Revenue Management: Research Overview and Prospects

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This survey reviews the forty-year history of research on transportation revenue management (also known as yield management). We cover developments in forecasting, overbooking, seat inventory control, and pricing, as they relate to revenue management, and suggest future research directions. The survey includes a glossary of revenue management terminology and a bibliography of over 190 references.

In the forty years since the first publication on overbooking control, passenger reservations systems have evolved from low level inventory control processes to major strategic information systems. Today, airlines and other transportation companies view revenue management systems and related information technologies as critical determinants of future success. Indeed, expectations of revenue gains that are possible with expanded revenue management capabilities are now driving the acquisition of new information technology [see, for example, GARVEY (1997)]. Each advance in information technology creates an opportunity for more comprehensive reservations control and greater integration with other important transportation planning functions. There is now a substantial literature of journal articles, technical reports, and conference proceedings describing the practice and theory of revenue management. This paper provides a survey and bibliography of work in this important area.

The paper is organized as follows. In Section 1, we outline the history and current nature of the revenue management problem and discuss some of the complexities that make solution and implementation so challenging. Sections 2 through 5 contain reviews of revenue management research in four key areas—forecasting, overbooking, seat inventory control, and pricing. Most of this review deals with airline revenue management because the airlines have the longest history of development in revenue management. In Section 6, we discuss important areas for future research and conclude our review. A glossary of revenue management terminology is provided in an appendix. Terminology used in this survey that can be found in the glossary is *highlighted* when first used.

1. REVENUE MANAGEMENT

1.1 Background

We begin with an abbreviated history of revenue management to provide a context for the survey to follow. More detailed accounts of the origins of revenue management can be found in BELOBABA (1987a), SMITH, LEIMKUHLER, and DARROW (1992), CROSS (1995), DUNLEAVY (1995), and VINOD (1995).

Before 1972, almost all quantitative research in reservations control focused on controlled *overbooking*. The overbooking calculations depended on predictions of the probability distributions of the number of passengers who appeared for boarding at flight time, so overbooking research also stimulated useful research on *disaggregate* forecasting of passenger *cancellations, no-shows*, and *go-shows*. Both forecasting and controlled overbooking achieved a moderate degree of success and established a degree of credibility for scientific approaches to reservations control.

In the early 1970s, some airlines began offering restricted discount *fare products* that mixed dis-

TABLE I
Revenue Management Research in Non-Airline Service Sectors

Sector	References
Automobile rental	Geraghty and Johnson (1997), Carol and Grimes (1995)
Broadcasting	Cross (1998)
Cruise lines	Kimes (1989), Ladany and Arbel (1991), Gallego and van Ryzin (1994)
Internet service provision	Nair, Bapna, and Brine (1997), Paschalidis and Tsitiklis (1998)
Lodging and hospitality	Rothstein (1974), Ladany (1977), Liberman and Yechiali (1977, 1978), Kimes (1989), Bitran and Mondschein (1995), Feng and Gallego (1995), Bitran and Gilbert (1996)
Nonprofit sector	Metters and Vargas (1999)
Passenger railways	Kimes (1989), Strasser (1996), Ciancimino et al. (1999)

count and higher fare passengers in the same aircraft compartments. For example, BOAC (now British Airways) offered earlybird bookings that charged lower fares to passengers who booked at least twenty-one days in advance of flight departure. This innovation offered the airline the potential of gaining revenue from seats that would otherwise fly empty; however, it presented them with the problem of determining the number of seats that should be *protected* for late booking, full fare passengers. If too few seats were protected, the airline would *spill* full fare passengers; if too many were protected, flights would depart with empty seats. No simple rule, like protecting a fixed percentage of capacity, could be applied across all flights because passenger booking behavior varied widely with relative fares, *itineraries*, season, day of week, time of day, and other factors.

It was evident that effective control of discount seats would require detailed tracking of booking histories, expansion of information system capabilities, and careful research and development of *seat inventory control* rules. LITTLEWOOD (1972) of BOAC proposed that discount fare bookings should be accepted as long as their revenue value exceeded the *expected revenue* of future full fare bookings. This simple, two fare, seat inventory control rule (henceforth, *Littlewood's rule*) marked the beginning of what came to be called *yield management* and, later, *revenue management*. In North America, the beginning of intensive development of revenue management techniques dates from the launch of American Airlines' Super Saver fares in April of 1977, shortly before the deregulation of U.S. domestic and international airlines.

Over the last twenty years, development of revenue management systems has progressed from simple *single leg control*, through *segment control*, and finally to *origin–destination control*. Each of these advances has required investment in more sophisticated information systems, but the return on these investments has been excellent [see, for example, Smith, Leimkuhler, and Darrow (1992), Cross

(1995)]. In 1999, most of the world's major air carriers and many smaller airlines have some level of revenue management capability. Other small airlines and international airlines in newly deregulated markets are beginning the development process.

The success of airline revenue management was widely reported, and this stimulated development of revenue management systems for other transportation sectors and in other areas of the service sector. A sample of related literature is listed in Table I.

1.2 The Airline Revenue Management Problem

The objective in revenue management is to maximize profits; however, airline short-term costs are largely fixed, and variable costs per passenger are small; thus, in most situations, it is sufficient to seek *booking policies* that maximize revenues. Also, although there is lower risk in accepting a current booking request than in waiting for later possible bookings, booking decisions are repeated millions of times per year; therefore, a risk-neutral approach is justified. All of our discussion in this paper will assume risk-neutral maximization of expected revenues as the objective.

Consider the arrival of a booking request that requires seats in an itinerary—one or more flights departing and arriving at specified times, within a specific *booking class*, at a given fare. The fundamental revenue management decision is whether or not to accept or reject this booking. DURHAM (1995) reports that a large computer reservations systems must handle five thousand such transactions per second at peak times, thus the decision must be reached within milliseconds of the request's arrival. Not surprisingly, no current revenue management system attempts full assessment of each booking request in real time. Instead, precomputed aggregate control limits are set that will close the system for further bookings of specific types while leaving it open for others. The reservations system can quickly determine the open or closed status of a booking

TABLE II
Elements of Airline Revenue Management

Customer Behavior and Demand Forecasting	Revenue Factors
Demand volatility	Fare values
Seasonality, day-of-week variation	Uncertainty of fare value
Special events	Frequent flyer redemptions
Sensitivity to pricing actions	Company or travel agent special vouchers
Demand dependencies between booking classes	*Cancellation* penalties or *restrictions*
Return itineraries	Variable Cost Factors
Batch bookings	Marginal costs per passenger
Cancellations	*Denied boarding* penalties
Censorship of historical demand data	*Goodwill costs*
Defections from delayed flights	Fare Products
Diversions	Number of products
Go-Shows	*Fences* (restrictions)
Group bookings	Problem Scale
Interspersed arrivals	Large airline or airline alliance; e.g., United/Lufthansa/SAS ORION System:
No-shows	4,000 flights and 350,000 passenger itineraries/day [see GARVEY (1997),
Recapture	BOYD (1998)]
Upgrades	Problem Interfaces
Control System	Market strategy
Booking lead time (often 300 days or more)	*Code-sharing* alliances
Number of *controllable booking classes*	Routing
Leg-based, segment-based, or full *ODF* control	Gate acquisition and schedule planning
Distinct *buckets, parallel nesting*, or *full nesting*	*Fleet assignment*
Reservations systems *connectivity*	
Frequency of control updates	
Overbooking	

category and report back to an agent or customer without actually evaluating the request.

The accept–reject decision can be restated as a question of valuation: What is the expected displacement cost of closing the incremental seats in the requested itinerary? To maximize expected revenues, the request should be satisfied only if the fare value of the requested itinerary equals or exceeds the expected displacement cost. [See TALLURI and VAN RYZIN (1999b) for an analysis of such displacement cost controls.]

The apparent simplicity of this valuation problem is deceptive—a complete assessment must allow for all possible future realizations of the reservations process that could be influenced by the availability of any of the seats on any of the legs in the booking. Fully traced, this influence propagates across the entire airline network because a booking can displace potential bookings that will have subsequent impacts of their own. This influence also propagates forward in time because many affected itineraries will terminate later than the booking being considered. Also, a booking will normally have a return component at a later date with its own set of concurrent and downstream effects. Many other factors increase the complexity of the evaluation process. Table II lists some of them.

