KTH Engineering Sciences

Computer Exercise 2

Numerical Solution of a Boundary Value Problem

Consider a pipe of length L with with small cylindrical cross section. In the pipe there is a fluid heated by an electric coil. The heat is spreading along the pipe and the temperature $T(z)$ at steady state is determined by the convection–diffusion ODE

$$-\frac{d}{dz}\left(\kappa\frac{dT}{dz}\right)+v\rho C\frac{dT}{dz}=Q(z),$$

where all parameters are constant: κ is the heat conduction coefficient, v is the fluid velocity in the z-direction through the pipe, ρ is the fluid density and C is the heat capacity of the fluid. The driving function $Q(z)$, modeling the electric coil, is defined as

$$Q(z)=\begin{cases}0, & 0\le z<a,\\ Q_0\sin\left(\frac{(z-a)\pi}{b-a}\right), & a\le z\le b,\\ 0, & b<z\le L.\end{cases}$$

At $z=0$ the fluid has the inlet temperature T_0,

$$T(0)=T_0.$$

At $z=L$ heat is leaking out to the exterior, which has the temperature T_{out}. This assumption gives the following boundary condition (BC):

$$-\kappa\frac{dT}{dz}(L)=k(T(L)-T_{\text{out}}),$$

where k is a constant heat convection coefficient. Use the following values of the parameters: $L=10$, $a=1$, $b=3$, $Q_0=50$, $\kappa=0.5$, $k=10$, $\rho=1$, $C=1$, $T_{\text{out}}=300$, $T_0=400$ and $v=0$, 0.1, 0.5, 1, 10.

Solve this boundary value problem with the finite difference method using MATLAB. Discretize the z-interval $[0,L]$ with constant stepsize and use a node-numbering where $z_0=0$ and $z_N=L$. Use at least second order accurate approximations for the ODE and the BCs.

Consider first the case $v=0$, which corresponds to no convection, only diffusion. Plot the solution $T(z)$ for $v=0$ with $N=10$, 20, 40, 80 in the same graph. Note the convergence of the curves in the graph.

With $N=40$ the solution is accurate enough for our purposes. Use this discretization to solve the problem for $v=0.1$, 0.5, 1, 10 in the same graph. When $v=10$ spurious oscillations occur! Make another plot when $v=10$, showing the solution for $N=10$, 20, 40 in the same graph. The oscillations become more pronounced when h is increasing. We have an oscillation problem! Suggest how to get rid of these oscillations. (Hint: Check e.g. Section 4.2.4 in the text book.)