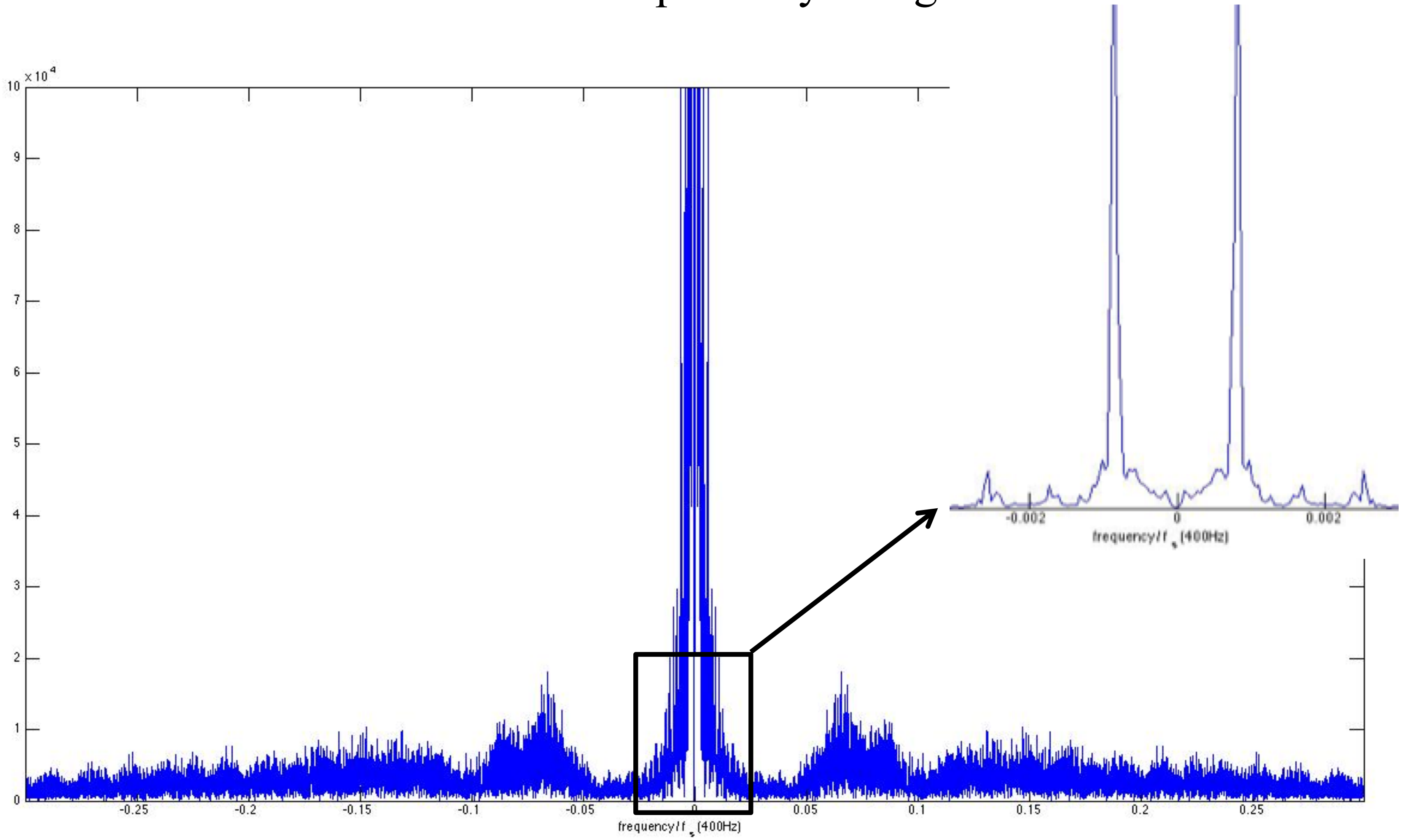
- Electrical ground is the common reference point. Thus, sometime it is also labeled as COMMON. Then there is also the GROUND, the voltage level of the earth.

- **Best Practice**
  - connect the signal grounds together and then directly connect it to earth ground via a good GROUND connection.
  - connect the power grounds together and then directly connect it to the earth ground via a separate GROUND connection.

  - Connecting power ground and signal ground together and then run a long wire to the GROUND is not a good idea. The resistance in the ground wire would allow noise in the power equipment to interfere the signal measurement. This is acceptable only if the power equipment is not very noisy.

Why are we discussing signal processing?
is the touchscreen noisy?

Sampling frequency: 400 Hz. The ball was oscillating along x-axis with period ~2.9675 s. Fast Fourier Transform was computed by using Matlab.