As can be seen in Table II, the practical complex-

ities of revenue management are daunting—we do not have space here to discuss all of them. As is always true, modeling, theoretical analyses, and implementation rely on assuming away many of these complicating factors and approximating others. It is important to remember that such approximations have yielded enormous revenue benefits for airlines and other enterprises.

The performance of a given revenue management system depends, in large part, on the frequency and accuracy of updates to control limits and the number of distinct booking classes that can be controlled. The determination of suitable control limits and characterization of their *structural* properties over time has been the principal focus of academic research, whereas the need for practical and implementable approximations to optimal limits has driven much of the practitioner research.

A readable account of the practical challenges in airline revenue management and related information systems developments can be found in Sections 6 and 7 of JENKINS (1995). Previous research surveys of airline operations research and revenue management are available in BELOBABA (1987b) and ETSCHMAIER and ROTHSTEIN (1974). A categorization of revenue management and more general *perishable asset revenue management* problems is provided in WEATHERFORD and BODILY (1992).

TABLE III
Airline Forecasting Research 1958–1999

Year	Reference	Year	Reference
1958	Beckmann and Bobkowski	1987	Sa
1961	Thompson	1988	Lee
1962	Taylor	1989	McGill
1967	Rothstein and Stone	1990	Bohutinsky
1970	Lyle	1990	Lee
1970	Martinez and Sanchez	1990	Swan
1972	Lennon	1991	Weatherford
1972	Littlewood	1993	Smith
1976	Conrad	1993a,b,c	Swan
1977	Kanafani and Sadoulet	1993	Weatherford, Bodily and Pfeifer
1978	Taneja	1994	Hopperstad
1982	Brenner	1994	Nahmias
1982	Wang	1994	Weatherford
1983	Harris and Marucci	1995	McGill
1985	Ben-Akiva and Lerman	1996	Belobaba and Weatherford
1986	L'Heureux	1996	Gallego
1987a,b	Belobaba	1997	Botimer
1987	Adams and Michael	1999	Belobaba and Farkas
1987	Ben-Akiva		

Graduate theses often contain thorough reviews of past research literature. We are aware of five master's theses and twelve doctoral dissertations in revenue management or related areas. The master's theses are those of SA (1987), WILLIAMSON (1988), BOHUTINSKY (1990), SUN (1992), SHAYKEVICH (1994). The doctoral dissertations are those of ROTHSTEIN (1968), BELOBABA (1987a), McGILL (1989), LEE (1990), WONG (1990), WEATHERFORD (1991), WILLIAMSON (1992), CHATWIN (1993), NOONAN (1993), BOTIMER (1994), LI (1994) and ZHAO (1999).

In the following sections, we review revenue management research in four key areas—forecasting, overbooking, seat inventory control, and pricing. Each section contains a listing of references that are relevant to the topic of the subsection and included in the bibliography. For completeness, we have included published articles, conference proceedings, working papers, industrial technical reports, and graduate theses. We cannot discuss all of the listed works, so limit ourselves to reviewing a subset that are illustrative of the type of work that has been done.

We make no claim to having identified all revenue management publications and regret any omissions. The authors would be delighted to receive copies of or references to any work that has not been included.

2. FORECASTING

FORECASTING IS AN important component of planning in any enterprise; but it is particularly critical in airline revenue management because of the direct influence forecasts have on the booking limits that determine airline profits. Not surprisingly, publication of approaches to airline forecasting are concurrent with the literature on overbooking because overbooking calculations depend on predictions of ultimate demand, cancellations, and no-shows. Table III lists references from the bibliography.

Unfortunately, the disaggregate forecasting required for both overbooking and revenue management is extremely difficult. The list of passenger behaviors and other complicating factors contained in Table II should make clear the reasons for this difficulty. Simply accounting for the effects of price volatility is a significant challenge in itself—in 1989, there were a reported 30,000 daily price changes in the U.S. domestic airline industry alone [see Williamson (1992, p. 42)].

In the following subsections, we review models for demand distributions, models for arrival processes, uncensoring of demand data, aggregate versus disaggregate forecasting, and current practices.

2.1 Models for Demand Distributions

Early descriptions of statistical models of passenger booking, cancellation, and no-show behavior directed toward overbooking calculations can be found in BECKMANN and BOBKOWSKI (1958). In that paper, the authors compare Poisson, Negative Binomial, and Gamma models of total passenger arrivals and offer evidence of a reasonable fit for the Gamma distribution to airline data. BECKMANN (1958) uses Gamma distributions to model the components of show-ups and develops an approximate optimality condition for the overbooking level. TAYLOR (1962)

TABLE IV
Stochastic Process Models for Airline Arrivals 1958–1999

Arrivals Process	Reference
Homogeneous, nonhomogeneous and/or compound Poisson processes	Gerchak, Parlar, and Yee (1985), Alstrup et al. (1986), Lee (1990), Virtamo and Aalto (1991), Stone and Diamond (1992), Sun (1992), Lee and Hersh (1993), Weatherford, Bodily, and Pfeifer (1993), Gallego and van Ryzin (1994), Lautenbacher and Stidham (1999), Subramanian, Lautenbacher, and Stidham (1999), Zhao (1999)
Stuttering Poisson processes	Beckmann and Bobkowski (1958), Rothstein (1968, 1971a)
Censored Poisson processes	Lee (1990)
General point processes	Gallego and van Ryzin (1994, 1997)

determines empirical probability-generating functions for booking behaviors that determine show-ups. Allowance is made for single and *batch bookings*, cancellations, and go-shows. The generating function is then used to estimate the parameters of a distribution for final show-ups. LYLE (1970) models demand as composed of a Gamma systematic component with Poisson random errors. This model leads to a negative binomial distribution for total demand, as in Beckmann and Bobkowski (1958), which is then truncated for demand censorship. MARTINEZ and SANCHEZ (1970) give a detailed analysis of booking and cancellation data from Iberia Airlines and discuss a convolution methodology similar, in part, to Taylor's approach for obtaining empirical demand and cancellation probability distributions.

Empirical studies have shown that the normal probability distribution gives a good continuous approximation to aggregate airline demand distributions [see, for example, Belobaba (1987a) and SHLIFER and VARDI (1975)]. Given the central limit theorem and the role of the normal distribution as the limiting distribution for both binomial and Poisson distributions, this is not surprising. However, many researchers have pointed out that the normal distribution becomes increasingly inappropriate at greater levels of disaggregation. Section 2.4 discusses this more fully.

2.2 Models for Arrival Processes

Many of the references cited above use a model for the stochastic arrival process of individual booking requests to construct distributions of total flight demand. In other work that seeks dynamic booking rules, specification of the arrivals process is an essential starting point. Examples of stochastic processes that have been used to model arrivals and the related references are listed in Table IV.

The use of the Poisson process, when appropriate, is useful in dynamic treatments because of the memoryless property of the exponential interarrival distribution; however, both the homogeneous and nonhomogeneous Poisson processes lead to Poisson cumulative arrival distributions. This is problematic for total demand modeling because the *coefficient of variation* of the Poisson distribution is the reciprocal of the square root of the mean. Thus, for example, the coefficient of variation for a booking class with mean demand of 100 would be 0.10—much lower than the 0.25 to over 1.0 encountered in practice. Fortunately, cumulative arrivals from compound Poisson processes can provide a reasonable fit to the coefficients of variation of real world arrival data. For example, the stuttering Poisson process used by ROTHSTEIN (1968, 1971a) and Beckmann and Bobkowski (1958) is a compound process that allows for batch arrivals at each occurrence of a Poisson arrival event and achieves more realistic variances.

2.3 Uncensoring Demand Data

Data contained in historical booking records are *censored* by the presence of booking and capacity limits on past demands. SWAN (1990) addresses the downward bias of censoring on late booking data and suggests simple statistical remedial measures. An earlier *spill formula* developed by Swan has been used for many years by practitioners to *unconstrain* demand. Lee (1990) presents a detailed stochastic model of passenger arrivals based on a censored Poisson process and develops maximum likelihood methods for estimating the parameters of these models. MCGILL (1995) develops a multivariate multiple regression methodology for removing the effects of censorship in multiple booking classes, and describes a bootstrapping approach to testing for correlations between fare class demands. A treatment of censored data in general inventory contexts can be found in NAHMIAS (1994). Smith, Leimkuhler, and Darrow (1992) mention experiments with censored regression at American Airlines, but, as far as we know, there have been no follow-up reports on these experiments.

