SF2822 Applied Nonlinear Optimization, 2018/2019
Project assignment 1C
Due Wednesday April 24 2019 23.59

Instructions for the project assignments are given in the course PM. Additional clarifications are given below.

- Discussion between the groups is encouraged, but each group must individually solve the assignments. It is *not* allowed to use solutions made by others in any form.
- Instructions for the report:
  - The report should have a leading title page where the project name and the group members' names, personal number and e-mail addresses are clearly stated.
  - The report should be written using a suitable word processor.
  - The contents should be such that another student in the course, who is not familiar with the project, should be able to read the report and easily understand:
    1. What is the problem? What it the problem background? This does *not* mean a copy of the project description, but rather a suitable summary of necessary information needed in order to understand the problem statement.
    2. How has the group chosen to formulate the problem mathematically? What assumptions have been made? If these assumptions affect the solution, this should be noted.
    3. What is the meaning of constraints, variables and objective function in the mathematical formulation?
    4. What is the solution of the formulated optimization problem? If suitable, refer the mathematical solution to the terminology of the (non-mathematical) problem formulation. (There could be more than one optimization problem.)
  - Most project descriptions contain a number of questions to be answered in the report. The report *must* contain the answers to these questions. They should, however, in a natural way be part of the content of the report and not be given in a "list of answers". The purpose of the questions is to suggest suitable issues to consider in the part of the report where the results are interpreted and analyzed. Make use of your knowledge from the course when formulating the problem and analyzing the results. Additional interpretations are encouraged as well as generalizations and other ways of modeling the problem.
  - A suggested outline of the report is as follows:
    1. Possibly a short abstract.
    2. Problem description and background information.
    3. Mathematical formulation.
    4. Results and analysis (interpretation of results).
    5. A concluding section with summary and conclusions.

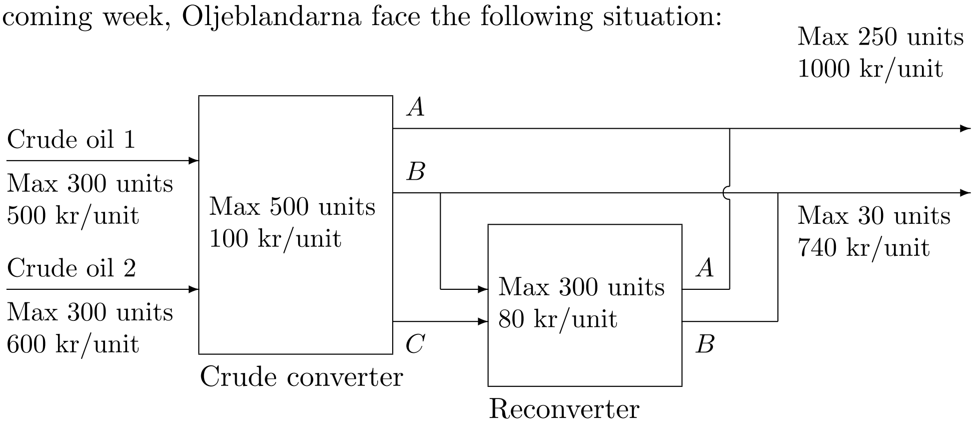
    Deviations from the outline can of course be done.
  - GAMS code should not be part of the report, and should not be referred to in the report.

- Each group should upload the following documents via the Canvas page of the course no later than by the deadline of the assignment:
  - The report as a pdf file.
  - GAMS files.

  Please upload your documents as individual pdf and gms files, and not as zip files.

- Each student should fill out a paper copy of the self assessment form and hand in at the beginning of the presentation lecture.

An important application area of mathematical programming is within different types of "blending problems", for example within the petrochemical industry. We will here consider the distillation of crude oil performed by a petrochemical company, Oljeblandarna, which has previously been considered in the course *SF2812 Applied linear optimization*. However, here we will deal with nonlinearities explicitly.
The coming week, Oljeblandarna face the following situation:



In a first step (the crude converter) three different products are obtained, here called A, B and C. The exchange (in percent) for the two crude oils is given in the following table:

	Prod. A	Prod. B	Prod. C
Crude oil 1	50	30	20
Crude oil 2	70	20	10

Product C is always refined directly in a second step (the reconverter) whereas product A may be sold directly. Product B may either be sold directly or alternatively refined in a second step (the reconverter). The exchange (in percent) for products B and C in the reconverter is given by the following table:

	Prod. A	Prod. B
Prod. B	90	10
Prod. C	40	60

The company wants to plan the production for the following three weeks, with the following conditions.
Each week one may buy at most 300 units of crude oil 1 to the price 500 kr/unit and at most 300 units of crude oil 2 to the price 600 kr/unit. Furthermore, on may each week sell at most 250 units of product A to the price 1000 kr/unit. Product B may each week be wold to the price 740 kr/unit. The first week, only 30 units may be sold, whereas the

second week and the third week, 130 units may be sold. The running cost for step 1 (the crude converter) is 100 kr/unit and the capacity is 500 units (for each of the weeks). The corresponding figures for step 2 (the reconverter) is 80 kr/unit and 300 units respectively. Produce A must be sold the same week that is is made, but product B may be stored during the weeks for which the planning is made. The storage cost for product B from one week to the next is 20 kr per unit. Product B that has been stored may be run through the reconverter.

The question is what production plan the company should hold during the three-week period in question to maximize its profit (sales profit minus purchase cost and running cost.)

The above model, which may be formulated as a linear program, is however not quite satisfactory. The quantity of Crude oil 2 which will be available through the regular sources is expected to remain fixed during the time period in question. The situation for Crude oil 1 is less certain. After a more careful investigation, the analysts at Oljeblandarna believe that the supply of Crude oil 1 for each week is approximately normally distributed with mean 300 units and standard deviation 20 units. Because of technical reasons, the decision on purchased quality and production plan for all three weeks has to be determined before the time period in question. In case of possible shortage of Crude oil 1, Oljeblandarna may purchase additional quantities of Crude oil 1 to the price 700 kr/unit.

Basic exercises

1. Formulate the problem of maximizing the expected profit as a nonlinear programming problem. (The only compensation that can be made is to buy additional quantities of Crude oil 1. The supply of Crude oile 1 from one week to another may be assumed uncorrelated.)

2. Solve the above optimization problem, preferably by making a GAMS model.

3. What does the optimal production plan look like? We recommend that you illustrate in a figure by drawing the "production flow" in the different links. Is the production plan globally optimal?

Advanced exercises

4. The linear relations that have been used above are approximations. In particular, the storage has a nonlinear dependency that should be included in the model. If x units of product B are stored one week, one obtains $x - 0.01x^2$ units of product B the following week. This is due to the fact that some of the oil becomes waste during the storage. (For $x > 100$, the above model is bad, but such large quantities are not expected in the storage.)

Include these storage losses in your model, and solve the problem again. Have you found a global optimium, or are you able to give a bound on how large the deviation from globally optimal objective function value might be?

5. Discuss potential model improvements.

Good luck!