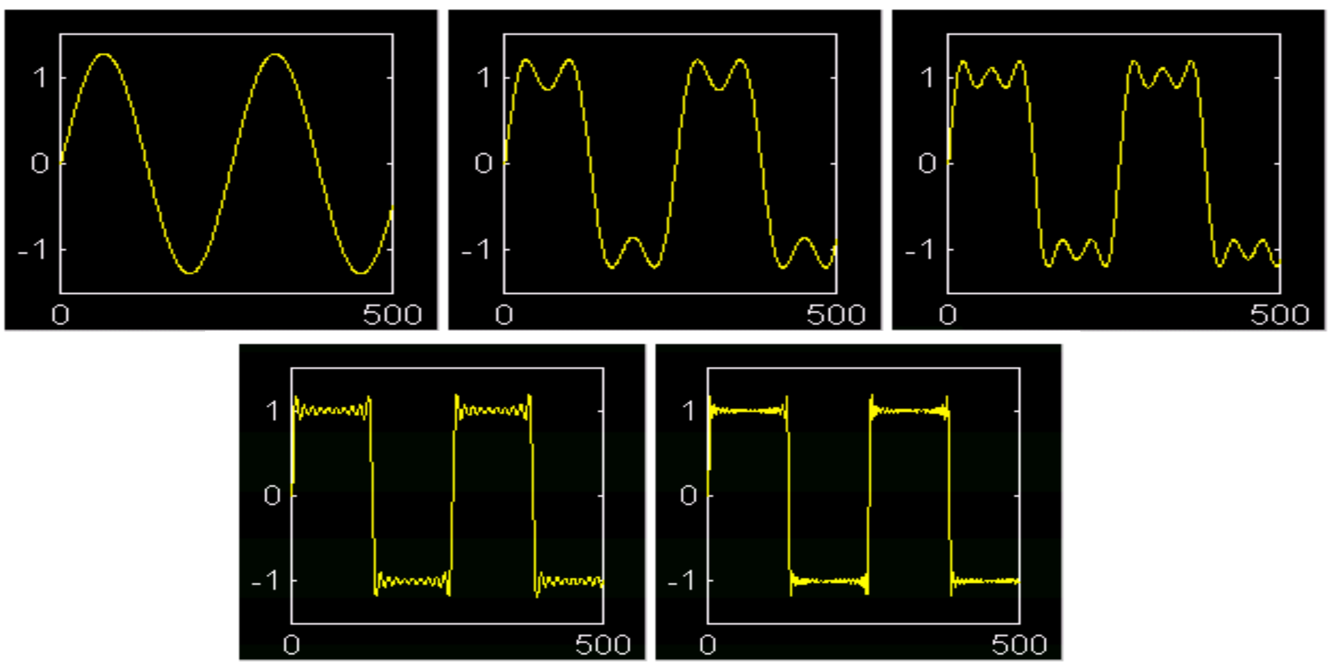


Basic Signal Processing Concepts: Fourier Series

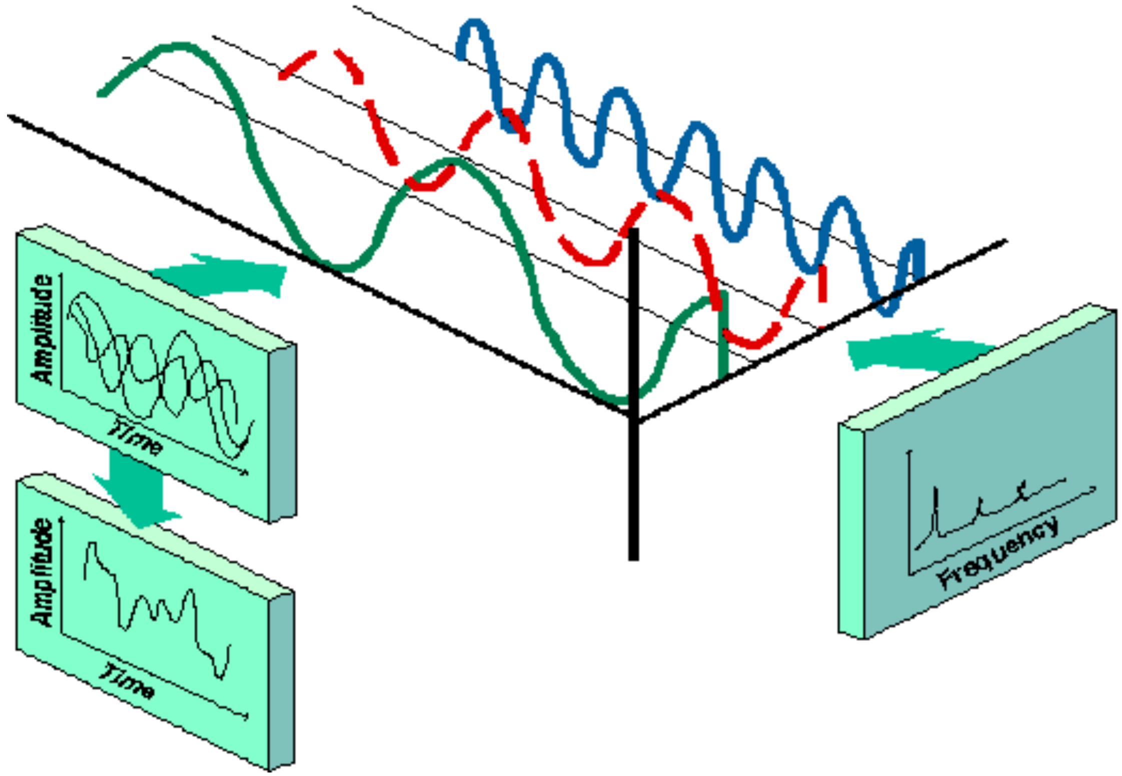
•Fourier discovered that a signal can be decomposed into sum of sinusoids.

•According to the Fourier series, a periodic signal x(t), whose period is $T_0=1/f_0$, can be expressed as:

$$x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos\left(\frac{2\pi nt}{T_0}\right) + \sum_{n=1}^{+\infty} b_n \sin\left(\frac{2\pi nt}{T_0}\right)$$



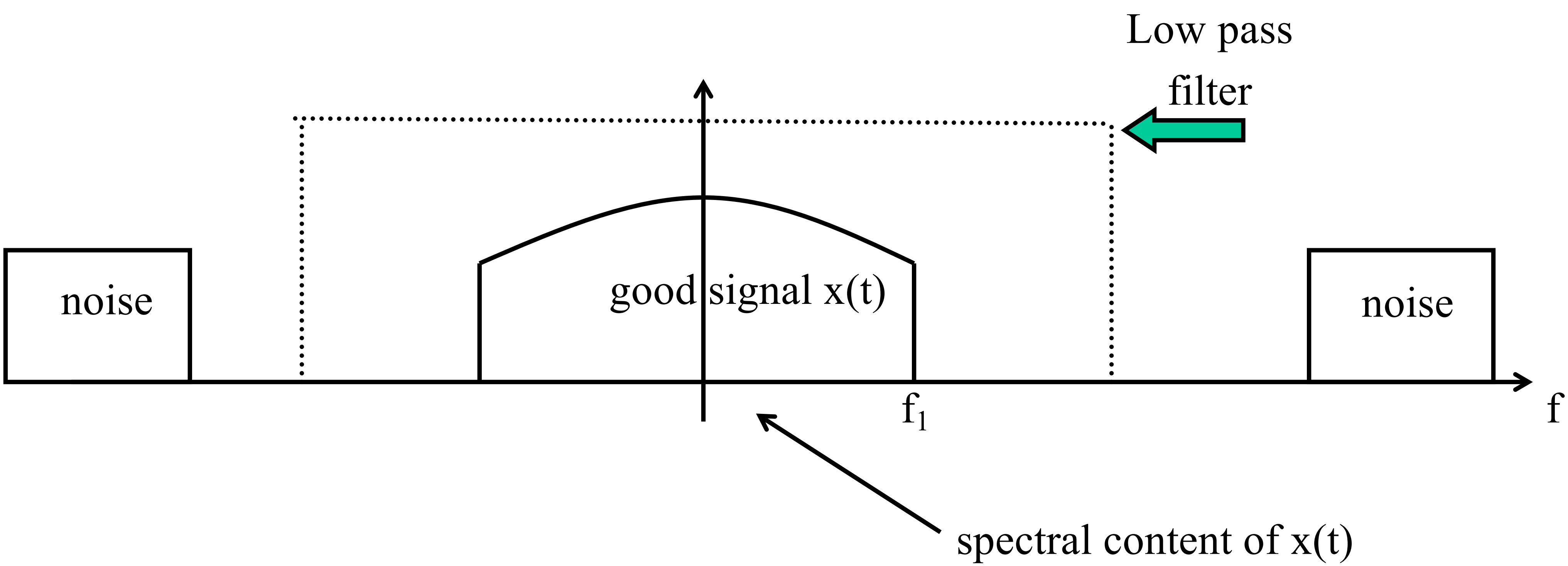
Basic Signal Processing Concepts: Fourier Series



$$x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos\left(\frac{2\pi nt}{T_0}\right) + \sum_{n=1}^{+\infty} b_n \sin\left(\frac{2\pi nt}{T_0}\right)$$

Good signal vs. Noise

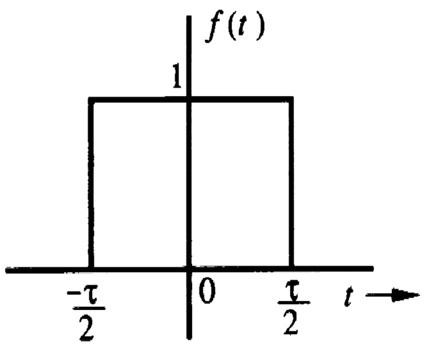
- Let's assume we have an input signal x(t) along with high frequency noise. We would like to filter the noise preserving the good signal!