2.4 Aggregate and Disaggregate Forecasting

Traditional regression techniques for aggregate airline forecasting are described in a book by TANEJA (1978). A more recent account of regression experiments with airline data can be found in the master's thesis of Sa (1987). Sa concludes that use of regression techniques can improve the performance of revenue management systems when compared to time series analysis or historical averages. BOTIMER (1997) discusses the effects of promotional seat sales on forecasting and revenue management.

Williamson (1992) points out that there are thousands of potential itineraries across a *hub-and-spoke* airline network. Some itineraries between major centers are traversed frequently enough that reasonable estimates of probabilities for those itineraries can be obtained. Many others are rarely traveled; thus, probabilities for them based on historical data are at or near zero. Unfortunately, taken together, rare itineraries form an important market component from a revenue standpoint. This is particularly true because their scarcity often reflects the large number of flight legs and correspondingly high revenue associated with them. It is unlikely that any way can be devised of predicting the probability of individual rare itineraries. The only recourse is to aggregate such itineraries into larger groups, average their fare values, and seek relative frequencies for the occurrence of any bookings from the group.

Some of the best information on potential bookings for a given flight is contained in the current booking profiles for the same or similar flights in earlier weeks. The use of such short-term booking information has been discussed by airline practitioners: HARRIS and MARUCCI (1983) at Alitalia, L'HEUREUX (1986) at Canadian Airlines International, ADAMS and MICHAEL (1987) at Quantas, and Smith, Leimkuhler, and Darrow (1992) at American Airlines. Typical applications use simple smoothing techniques to incorporate partial booking data from related flights at different phases in their booking process.

The doctoral dissertation of Lee (1990) discusses many issues in disaggregate airline demand forecasting and incorporates censoring in estimation of Poisson models for booking arrival processes.

There has been significant research activity in many disciplines on discrete choice behavior modeling using, primarily, multinomial logit estimations. A basic reference specifically directed at transportation demand modeling is that of BEN-AKIVA (1987). HOPPERSTAD (1994) discusses the potential of path preference models for detailed prediction of passenger behaviors.

GALLEGO (1996) presents a deterministic model of demand behavior under price changes that incorporates *diversion* and *recapture* of passengers in different booking classes. WEATHERFORD, BODILY, and PFEIFER (1993) incorporate diversion in a stochastic model of booking arrivals for two classes. They use beta functions for the intensity functions of nonhomogeneous Poisson processes for each class and scale the intensity functions with a Gamma distribution for total arrivals.

2.5 Current Practice

Airlines are understandably reluctant to share information about their forecasting methodologies because their revenue management activities are so heavily dependent on accurate forecasting. As far as we know at this time, most disaggregate forecasting systems depend on relatively simple moving average and smoothing techniques augmented with careful analysis of recent booking profiles, as mentioned above. Manual intervention is required on an exception basis for critical markets or to anticipate the impact of changes in prices or other important aspects of market structure. Regression and time series techniques have proven of some use for forecasts of aggregate demand, but not at the disaggregate level. This mirrors the general state of forecasting methodology in inventory control applications reported widely throughout industry.

3. OVERBOOKING RESEARCH

AS DISCUSSED IN SECTION 1.1, overbooking has the longest research history of any of the components of the revenue management problem. Table V lists publications in airline overbooking.

Discussions of policy issues relating to passenger overbooking and equitable *bumping* are found in SIMON (1968, 1972), FALKSON (1969), BIERMAN and THOMAS (1975), ROTHSTEIN (1971a, b, 1975, 1985), VICKREY (1972), and NAGARAJAN (1979). These papers are interesting from a historical perspective because they contain suggestions from academic economists, including one Nobel laureate (Vickrey), that *oversold* conditions could be resolved with auctions. These suggestions were evidently dismissed as unrealistic by airline managers of the time, but they proved prophetic. The Vickrey paper also contains a conceptual description of a multiple fare class reservations system bearing a strong resemblance to those now in widespread use.

The objective of most of the early technical research on airline overbooking was to control the probability of *denied boardings* within limits set by airline management or external regulating bodies.

TABLE V
Overbooking Research 1958–1999

Year	Reference	Year	Reference
1958	Beckmann	1975	Shlifer and Vardi
1960	Kosten	1979	Nagarajan
1962	Taylor	1983	Ruppenthal and Toh
1964	Deetman	1985	Rothstein
1967	Rothstein and Stone	1986	Alstrup et al.
1968	Rothstein	1987a,b	Belobaba
1968	Simon	1989	Alstrup
1969	Falkson	1989	Brumelle and McGill
1971a,b	Rothstein	1989	McGill
1972	Andersson	1989	Alstrup
1972	Simon	1993	Chatwin
1972	Vickrey	1995	Dunleavy
1974	Etschmaier and Rothstein	1998	Karaesman and van Ryzin
1975	Bierman and Thomas	1999a,b	Chatwin
1975	Rothstein	1999	Subramanian, Lautenbacher and Stidham

In this setting, an early, nondynamic optimization model for overbooking is that of Beckmann (1958). Statistical models of various levels of sophistication are described by THOMPSON (1961), Taylor (1962), ROTHSTEIN and STONE (1967), Martinez and Sanchez (1970), and Littlewood (1972). An overbooking model extended to allow for two fare classes and a two-leg flight is described by Shlifer and Vardi (1975). In part of his Ph.D. dissertation, Belobaba (1987a) discusses the problem of overbooking in multiple fare classes and suggests a heuristic approach to solving the problem. BRUMELLE and MCGILL (1989) present a static formulation of the overbooking problem and show that it is a special case of a general model of the two fare class seat allocation problem. None of these studies allow for the dynamics of the passenger cancellation and reservation process subsequent to the overbooking decision.

A number of researchers have developed dynamic optimization approaches to the airline overbooking problem and the related problem in the hotel/motel industry. The usual objective in these formulations is to determine a booking limit for each time period before flight departure that maximizes expected revenue, where allowance is made for the dynamics of cancellations and reservations in subsequent time periods and for penalties for oversold seats. KOSTEN (1960) develops a continuous time approach to this problem, but this approach requires solution of a set of simultaneous differential equations that make implementation impractical. Rothstein (1968), in his Ph.D. thesis, describes the first dynamic programming (DP) model for overbooking and reviews the results of test runs of the model at American Airlines. ALSTRUP et al. (1986) describe a DP treatment of overbooking for a two-class, nonstop flight and describe computational experience with the approach at Scandinavian Airlines. A DP analysis similar to Rothstein's but developed for the hotel/motel industry and extended to two fare classes is described in LADANY (1976, 1977) and LADANY and ARBEL (1991). A control-limit type structural solution to the (one class) hotel overbooking problem is described in LIBERMAN and YECHIALI (1977, 1978).

The dissertation of Chatwin (1993) deals exclusively with the overbooking problem and provides a number of new structural results. Chatwin's paper in this issue and CHATWIN (1999b) contain excerpts and refinements based on his dissertation. KARAESMAN and VAN RYZIN (1998) address the problem of jointly setting overbooking levels when there are multiple inventory classes that can serve as substitutes for one another; for example, first class and coach travel service, or compact and full size rental cars.

4. SEAT INVENTORY CONTROL

THE PROBLEM OF seat inventory control across multiple fare classes has been studied by many researchers since 1972. There is a progression from Littlewood's rule for two fare classes, to *expected marginal seat revenue* (EMSR) control for multiple classes, to *optimal booking limits* for single-leg flights, to segment control and, more recently, to *ODF control*. Each step in this progression can be viewed as a refinement of the displacement cost valuation discussed in Section 1.2. We review research on the single leg and network problems separately below. Table VI list publications on single-leg control.

