KTH Engineering Sciences

Project 1

Signal propagation in a long electrical conductor

We want to study the signal propagation in a conductor of length $X = 10000$ [m] with resistance R, inductance L and capacitance C. At $x = 0$ signals of amplitude 1 [V] are sent during repeated time intervals of different length. This time-dependent signal function $u_0(t)$ is available as a MATLAB-function in Canvas. The voltage in the conductor is a function $u(x,t)$ of the position x and time t. In our model, it solves the following hyperbolic PDE,

$$\frac{\partial^2 u}{\partial t^2} + \frac{R}{L}\frac{\partial u}{\partial t} = \frac{1}{LC}\frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le X, \quad t > 0,$$

which is a damped wave equation. The initial conditions are given by

$$u(x,0) = 0, \qquad \frac{\partial u}{\partial t}(x,0) = 0, \quad 0 < x \le X.$$

The boundary condition at $x = 0$ is the given signal $u_0(t)$,

$$u(0,t) = u_0(t).$$

At the other end $x = X$, the conductor is open, i.e. the signal exits the conductor without any reflection. The boundary condition fullfilling this condition is the advection equation, i.e.,

$$\frac{\partial u}{\partial t}(X,t) + \frac{1}{\sqrt{LC}}\frac{\partial u}{\partial x}(X,t) = 0.$$

Use the Finite Difference Method (FDM) and discretize the x-axis into $N = 100$ intervals. Use central difference approximations for both the space and time derivatives. To approximate the boundary condition at $x = X$, the upwind discretization (FTBS) is appropriate.

The following parameter values are given for the conductor: $R = 0.004$ [Ω], $L = 10^{-6}$ [H] and $C = 0.25 \cdot 10^{-8}$ [F]. Simulate the signal propagation during a sufficiently long time, e.g., during 3 ms. Use the maximum time step fulfilling the stability condition. Present your results for $u(x,t)$ graphically as an animation. Verify that the propagation speed is $c = 1/\sqrt{LC}$. Also plot the signal at the end of the conductor $u(X,t)$ as a function of t and, on top of it, plot the input signal $u_0(t)$, suitably shifted by the propagation time from $x = 0$ to $x = X$. Compare the input and output signals.

Suppose the signal is noisy. How much noise can be added, before the signal is no longer recognizable at $x = X$? Simulate this in MATLAB by adding a normally distributed random number with mean zero and variance η to u at $x = 0$ in each time step. How large can η be?

Finally, go back to the noiseless signal and try changing the time step to 80% of the stability limit. What are your comments on the results using the two different time steps? Compare your results with solutions obtained with larger N. Make the same experiment with a smooth signal, of your choice.