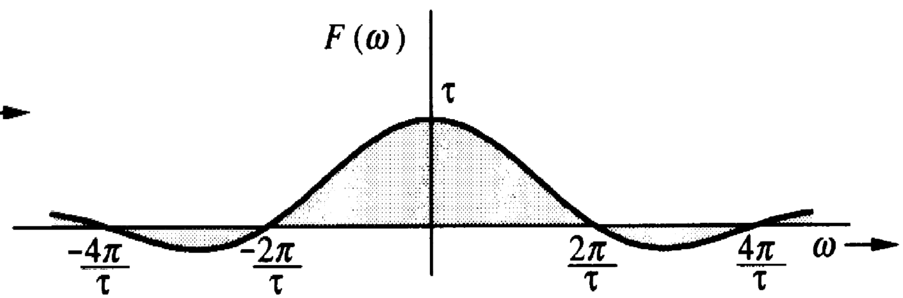


Basic Signal Processing Concepts: Continuous Fourier Transform

- An aperiodic signal x(t) with finite energy can be represented by its continuous fourier transform.

$$\int \left| x(t) \right| dt < +\infty$$





Key observation: the narrower the pulse, the wider is the frequency range due to the role played by pulse width in the transformation.

Basic Signal Processing Concepts: Continuous Fourier Transform

- continuous fourier transform.

$$x(t) \overset{FT}{\Leftrightarrow} X(\omega)$$

Frequency domain!

ω=2πf

Fourier transform ➔

$$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$$

Inverse transform ➔

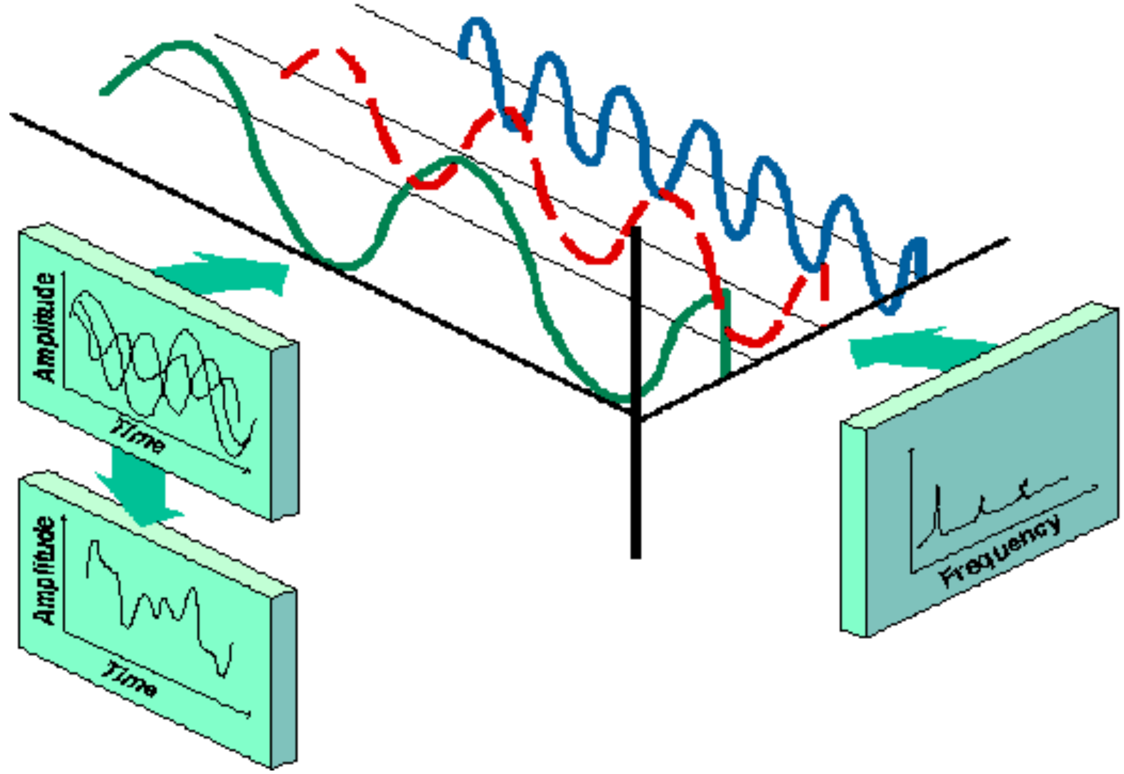
$$\mathcal{F}^{-1}[X(\omega)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega$$

Time domain!

Basic Signal Processing Concepts: Continuous Fourier Transform

- {x(t),X(ω)} are a Fourier transform pair. As they are uniquely constructable from each other, they must both encode the same information, but in different domains. X(ω) expresses the frequency content of x(t); that is, its spectrum.

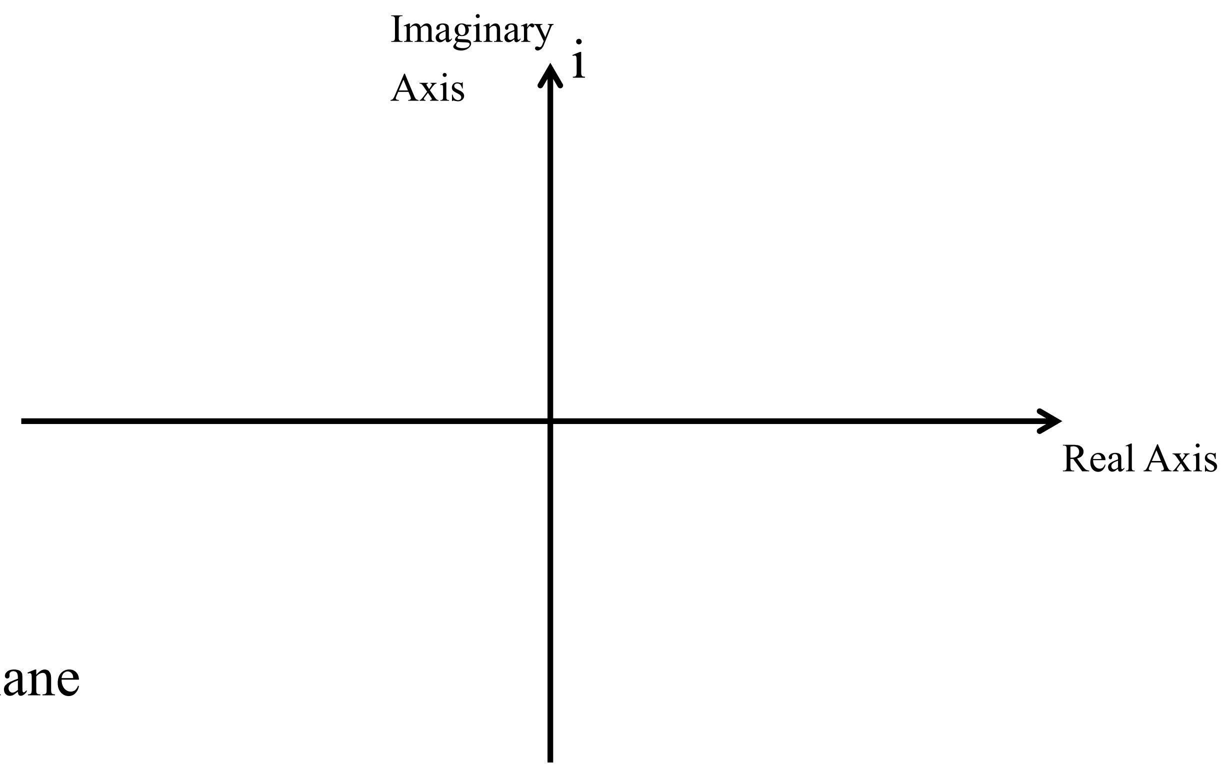
$$x(t) \overset{FT}{\Leftrightarrow} X(\omega)$$