TABLE VI
Single-leg Seat Inventory Control 1972–1999

Year	Reference	Year	Reference
1972	Littlewood	1992	Stone and Diamond
1973	Bhatia and Parekh	1992	Sun
1976	Mayer	1992	Wollmer
1977	Ladany and Bedi	1993	Weatherford, Bodily, and Pfeifer
1978	Hersh and Ladany	1993	Brumelle and McGill
1982	Wang	1993	Lee and Hersh
1982	Buhr	1994	Weatherford
1982	Richter	1994	Shaykevich
1983	Titze and Griesshaber	1994	Young and Van Slyke
1985	Simpson	1995	Bodily and Weatherford
1986	Alstrup et al.	1995	Robinson
1986	Kraft, Oum and Tretheway	1996	Belobaba and Weatherford
1986	Pratte	1997	Brumelle and Walczak
1986a,b	Wollmer	1998	Kleywegt and Papastavrou
1985	Gerchak, Parlar, and Yee	1998	Li and Oum
1987	Gerchak and Parlar	1998	Li
1989	McGill	1998	Van Ryzin and McGill
1989	Belobaba	1998a,b	Zhao and Zheng
1989	Pfeifer	1999	Subramanian, Lautenbacher and Stidham
1990	Brumelle et al.	1999	Lautenbacher and Stidham
1991	Weatherford		

4.1 Single-Leg Seat Inventory Control

Most early seat inventory control research required most or all of the following simplifying assumptions: 1) *sequential booking classes*; 2) *low-before-high fare* booking *arrival pattern*; 3) statistical independence of demands between booking classes; 4) no cancellations or no-shows (hence, no overbooking); 5) single flight leg with no consideration of *network effects*; and, 6) no batch booking.

Littlewood's rule can be viewed as an early expression of the displacement cost rule for two booking classes under all six assumptions. Derivations of Littlewood's rule are given by BHATIA and PAREKH (1973) and, by a different method, RICHTER (1982). MAYER (1976) describes a simulation study of the performance of Littlewood's rule and offers evidence that, if it is used more than once before flight departure, the rule can perform as well as a more complex DP model in which the low-before-high fare arrival assumption is relaxed. He also suggests that the seat allotment and overbooking analyses can be done independently with little revenue loss. TITZE and GRIESSHABER (1983) offer additional simulation evidence that Littlewood's rule is robust to modest departures from the low-before-high fare assumption.

Belobaba (1987a) extends Littlewood's rule to multiple fare classes and introduces the term EMSR for the general approach. The EMSR method does not produce optimal booking limits except in the two-fare case; however, it is particularly easy to implement. McGill (1989) and WOLLMER (1992) fur-

nish evidence that EMSR provides reasonable approximations with typical airline demand distributions. ROBINSON (1995) shows that, for more general demand distributions, the EMSR method can produce arbitrarily poor results. A later refinement of EMSR, called EMSRb, apparently produces better approximations to optimal booking limits and has been widely implemented. For a description of EMSRb, see VAN RYZIN and MCGILL (1998).

Methods for obtaining optimal booking limits for single-leg flights are provided in McGill (1989), CURRY (1990), Wollmer (1992), BRUMELLE and MCGILL (1993). All of these results require assumptions 1 through 6. Curry also proposes an approximation for the network problem, a relaxation of assumption 5. Brumelle and McGill show that, under all six assumptions, the seat inventory control problem is a monotone optimal stopping problem (CHOW, ROBBINS, and SIEGMUND, 1971), and, consequently, that *static control limit* policies are optimal over the class of all control policies for this restricted problem, including *dynamic* ones. They also characterize optimal booking limits with a set of probability conditions that are the same as the EMSR conditions for the first two fare classes but include joint probabilities that are lacking from the EMSR method for additional fare classes. Robinson (1995) generalizes the probability conditions to relax the low-before-high fare assumption.

Van Ryzin and McGill (1998) show that, when demand distributions are stationary across multiple flights, the optimality conditions of Brumelle and

McGill (1993) can be exploited in an adaptive stochastic approximation method that requires no separate forecasting or uncensoring of demands and no direct reoptimization of protection levels.

LI and OUM (1998) provide a preliminary analysis of the seat allocation problem for two airlines competing with identical aircraft and fares on a single-leg route. Their game theoretic formulation demonstrates the existence of equilibrium seat allocations under certain assumptions on the demand distributions.

Belobaba (1987a) proposes an optimality condition for the two fare class problem that allows for the possibility of upgrades from the lower fare class to the higher in the event that the lower class is closed for bookings. PFEIFER (1989) presents a proof of the result. McGill (1989) and BRUMELLE et al. (1990) show that a variant of Littlewood's rule is optimal for the two fare case when discount and full fare demands are statistically dependent, subject to a mild monotonicity assumption on the nature of the dependency. The upgrade result is obtained as a corollary. They further show that, with the same monotonicity assumption, this problem is a monotone optimal stopping problem.

No optimal static control limit policies exist when any assumption other than the low-before-high or independent demand assumptions is relaxed. DP treatments of the single leg problem are presented in Mayer (1976), LADANY and BEDI (1977), STONE and DIAMOND (1992), Sun (1992), LEE and HERSH (1993), Shaykevich (1994), YOUNG and VAN SLYKE (1994), BRUMELLE and WALCZAK (1997, 1998b), ZHAO and ZHENG (1998a), LAUTENBACHER and STIDHAM (1999), SUBRAMANIAN, LAUTENBACHER, and STIDHAM (1999), and Zhao (1999). Space does not permit a review of all of these contributions, and there is some overlap among them. GERCHAK, PARLAR, and YEE (1985) contains the earliest dynamic structural results for this type of problem, but this work is often overlooked because it deals with optimal discounting of unsold bagels—a classic example of the generality of revenue management problems. Ladany and Bedi (1977) and HERSH and LADANY (1978) deal with a two-leg flight; however, we include it as a single-leg model here because their models assume no boarding of passengers at the intermediate stop.

Lee and Hersh (1993) report a generalization of monotonicity results to the batch booking case; however, these results are shown to be in error in Brumelle and Walczak (1997). Both KLEYWEGT and PAPASTAVROU (1998) and VAN SLYKE and YOUNG (1994) characterize seat inventory control problems as special cases of certain stochastic knapsack problems. Both address nonmonotonicity of bid-prices (see below) under batch booking.

4.2 Segment and Origin–Destination Control

Since the 1980s, network effects in revenue management have become increasingly significant because the expansion of hub-and-spoke networks has dramatically increased the number of passenger itineraries that involve connections to different flights. It has been recognized for some time that revenue management should account for these network effects but that this cannot be accomplished effectively with single-leg control. Progress in this area has been impeded not by the lack of approaches to network inventory control so much as by the limitations of older reservations systems. Even if optimal solutions existed for the many hundreds of itineraries that traversed a single leg, those solutions had to be mapped into a much smaller number of *controllable booking classes*. This situation is changing—the most advanced reservations systems are now capable of incorporating network information, and the emergence of *seamless availability* will allow for much finer control of seat availability.

We review, below, four approaches that have been taken to network revenue management. Table VII contains relevant references.

4.2.1 *Mathematical Programming Formulations*

GLOVER et al. (1982) describe a minimum cost network flow formulation for the passenger O–D problem and an implementation at Frontier airlines. Passenger demands are assumed deterministic in this formulation, so the focus of the formulation is on network effects rather than the stochastic elements. DROR, TRUDEAU, and LADANY (1988) propose a similar deterministic network minimum cost flow formulation that allows for cancellations as deterministic losses on arcs in the network. Wong (1990), in his Ph.D. dissertation, develops a network formulation for the single fare class, multiple itinerary problem and extends it to approximations for the multiple booking class case. The single fare case and some comparisons of different *cabin* assignment methods are discussed in WONG, KOPPELMAN, and DASKIN (1993). WOLLMER (1986c) proposes a linear programming (LP) network formulation that allows for stochastic demand by incorporating expected marginal seat values as coefficients in the objective function. Each ODF generates a set of zero–one decision variables for each flight, with a corresponding set of monotonically decreasing objective function coefficients determined by the marginal expected values. Wollmer shows that this formulation can be converted to a minimum cost network formulation for

TABLE VII
Origin–Destination Control 1982–1999

Year	Reference	Year	Reference
1982	D'Sylva	1990	Vinod
1982	Glover et al.	1990	Wong
1983	Wang	1991	Phillips, Boyd, and Grossman
1985	Simpson	1991	Vinod
1986a,b	Wollmer	1992	Williamson
1987a,b	Belobaba	1993	Talluri
1988	Dror, Trudeau and Ladany	1993	Wong, Koppelman, and Daskin
1988	Smith and Penn	1994a,b	Talluri
1988	Williamson	1995	Vinod
1988	Wysong	1996	Talluri and van Ryzin
1989	Simpson	1997	Garcia-Diaz and Kuyumcu
1989	Vinod	1999	Ciancimino et al.
1990	Curry	1999a,b	Talluri and van Ryzin
1990	Vinod and Ratliff		

greater efficiency in solution; however, the size of the problem for a typical airline network is extremely large.

