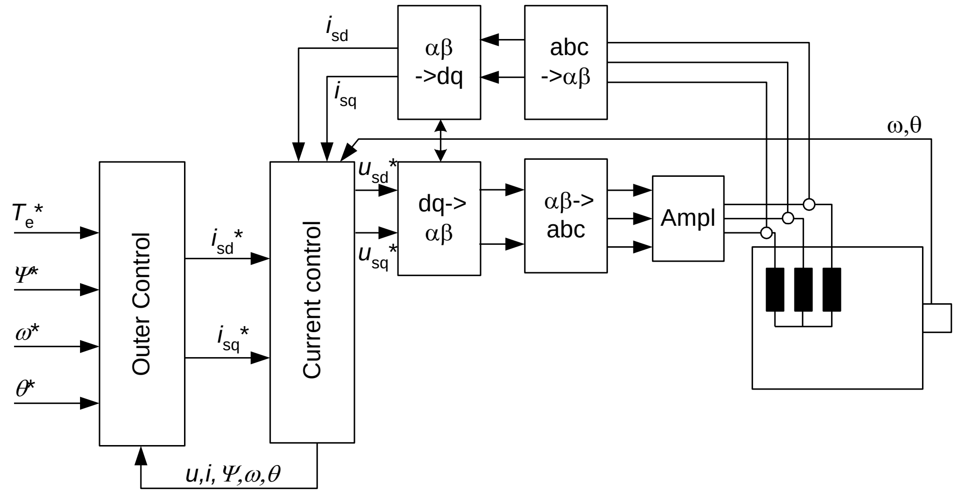
Course outline:

- W1: RL circuit current control, PMDC current and speed control, control analysis, PMSM modelling, transformation theory.
- W2: PMSM-drive, Induction machine modelling, state-space modelling, inverse Γ-induction machine model
- W3: Field-oriented control using internal rotor flux sensors
- W4: Field-oriented control using estimated rotor flux with both DFO and IFO, speed and position control, control analysis, sensorless control
- W5: three-phase converter, digital controller
- W6: Field weakening, V/Hz control
- W7: other types of control, signal injection
- W8: Old exams



Examination: Grading 5, 4, 3, Not passed
Written exam, max 50p, limits 3 20p, 4 30p, 5 40p.
+ approved project and laboratory tasks (7 in total)

Content on written exam: Home assignments, project, laboratory, tutorials, lectures

Only CTH-approved calculators are allowed at the exam.
The formula paper will be attached to the exam

Learning outcomes (After the course the students should be able to):

- design current, speed and position controllers of electric machines, based on bandwidth requirements of their performance, the parameters of the machine together with the load and the supplying power electronic converter.
- construct/develop a control system of a DC-machine and to judge the performance of the current and speed controller using a linear power amplifier.
- construct/develop a field-oriented control system of an induction machine and a PM synchronous machine and to judge the performance of the current and speed controllers.
- implement and evaluate active damping, feed-forward and anti-windup of the regulators.
- present currents, voltages and fluxes in 3- and 2-phase stationary systems as well as in the rotating 2-phase system, and to be able to move between these representation systems.
- derive, implement and judge the performance of the current model flux estimator in direct and indirect field orientation.
- derive the base equations of the voltage model flux estimator and evaluate the performance of the voltage model.
- derive the base equations for estimating the rotor position with signal injection for a salient PM synchronous machine and evaluate its performance.
- use the state-space representation for simulation of electric machines and be able to derive the state-space equations from the standard equation set-up describing an electric machine.
- describe how a three-phase converter operates and to determine the switching pattern that is created by the converter and the impact that this pattern has on the machine.
- design a field weakening controller for the machines.
- implement the developed control system on a drive system with a dSPACE real time control system and evaluate the drive system performance.
- describe how a Volt/Hz control operates.
- choose the relevant (environmental friendly) drive system for a given application with given specifications.

Teaching activities SW1

V3	Monday 16/1	Tuesday 17/1	Wednesday 18/1	Thursday 19/1	Friday 20/1
08-09	Lecture 1			Project intro. ½ No supervision	
09-10					
10-11	Lecture 2		Lecture 3	Project intro. ½ No supervision	
11-12					
12-13					
13-14			Lecture 4	Computer lab supervision	
14-15					
15-16			Tutorial 1		
16-17					

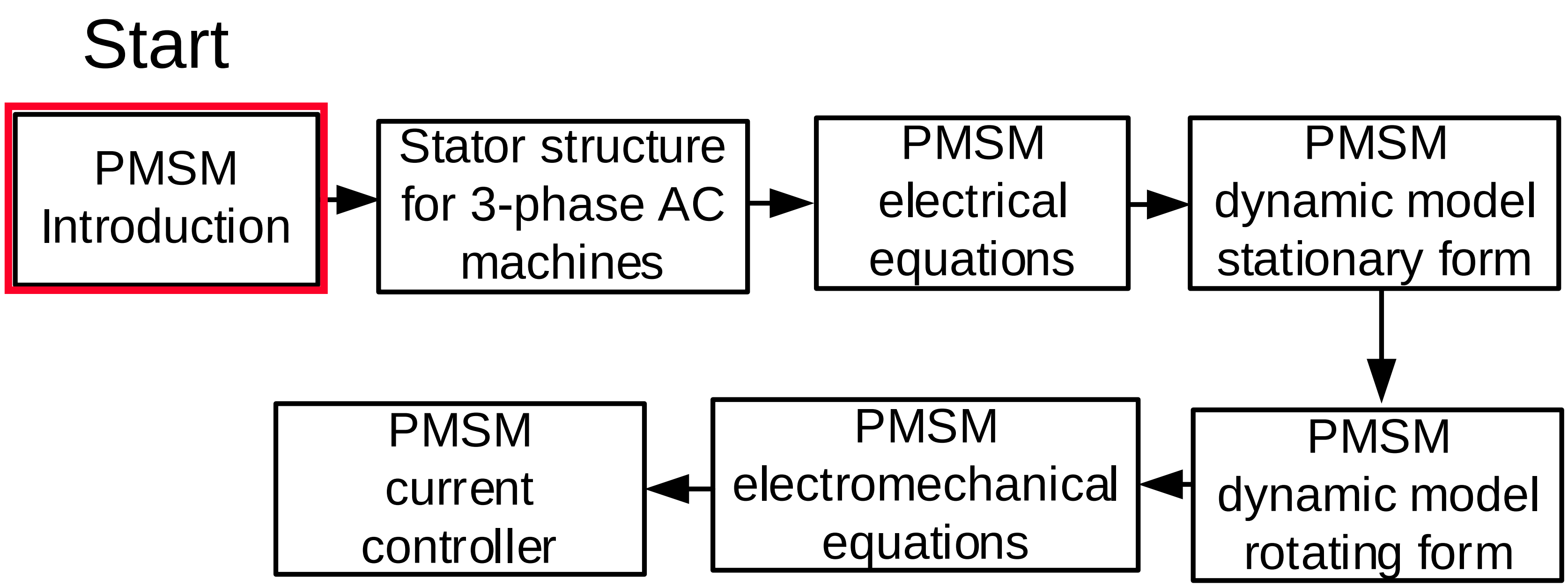
L1. Introduction to electric drives. Current control derivation for an RL circuit. Current controller analysis.

L2. Current and speed control derivation for permanent magnet DC machine.

L3. Transformation theory, rotating flux linkage.

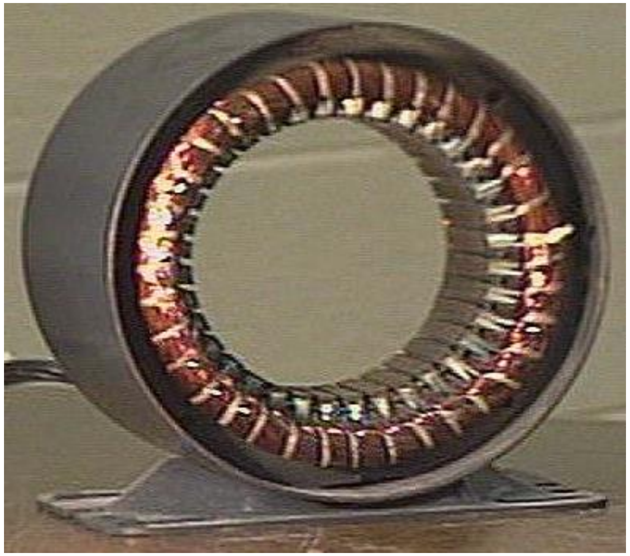
L4. PMSM modeling and implementation.

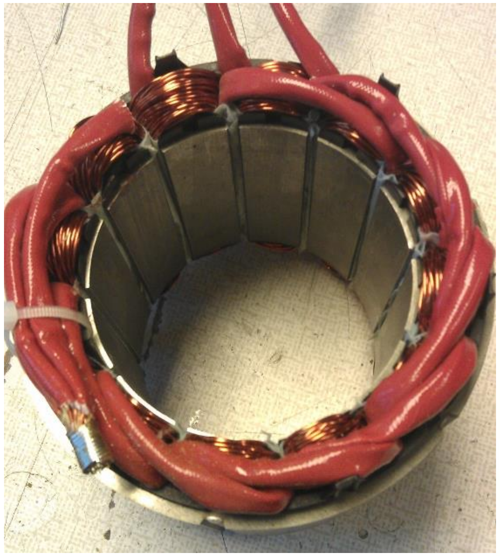
Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system



Permanent Magnetized Synchronous Machine (PMSM)

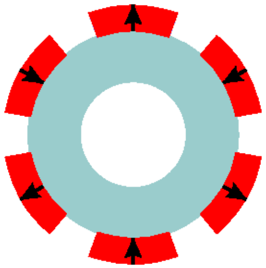
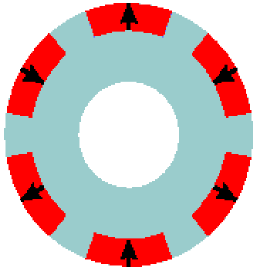
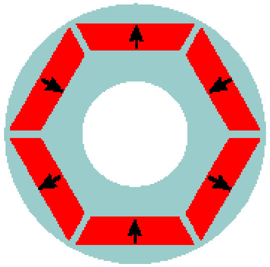
- Stator windings generates a rotating magnetic field.
- Machine magnetized by magnets attached to the rotor.
- The (NeFeB (Neodymium-Iron-Boron)) magnets of today are:
  - Powerful (~1 Tesla) and is not so easily demagnetized
  - Low dependence on temperature (~200 degr C).





PMSM – Rotor design

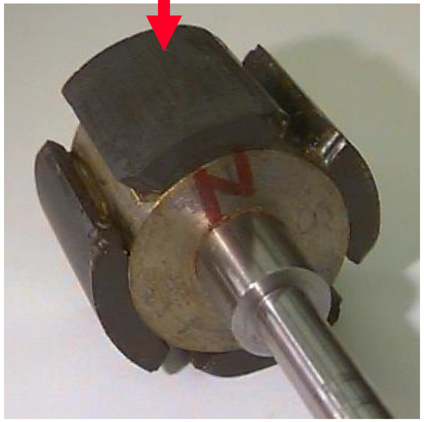
Some possible rotor constructions

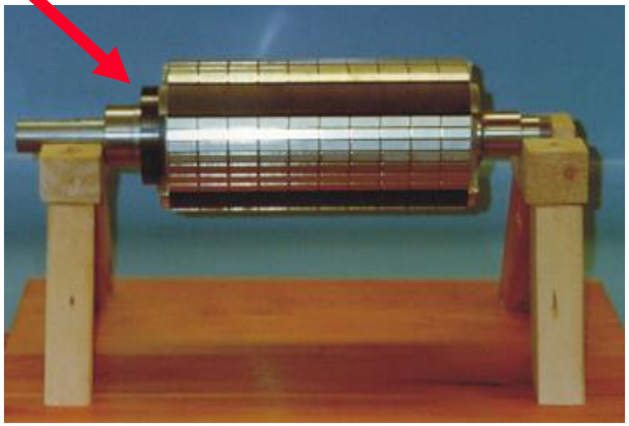




Surface mounted

Inset mounted

Interior mounting

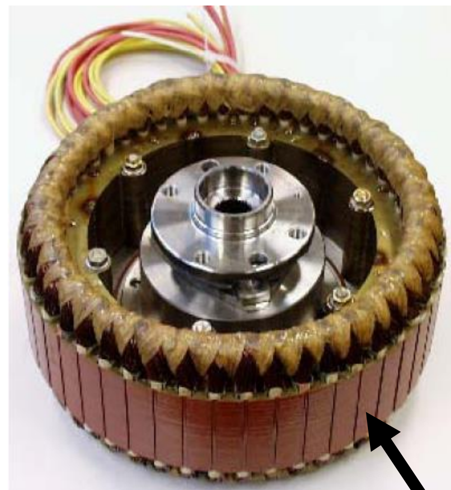




PMSM - Rotordesign

The rotor can also be mounted on the outside:

+



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Outer rotor

Inner stator

Wheel motor

NeFeB-magnet

Laminated steel

PMSM - Properties

+high efficiency (>95%).

”No” rotor losses.

+Quick mechanical dynamics (Ex. 8 kW machine, bandwidth, speed control ~100 Hz).

- Position of rotor must be well known.

Resolver (absolute encoder)

Encoder

Estimation (sensorless control)

-Open loop usually not possible (grid connected usually not possible)

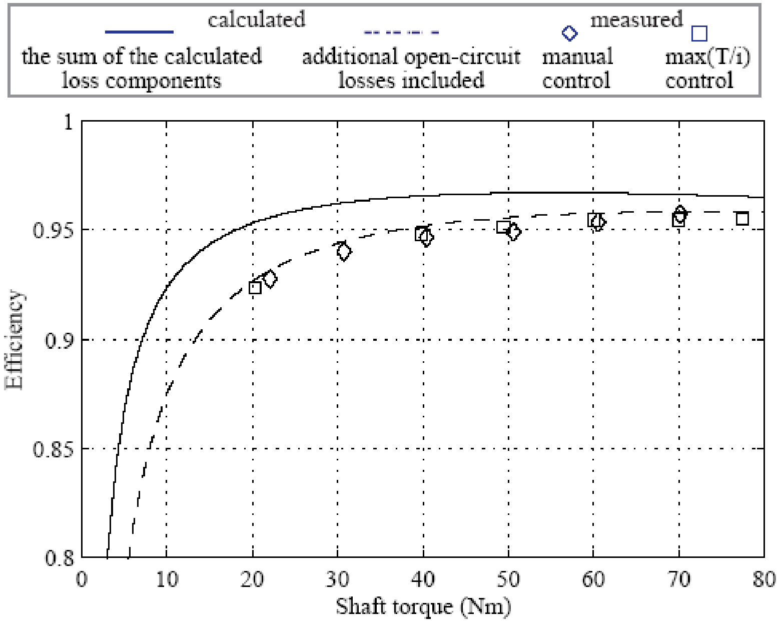
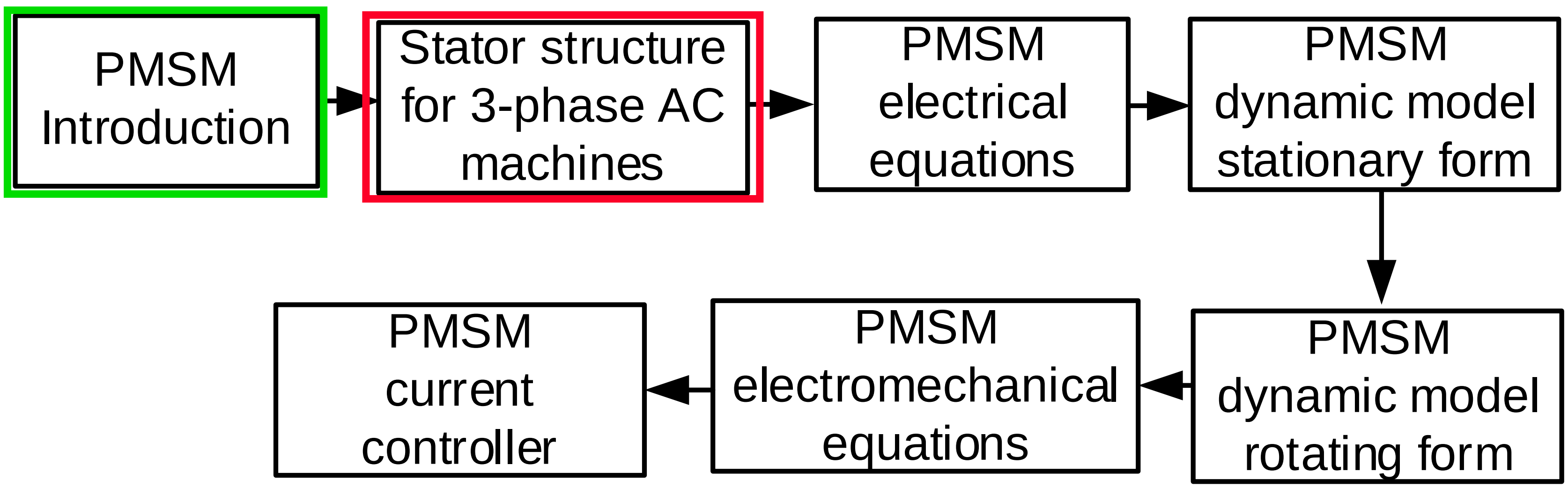
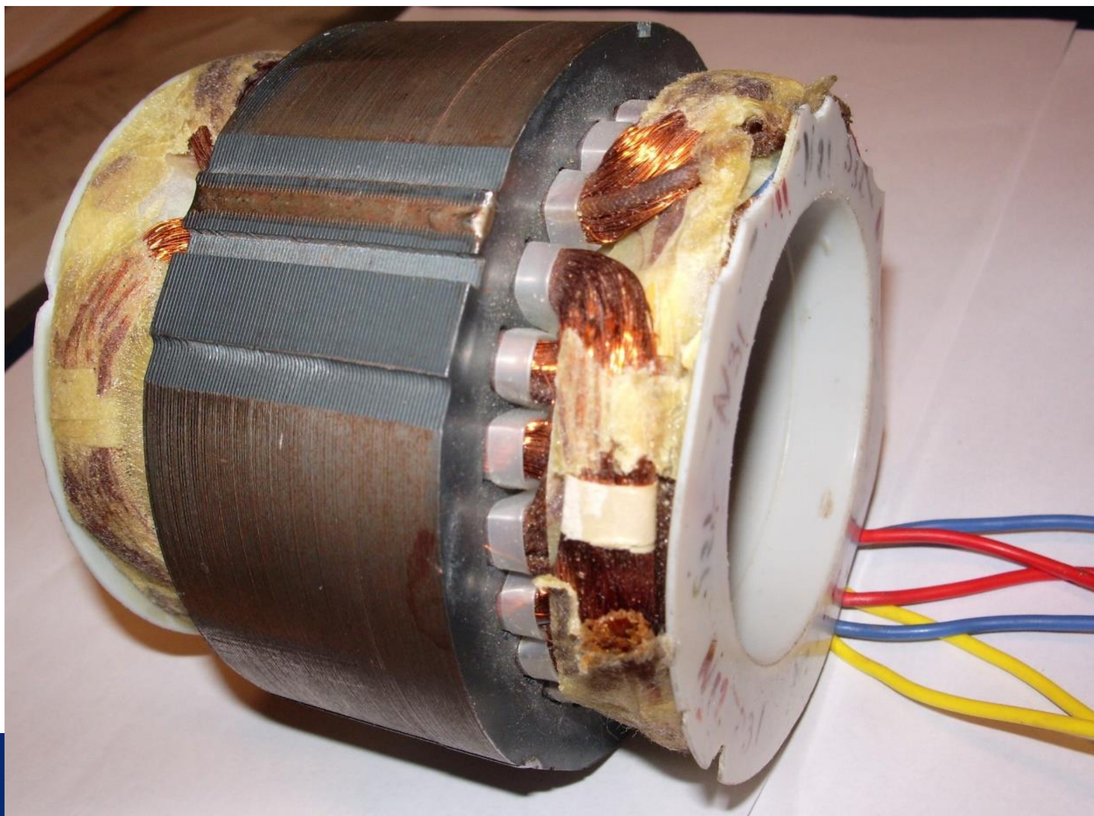