Appendix: a graphical intuition of $e^{i\omega t}$

$\omega=2\pi f$

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$



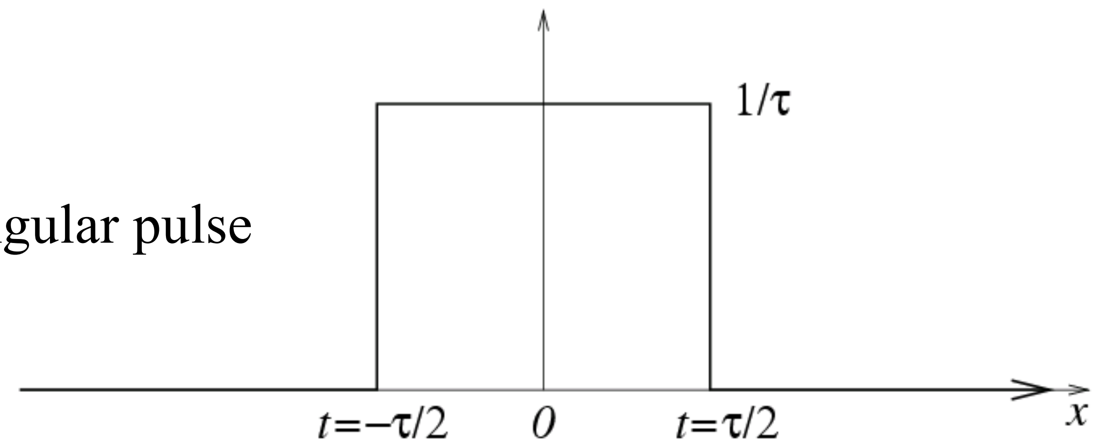
complex plane

Quiz: what is the graphical representation of $e^{i\omega t}$?

Continuous Fourier Transform: an example

- Let's analyze an example: fourier transform of PULSE.

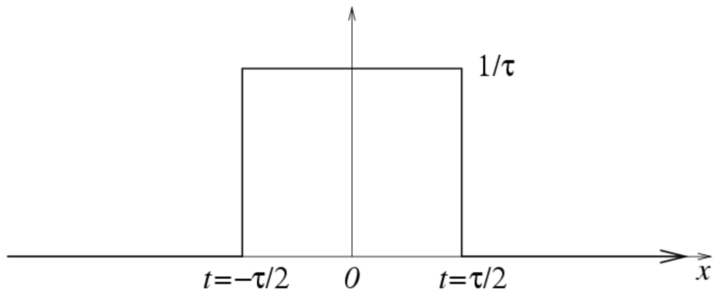
Rectangular pulse



$$P(t) = \begin{cases} \frac{1}{\tau} & \text{if} \quad |t| < \frac{\tau}{2} \\ 0 & \text{if} \quad |t| \geq \frac{\tau}{2} \end{cases}$$

The fourier transform is: $$X(\omega) = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-i\omega t}dt$$

Continuous Fourier Transform: an example



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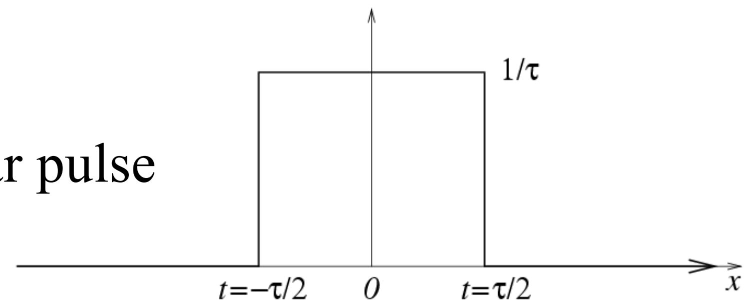
Continuous Fourier Transform: an example

Rectangular pulse



even function

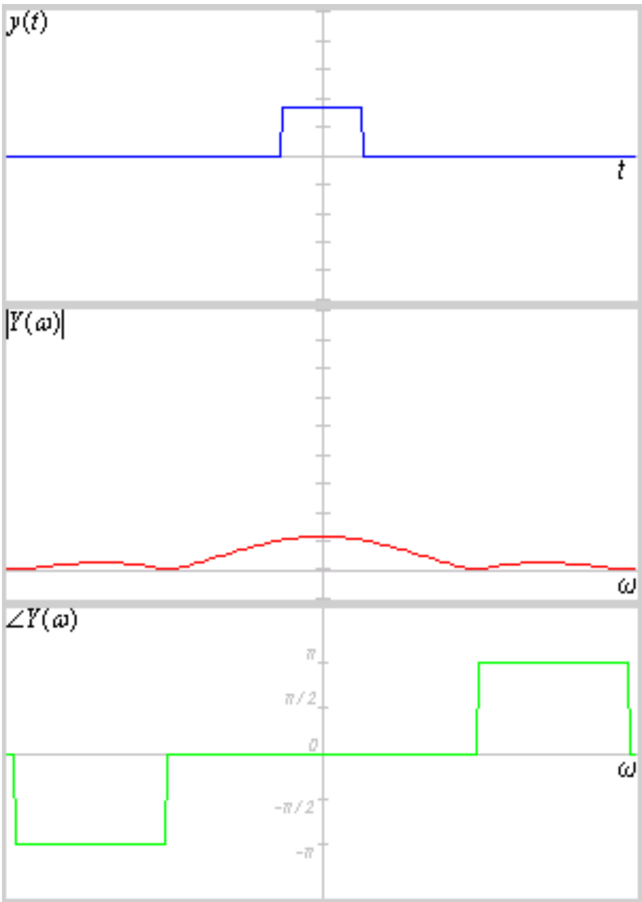
odd function

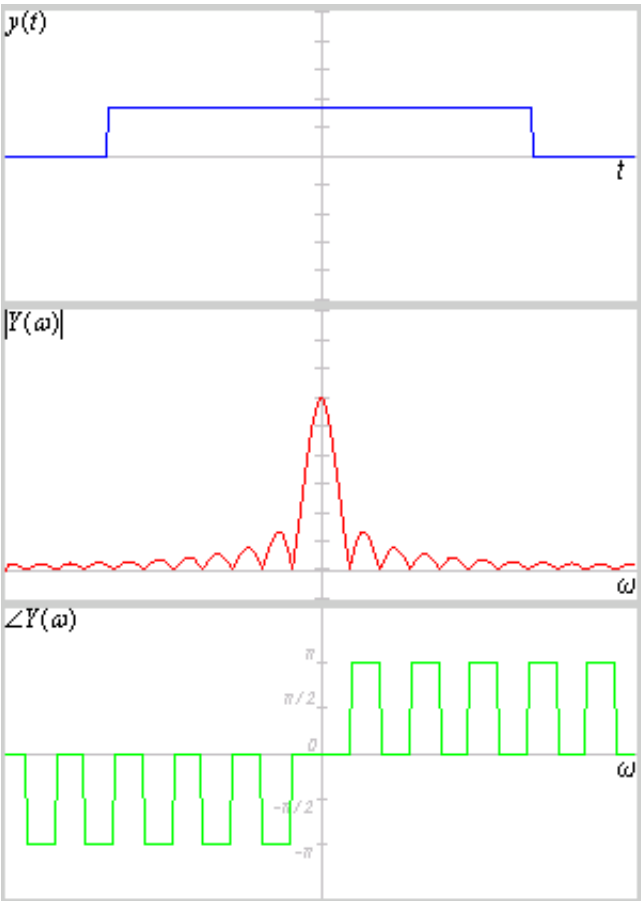
Making the following substitution: $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$