One of the drawbacks of these mathematical programming formulations is that they produce non-nested allocations. The formulations have seen some use for planning purposes but have not been implemented for day-by-day seat inventory control. Their main potential seems to be as components of the bid-price approaches that are discussed below.

Curry (1990) describes a combined mathematical programming/marginal analysis formulation for the O–D problem that uses piecewise linear approximations to the revenue function in a linear program that obtains distinct bucket allocations for different O–Ds. The different fare classes for each O–D are then nested, and each O–D nest is separately optimized for single-leg, nested booking limits. This approach has been implemented in some revenue management systems.

4.2.2 *Segment Control*

The earliest implementations of partial O–D control were at the flight *segment* level. These implementations allow for the revenue value of a multileg itinerary as long as the itinerary does not involve connections between different flights. The motivation for this partial solution to the ODF control problem was the feasibility of exploiting *segment-closed indicators* that were available in the reservation control system. Descriptions of these initial developments in O–D control are provided in Smith and Penn (1988) and Vinod (1995). The methods available to determine seat/segment allocation rules are similar to those available for the broader O–D control discussed below, so we will defer that discussion to there.

4.2.3 *Virtual Nesting*

The first systems that addressed the broader O–D control problem were developed to accommodate the limited number of controllable booking classes that the CRS provided. Belobaba (1987a), Smith and Penn (1988), Williamson (1988, 1992), and Vinod (1989, 1995) all outline techniques for clustering ODFs into single-leg booking classes to achieve an approximation to network control. Such methods assign ODFs to booking classes on the basis of some measure of their total value to the airline instead of just their fare class. A variety of options are available for the clustering process (often called *indexing*), including assignment by total value, assignment by estimated leg value (prorated, for example, by leg distance), and assignment by estimated net value after allowance for displacement effects (dual prices from a deterministic network LP). Both Smith and Penn (1988) and Williamson (1988) report on simulation tests of the relative merits of the different approaches. These two studies reach conflicting conclusions regarding the comparative merits of some of the clustering approaches but agree in the finding that the use of dual prices dominates the other methods.

4.2.4 *Bid-Price Methods*

Smith and Penn (1988), Simpson (1989), and Williamson (1992) incorporate information from LP/network models into the detailed accept–deny decision process of seat inventory control. They use dual prices from a deterministic LP model to establish marginal values for incremental seats on different legs in an airline network. Typically, expected demands replace random demands as constraints in the LP formulations. The dual prices are summed across legs in a passenger itinerary to establish an

TABLE VIII
Pricing Research 1971–1999

Year	Reference	Year	Reference
1971	Kostyrsky	1994	Gallego and van Ryzin
1981	Dolan and Jeuland	1994	Inzerilli and Jara-Diaz
1984	Reibstein and Gatignon	1994	Li
1988	Lau and Lau	1995	Kretsch
1990	Oram	1995	Oum
1991	Weatherford	1995	Feng and Gallego
1991	Carpenter and Hanssens	1996	Dana
1993	Oum, Zhang, and Zhang	1996	Gallego
1994	Borenstein and Rose	1997	Gallego and van Ryzin
1994	Botimer	1998	Dana
1994	Carpenter and Hanssens	1999	You

approximate displacement cost, or *bid-price*, for that itinerary. A booking request for a passenger itinerary is rejected if the bid price exceeds the fare for the itinerary, and is accepted otherwise. Thus, this approach attempts to directly incorporate the estimated displacement cost as a cutoff value for acceptable fares.

The disadvantages and advantages of this approach are discussed in Williamson (1992) and Smith and Penn (1988). A more recent account from a managerial viewpoint can be found in Vinod (1995). Despite potential theoretical drawbacks to the bid-price approach, it has a very convincing advantage—it replaces multiple booking limits and complex nesting schemes with a single bid-price value for each flight leg and a simple rule for rejecting or accepting itinerary requests.

5. PRICING

THERE IS AN EXTENSIVE literature on airline pricing from an economic perspective that we cannot review here. For the most part, that literature deals with pricing and price competition at an industry level rather than the operational, revenue management decision level. This literature is nonetheless relevant to strategic and marketing decisions that are important in revenue management. We discuss here a few examples. Table VIII contains a listing of some relevant articles.

KRETSCH (1995) describes fare management policy from a managerial standpoint. DANA (1998) explains price dispersion as a competitive market reaction to consumers' uncertainty about travel and the risk of rationing due to capacity constraints. The work explains how airline pricing, which looks like classic second-degree monopoly price discrimination, can, in fact, be the result of a perfectly competitive market. DANA (1996) shows that a firm that offers high and low prices and then rations the capacity sold at the low price, as is done in yield

management practice, is in a unique competitive equilibrium. BORENSTEIN and ROSE (1994) provide empirical tests of airline competition and its relationship to the degree of price dispersion observed in fares. They also address the question of whether price dispersion is the result of monopoly or second-degree price discrimination. OUM, ZHANG, and ZHANG (1993) and OUM (1995) deal with aspects of pricing in deregulated airline markets and the influence of *code-sharing* agreements on international fares.

It is now common for airline practitioners to view pricing as part of the revenue management process. The reason for this is clear—the existence of differential pricing for airline seats is the starting point for revenue management, and price is generally the most important determinant of passenger demand behavior. There is also a natural duality between price and seat allocation decisions, as pointed out in GALLEGO and VAN RYZIN (1997). If price is viewed as a variable that can be controlled on a continuous basis, a booking class can be shut down by raising the price sufficiently high. Also, when there are many booking classes available, shutting down a booking class can be viewed as changing the price structure faced by the customer. Treatments of revenue management as a dynamic pricing problem can be found in Ladany and Arbel (1991), GALLEGO and VAN RYZIN (1994, 1997), FENG and GALLEGO (1995), and YOU (1999).

The single-leg revenue management problem with two fare classes is essentially equivalent to the much-studied single period inventory or *newsvendor* problem. Thus, research on pricing in the single-period inventory setting has some relevance to revenue management. LAU and LAU (1988) provide a joint pricing/inventory control analysis for the newsvendor problem. A more general treatment of price, capacity, and technology decisions for profit maximizing firms is provided in GAIMON (1988).

There has been very little published research on joint capacity allocation/pricing decisions in the revenue management context. Botimer (1994), in his doctoral dissertation, uses a model for unrestricted demand for airline seats to derive the demand functions for restricted fare products by incorporating the cost of restrictions to passengers. He also considers modifications to allow for passenger diversions. Li (1994) proves that it is optimal to offer relatively small numbers of fare classes (three in one case, four in another) when the restrictions applied to differentiate the fare classes satisfy certain regularity conditions. Weatherford (1994) presents a formulation of the simultaneous pricing/allocation decision that assumes normally distributed demands, and models mean demand as a linear function of price. The corresponding expressions for total revenue as a function of both price and allocation are extremely complex, and no structural results are obtained. Computational results from a variety of test problems are supplied along with general comments on when inclusion of prices as decision variables justifies the greater computational effort required.

Recent work by Gallego (1996) uses a simple deterministic model to examine pricing and market segmentation decisions. His model takes into account both demand diversion and demand recapture. He gives precise conditions to guarantee the optimality of low to high pricing and lower and upper bounds on the optimal revenue.

6. CONCLUSION AND RESEARCH PROSPECTS

Research and development of revenue management systems is far from over. We close our survey with some suggested directions that future research may take in four areas—forecasting, dynamic programming, ODF revenue management, and systems integration.