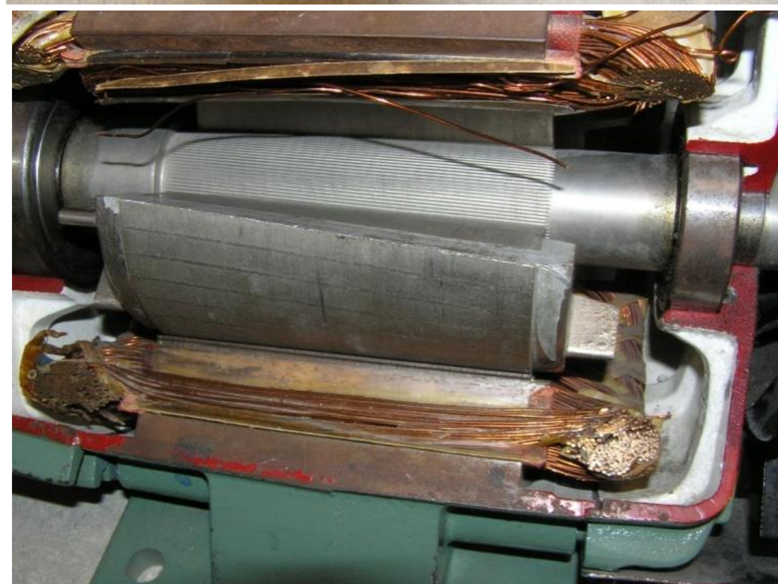
Figure 17: Predicted and measured efficiencies at 6000 rpm

Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system



Stator structure
Same for all 3-phase AC machines





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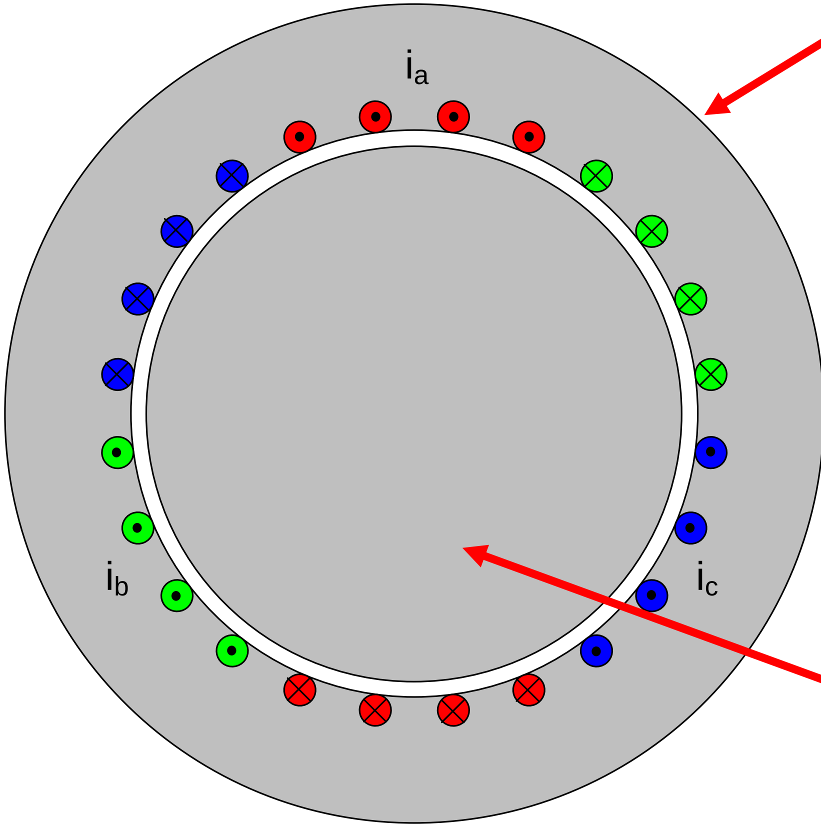
Stator structure
Same for all 3-phase AC machines



Stator (the stationary part)

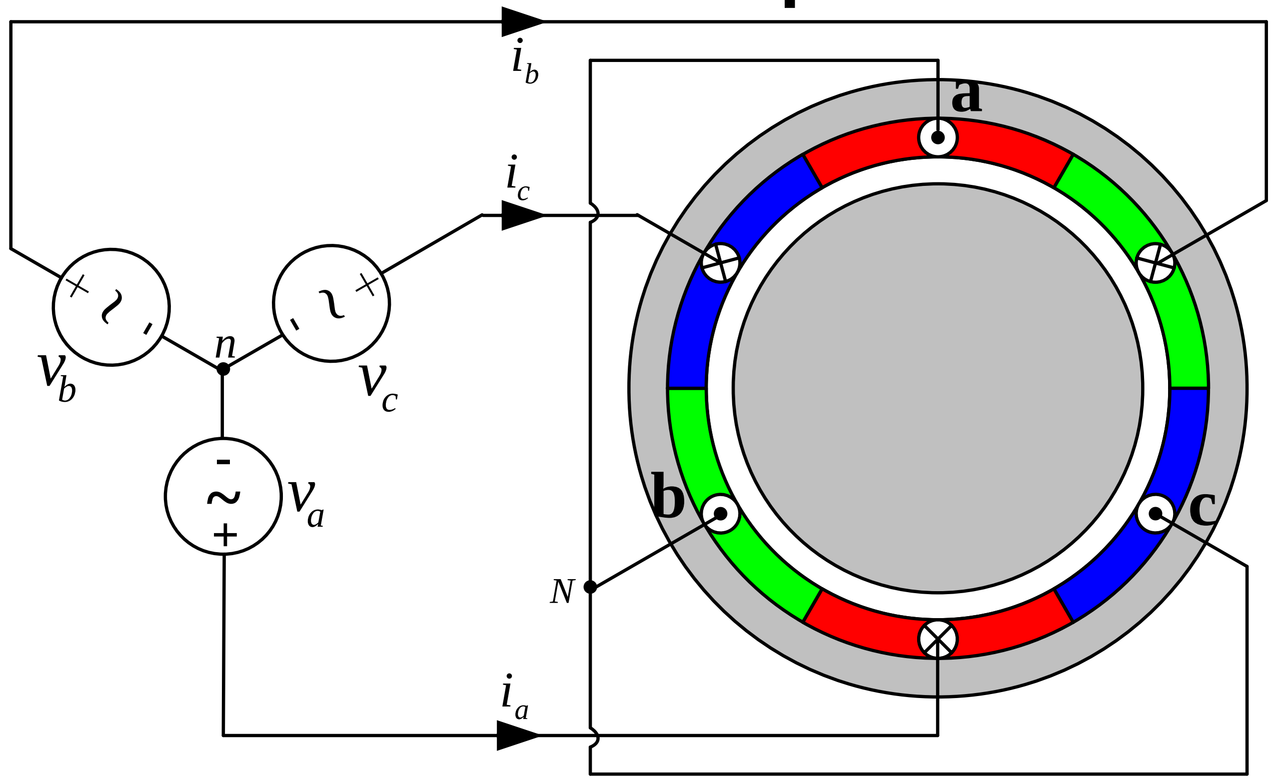
- 3 windings (one per phase)
  - a-phase red
  - b-phase green
  - c-phase blue
- 120º twisted apart in the room

The type of machine you get depends on which type of rotor (the rotating part) you put in the machine:

- here just a round iron, no machine
- Magnets → synchronous machine
- Windings → induction machine
- Not round iron → synchronous reluctance machine

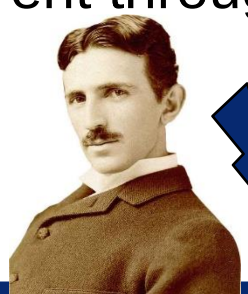
Let us now study the function of the stator in detail, because this function is the same for all 3-phase AC machines

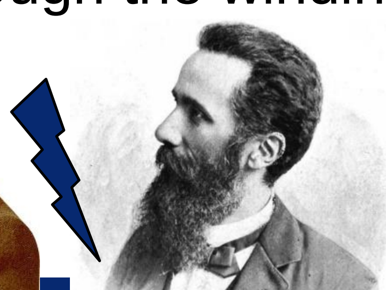
Stator structure
Same for all 3-phase AC machines



We connect the 3 windings to a 3-phase AC voltage. This will drive a 3-phase AC current through the windings.

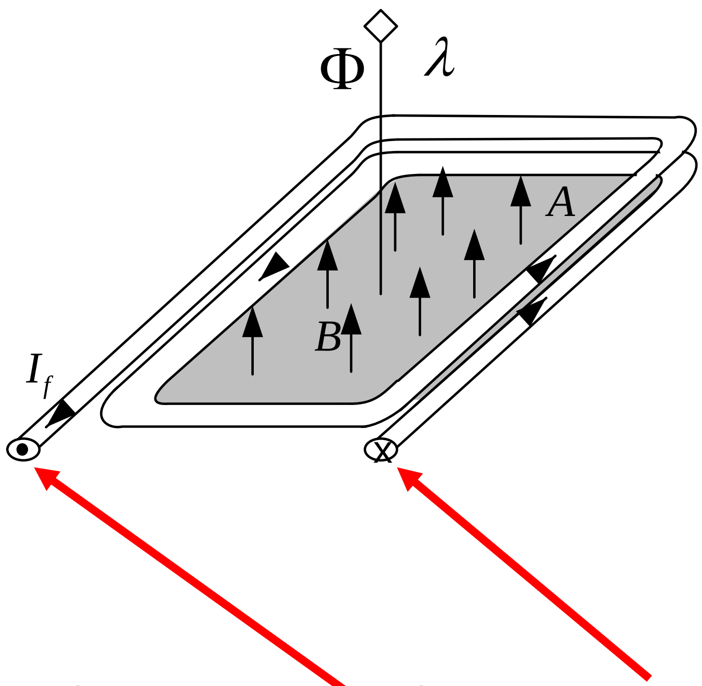
Nicola Tesla
US patent 1888

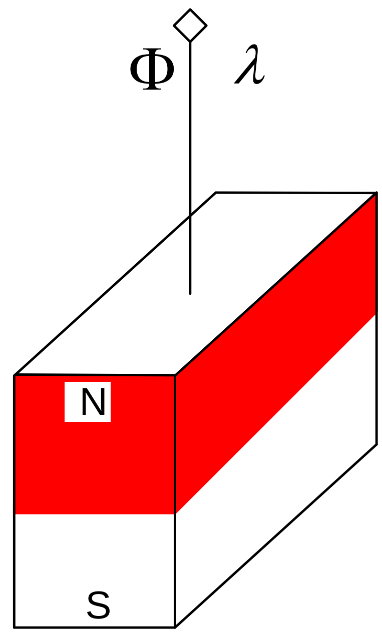




Galileo Ferraris
first prototype 1885

How is the magnetic flux created? Electromagnet or permanent magnet





$B \propto i$ Magnetic flux density [Vs/m²]

$\Phi = AB$ Magnetic flux [Vs]

A Area [m²]

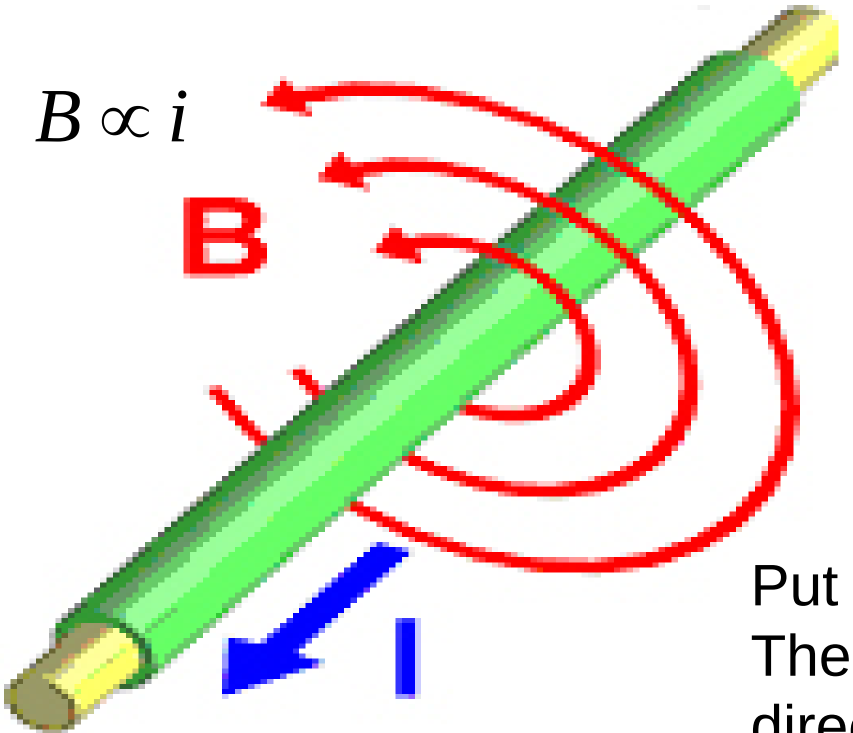
$\Psi = N\Phi$ Flux linkage [Vs]

N Number of turns

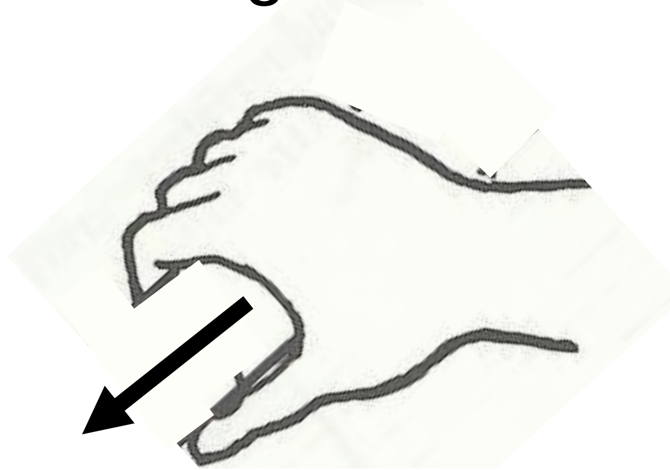
- Current out •, Current in X,
- Flux direction: Use your right hand, thumb in the current direction and fold the other fingers around the conductor, the fingers then point in the magnetic flux direction.

The magnetic flux around a current-carrying conductor

$B \propto i$



The right-hand rule



Put the thumb in the direction of the current. The fingers then follow the magnetic flux direction.

In which directions does the three windings generate flux?