$$X(\omega) = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-i\omega t}\,dt \; = \left[\frac{\sin(\omega t)}{\omega\tau}\right]_{\frac{-\tau}{2}}^{\frac{\tau}{2}} = \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\omega\tau} - \frac{\sin\left(-\frac{\omega\tau}{2}\right)}{\omega\tau} = \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}$$

Continuous Fourier Transform: an example

Rectangular pulse x(t)





Appendix: Fourier Series

S_n are complex numbers and represent the **spectrum** of x(t)

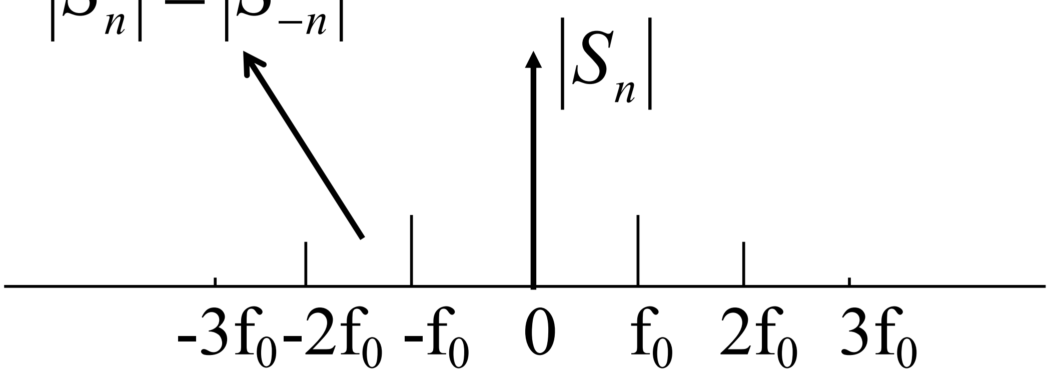
$$x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos\left(\frac{2\pi nt}{T_0}\right) + \sum_{n=1}^{+\infty} b_n \sin\left(\frac{2\pi nt}{T_0}\right) = \sum_{n=-\infty}^{+\infty} S_n \cdot e^{\frac{i2\pi nt}{T_0}}$$

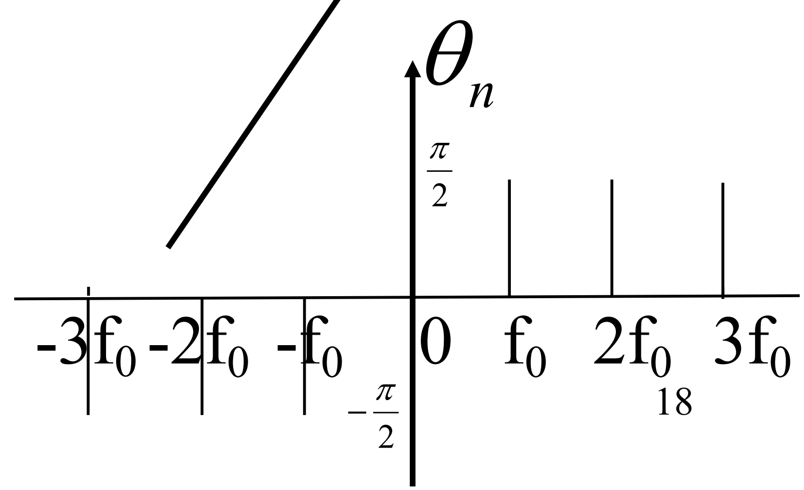
S_n completely represent the signal x(t) in the frequency domain.

$$S_n = |S_n| \cdot e^{i\theta_n} = |S_n|(\cos\theta_n + i\sin\theta_n)$$

$$\theta_n = -\theta_{-n}$$

$$|S_n| = |S_{-n}|$$

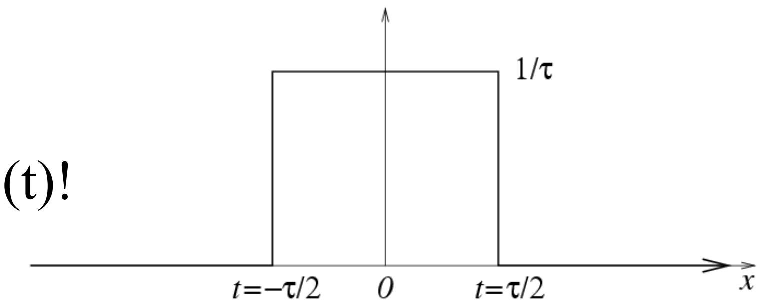




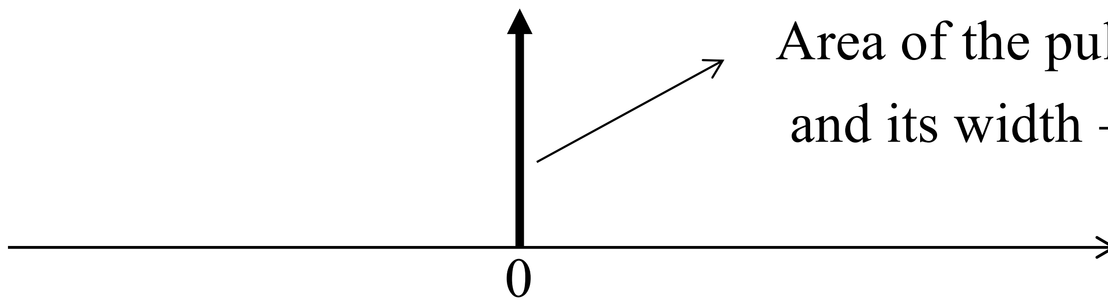
Appendix: delta function

What does happen if $\tau \to 0$?