6.1 Forecasting

It is difficult to be optimistic about breakthroughs in airline disaggregate forecasting because of the slow progress of effective forecasting technology in less complex areas elsewhere in industry. Nonetheless, a need for reliable airline demand forecasts at increasingly disaggregate levels will parallel increases in sophistication of ODF revenue management systems. The most promising direction for improvement of airline forecast accuracy is in detailed empirical studies of the behaviors of different passenger types in response to changes in fare product offerings. Tracking of individual behavior of passengers who fly frequently could lead to improved prediction of cancellation and no-show behavior in different passenger categories. Smith, Leimkuhler, and Darrow (1992) discuss the potential of discrete choice modeling [see, for example, Ben-Akiva and Lerman (1985), Lee (1990)]. Much work remains to be done—the potential benefits of sharper forecasts certainly justify substantial investments in forecasting methodology and market analysis.

6.2 Implementable Dynamic Programming Approaches

Dynamic formulations of the revenue management problem are required to properly model real-world factors like cancellations, overbooking, batch bookings, and interspersed arrivals. Unfortunately, DP formulations, particularly stochastic ones, are well known for their unmanageable growth in size when real-world implementations are attempted. Usually, the only hope for dynamic optimization in these settings lies in identification and exploitation of structural properties of optimal or near optimal solutions. Knowledge that an optimal solution must be of a control-limit type or be represented by a monotonic threshold curve can be invaluable in development of implementable systems. The existing literature has already identified such structures in special cases of the revenue management problem; however, there are difficult areas still requiring work. The inclusion of batch bookings, which influences the regularity properties required for existing structural results, is a particularly critical area for research—batch bookings are common in airline reservations. The work of Young and Van Slyke (1994), Brumelle and Walczak (1997), and Kleywegt and Papastavrou (1998) provide reference points for this line of research.

It seems unrealistic at this time to imagine that exact DP algorithms could be used for real-time revenue management in the airline industry. However, DP can be used in two important ways: first, on an exception basis for flight or network segments that are identified as being particularly critical, and second, as a calibration tool for checking the performance of less accurate but more efficient solution methods.

Recently developed approximation methods for DP and stochastic programming may be useful in revenue management. Good references for these approximation methods are the books by Bertsekas and Tsitsiklis (1996) and Birge and Louveaux (1997).

6.3 ODF Revenue Management

Bid-price methods appear to be the most promising method for the ODF revenue management prob-

lem because they are simple to implement, and there is empirical evidence that they lead to increased revenues for airlines. However, at present, it is not known how close the total dual prices are to the true expected displacement costs of booked seats in an itinerary in a large airline network. One surprising finding is how well the deterministic dual price approach appears to work despite its obvious drawbacks. Talluri and van Ryzin (1999) provide a number of findings on the accuracy (and potential inaccuracies) of the bid-price technique and furnish some simple counterexamples to its optimality. There is room for further analysis of this approach.

A particularly interesting question is how bid prices can be applied to the common batch and group booking problems. For example, even if we assume that the sum of the dual prices along an itinerary are approximately equal to the displacement cost for one seat, it is unlikely that the displacement cost of a batch booking for the same itinerary will be a simple multiple of the individual displacement costs.

Assuming that the suitability of bid-prices can be verified, there is an interesting computational challenge in increasing the speed with which the dual prices can be updated. Is it possible to accomplish near real time reoptimizations of the LP model so that dual prices reflect the latest information on fares and bookings in the network?

As mentioned in Williamson (1992), one useful by-product of the dual-price calculation is the identification of flights with unusually high dual prices. Such flights correspond to bottlenecks in the airline network. There is a direct analogy here with the concept of bottleneck in the manufacturing industry—bottleneck flights can constrain flows through major segments of the airline network and should be managed intensely. In the industrial setting, the recommended treatment for bottlenecks is: 1) to expand their capacity if possible, and 2) ensure maximum use of the bottleneck by placing buffer inventory in front (and sometimes behind) the bottleneck. In the airline setting, the bottleneck capacity can be increased by assignment of more or larger aircraft, and the buffer inventory can be achieved with overbooking. The interesting research question is how to systematically incorporate bottleneck dual price information into overbooking and fleet assignment processes. BERGE and HOPPERSTAD (1993) offer evidence that there is significant revenue potential in dynamic fleet assignment.

6.4 Integration with Other Planning Functions

Revenue management decisions are highly interdependent with decisions made in other key areas of airline planning; in particular, pricing, fare product design, fleet assignment, and route planning. For example, although pricing and fare restriction decisions occur at a much slower rate than seat inventory control decisions, the revenue impact of a pricing decision is ultimately determined at the seat allocation level, thus there is a clear need for integration of these two decision processes. Also, as discussed above, dual prices at the seat inventory level are relevant to fleet assignment decisions. They are also relevant to longer-term route planning and market development decisions. [See, for example, DOBSON and LEDERER (1993).] Each of these different decision levels present significant theoretical and computational challenges. Mathematical programming and other optimization techniques have been developed for each of them, but these formulations typically treat the problems in isolation.

There is emerging the prospect of an integrated hierarchy of decision systems in which decisions and information obtained at one level are smoothly available to other levels. There are a number of interesting associated research questions. Is there a natural problem/subproblem structure for such an integrated planning system that can exploit decomposition techniques from network optimization and mathematical programming? What information should be exchanged? How frequently? What is the reliability and stability of such a system? There are other areas of interest to revenue management practitioners and researchers; for example, the long-term implications of ticket sales or auctioning through the Internet.

It is clear that revenue management will continue to generate interesting applications and research questions for years to come. We hope that this survey will serve as a stimulus for future research in this challenging and important area.

APPENDIX

REVENUE MANAGEMENT GLOSSARY

WE PROVIDE HERE a glossary of the sometimes-confusing terminology of revenue management. Our aim in supplying a separate glossary is to avoid needless definitions for readers familiar with revenue management while assisting others who are new to the field. Many of the terms described here have different meanings in more general contexts but are presented here with their usual meanings in revenue management.

Aggregation of demand: The level of summarization of passenger demand data. The trend has been toward increasing levels of disaggregation in seat

inventory optimization; however, pricing, forecasting, and booking control processes often operate at different levels of aggregation. Possible dimensions for disaggregation include: market, season, month, week, section of week (e.g., midweek versus weekend), day of week, time of day, *flight number*, booking class, fare, *flight leg*, segment, and itinerary.

Arrival pattern: The pattern of arrivals of booking requests. In the airline context, some possible arrival patterns are: sequential booking classes, low-before-high fares, or *interspersed arrivals*.

Batch booking: (also *multiple booking*, or *bulk arrival*) A booking request that arrives through normal reservation channels for two or more seats to be booked for the same itinerary. Contrast with *group bookings*.

Bid price: A net value (bid-price) for an *incremental seat* on a particular flight leg in the airline network. Also referred to as *minimum acceptable fare*, hurdle price, probabilistic shadow price, displacement cost, or probabilistic dual cost.

Bid price control: A method of network seat inventory control that assesses the value of an ODF itinerary as the sum of the bid-prices assigned to individual legs in the itinerary. Typically, an ODF request is accepted if its fare exceeds the total bid-prices. Also referred to as *continuous nesting*.

Booking class: A category of bookings that share common features (e.g., similar revenue values or restrictions) and are controlled as one class. This term is often used interchangeably with *fare class* or *bucket*.

Booking limit: The maximum number of seats that can be sold to a particular booking class. In *nested booking* systems, booking limits apply to the total number of seats sold to a particular booking class and any lower fare booking classes.

Booking policy: A booking policy is a set of rules that specify at any point during the booking process whether a booking class should be open. In general, such policies may depend on the pattern of prior demands or be randomized in some manner and must be generated dynamically as the booking process unfolds for each flight. In some circumstances, optimal or approximately optimal booking policies can be defined by a set of fixed *protection levels* or *threshold curves*.

Buckets: This term is used in two related ways. First, in older reservations systems, seats for different fare classes or groups of classes are pre-assigned to distinct buckets. These seats are available exclusively to bookings in that fare class. This method simplifies reservations control but is clearly undesirable from a revenue standpoint because seats could fly empty in a discount bucket even if there is higher fare demand available to fill them. Second, buckets also refer to clusters of different fare classes or ODFs that are grouped together for control purposes in a *virtual nesting* system. A single booking limit is set for all classes in the bucket or lower value buckets.

Bulk arrival: See *batch booking*.

Bumping: See *denied boarding*.

Cabin: The physical compartment of an aircraft containing a particular type of seating. For example, an aircraft may be equipped with a first class cabin and a coach cabin, each with different seating and separated by a partition. Multiple fare classes are usually available in each cabin of the aircraft.