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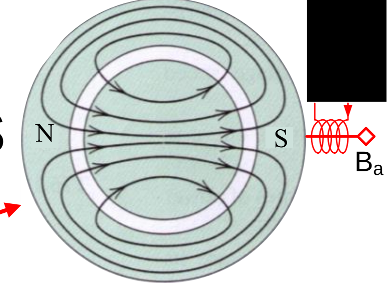
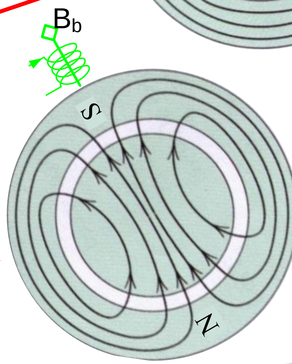
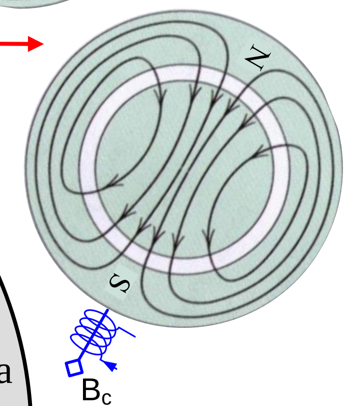
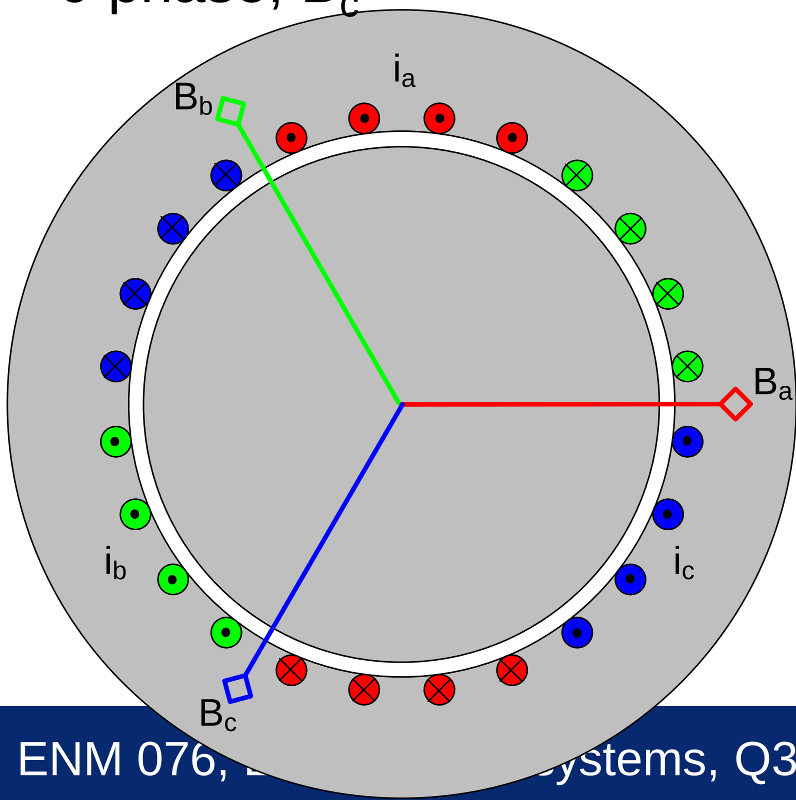
Stator structure
Same for all 3-phase AC machines

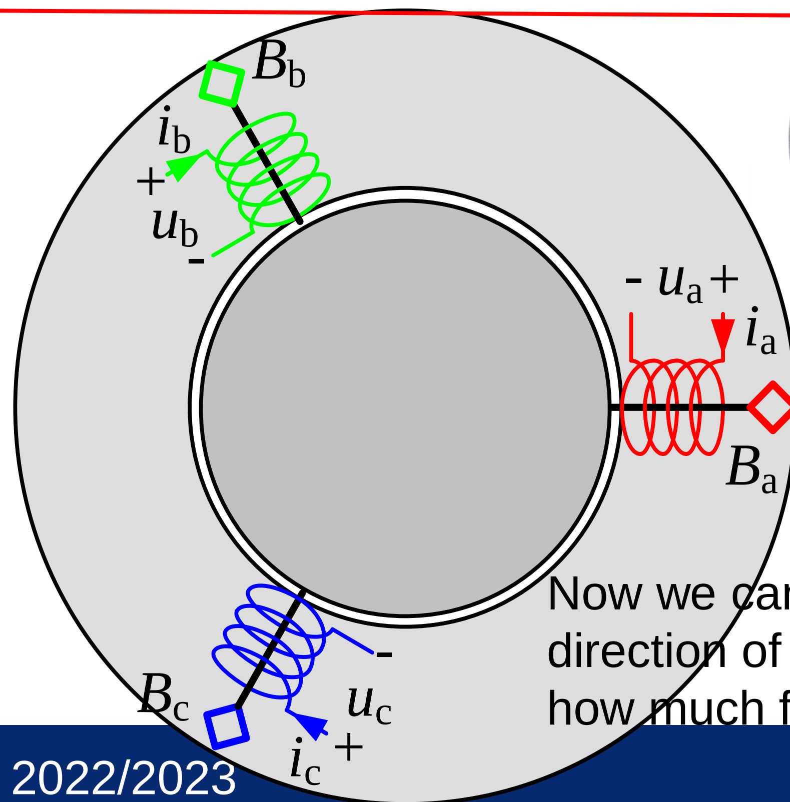
- In which direction in the machine will the flux generated by a positive phase current be?
- A positive current is according to the * and x in the figure.
- Start with the a-phase, B_a
- b-phase, B_b
- c-phase, B_c











Now we can determine the direction of the flux, but how much flux do we get?

Relation between voltage and flux linkage

”Faraday-Henry’s law”

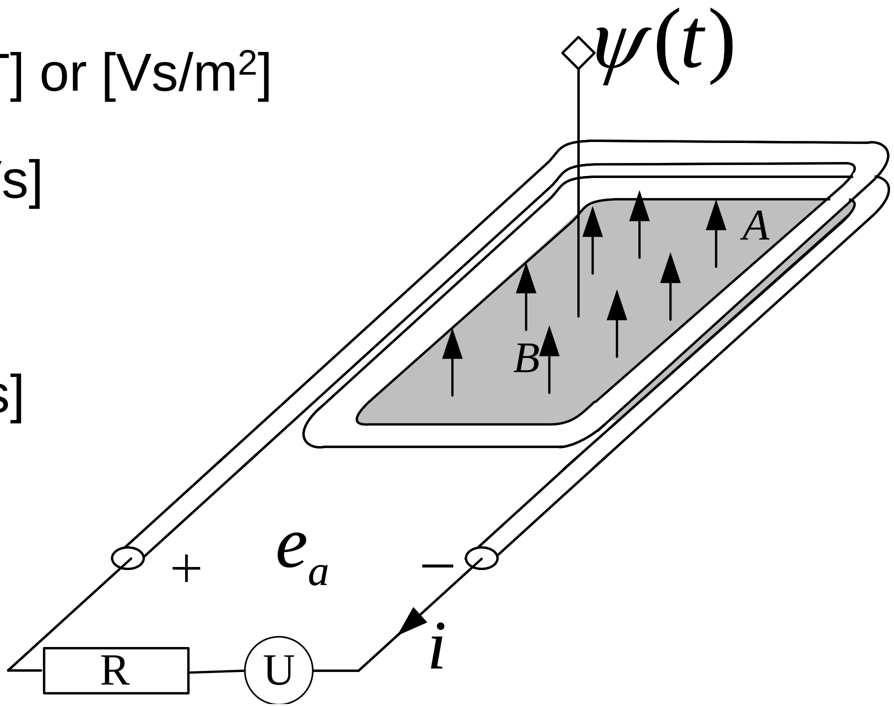
B Magnetic field density [T] or [Vs/m²]

$\Phi \propto AB$ Magnetic flux [Wb] or [Vs]

A Area [m²]

$\psi = N\Phi$ Flux linkage [Wb] or [Vs]

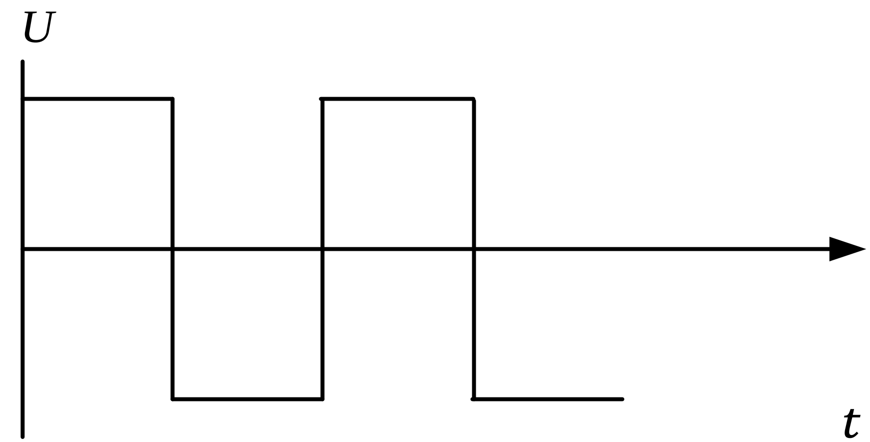
N Number of turns

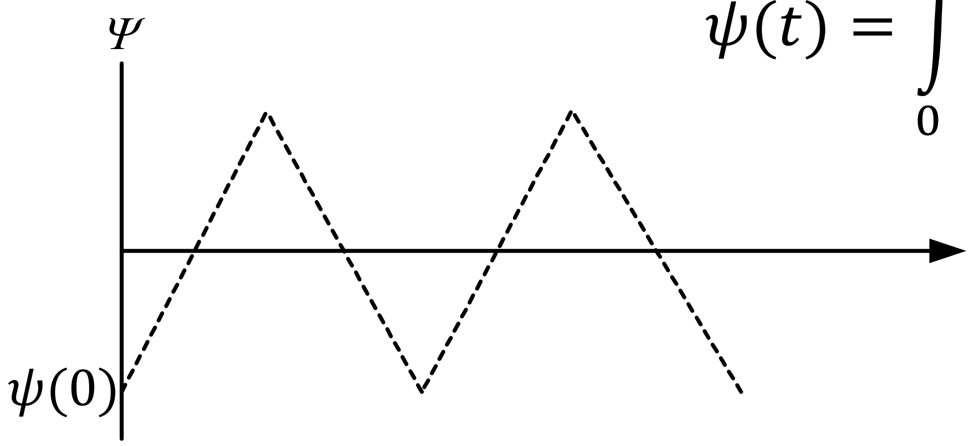


For a winding with several turns N

$$e = \frac{Nd\Phi}{dt} = \frac{d\psi}{dt}, \quad u - Ri = \frac{d\psi}{dt}, \quad \psi = \int (u - Ri)dt$$

Formation of flux linkage from voltage:





$$\psi(t) = \int_0^t (u - Ri)dt + \psi(0)$$

The flux linkage from the three coils

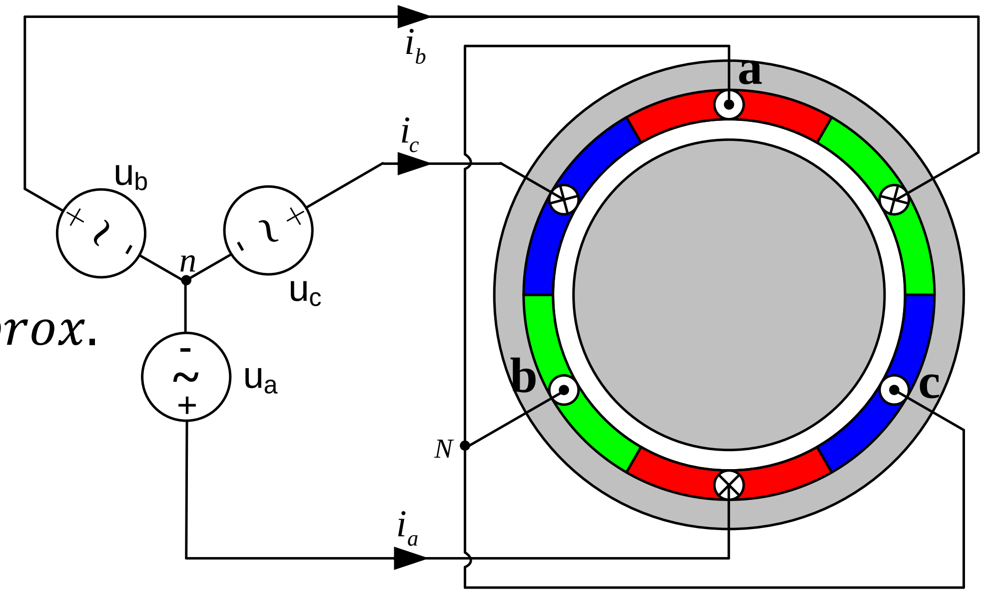
A three-phase voltage connected to three coils arranged as in figure

Three-phase voltage

$$u_a(t) = \hat{U}\cos(\omega t + \phi)$$

$$u_b(t) = \hat{U}\cos\left(\omega t + \phi - \frac{2\pi}{3}\right) \rightarrow approx.$$

$$u_c(t) = \hat{U}\cos\left(\omega t + \phi - \frac{4\pi}{3}\right)$$



$$\psi_a(t) = \hat{\psi}\cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

$$\psi_b(t) = \hat{\psi}\cos\left(\omega t + \phi - \frac{2\pi}{3} - \frac{\pi}{2}\right)$$

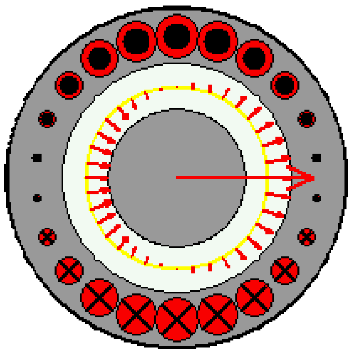
$$\psi_c(t) = \hat{\psi}\cos\left(\omega t + \phi - \frac{4\pi}{3} - \frac{\pi}{2}\right)$$

The flux linkage we get if we integrate the voltage-Ri, if we neglect the resistance of the winding the flux linkage will be lagging the voltage by 90º

Lett us start by looking on the flux linkage from phase a.

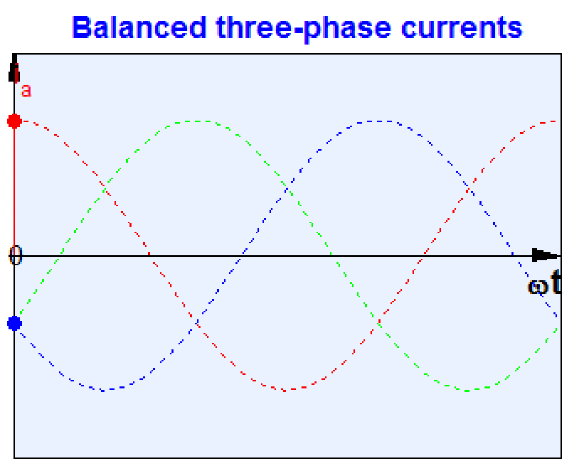
Flux linkage from phase a

$$\psi_a(t)$$



Phase A

$$B \propto i \rightarrow \psi \propto i$$

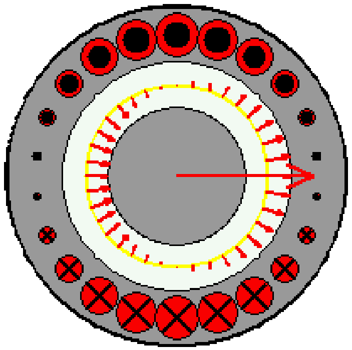


What is the flux linkage from phase b and c?

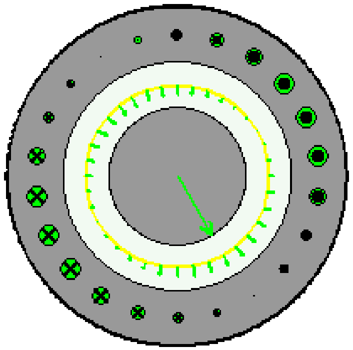
http://people.ece.umn.edu/users/riaz/animations/abcvec.html

Flux linkage from each phase

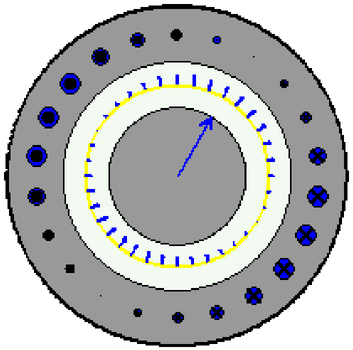
$\psi_a(t)$



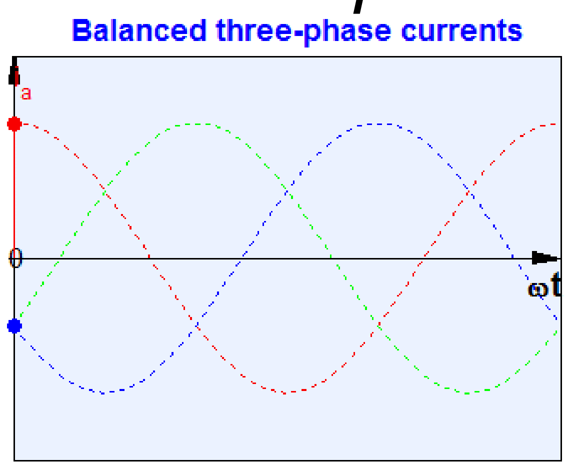
$\psi_b(t)$



$\psi_c(t)$



$$B \propto i \;\rightarrow \psi \propto i$$



We get 3 flux linkage that are

- 120º twisted apart in the room
- and 120º phase shifted in time

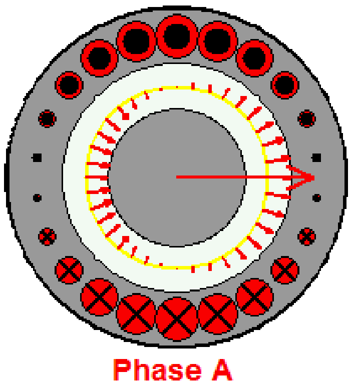
What will the total resultant flux linkage be, the air gap flux linkage $\psi_{gap}(t) = \psi_a(t) + \psi_b(t) + \psi_c(t)$?