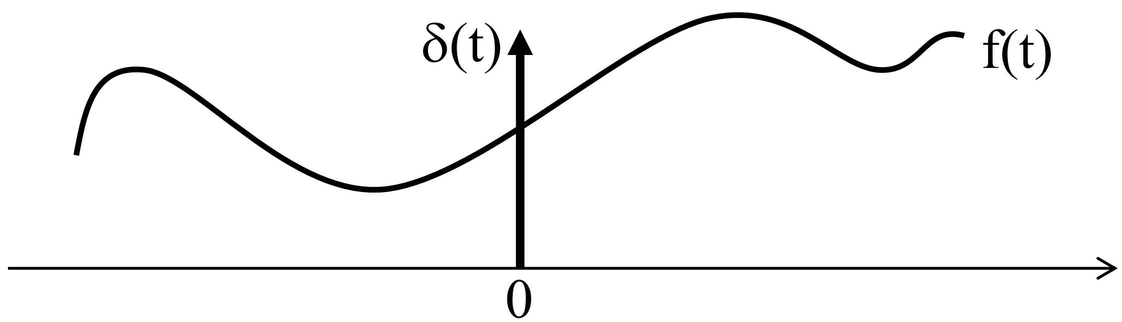
We obtain the delta function $\delta(t)$!



$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$



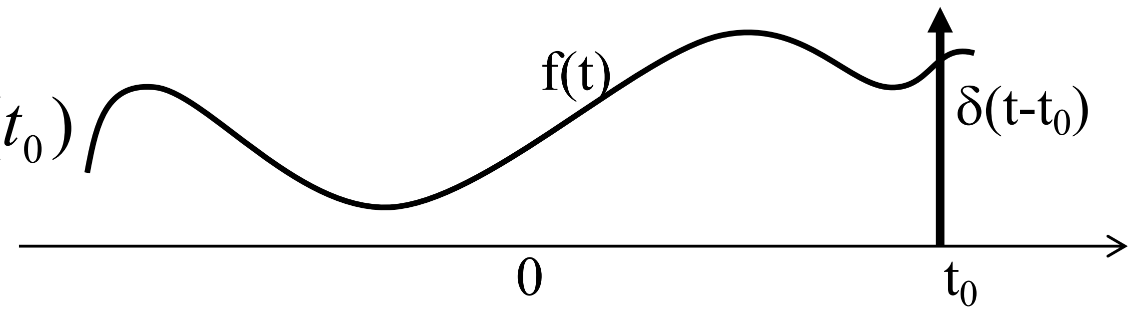
$$\int_{-\infty}^{+\infty} \delta(t) f(t) dt = f(0)$$



Appendix: delta function

$$\delta\left(t-t_0\right)$$

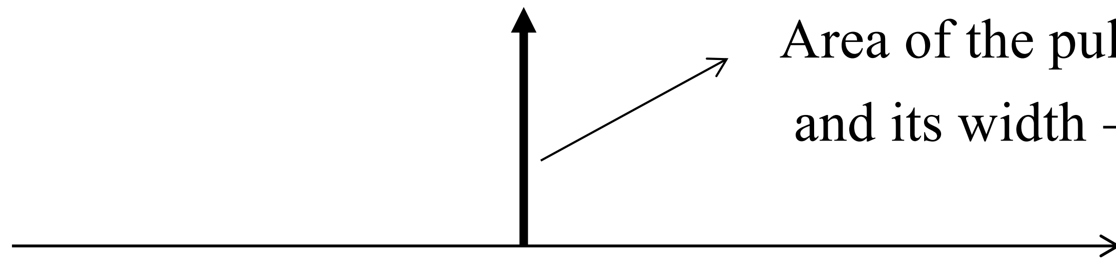
$$\int_{-\infty}^{+\infty}\delta(t-t_0)f(t)dt=f(t_0)$$



$$\delta\left(t\right)=\delta\left(-t\right)\Rightarrow\delta\left(t_0-t\right)=\delta\left(t-t_0\right)$$

Appendix: delta function

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$



Area of the pulse δ(t) is 1
and its width →0

$$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$$

Quiz: what is the FT of a delta function?

Continuous Fourier Transform of a periodic function

- After defining the continuous fourier transform specifically for non-periodic functions, what does the continuous FT of a periodic function look like?
- Let's consider the simplest possible:

$$x(t) = \sin(\Omega t)$$

The continuous fourier transform is: $$X(\omega) = \int_{-\infty}^{\infty} \sin(\Omega t) e^{-i\omega t} dt$$

$$\mathcal{F}[\sin(\Omega t)] = \frac{\pi}{i}\delta(\omega - \Omega) - \frac{\pi}{i}\delta(\omega + \Omega)$$

We derive it step by step in the appendix!!!

Continuous Fourier Transform of a periodic function

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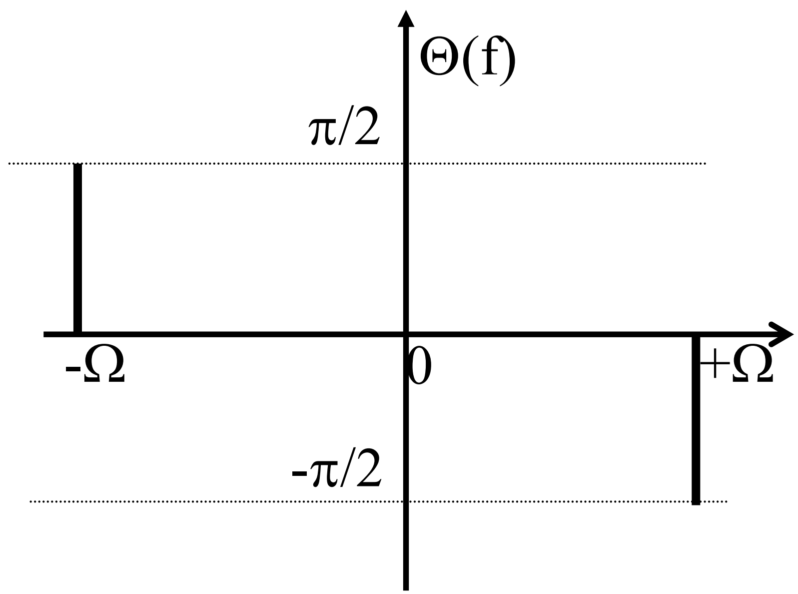
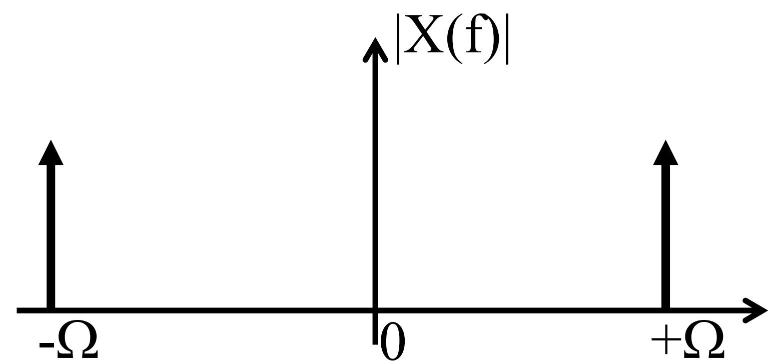
$$\mathcal{F}[\sin(\Omega t)] = \frac{\pi}{i}\delta(\omega - \Omega) - \frac{\pi}{i}\delta(\omega + \Omega)$$





Appendix: Continuous Fourier Transform of sin(Ωt)

- Having defined the continuous fourier transform specifically for non-periodic functions, what does the continuous FT of a periodic function look like?