Cancellations: Returns or changes in bookings that occur early enough in the booking period to permit subsequent rebooking through the reservations system.

Censorship of demand data: Typically, no record can be kept of booking requests that occur after a fare class is closed down. Thus, in booking histories, the number of flights on which demand reached a booking limit can be determined but not the amount by which demand exceeded the limit. Formally, this condition is known as multiple Type I censorship of the data—the censorship points are known (booking limits), but may vary between observations (flights).

Code-sharing: It is now relatively common for small groups of domestic and international airlines to form alliances in which the members interlist some or all of their flights. The two character airline designator code from one airline is applied to flight numbers of other alliance airlines so that there is an apparent expansion of participating airlines' networks.

Coefficient of variation: The standard deviation expressed as a proportion of the mean of a probability or relative frequency distribution. Thus a *demand distribution* with mean demand 100 and standard deviation 40 would exhibit a coefficient of variation of 0.40. Airline demand data typically display coefficients of variation in the range 0.25 to over 1.0, depending on the level of aggregation of the data.

Connectivity (in reservations systems): The degree to which the elements of the reservations sys-

tem are electronically interconnected. See *seamless availability*.

Continuous nesting: (see *bid-price control*)

Controllable booking classes: All early reservations systems and many existing systems offer only a small number of distinct booking categories (five to ten) that can actually be controlled at booking outlets. Thus, regardless of the number of booking classes or distinct passenger itineraries that can be handled by the revenue management optimization process, the controls in such systems can only be applied to a small number of aggregate booking classes or buckets.

Control limit policy: A structural solution that specifies an upper bound (limit) on the number of seats sold in each fare class (or collection of fare classes) for each time before flight departure.

CRS: Computer reservations system.

Defections: It can occur that a confirmed passenger who shows up for a flight switches to a flight with another airline (usually because of a delay in the original flight departure). Defections constitute a relatively small component of lost passengers and are normally counted as part of no-shows. However, they are distinct from no-shows, and any attempt to predict their occurrence requires an estimation of the probability distribution for departure delays.

Demand distribution: An assignment of probabilities (probability distribution) to each possible level of demand for a flight or booking class. A preliminary estimate of such a demand distribution can be obtained by calculating the proportion of each demand level seen on comparable past flights; i.e., a relative frequency distribution.

Demand factor: The ratio of demand over capacity for a flight or booking class. (Contrast with *load factor*.)

Denied boarding: Turning away ticketed passengers when more passengers show-up at flight time than there are seats available on the flight, usually as a result of overbooking practices. Denied boardings can be either **voluntary**, when passengers accept compensation for waiting for a later flight, or **involuntary**, when an insufficient number of passengers agree to accept compensation. In the latter case, the airline will be required to provide compensation in a form mandated by civil aviation law.

Disaggregate: See *aggregation of demand*.

Displacement cost: In revenue management, the displacement (or *opportunity*) cost of a booking in-

cludes all future revenues that may be lost if the booking is accepted. Taken to the extreme, these include the revenue value of potential displaced future bookings anywhere in the airline network and *goodwill costs* from those displacements. Assessment of the costs and probabilities of such displacements should allow for the dynamics of cancellations and overbooking and the expected costs of oversold conditions.

Diversion: The booking of a customer at a fare level lower than one they would have been prepared to pay. This occurs, for example, when a business traveler has sufficient advance notice of a trip to book in a discount class intended primarily for leisure travelers. *Restrictions* are designed to inhibit such diversion.

Dual prices (also shadow prices): The marginal value of one additional unit of a constrained resource, as determined by a mathematical programming solution to an optimization model. Dual prices are one source of the marginal seat values used in bid-price control.

Dynamic models: Models that take into account future possible booking decisions in assessing current decisions. Most revenue management problems are properly modeled as dynamic programming problems.

Expected marginal seat revenue (EMSR): The expected revenue of an incremental seat if held open. This is a similar concept to that of bid-price but generally used in a simpler context.

Expected revenue: The statistical expected revenue; that is, the sum of possible revenue values weighted by their probabilities of occurrence.

Fare basis code: An alphanumeric encryption of the conditions and restrictions associated with a given fare. Usually several fare basis codes are contained in a single fare class.

Fare class: A category of booking with a (relatively) common fare. Typical labels for such classes [see Vinod (1995)] are: **F** for first class (separate compartment); **J** for business class, **U** for business class frequent flyer redemption (often separate compartment); **Y** for full fare coach; **B**, **M**, **Q**, **V** for progressively more discounted coach bookings; and **T** for frequent flyer coach cabin redemptions. Often other fare products (such as travel agent or company travelers) are categorized under one of these designations for control purposes.

Fare product: The full set of attributes associated with a specific transportation service. The set in-

cludes the fare as well as any restrictions or benefits that apply to that service at that fare.

Fences: See *restrictions*.

Fleet assignment: Most airlines have a variety of aircraft types and sizes in their fleets. The fleet assignment process attempts to allocate aircraft to routes in the airline network to maximize contribution to profit. There are strong potential linkages between fleet assignment and revenue management processes because aircraft assignments determine leg capacities in the network.

Flight leg: A section of a flight involving a single takeoff and landing (or no boarding or deplaning of passengers at any intermediate stops). Also *leg*.

Flight number: A numeric or alphanumeric label for a flight service that involves (generally) a single aircraft departing from an origin airport, possibly making additional scheduled stops at one or more intermediate airports, and terminating at a destination airport.

Full Nesting: See *nested booking*.

Global distribution system (GDS): Computer and communications systems for linking booking locations with the computer reservation systems of different airlines. Examples are SABRE, Galileo, and Amadeus.

Goodwill costs: An airline's rejection of a booking request can affect a customer's propensity to seek future bookings from that airline. This cost is difficult to assess but is considered particularly acute in competitive markets and with customers who are frequent air travelers. An approximate assessment of the cost of a permanently lost customer is the expected net present value of all future bookings from the customer minus the opportunity costs of those bookings.

Go-show: Passengers who appear at the time of flight departure with a valid ticket for the flight but for whom there is no record in the reservation system. This no-record situation can occur when there are significant time lags in transferring booking information from reservations sources (e.g., travel agent's offices) to the CRS or when there are transmission breakdowns.

Group bookings: Bookings for groups of passengers that are negotiated with sales representatives of airlines; for example, for a large group from one company travelling to a trade show. These should be distinguished from batch bookings.

Hub-and-spoke network: A configuration of an airline's network around one or more major hubs that serve as switching points in passengers' itineraries to spokes connected to smaller centers. The proliferation of these networks has greatly increased the number of passenger itineraries that include connections to different flights.

Hub bank: A collection of inbound and outbound flights that are scheduled to arrive or depart within a time span that enables convenient passenger connections among flights. An airline hub will typically operate with several hub banks throughout the day.

Incremental seat: One additional seat, given the number of seats already booked.

Independence of demands: The assumption that demands in one customer category (e.g., booking class or ODF) are statistically independent of demands in other categories. It is widely believed that this assumption is not satisfied in practice. See, for example, Hopperstad (1994).

Indexing: The process of assigning individual ODF categories to virtual nesting buckets. Smith, Leimkuhler, and Darrow (1992) provide details.

Interspersed arrivals: Characteristic of an arrivals process in which booking requests in different booking classes do not arrive in any particular order. (Compare with sequential booking classes.)

Itinerary: For purposes of this paper, an itinerary is a trip from an origin to a destination across one or more airline networks. A complete specification of an itinerary includes departure and arrival times, flight numbers, and booking classes. The term is used ambiguously to include both one-way and round-trip travel. That is, used in the first way, a round-trip involves two itineraries and, in the second way, one itinerary.

Leg: See *flight leg*.

Leg based control: An older, but still common, method of reservations control and revenue management in which limits are set at the flight leg level on the number of passengers flying in each booking class. Such systems are unable to properly control *multileg* traffic, although virtual nesting provides a partial solution.

Littlewood's rule: This simple two-fare allocation rule was proposed by Littlewood (1972). Given average high fare f_1, average discount fare f_2, random full fare demand Y, and s seats remaining, Littlewood's rule stipulates that a discount seat should be sold as long as the discount fare equals or exceeds

the expected marginal return from a full fare booking of the last remaining seat; that is, discount demand should be satisfied as long as $f_2 \geq f_1 \Pr(Y > s)$. This is essentially equivalent to the classic optimal stocking rule for single period stochastic inventory (newsvendor) problems.