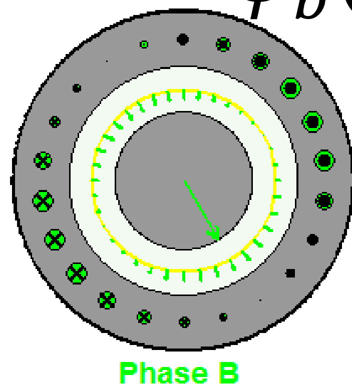
http://people.ece.umn.edu/users/riaz/animations/abcvec.html

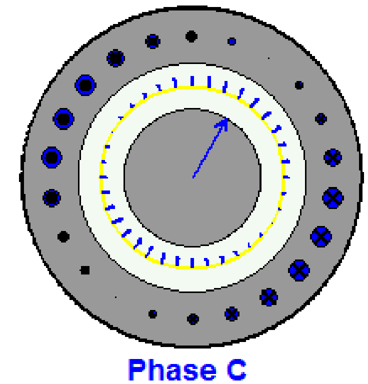
Rotating flux

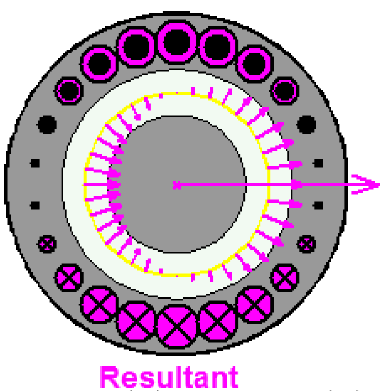
The total magnetic flux density becomes a rotating flux density with constant amplitude, and it makes one revolution per electrical period

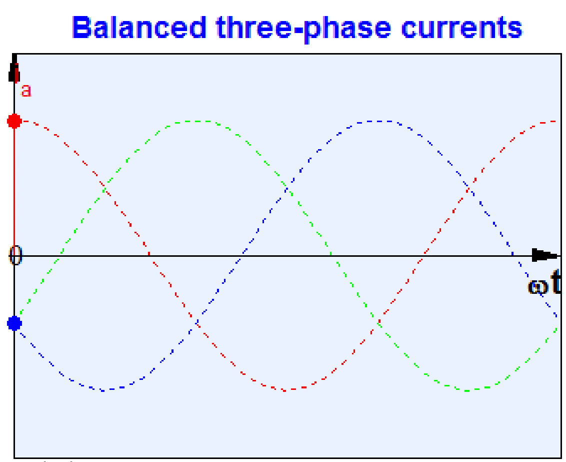
$\psi_a(t)$ $\psi_b(t)$ $\psi_c(t)$

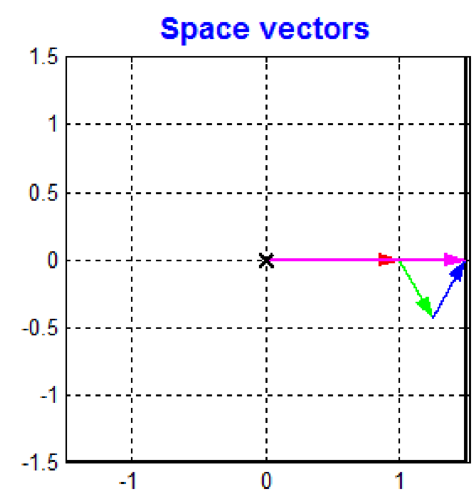












$$\psi_{gap}(t) = \psi_a(t) + \psi_b(t) + \psi_c(t)$$

http://people.ece.umn.edu/users/riaz/animations/abcvec.html

Rotating magnetic flux density in the airgap

This is the main principle for all 3-phase AC machines

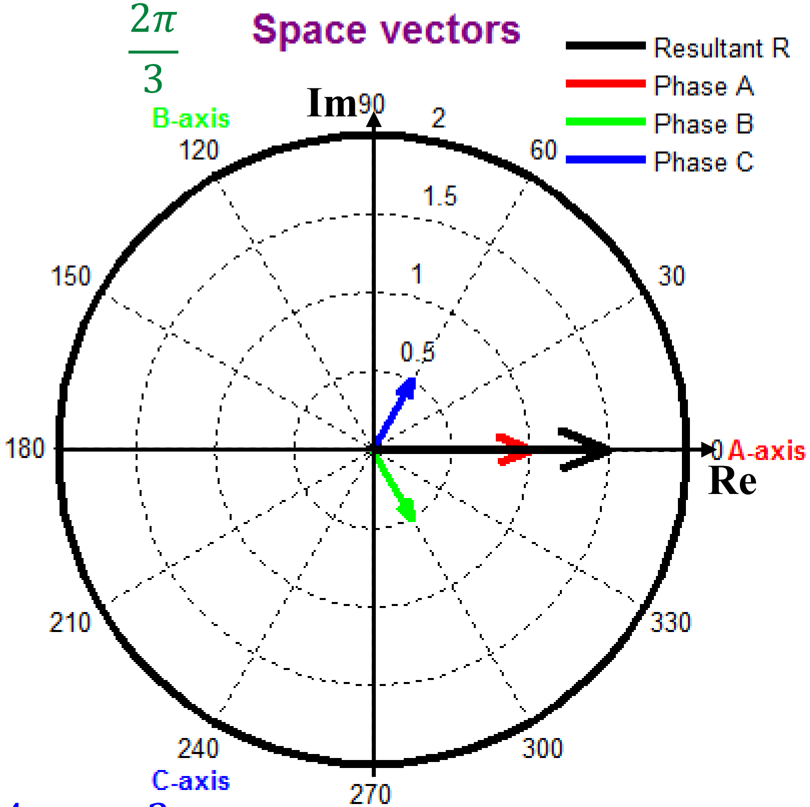
- Stator is constructed with 3 windings (one per phase) that are 120º twisted apart in the room
- The stator is connected to a symmetrical sinusoidal three-phase voltage, this means three voltages with the same magnitude and frequency, and they are 120º phase shifted in time
- The voltages will create three flux linkages, one from each phase. The flux linkages comes from the current flowing in the winding. The flux linkages from each phase will vary in size sinusoidal but they will have the same orientation in the room all the time, i.e. it is stationary. The 3 flux linkage are:
  - 120º twisted apart in the room
  - and 120º phase shifted in time
- If the three flux linkages are added together, we will get a rotating flux linkages which has the same amplitude all the time and which moves around at a speed determined by the frequency and number of poles of the windings. The consequence is that it seems that we have a magnet with constant flux linkages that moves around in the stator.
- We call the speed at which the flux linkages rotates at the synchronous speed.
- We can show that the total flow has a constant amplitude and rotates by adding the three fluxes at any given time and finding that we get a resultant flux linkages which has a constant amplitude, and which rotates around at a synchronous speed.

How can we mathematically model this?

CHALMERS How can we calculate the resultant field?

Tool: Space vectors, Three-phase to two-phase transformation

$\frac{2\pi}{3}$



$\frac{4\pi}{3} = -\frac{2\pi}{3}$

Complex number

http://people.ece.umn.edu/users/riaz/animations/spacevectors.html

What is a space vector?

We put in the complex plane on our machine and align the real axis with the direction of the a-phase.

Then the phase value is multiplied with the direction of the phase coil, for the a-phase the direction is 1 since it is aligned with the real axis and the space vector for the a-phase is

$\underline{\psi_a}(t) = \psi_\mathrm{a}(t) \cdot 1$ the red vector

For the b- and c-phase we get

$\underline{\psi_b}(t) = \psi_\mathrm{b}(t) \cdot e^{j\frac{2\pi}{3}}$ the green vector

$\underline{\psi_c}(t) = \psi_\mathrm{c}(t) \cdot e^{j\frac{4\pi}{3}}$ the blue vector

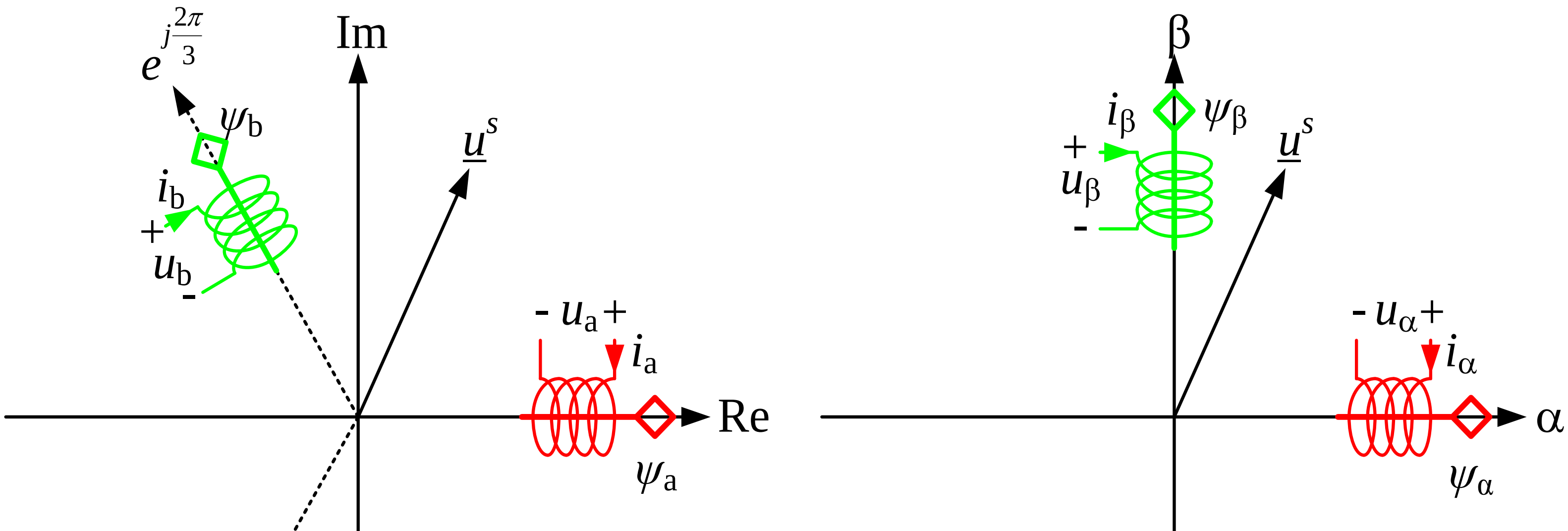
The total space vector is obtained by adding the three contributions together and scale the length

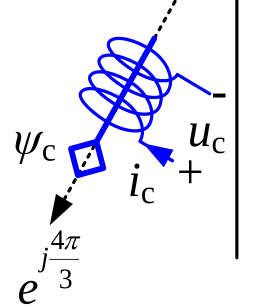
$$\underline{\psi}^s(t) = \frac{2}{3} K \left(\underline{\psi_a}(t) + \underline{\psi_b}(t) + \underline{\psi_c}(t) \right)$$
$$= \psi_\alpha(t) + j\psi_\beta(t)$$

Then we get the complex resulting black vector

Tool: Space vectors, Three-phase to two-phase transformation

Neglect the zero-sequence, which means that $u_a(t) + u_b(t) + u_c(t) = 0 \Rightarrow$ $u_c(t) = -(u_a(t) + u_b(t)) \Rightarrow$ **Only need a two-phase system**





$$\underline{\psi}^s(t) = \frac{2}{3}K\left(\psi_\mathrm{a}(t) + \psi_\mathrm{b}(t)e^{j\frac{2\pi}{3}} + \psi_\mathrm{c}(t)e^{j\frac{4\pi}{3}}\right) = \psi_\alpha(t) + j\psi_\beta(t)$$

The contribution of the phase flux linkages are multiplied with their respective direction and scaled with the constant 2/3K and summed together.

All phase quantities can be transformed in this way, u, i,…

Transforming a three phase voltage to the αβ-system

Three-phase voltage

$$u_a(t) = V\cos(\omega t + \phi)$$
$$u_b(t) = V\cos\left(\omega t + \phi - \frac{2\pi}{3}\right)$$
$$u_c(t) = V\cos\left(\omega t + \phi - \frac{4\pi}{3}\right)$$

Three-phase to two-phase transformation

$$\underline{u}^s(t) = u_\alpha(t) + ju_\beta(t) = K\frac{2}{3}\left(u_\mathrm{a}(t) + u_\mathrm{b}(t)e^{j\frac{2\pi}{3}} + u_\mathrm{c}(t)e^{j\frac{4\pi}{3}}\right)$$

Using Eulers formula $\cos(\varphi) = \dfrac{e^{j\varphi} + e^{-j\varphi}}{2} \quad \Rightarrow \quad u_a = \dfrac{V}{2}\left(e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}\right)\ldots.$

$$\underline{u}^s(t) = K\frac{V}{2}\frac{2}{3}\left(e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)} + \left(e^{j\left(\omega t+\varphi-\frac{2\pi}{3}\right)} + e^{-j\left(\omega t+\varphi-\frac{2\pi}{3}\right)}\right)e^{j\frac{2\pi}{3}} + \left(e^{j\left(\omega t+\varphi-\frac{4\pi}{3}\right)} + e^{-j\left(\omega t+\varphi-\frac{4\pi}{3}\right)}\right)e^{j\frac{4\pi}{3}}\right)$$

$$= \frac{KV}{3}\left(e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)} + e^{j(\omega t+\varphi)} + e^{-j\left(\omega t+\varphi-\frac{4\pi}{3}\right)} + e^{j(\omega t+\varphi)} + \underbrace{e^{-j\left(\omega t+\varphi-\frac{8\pi}{3}\right)}}_{=e^{-j\left(\omega t+\varphi-\frac{2\pi}{3}\right)}}\right)$$

$$= \frac{KV}{3}\left(3e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)}e^{j\frac{4\pi}{3}} + e^{-j(\omega t+\varphi)}e^{j\frac{2\pi}{3}}\right) = \frac{KV}{3}\left(3e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)}\left(1 + e^{j\frac{4\pi}{3}} + e^{j\frac{2\pi}{3}}\right)\right)$$

$$= \frac{KV}{3}\left(3e^{j(\omega t+\varphi)} + e^{-j(\omega t+\varphi)}\left(1 + \underbrace{\cos\frac{4\pi}{3}}_{-0.5} + \underbrace{\cos\frac{2\pi}{3}}_{-0.5} + j\left(\underbrace{\sin\frac{4\pi}{3}}_{=-\frac{\sqrt{3}}{2}} + \underbrace{\sin\frac{2\pi}{3}}_{\frac{\sqrt{3}}{2}}\right)\right)\right) \Rightarrow$$

$$\boxed{\underline{u}^s(t) = KVe}^{j(\omega t+\phi)}$$

Amplitude invariant scaling: $K = 1 \Rightarrow \left|\underline{u}^s\right| = V$

RMS-value scaling: $K = 1/\sqrt{2} \Rightarrow \left|\underline{u}^s\right| = V/\sqrt{2}$

Three-phase to two-phase transformation on matrix form

$$\underline{u}^s(t) = K\frac{2}{3}\left(u_\mathrm{a}(t) + u_\mathrm{b}(t)e^{j\frac{2\pi}{3}} + u_\mathrm{c}(t)e^{j\frac{4\pi}{3}}\right) =$$

$$K\frac{2}{3}\left(u_\mathrm{a}(t) + u_\mathrm{b}(t)\cos(\frac{2\pi}{3}) + ju_\mathrm{b}(t)\sin(\frac{2\pi}{3}) + u_\mathrm{c}(t)\cos(\frac{4\pi}{3}) + ju_\mathrm{c}(t)\sin(\frac{4\pi}{3})\right) =$$

$$K\frac{2}{3}\left(u_\mathrm{a}(t) - \frac{1}{2}u_\mathrm{b}(t) - \frac{1}{2}u_\mathrm{c}(t) + j\frac{\sqrt{3}}{2}\left(u_\mathrm{b}(t) - u_\mathrm{c}(t)\right)\right) =$$

$$K\left(\frac{2}{3}u_\mathrm{a}(t) - \frac{1}{3}u_\mathrm{b}(t) - \frac{1}{3}u_\mathrm{c}(t) + j\frac{1}{\sqrt{3}}\left(u_\mathrm{b}(t) - u_\mathrm{c}(t)\right)\right) = u_\alpha(t) + ju_\beta(t)$$

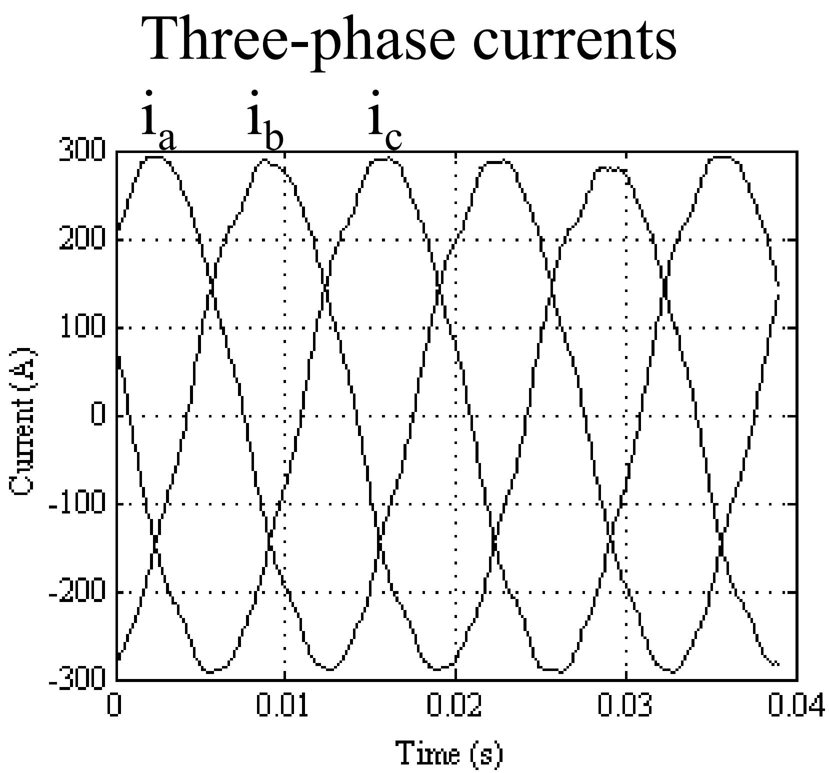
$$\begin{bmatrix} s_\alpha \\ s_\beta \end{bmatrix} = K\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} s_\mathrm{a} \\ s_\mathrm{b} \\ s_\mathrm{c} \end{bmatrix}$$

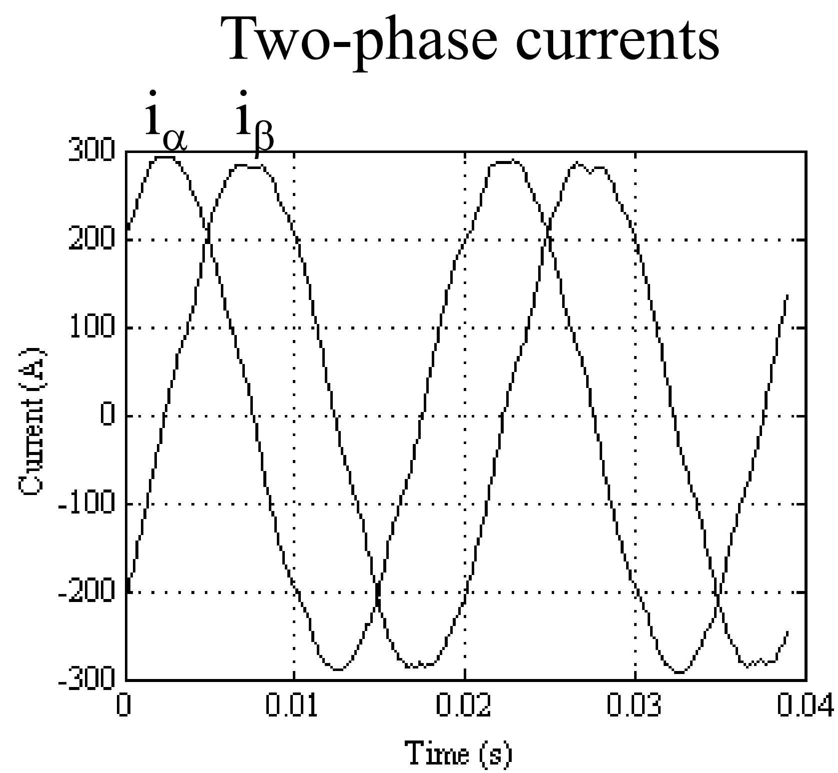
Clarke transformation

Edith Clarke

$$\begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} = \frac{1}{K}\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} s_\alpha \\ s_\beta \end{bmatrix}$$

Example of a three-phase to two-phase transformation





If there is no zero-sequence (i.e. if all phase values add to zero at all the time), no information is lost when transforming from three-phase to two-phase!