- Let's consider the simplest possible:

$$x(t) = \sin(\Omega t)$$

The fourier transform is:

$$X(\omega) = \int_{-\infty}^{\infty} \sin(\Omega t) e^{-i\omega t} dt$$

Appendix: Continuous Fourier Transform of sin(Ωt)

- Let' s apply de Moivre' s Theorem: $\cos(n\omega t) = \frac{1}{2}(e^{in\omega t} + e^{-in\omega t})$

$$\sin(n\omega t) = \frac{1}{2i}(e^{in\omega t} - e^{-in\omega t})$$

$$X(\omega) = \int_{-\infty}^{\infty} \sin(\Omega t) e^{-i\omega t} dt \quad = \int_{-\infty}^{\infty} \frac{1}{2i}(e^{i\Omega t} - e^{-i\Omega t}) e^{-i\omega t} dt$$

$$= \frac{1}{2i}\int_{-\infty}^{\infty} e^{i(\Omega-\omega)t} dt - \frac{1}{2i}\int_{-\infty}^{\infty} e^{i(-\Omega-\omega)t} dt$$

Appendix: Continuous Fourier Transform of sin(Ωt)

- Let's apply the following eq.: $$\delta(\omega-\omega_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{i(\omega-\omega_0)t}\,dt$$

$$\frac{1}{2i}\int_{-\infty}^{\infty}e^{i(\Omega-\omega)t}\,dt-\frac{1}{2i}\int_{-\infty}^{\infty}e^{i(-\Omega-\omega)t}\,dt=\frac{\pi}{i}\delta(\Omega-\omega)-\frac{\pi}{i}\delta(-\Omega-\omega)$$

$$=\frac{\pi}{i}\delta(\omega-\Omega)-\frac{\pi}{i}\delta(\omega+\Omega)$$

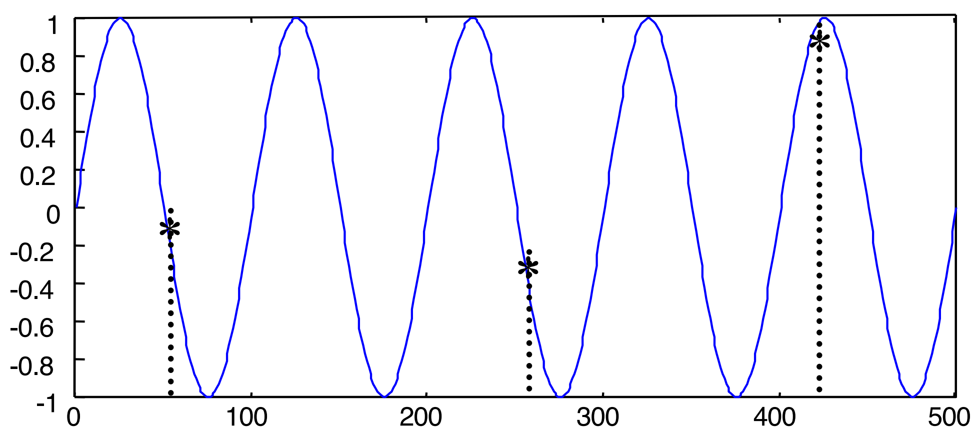
$$\delta(-t)=\delta(t)$$

Key Concepts in Signal Processing

- The term, **bandwidth**, refers to the range of frequency components of a signal that has non-negligible magnitudes with respect to the application at hand.
- The most common noises are high frequency noises and thus the popularity of low pass filters.
- Noise in the same frequency ranges as the signal can only be filtered by complex model based filtering, e.g, Kalman filters. If you have significant noises in your signal range, try to prevent them at the source (shielding, differential inputs, better transformers, proper grounding etc).
- INCORRECT SAMPLING WILL TRANSFORM NOISE THAT IS OUTSIDE OF SIGNAL FREQUENCY RANGE INTO A NEW NOISE THAT IS WITHIN THE SIGNAL FREQUENCY RANGE.

Key Concepts in Sampling

- Nyquist showed that to correctly capture an analog signal digitally, the sampling rate must be at least twice F_{max}, where F_{max} is the **highest** frequency of the original signal.
- This 2 times result assumes perfection in the sampling and filtering process. You won't have it in practice. So the practical rule of thumb is at least 3 times.
- Aliasing. If you sample too slow, the high frequency components will become irregular noise at the sampling frequency. They are noises that are in the same frequency range of your signal!!!



Applications of Nyquist Theorem - 1

What signal is in the eyes of the beholder. Nyquist was interested in digital samples that capture all the information in the electrical waveform.

The term signal in Nyquist theorem means the total information in the waveform. His signal means the sum of

the "real signal" that you love and
the noise that you hate

This "love and hate situation" makes the correct application of Nyquist theorem interesting.

Supposed that we have a signal whose frequency components are in the range of 10 to 60 Hz and noise ranges from 500 to 1000 Hz. What is a minimal and preferred practical sampling frequencies?

Applications of Nyquist Theorem - 2

- If you are sampling below 2 * 1000 Hz, a portion of the 1000 Hz noise will be transformed into lower frequency noises that you cannot filter away using classical frequency domain filters.
- If you use sampling frequencies in the order of kHz, it can be too fast for an embedded computer. You might need DSP hardware if you want to digitally filter out the noise after sampling.

Applications of Nyquist Theorem - 2

- If you are sampling below 2 * 1000 Hz, a portion of the 1000 Hz noise will be transformed into lower frequency noises that you cannot filter away using classical frequency domain filters.

- If you use sampling frequencies in the order of kHz, it can be too fast for an embedded computer. You might need DSP hardware if you want to digitally filter out the noise after sampling.

- Another way is to use what is known as anti-aliasing filter. A simple hardware analog filter before sampling process begins.

Appendix: an Understanding of Nyquist Theorem

Fig. 8.1

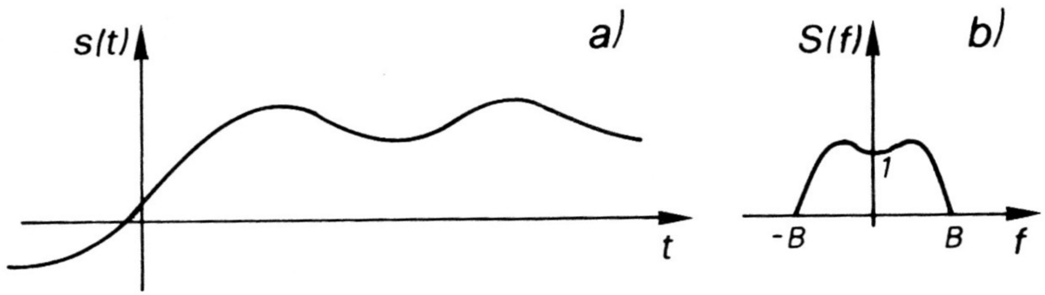
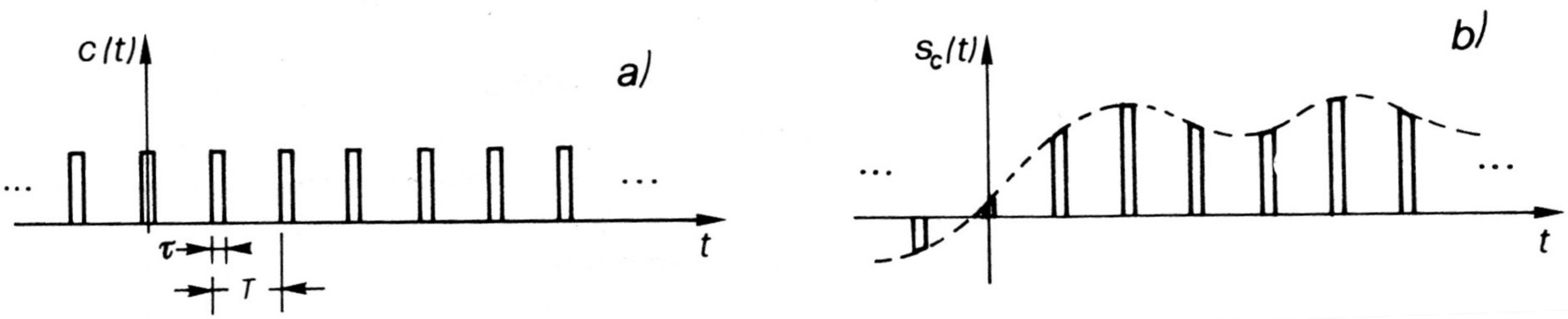
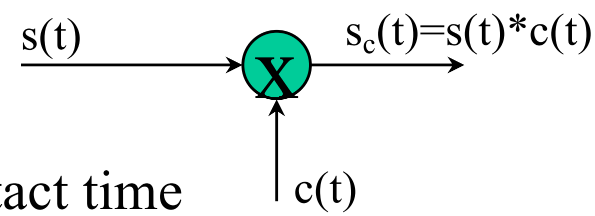