Load factor: The ratio of seats filled on a flight to the total number of seats available.

Low-before-high fares: (Also called *monotonic fares* or *sequential fares*.) The sequential booking class assumption is often augmented by the additional assumption that booking requests arrive in strict fare sequence, generally from lowest to highest as flight departure approaches. The existence of low *standby fares* violates this assumption.

Minimum acceptable fare (MAF): See *bid-price*.

Monotonic fares: See *low-before-high fares*.

Multileg: A section of an itinerary or network involving more than one *leg*.

Multiple booking: See *batch booking*.

Nested booking: In fully nested (also called *serially nested*) booking systems, seats that are available for sale to a particular booking class are also available to bookings in any higher fare booking class, but not the reverse. Thus, a *booking limit L* for a discount booking class defines an upper bound on bookings in that class and any lower valued classes and a corresponding protection level of $(C - L)$ for all higher classes; where C is the total capacity of the pool of seats shared by all classes. This should be contrasted with the older distinct bucket approach to booking control. See, also, *parallel nesting*.

Network effects: A booking on any leg in the airline network may block booking of any itinerary that includes that leg. Subsequent interactions of the blocked itinerary with other legs in the network can, in a similar fashion, propagate across the full network.

Newsvendor problem: The problem of choosing the quantity of a perishable item to stock (e.g., newspapers) given known cost, selling price, and salvage values, and subject to uncertain future demand. (Also called the newsboy or single period stocking problem.) This classic problem is essentially equivalent to the simple two-fare *seat allocation problem* with sequential arrivals.

No-shows: Booked passengers who fail to show up at the time of flight departure, thus allowing no time for their seat to be booked through normal reservations processes. No-shows are particularly common among full fare passengers whose tickets are fully refundable in the event of cancellation or no-show.

OBL: See *optimal booking limits*.

ODF control (O–D problem): Origin–destination fare control. An approach to revenue management that accounts for all possible passenger itineraries between origins and destinations in the airline network, at all fare levels. See *network effects*.

Opportunity cost: See *displacement cost*.

Optimal booking limits: This term is often used to refer to exact booking limits for the single leg seat inventory control under assumptions 1 through 6 in Section 4.1. They are only optimal within the context of that basic model. At present, there are no truly optimal booking limits for the full ODF revenue management problem, and likely never will be.

Origin–destination control: See *ODF control*.

Overbooking: The practice of ticketing seats beyond the capacity of an aircraft to allow for the probability of no-shows.

Oversold: An ambiguous term sometimes used when more passengers show up for a flight than there are seats available. Such situations must be resolved with denied boardings.

Parallel nesting: See *nested booking*. This is an approach to booking that is intermediate between simple distinct bucket control and *full nesting*. A number of lower fare classes are assigned to distinct buckets, but these buckets are nested in one or more higher fare classes. This approach reduces the revenue potential of the combined fare classes, but may facilitate control.

Perishable asset revenue management (PARM): A term introduced in Weatherford and Bodily (1992) for the general class of revenue management problems, which includes airline revenue management.

Protected seats: Seats that are restricted to bookings in one or more fare classes. In fully nested booking systems, seats are protected for bookings in a fare class or any higher fare class.

Protection levels: The total number of protected seats for a booking class. In fully nested booking systems the protection level for a fare class applies to that class and all higher fare classes.

RCS: Reservations Control System.

Recapture: The booking of a passenger who is unable to obtain a reservation for a particular flight or

set of flights with an airline onto alternative flights with the same airline.

Reservation system controls: The internal logic used by the reservation system for controlling the availability of seats. This logic is usually difficult to change and is often a significant constraint when implementing a yield management system. See *controllable booking classes.*

Restrictions: Sets of requirements that are applied to discount fare classes to differentiate them as fare products and discourage diversion. Examples are fourteen-day advance booking requirements, cancellation penalties, Saturday night stayover, and midweek departure requirements. Also referred to as booking *fences.*

Revenue management: The practice of controlling the availability and/or pricing of travel seats in different booking classes with the goal of maximizing expected revenues or profits. This term has largely replaced the original term yield management.

Rules: See *restrictions.*

Seamless availability: A capability of reservation and information systems that allows for direct transmission of availability requests from ticket agents to airlines. With this capability, airlines may be able to provide unrestricted origin–destination control of their seat inventory.

Seat allocation: See *seat inventory control.*

Seat inventory control: The component of a revenue management system that controls the availability of seats for different booking classes.

Segment: One or more flight legs covered by a single flight number. Thus, if a flight originates at airport **A**, makes an intermediate stop at **B**, and terminates at **C**; the possible flight segments are **AB**, **BC**, and **ABC**.

Segment closed indicator (SCI): A flag in reservations control systems that indicates that a booking class is closed to bookings over a particular segment. The same booking class may be open for bookings over other segments of the same flight. This allows for O–D control at the segment level.

Segment control: A level of itinerary seat inventory control that accounts for the revenue value of flight segments, but does not account for itineraries that involve other flight segments. In the case of a two leg flight **A** to **B** to **C**, segment control would permit closing the **AB** segment to a discount booking class but leaving the **ABC** segment open for the same class. This system fails to account for the pos-

sibly high revenue value of a booking that includes, for example, the segment **AB** in its itinerary but switches to a different flight at **B**.

Sequential booking classes: The assumption that requests for bookings in particular classes are not interleaved; for example, all B-class requests arrive before any Y-class requests. This assumption is rarely satisfied in practice; however, it is close enough to permit significant revenue gains from methods based on the assumption. Also, early booking restrictions on many discount booking classes ensure a degree of compliance.

Sequential fares: See *low-before-high fares.*

Serial nesting: See *nested booking.*

Show-ups: Passengers who appear for boarding at the time of flight departure. The number of show-ups is (final bookings + go-shows + standbys − no-shows).

Single-leg control: See *leg based control.*

Space control: See *seat inventory control.*

Spill: Unsatisfied demand that occurs because a capacity or booking limit has been reached. See *censorship of demand data.*

Spill formula: A formula or algorithm that estimates the amount of spill that has occurred on past flights.

Spoilage: Seats that travel empty despite the presence of sufficient demand to fill them. This will occur, for example, if discount booking classes are closed too early, and full fare demands do not fill the remaining seats. This should be distinguished from excess capacity—seats that are empty because of insufficient total demand.

Standby fares: Some airlines will sell last minute discount seats to certain categories of travelers (e.g., youth or military service personnel) who are willing to wait for a flight that would otherwise depart with empty seats.

Static models: Models that set current seat protection policies without consideration of the possibility of adjustments to the protection levels later in the booking process. (Compare with *dynamic models.*)

Structural solution: A solution to an optimization problem in the form of specifications (frequently equations) that reveal the pattern of behavior of optimal solutions. These are important because they lead to a deeper understanding of the nature of optimal solutions and can lead to development of efficient solution algorithms.

Threshold curves: Threshold curves are functions that return time-dependent booking limits for overbooking or seat inventory control.

Unconstrained demand: An estimate of the demand for a past flight or fare class that has been corrected for censorship.

Upgrade: This term is used in two ways. First, it refers to an offer to a passenger to fly in a higher service class without additional charge (e.g., in exchange for frequent flyer points, or to avoid a denied boarding). Second, it refers to a decision by a customer to book in a higher fare class than originally intended when he or she is advised that no seats are available at their preferred fare.

Virtual nesting/virtual classes: This is one approach to incorporating origin–destination information into leg or segment based control systems. Multiple ODFs are grouped into virtual buckets on the basis of similar revenue characteristics (e.g., comparable total fare values, or similar total bid prices). The virtual buckets may easily contain a mixture of traditional fare classes. The buckets are then nested and assigned to traditional booking classes for control in a leg based reservation system.

Yield management: The early term used for what is now more commonly called revenue management. Cross (1995) attributes the original term to Robert L. Crandall when he was Senior Vice President for Marketing (later CEO) at American Airlines. Weatherford and Bodily (1992) introduced the general term, perishable asset revenue management, for the general class of inventory control problems of which airline revenue management is an example.

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