Active and reactive power

The instantaneous active and reactive power can be calculated as

$$p(t)=\left(u_{\mathrm{a}}(t)i_{\mathrm{a}}(t)+u_b(t)i_b(t)+u_c(t)i_c(t)\right)=$$

$$\frac{3}{2K^2}\left(u_\alpha(t)i_\alpha(t)+u_\beta(t)i_\beta(t)\right)=\frac{3}{2K^2}\mathrm{Re}\left\{\underline{u}^s\,\underline{i}^{s*}\right\}$$

Complex conjugate

$$\underline{i}^s=i_\alpha+ji_\beta\Rightarrow$$

$$\underline{i}^{s*}=i_\alpha-ji_\beta$$

$$q(t)=\frac{1}{\sqrt{3}}\left(u_{\mathrm{a}}(t)\left(i_{\mathrm{c}}(t)-i_b(t)\right)+u_b(t)\left(i_{\mathrm{a}}(t)-i_c(t)\right)+u_c(t)\left(i_{\mathrm{b}}(t)-i_a(t)\right)\right)=$$

$$\frac{3}{2K^2}\left(u_\beta(t)i_\alpha(t)-u_\alpha(t)i_\beta(t)\right)=\frac{3}{2K^2}\mathrm{Im}\left\{\underline{u}^s\,\underline{i}^{s*}\right\}$$

But when we say active and reactive power, we do not mean the instantaneous values we mean the average over one period

$$P=\mathrm{AVG}\left\{p(t)\right\}\qquad Q=\mathrm{AVG}\left\{q(t)\right\}$$

Selection of scaling constant K

The scaling constant can be selected arbitrarily $\neq 0$.

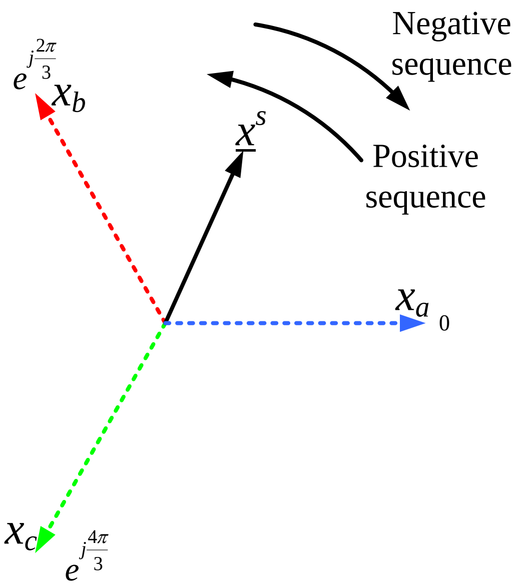
For a three-phase sinusoidal quantity $x_a(t), x_b(t), x_c(t)$

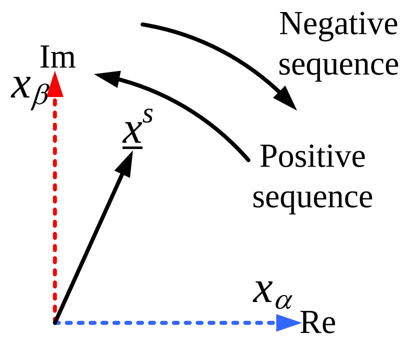
Amplitude invariant scaling: $K = 1 \Rightarrow \left|\underline{x}^s\right| = \hat{x}_a = \hat{x}_b = \hat{x}_c$

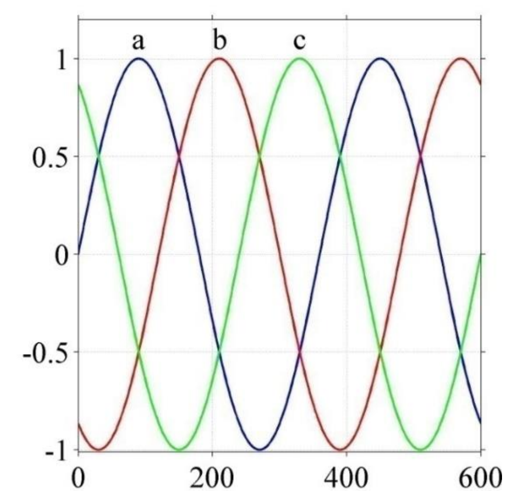
We will use this

RMS-value scaling: $K = 1/\sqrt{2} \Rightarrow \left|\underline{x}^s\right| = X_{a,RMS} = X_{b,RMS} = X_{c,RMS}$

Power-invariant scaling: $K = \sqrt{3/2} \Rightarrow p = u_\alpha i_\alpha + u_\beta i_\beta$







Representation of currents, voltages and flux linkages using space vectors

The stationary system (αβ system)

$$\underline{i}_s^s = i_\alpha + j i_\beta$$

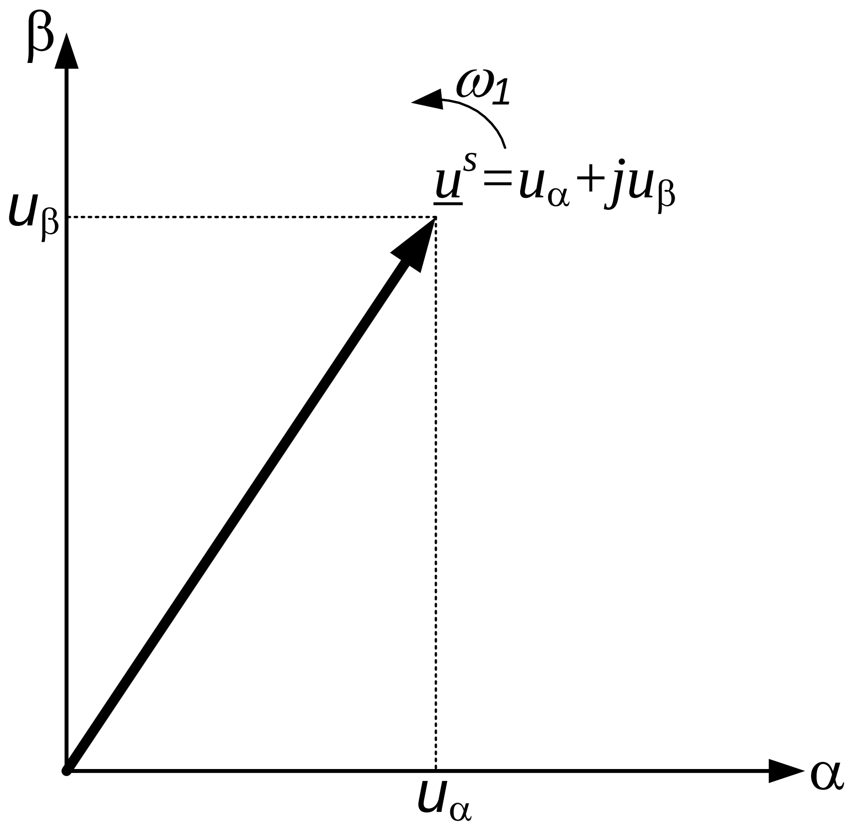
$$\underline{u}_s^s = u_\alpha + j u_\beta$$

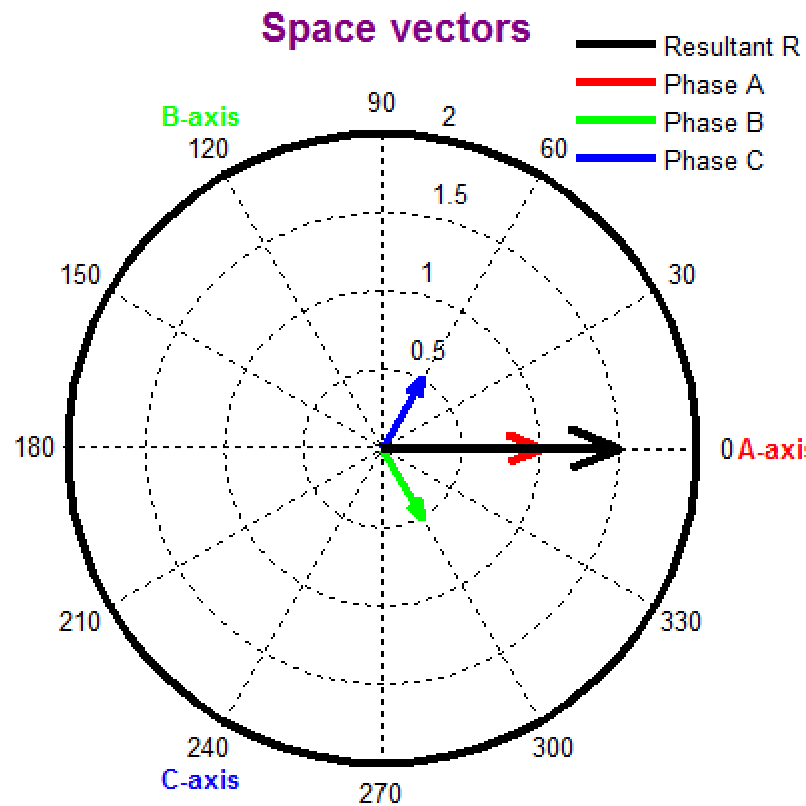
$$\underline{\psi}_s^s = \psi_\alpha + j \psi_\beta$$

Stator (flux linkage)

Complex number

Voltage rotating in the $\alpha\beta$-system





Rotating coordinate system

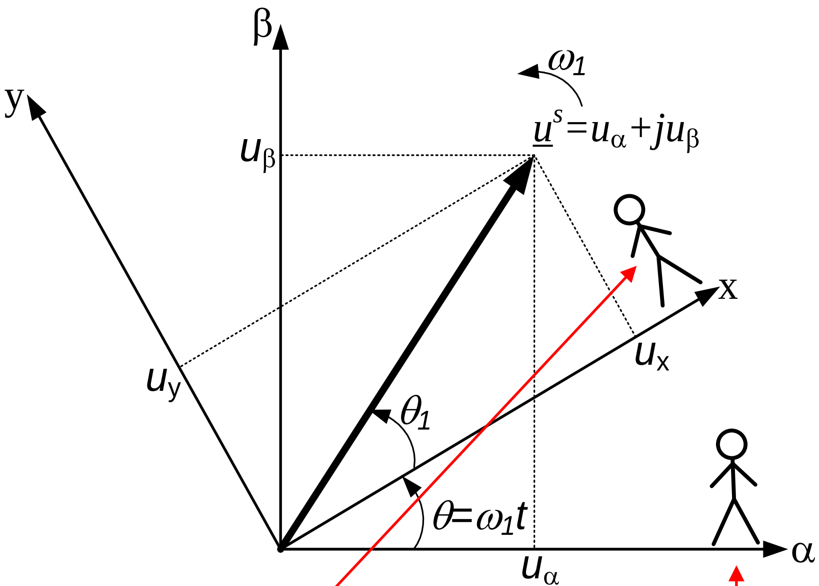
$$\underline{u}^{s} = \left|\underline{u}^{s}\right| e^{j(\theta_1+\theta)} = \underbrace{\left|\underline{u}^{s}\right| e^{j\theta_1}}_{\underline{u}^{xy}} e^{j\theta} = \underline{u}^{xy} e^{j\theta} =$$

$$= \left(u_x + ju_y\right)\left(\cos\theta + j\sin\theta\right) =$$

$$= \left(u_x\cos\theta - u_y\sin\theta\right) + j\left(u_y\cos\theta + u_x\sin\theta\right) =$$

$$= u_\alpha + ju_\beta \quad \Rightarrow$$

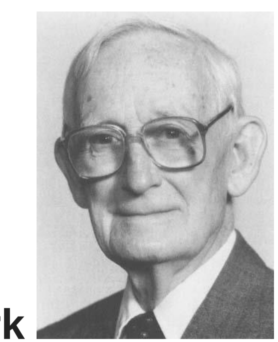
$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$



If we instead observe the vector in a system that rotates with the same speed as the vector, we see a stationary vector, i.e an DC signal

Here we are observing the vector in the stationary system (αβ), we then see the vector rotating, i.e an AC signal

Park transformation



Robert H. Park

Moving between stationary and rotating system

From stationary to rotating Multiplying the quantities with $e^{-j\theta}$

$$\underline{u}^{xy} = \underline{u}^{s} e^{-j\theta}$$

From rotating to stationary Multiplying the quantities with $e^{j\theta}$

$$\underline{u}^{s} = \underline{u}^{xy} e^{j\theta}$$

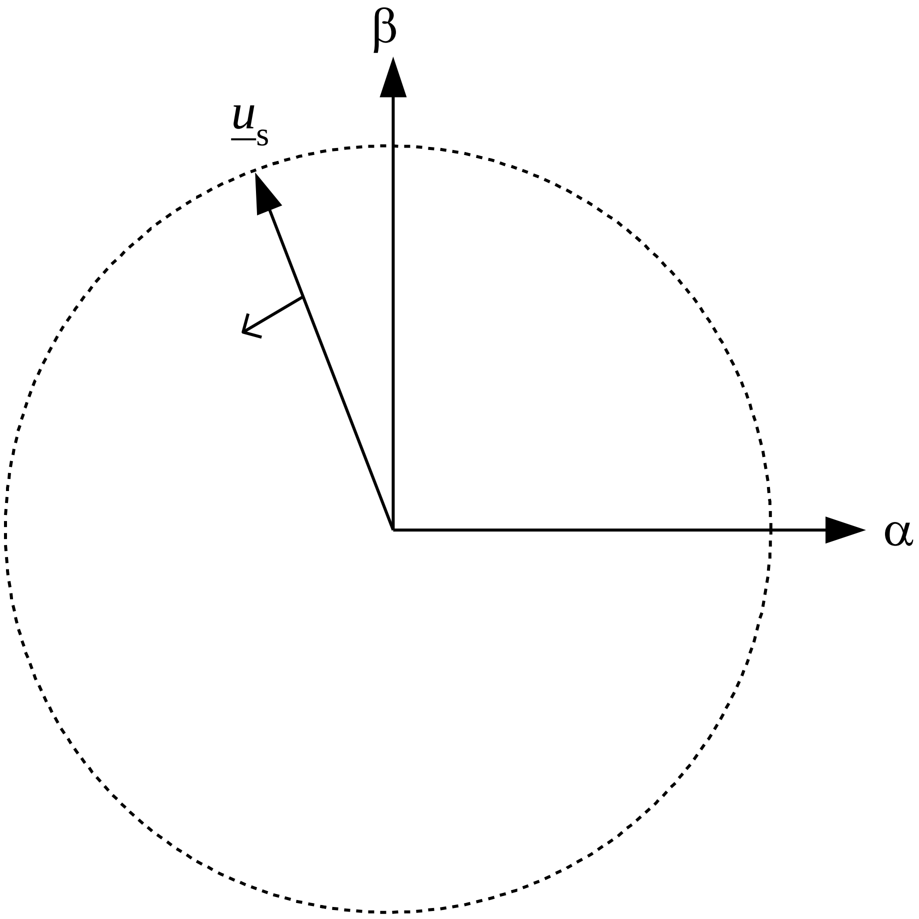
The calculation of active power (and reactive power) is the same, for αβ we hade that

$p(t) = \frac{3}{2K^2}\mathrm{Re}\{\underline{u}^s \underline{i}^{s*}\}$ If we move this to dq $\underline{u}^s = \underline{u}e^{j\theta}$ *and* $\underline{i}^s = \underline{i}e^{j\theta}$

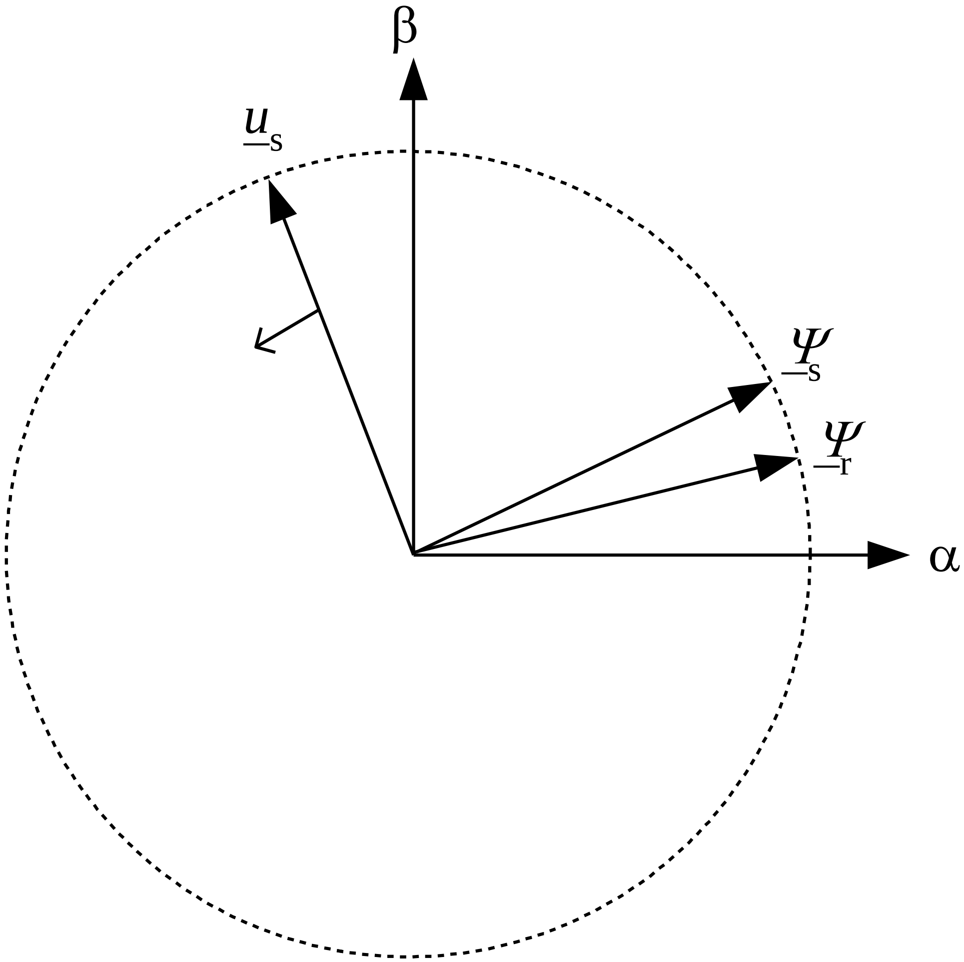
$$p(t) = \frac{3}{2K^2}\mathrm{Re}\{\underline{u}e^{j\theta}\left(\underline{i}e^{j\theta}\right)^*\} = \frac{3}{2K^2}\mathrm{Re}\{\underline{u}e^{j\theta}\underline{i}^* e^{-j\theta}\} = \frac{3}{2K^2}\mathrm{Re}\{\underline{ui}^*\}$$

What should be the θ be for our modeling?