Fig. 8.2

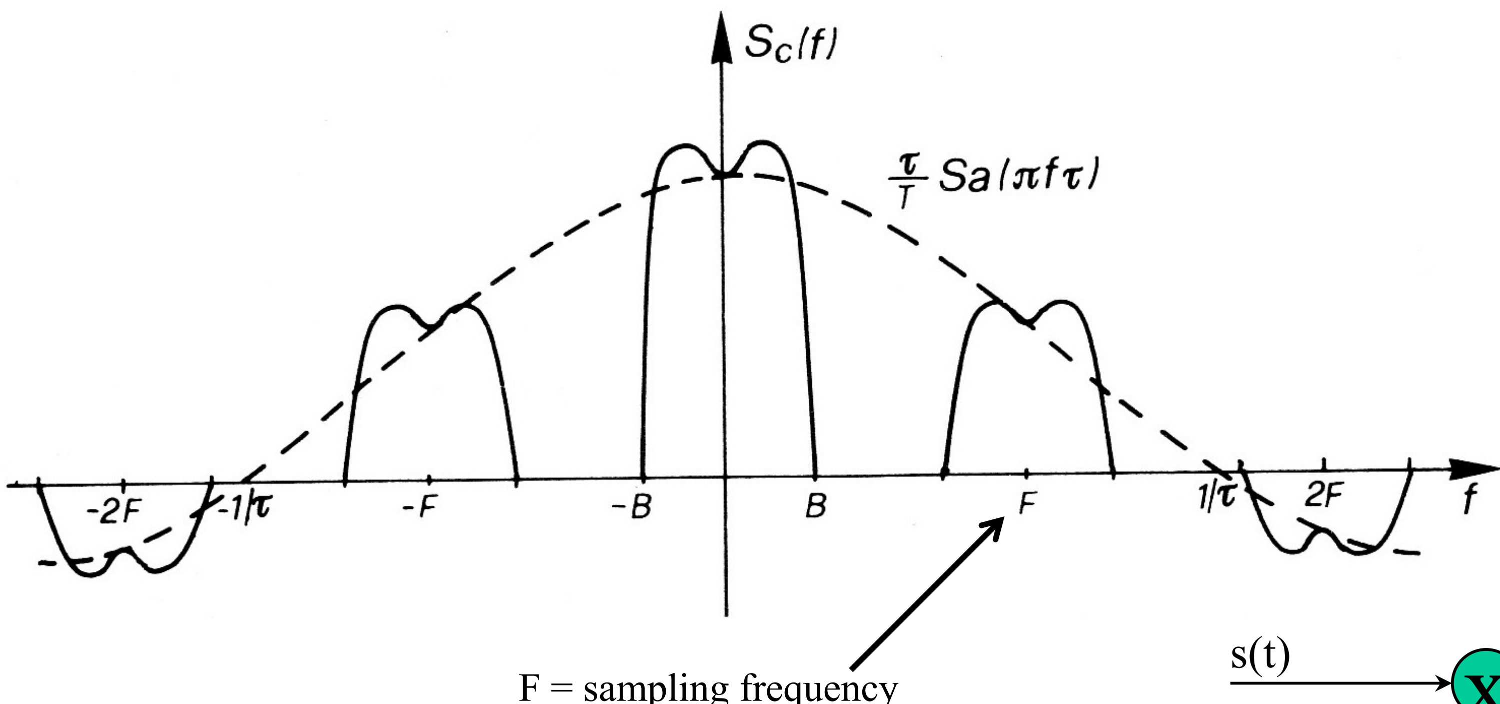


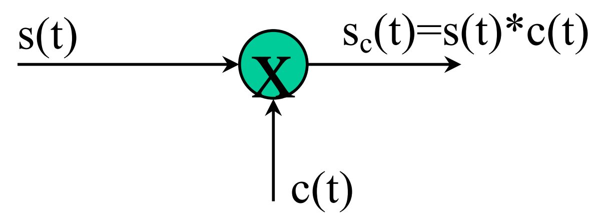


c(t) is a rectangular pulse train, where τ is the contact time and ω_c is the sampling rate.

Appendix: an Understanding of Nyquist Theorem

After the sampling process, the resulting signal $S_c(f)$ has the following spectrum:





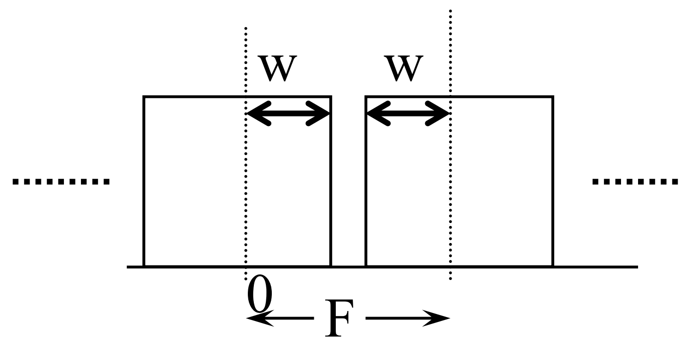
- **Quiz: How can we re-obtain the original signal after performing signal sampling?**

Basic Signal Processing Concepts:
Nyquist Rate and Aliasing

For a baseband signal with bandwidth W, the sampling frequency F (amount of frequency shift) must be greater than 2W (where W is the bandwidth of the signal) or there will be overlaps between the spectrums of the shifted signals. The overlap, if any, is called aliasing.

$$F > 2W$$

Nyquist rate!



It follows that the sampling rate F must be greater than two times the signal bandwidth W (**<u>the highest frequency in the signal</u>**) to avoid aliasing. This is known as the Nyquist rate. If there is no aliasing, the signal can be recovered perfectly in theory by using an ideal low pass filter.