Rotating voltage vector

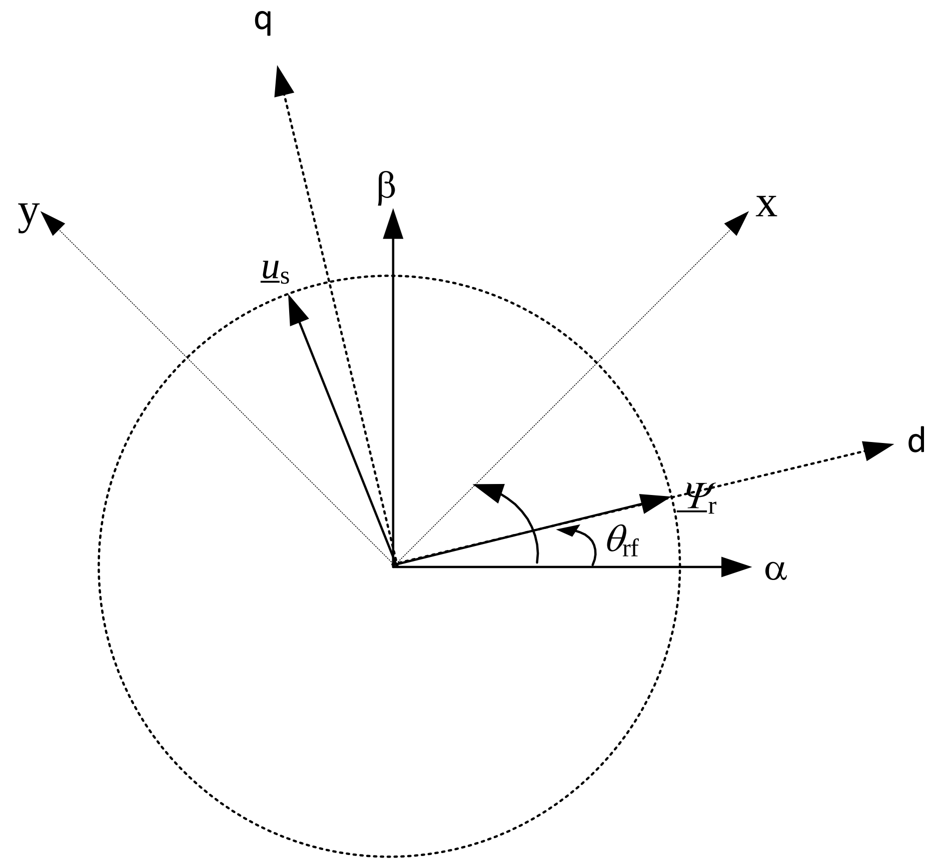


Rotating voltage vector, formation of flux linkage



$$\underline{\psi}_s(t) = \int_0^t (\underline{u}_s - R_s \underline{i}_s) dt$$

dq coordinate system, rotor flux in d-direction



xy-system: θ arbitrarily selected

dq-system: Rotor flux oriented in x-direction (d-direction)

Representation of currents, voltages and flux linkages using space vectors

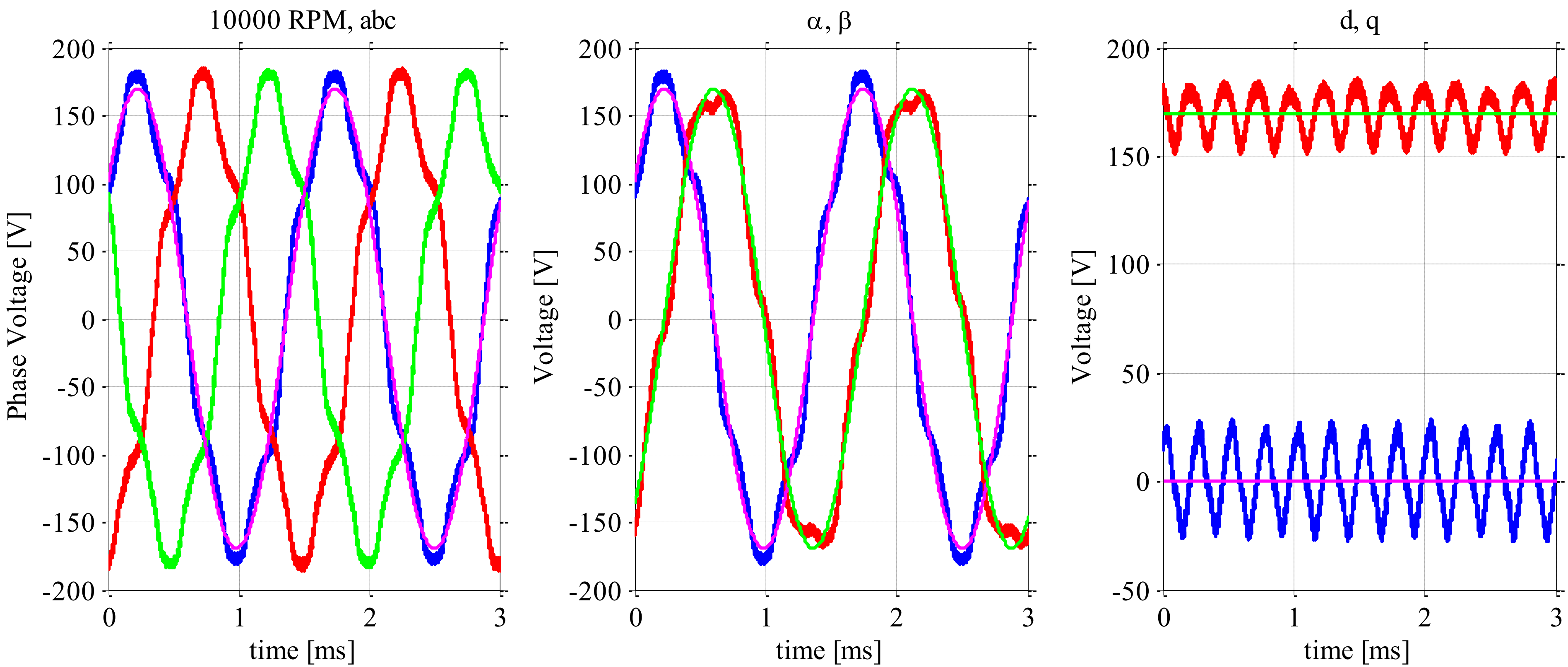
$$\underline{i}_s^s = i_{s\alpha} + j i_{s\beta} \quad \Leftrightarrow \quad \underline{i}_s \quad \Leftrightarrow \quad i_{sd} + j i_{sq} = \underline{i}_s^{\cancel{dq}}$$

$$\underline{u}_s^s = u_{s\alpha} + j u_{s\beta} \quad \Leftrightarrow \quad \underline{u}_s \quad \Leftrightarrow \quad u_{sd} + j u_{sq} = \underline{u}_s^{\cancel{dq}}$$

$$\underline{\psi}_s^s = \psi_{s\alpha} + j \psi_{s\beta} \quad \Leftrightarrow \quad \underline{\psi}_s \quad \Leftrightarrow \quad \psi_{sd} + j \psi_{sq} = \underline{\psi}_s^{\cancel{dq}}$$

These I will not write in the course

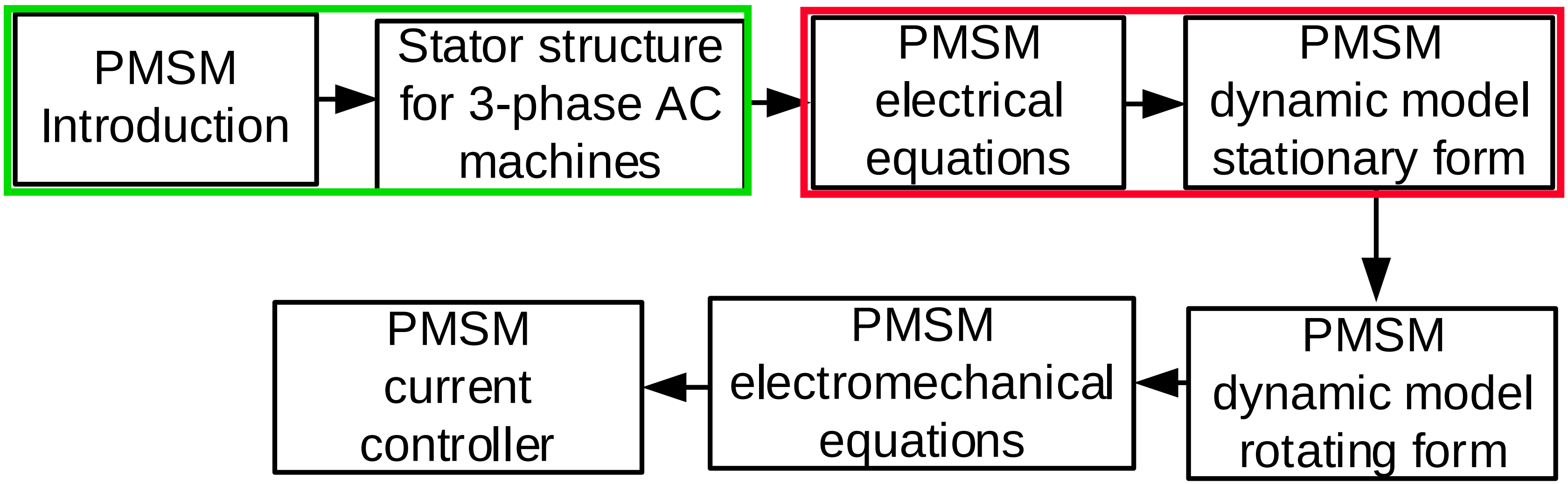
Example of transforming a 3-phase voltage with harmonics to αβ and to dq



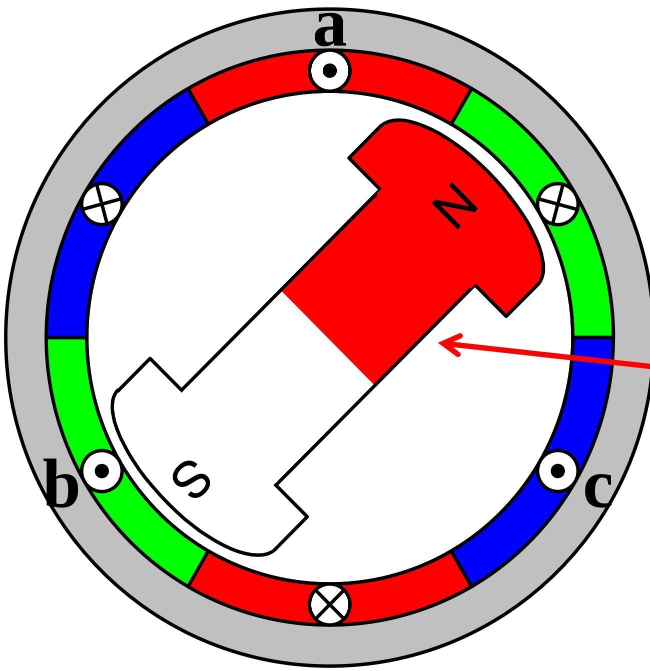
If there is no zero-sequence, no information is lost when transforming from three-phase to two-phase to dq!

The fundamental part (the sine) of the voltage will be a constant in dq, the harmonics will still be harmonics with another frequency

Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system



Dynamic modelling of the PMSM

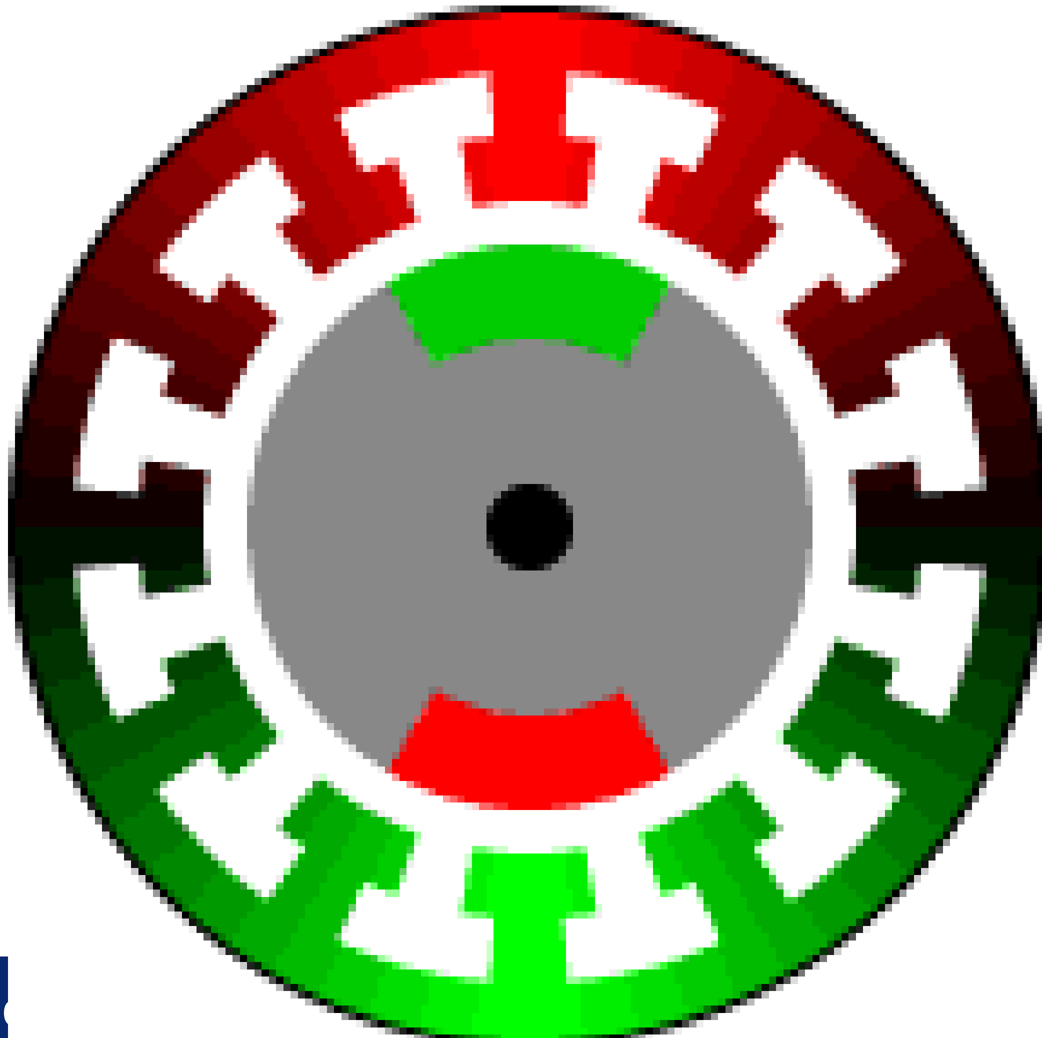


The PMSM:
- 3-phase stator
- A magnet in the rotor

Assumptions:
- No zero-sequence component in the three-phase quantities
- Sinusoidal distribution of the stator windings
- Linear magnetization characteristics
- No losses in the iron
- Resistances independent of temp. and freq.

When the stator creates the rotating field, what will the rotor magnet do?

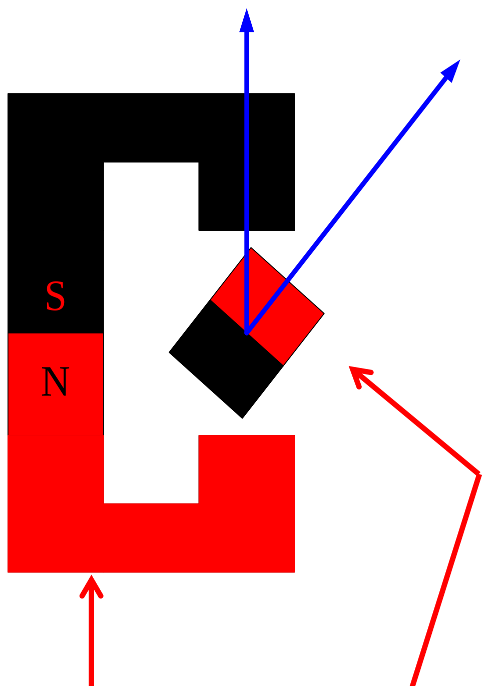
The magnet in the rotor will follow the resultant magnetic field produced by the stator, i.e synchronous with the stator rotating field -> a synchronous machine

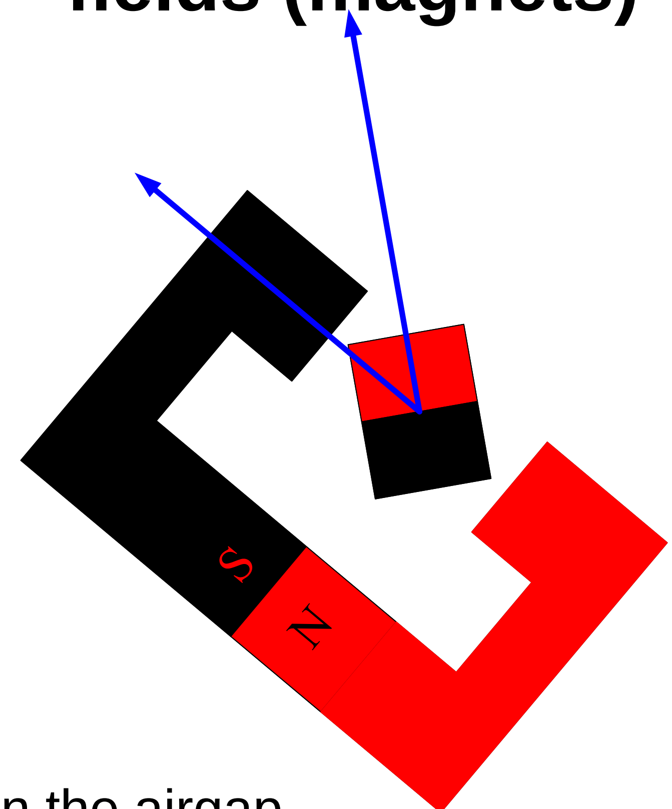


Principal operation of the PMSM

Static behaviour of two magnets

Rotation of both fields (magnets)





With the stator windings we create a flux in the airgap.

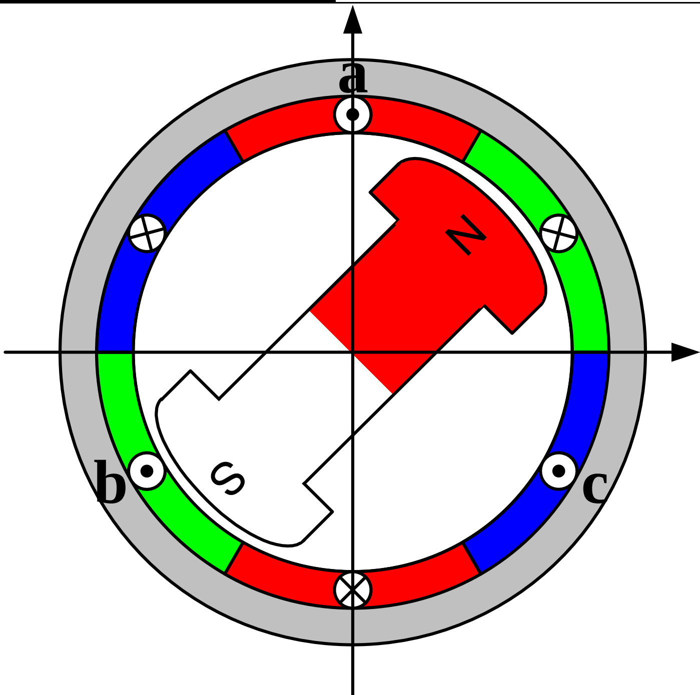
The magnet in the rotor wants to align its flux with this flux

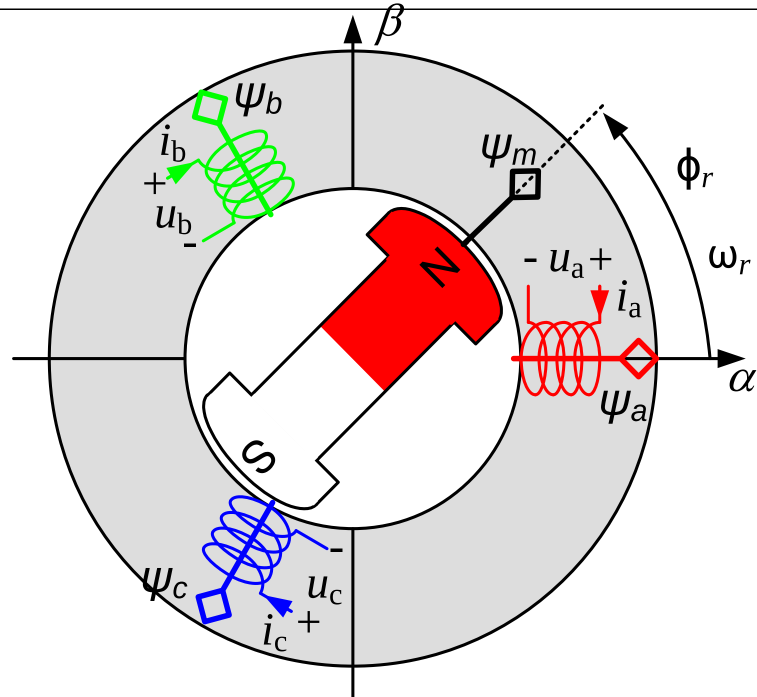
If we load the rotor (put a torque on the shaft) we will pull these tow fluxes apart

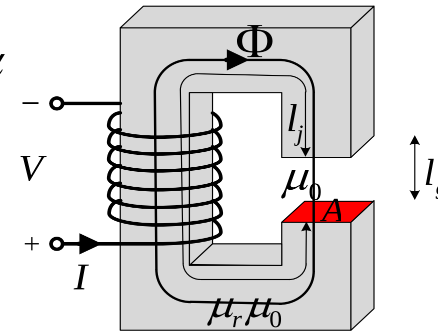
When the stator flux rotates it will drag the rotor with it

CHALMERS

PMSM– derivation of dynamic model







We define the position of the rotor ϕ_r = direction of the rotor flux (ψ_m) and ω_r is the rotational speed of the rotor in rad/s

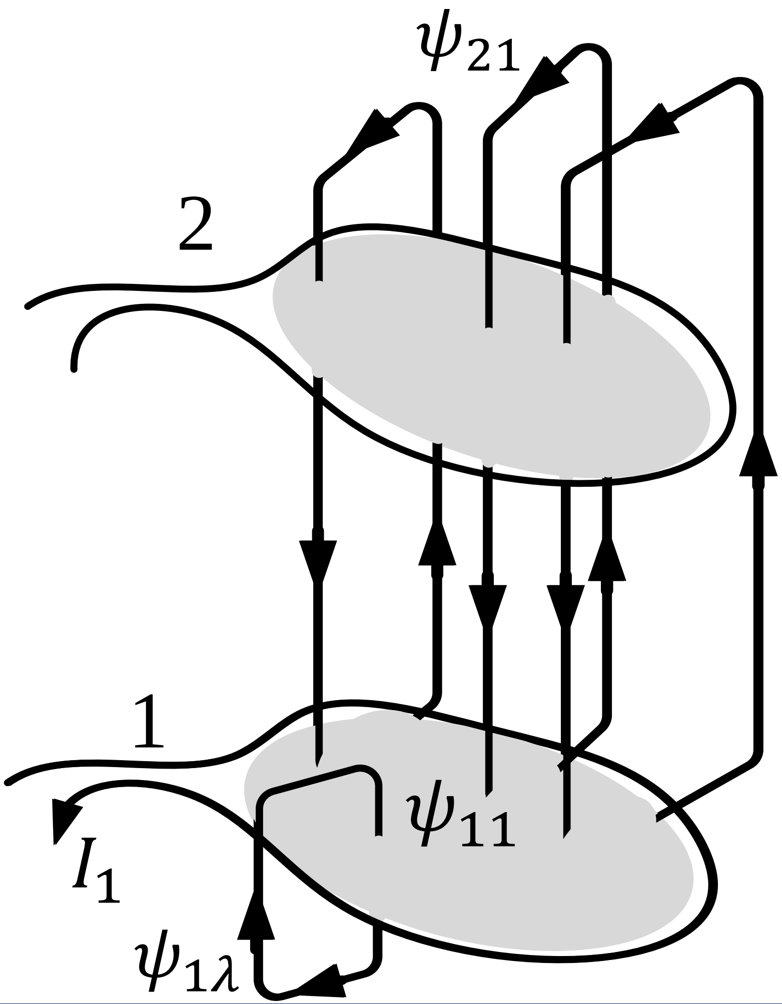
For the derivation we assume that we do not have a ”magnetic” round rotor, we assume that the rotor looks like the one in the figure. This means that the length of the air-gap will be different.

The inductance of an inductor with an airgap can be calculated as $L = \frac{\mu_0 A N^2}{\frac{l_j}{\mu_r} + l_g}$

Along the rotor direction the length of the air-gap, l_g, is short → L large, perpendicular to the rotor direction l_g is longer → L lower

How can we model the fluxes generated by the currents in the 3 coils?

Inductance for a linear magnetic circuit: Mutual, self and leakage inductance



If we run a current I_1 through coil 1 we get the self flux linkage of coil 1 as

$$\psi_{11} = L_{11} I_1$$

This is the total flux linkage created in coil 1 by the current I_1.The subscript is coil-current 11. The relation between the self flux linkage and the current is the self inductance L_{11} of coil 1.

We now introduce coil 2

As we can see in the figure the main part of the self flux linkage of coil 1 also links with coil 2, this part of the flux is the mutual flux

$$\psi_{21} = L_{21} I_1$$

This is the flux linkage in coil 2 caused by I_1. The inductance L_{21} is the mutual inductance and for the mutual inductance we have that $L_{21} = L_{12} = M$ the magnetic connection between coil 1 and 2 is the same as the one between coil 2 and 1.

The part of the self flux linkage of coil 1 that does not link with coil 2 is

$\psi_{1\lambda} = L_{1\lambda} I_1$

This is called the leakage flux and the inductance is the leakage inductance.

In total we have that $\psi_{11} = \psi_{1\lambda} + \psi_{21}$

PMSM– derivation of dynamic model

3-phase model

For the coupled-circuit we can express the phase voltage as:

$$u_a = R_a i_a + \frac{d\psi_a}{dt}$$

$$u_b = R_b i_b + \frac{d\psi_b}{dt}$$

$$u_c = R_c i_c + \frac{d\psi_c}{dt}$$

The applied phase voltage is equal to the resistive voltage drop in the winding + the change of the flux linkage in the winding. This is valid for each of the 3 windings.

The flux linkages for the phases, the windings, can then be expressed as:

$$\psi_a = \boxed{l_{aa}} i_a + l_{ab} i_b + l_{ac} i_c + \psi_m \cos(\theta_r)$$

$$\psi_b = l_{ba} i_a + \boxed{l_{bb}} i_b + l_{bc} i_c + \psi_m \cos(\theta_r - 120°)$$

$$\psi_c = l_{ca} i_a + l_{cb} i_b + \boxed{l_{cc}} i_c + \psi_m \cos(\theta_r + 120°)$$

The total flux linkage of the winding is built up by the current in the winding itself and by the current in the 2 other windings and the magnet in the rotor (ψ =Li)

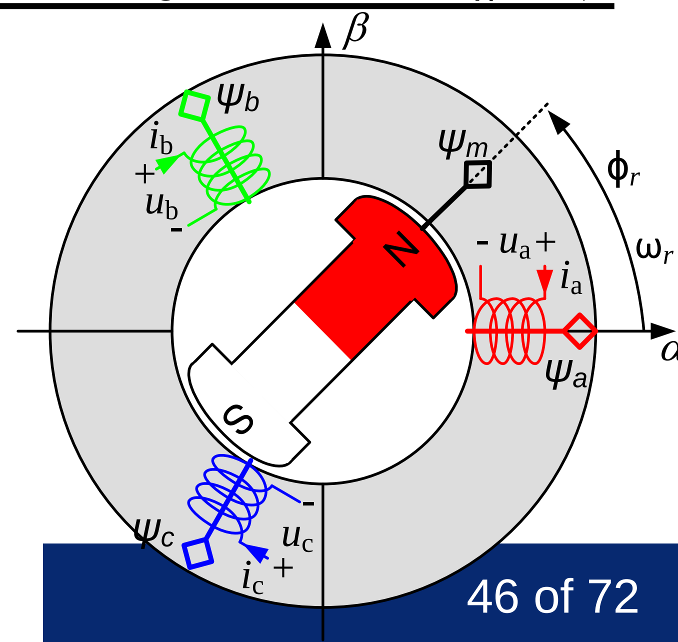
The self-inductance of the stator windings can due to the saliency be approximated as :

$$l_{aa} = L_{aa0} + L_{aa2}\cos(2\theta_r)$$

$$l_{bb} = L_{aa0} + L_{aa2}\cos\big(2(\theta_r - 120°)\big)$$

$$l_{cc} = L_{aa0} + L_{aa2}\cos\big(2(\theta_r + 120°)\big)$$

where L_{aa0} is the constant inductance and L_{aa2} is the variation in the inductance due to the saliency. It varies 2 times per turn and it is highest when the N or S is aligned with the coil



PMSM– derivation of dynamic model

3-phase model

For the coupled-circuit we can express the phase voltage as:

$$u_a = R_a i_a + \frac{d\psi_a}{dt}$$

$$u_b = R_b i_b + \frac{d\psi_b}{dt}$$

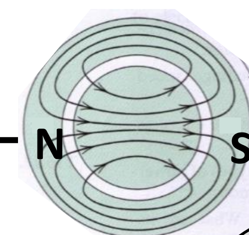
$$u_c = R_c i_c + \frac{d\psi_c}{dt}$$

The flux linkages for the phases can then be expressed as:

$$\psi_a = l_{aa} i_a + \boxed{l_{ab}} i_b + \boxed{l_{ac}} i_c + \psi_m \cos(\theta_r)$$

$$\psi_b = \boxed{l_{ba}} i_a + l_{bb} i_b + \boxed{l_{bc}} i_c + \psi_m \cos(\theta_r - 120°)$$

$$\psi_c = \boxed{l_{ca}} i_a + \boxed{l_{cb}} i_b + l_{cc} i_c + \psi_m \cos(\theta_r + 120°)$$

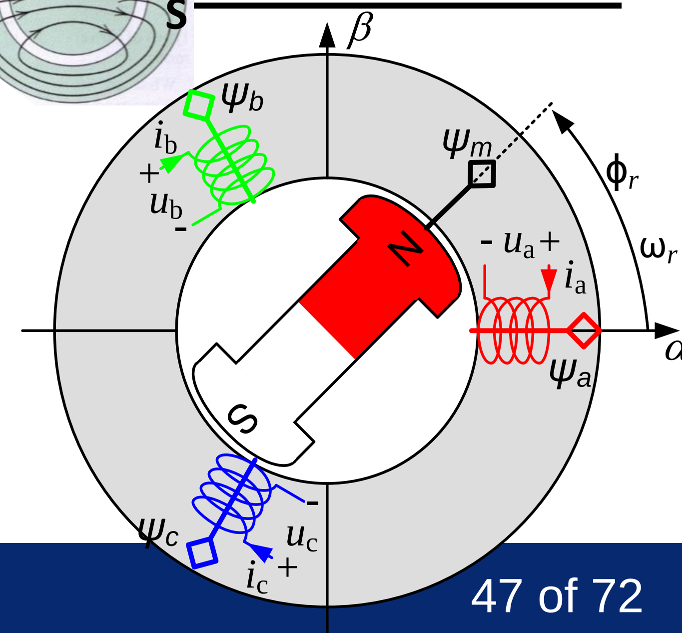


The mutual inductances between the stator windings can be approximated as :

$$l_{ab} = l_{ba} = -\left[L_{ab0} + L_{aa2}\cos\left(2(\theta_r + 30°)\right)\right]$$

$$l_{bc} = l_{cb} = -\left[L_{ab0} + L_{aa2}\cos\left(2(\theta_r - 90°)\right)\right]$$

$$l_{ca} = l_{ac} = -\left[L_{ab0} + L_{aa2}\cos\left(2(\theta_r + 150°)\right)\right]$$



The mutual inductance has a negative value because a positive flux in one phase will be negative in the other two phases due to that the magnetic flux goes round in closed paths.

PMSM– derivation of dynamic model

3-phase model

By using that there is no zero-component in the three-phase quantities (assuming that they always add to zero) and by using some cosine calculation rules to change the phase shift of the mutual inductances to be 0 or 120 deg we get

$$\psi_a = \left(L_{aa0} + L_{ab0} + \frac{3}{2} L_{aa2} \cos(2\phi_r)\right) i_a + \frac{\sqrt{3}}{2} L_{aa2} \sin(2\phi_r)(i_b - i_c) + \Psi_m \cos(\phi_r)$$

$$\psi_b = \left(L_{aa0} + L_{ab0} + \frac{3}{2} L_{aa2} \cos\left(2\left(\phi_r - 120^\circ\right)\right)\right) i_b + \frac{\sqrt{3}}{2} L_{aa2} \sin\left(2\left(\phi_r - 120^\circ\right)\right)(i_c - i_a) + \Psi_m \cos\left(\phi_r - 120^\circ\right)$$

$$\psi_c = \left(L_{aa0} + L_{ab0} + \frac{3}{2} L_{aa2} \cos\left(2\left(\phi_r + 120^\circ\right)\right)\right) i_c + \frac{\sqrt{3}}{2} L_{aa2} \sin\left(2\left(\phi_r + 120^\circ\right)\right)(i_a - i_b) + \Psi_m \cos\left(\phi_r + 120^\circ\right)$$

$$u_a = R_a i_a + \frac{d\psi_a}{dt}$$

$$u_b = R_b i_b + \frac{d\psi_b}{dt}$$

$$u_c = R_c i_c + \frac{d\psi_c}{dt}$$

The 3-phase model can be used for simulation purpose, but it is not suitable model for analyzing the behavior and for controlling the machine. If we want to use this model for analyzing the PMSM, we need to solve the differential equations for a sinusoidal excitation…..

Solution: derive the model in the rotating frame (dq) instead, where the excitation is constant in steady-state

PMSM– derivation of dynamic model

3-phase model to αβ-model

Neglect the zero sequence and uses amplitude invariant transformation

We have the three phase to two phase transformation

$$\underline{u}_s^s = u_{s\alpha} + ju_{s\beta} = \frac{2}{3}\left(u_\mathrm{a} + u_\mathrm{b}e^{j\frac{2\pi}{3}} + u_\mathrm{c}e^{j\frac{4\pi}{3}}\right) = \frac{2}{3}\left(\underbrace{R_a i_a + \frac{d\psi_a}{dt}}_{u_\mathrm{a}} + \underbrace{\left(R_b i_b + \frac{d\psi_b}{dt}\right)}_{u_\mathrm{b}}e^{j\frac{2\pi}{3}} + \underbrace{\left(R_c i_c + \frac{d\psi_c}{dt}\right)}_{u_\mathrm{c}}e^{j\frac{4\pi}{3}}\right) =$$

$$= R_s\underbrace{\frac{2}{3}\left(i_\mathrm{a} + i_\mathrm{b}e^{j\frac{2\pi}{3}} + i_\mathrm{c}e^{j\frac{4\pi}{3}}\right)}_{\underline{i}_s^s} + \frac{d}{dt}\underbrace{\left(\frac{2}{3}\left(\psi_\mathrm{a} + \psi_\mathrm{b}e^{j\frac{2\pi}{3}} + \psi_\mathrm{c}e^{j\frac{4\pi}{3}}\right)\right)}_{\underline{\psi}_s^s} = R_s\underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt}$$

$$\underline{\psi}_s^s = \frac{2}{3}\left(\psi_\mathrm{a} + \psi_\mathrm{b}e^{j\frac{2\pi}{3}} + \psi_\mathrm{c}e^{j\frac{4\pi}{3}}\right)$$

The applied stator voltage vector is equal to the resistive voltage plus the change in the stator flux linkage vector. We now need to calculate the stator flux linkage vector

$$\psi_a = \Big(\overbrace{L_{aa0} + L_{ab0}}^{1} + \overbrace{\frac{3}{2}L_{aa2}\cos(2\phi_r)}^{2}\Big)i_a + \overbrace{\frac{\sqrt{3}}{2}L_{aa2}\sin(2\phi_r)(i_b - i_c)}^{3} + \overbrace{\Psi_m\cos(\phi_r)}^{4}$$

$$\psi_b = \left(L_{aa0} + L_{ab0} + \frac{3}{2}L_{aa2}\cos\left(2\left(\phi_r - 120^\circ\right)\right)\right)i_b + \frac{\sqrt{3}}{2}L_{aa2}\sin\left(2\left(\phi_r - 120^\circ\right)\right)(i_c - i_a) + \Psi_m\cos\left(\phi_r - 120^\circ\right)$$

$$\psi_c = \left(L_{aa0} + L_{ab0} + \frac{3}{2}L_{aa2}\cos\left(2\left(\phi_r + 120^\circ\right)\right)\right)i_c + \frac{\sqrt{3}}{2}L_{aa2}\sin\left(2\left(\phi_r + 120^\circ\right)\right)(i_a - i_b) + \Psi_m\cos\left(\phi_r + 120^\circ\right)$$

PMSM– derivation of dynamic model

3-phase model to αβ-model

When calculate the stator flux linkage vector we take the 4 parts one and one

$$\underline{\psi}_{s,1}^{s}=\frac{2}{3}\left(\left(L_{aa0}+L_{ab0}\right)i_a+\left(L_{aa0}+L_{ab0}\right)i_b e^{j\frac{2\pi}{3}}+\left(L_{aa0}+L_{ab0}\right)i_c e^{j\frac{4\pi}{3}}\right)=\left(L_{aa0}+L_{ab0}\right)\underline{i}_s^s$$

$$\underline{\psi}_{s,2}^{s}=\frac{2}{3}\left(\frac{3}{2}L_{aa2}\cos(2\phi_r)i_\mathrm{a}+\frac{3}{2}L_{aa2}\cos\left(2\left(\phi_r-120^\circ\right)\right)i_\mathrm{b}e^{j\frac{2\pi}{3}}+\frac{3}{2}L_{aa2}\cos\left(2\left(\phi_r+120^\circ\right)\right)i_\mathrm{c}e^{j\frac{4\pi}{3}}\right)=$$

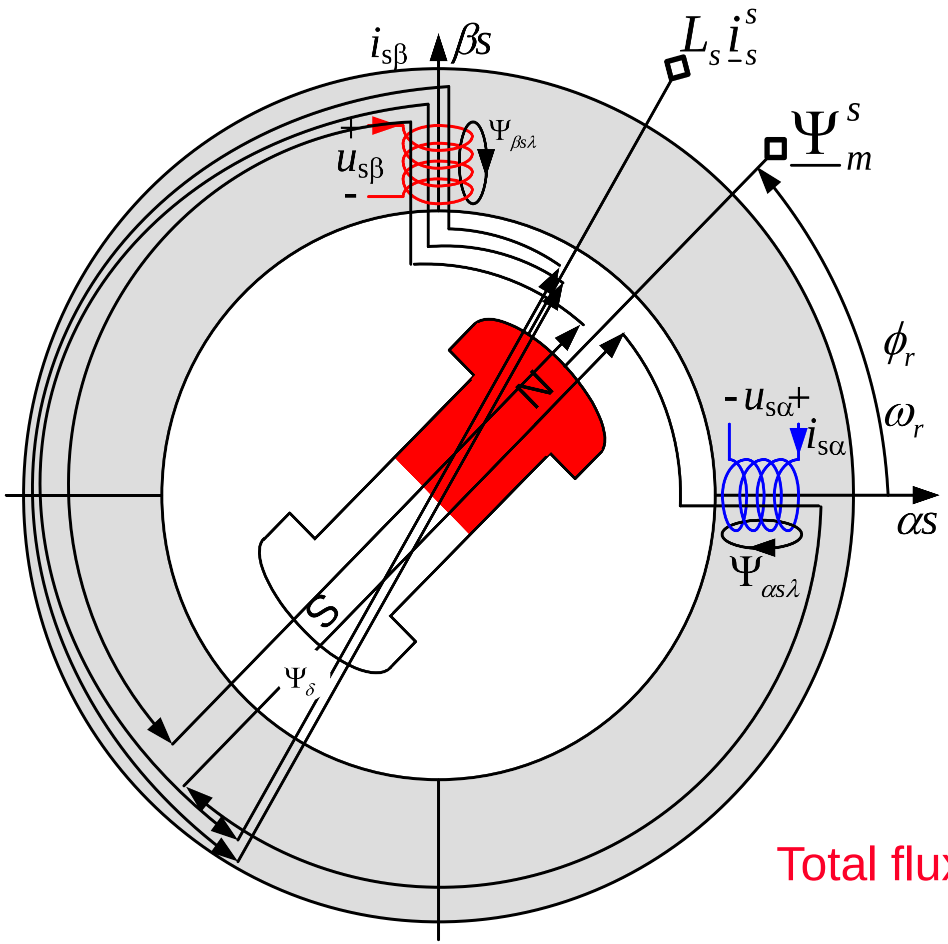
$$=\frac{3}{4}L_{aa2}e^{j2\phi_r}\frac{2}{3}\left(i_\mathrm{a}+i_\mathrm{b}e^{-j\frac{2\pi}{3}}+i_\mathrm{c}e^{j\frac{2\pi}{3}}\right)=\frac{3}{4}L_{aa2}e^{j2\phi_r}\underline{i}_s^{s*}$$

$$\underline{\psi}_{s,3}^{s}=\frac{2}{3}\left(\frac{\sqrt{3}}{2}L_{aa2}\sin(2\phi_r)(i_b-i_c)+\frac{\sqrt{3}}{2}L_{aa2}\sin\left(2\left(\phi_r-120^\circ\right)\right)(i_c-i_a)e^{j\frac{2\pi}{3}}+\frac{\sqrt{3}}{2}L_{aa2}\sin\left(2\left(\phi_r+120^\circ\right)\right)(i_a-i_b)e^{j\frac{4\pi}{3}}\right)=$$

$$=L_{aa2}e^{j2\phi_r}\frac{\sqrt{3}}{2}\underbrace{\frac{e^{j\frac{2\pi}{3}}-e^{-j\frac{2\pi}{3}}}{2j}}_{\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}}\underline{i}_s^{s*}=\frac{3}{4}L_{aa2}e^{j2\phi_r}\underline{i}_s^{s*}$$

$$\underline{\psi}_{s,4}^{s}=\frac{2}{3}\left(\Psi_m\cos(\phi_r)+\Psi_m\cos\left(\phi_r-120^\circ\right)e^{j\frac{2\pi}{3}}+\Psi_m\cos\left(\phi_r+120^\circ\right)e^{j\frac{4\pi}{3}}\right)=\frac{2}{3}\frac{3}{2}\Psi_m e^{j\phi_r}=\Psi_m e^{j\phi_r}$$

PMSM modeling in αβ-system



$$\underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt}$$

$$\underline{\psi}_s^s = \underline{\psi}_{s,1}^s + \underline{\psi}_{s,2}^s + \underline{\psi}_{s,3}^s + \underline{\psi}_{s,4}^s =$$

$$= \left(L_{aa0} + L_{ab0}\right)\underline{i}_s^s + \frac{3}{2} L_{aa2} e^{j2\phi_r} \underline{i}_s^{s*} + \Psi_m e^{j\phi_r}$$

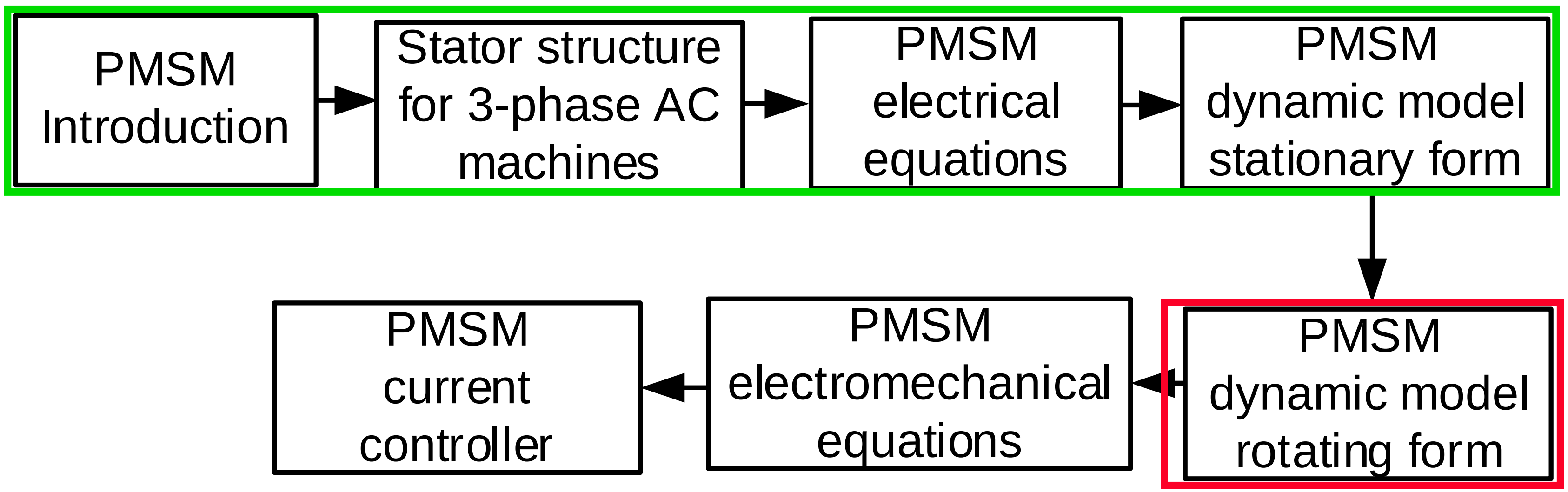
$$\underline{\psi}_s^s = "\underline{L}_s \underline{i}_s^s" + \Psi_m e^{j\phi_r}$$

Magnet flux linkage

Total flux linkage

Leakage and mutual inductance

Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system



Now we have a two phase machine, but we still have sinusoidal excitation. We now transform this to the rotating frame (dq) instead, where the excitation is constant in steady-state

PMSM modeling, moving to dq-representation

Stator voltage equation in stationary form:

electrical rotor angle

$$\underline{\psi}_s^s = (L_{aa0} + L_{ab0})\underline{i}_s^s + \frac{3}{2}L_{aa2}e^{j2\phi_r}\underline{i}_s^{s*} + \Psi_m e^{j\phi_r}$$

$$\underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt}$$

Calculate derivatives

Input

Relations between rotating and stationary quantities:

$$\boxed{\underline{i}_s^s = \underline{i}_s e^{j\theta_r} \quad \underline{u}_s^s = \underline{u}_s e^{j\theta_r} \quad \underline{\psi}_s^s = \underline{\psi}_s e^{j\theta_r}}$$

dq-transformation angle

Chain rule for obtaining relation for current derivative relation:

$$\frac{d\underline{\psi}_s^s}{dt} = \frac{d}{dt}\left(\underline{\psi}_s e^{j\theta_r}\right) = \frac{d\underline{\psi}_s}{dt}e^{j\theta_r} + j\omega_r \underline{\psi}_s e^{j\theta_r}$$

Gives in rotating coordinates:

$$\underline{u}_s \cancel{e^{j\theta_r}} = R_s \underline{i}_s \cancel{e^{j\theta_r}} + \frac{d\underline{\psi}_s}{dt}\cancel{e^{j\theta_r}} + j\omega_r \underline{\psi}_s \cancel{e^{j\theta_r}}$$

$$\underline{\psi}_s^s = \underline{\psi}_s e^{j\theta_r} = (L_{aa0} + L_{ab0})\underline{i}_s e^{j\theta_r} + \frac{3}{2}L_{aa2}e^{j2\phi_r}\left(\underline{i}_s e^{j\theta_r}\right)^* + \Psi_m e^{j\phi_r} \quad \Rightarrow \text{Divide with } e^{j\theta_r}$$

$$\underline{\psi}_s = (L_{aa0} + L_{ab0})\underline{i}_s + \frac{3}{2}L_{aa2}e^{j(2\phi_r - 2\theta_r)}\underline{i}_s^* + \Psi_m e^{j(\phi_r - \theta_r)}$$

If we now assume perfect field orientation, $\phi_r = \theta_r$, rotor flux in d-direction →

$$\underline{\psi}_s = (L_{aa0} + L_{ab0})\underline{i}_s + \frac{3}{2}L_{aa2}\underbrace{e^{j(2\varphi_r - 2\theta_r)}}_{=1}\underline{i}_s{}^* + \Psi_m \underbrace{e^{j(\varphi_r - \theta_r)}}_{=1} \quad \Rightarrow$$

$$\underline{\psi}_s = (L_{aa0} + L_{ab0})\underline{i}_s + \frac{3}{2}L_{aa2}\underline{i}_s{}^* + \Psi_m \quad \Rightarrow$$ We know that $\underline{i}_s = i_{sd} + ji_{sq}$

$$\underline{\psi}_s = (L_{aa0} + L_{ab0})(i_{sd} + ji_{sq}) + \frac{3}{2}L_{aa2}(i_{sd} - ji_{sq}) + \Psi_m \quad \Rightarrow$$

$$\underline{\psi}_s = \underbrace{\left(L_{aa0} + L_{ab0} + \frac{3}{2}L_{aa2}\right)}_{L_{sd}} i_{sd} + j\underbrace{\left(L_{aa0} + L_{ab0} - \frac{3}{2}L_{aa2}\right)}_{L_{sq}} i_{sq} + \Psi_m \Rightarrow$$

$$\underline{\psi}_s = L_{sd}i_{sd} + jL_{sq}i_{sq} + \Psi_m$$

Due to that we do not have an a ”magnetic” round rotor we get different inductance in the d and q directions. From previous we hade that:

$$\underline{u}_s = R_s\underline{i}_s + \frac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} + j\omega_r\underline{\psi}_s \quad \Rightarrow$$ We know that $\underline{u}_s = u_{sd} + ju_{sq}$

We now split this into real and imaginary part

$$u_{sd} = R_s i_{sd} + L_{sd}\frac{di_{sd}}{dt} - \omega_r L_{sq} i_{sq}$$

$$u_{sq} = R_s i_{sq} + L_{sq}\frac{di_{sq}}{dt} + \omega_r L_{sd} i_{sd} + \omega_r \psi_m$$

CHALMERS

PMSM modeling, dq-system, amplitude invariant transformation

We now have the electrical part of our PMSM model in the dq-system

$$u_{\text{sd}} = R_s i_{\text{sd}} + L_{\text{sd}} \frac{\mathrm{d}i_{\text{sd}}}{\mathrm{d}t} - \omega_r L_{\text{sq}} i_{\text{sq}}$$

$$u_{\text{sq}} = R_s i_{\text{sq}} + L_{\text{sq}} \frac{\mathrm{d}i_{\text{sq}}}{\mathrm{d}t} + \omega_r L_{\text{sd}} i_{\text{sd}} + \omega_{\mathrm{r}} \psi_{\mathrm{m}}$$

What does this represent?

The back-emf, the voltage induced in the stator windings by the movement of the rotor

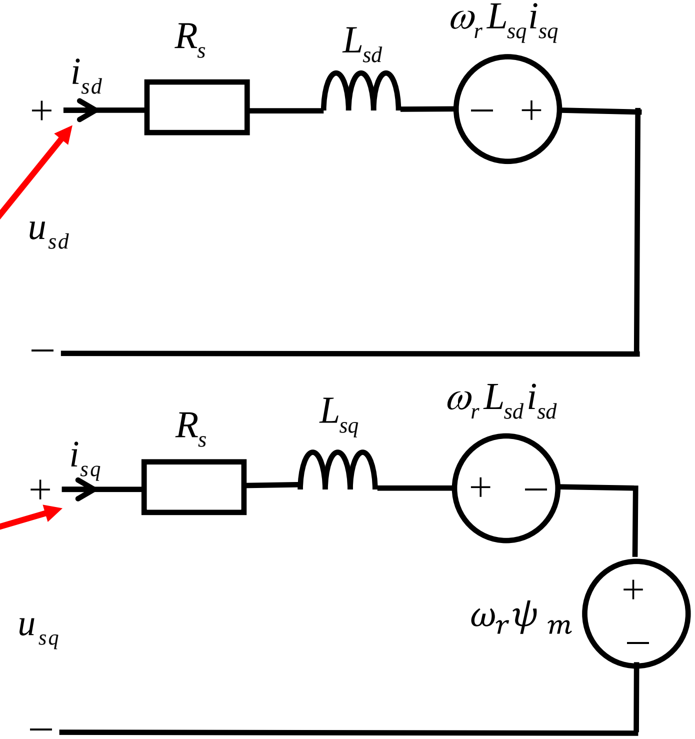
Transient voltage drop of the inductance

Steady-state voltage drop of the inductance

Resistive voltage drop in the winding R_s is the resistance of the phase winding

Applied stator voltage, constant in steady-state

Stator current, constant in steady-state

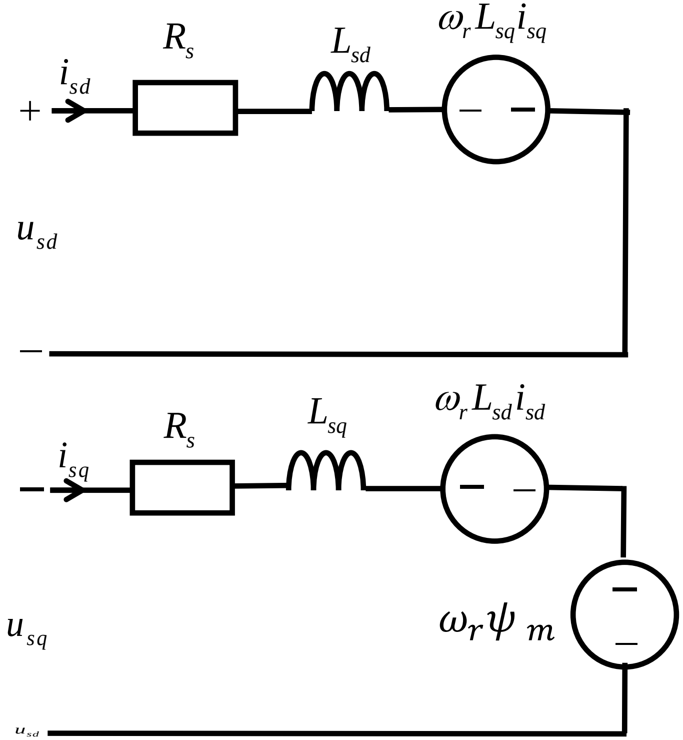


PMSM modeling, dq-system, amplitude invariant transformation

The electrical part of our PMSM model in the dq-system

$$\underline{u}_s = R_s \underline{i}_s + \frac{\mathrm{d}\underline{\lambda}_s}{\mathrm{d}t} + j\omega_r \underline{\lambda}_s$$

and the circuit model of it



How do we calculate the output power, the shaft power?

Lett us look on the power flow for this circuit (our model)

We remember that power in dq we calculate as

$$p(t) = \frac{3}{2K^2}\mathrm{Re}\{\underline{u}\underline{i}^*\}$$

Power flow in steady state

$$P_1 = \frac{3}{2K^2} Re\{\underline{u}_s \underline{i}_s{}^*\}$$

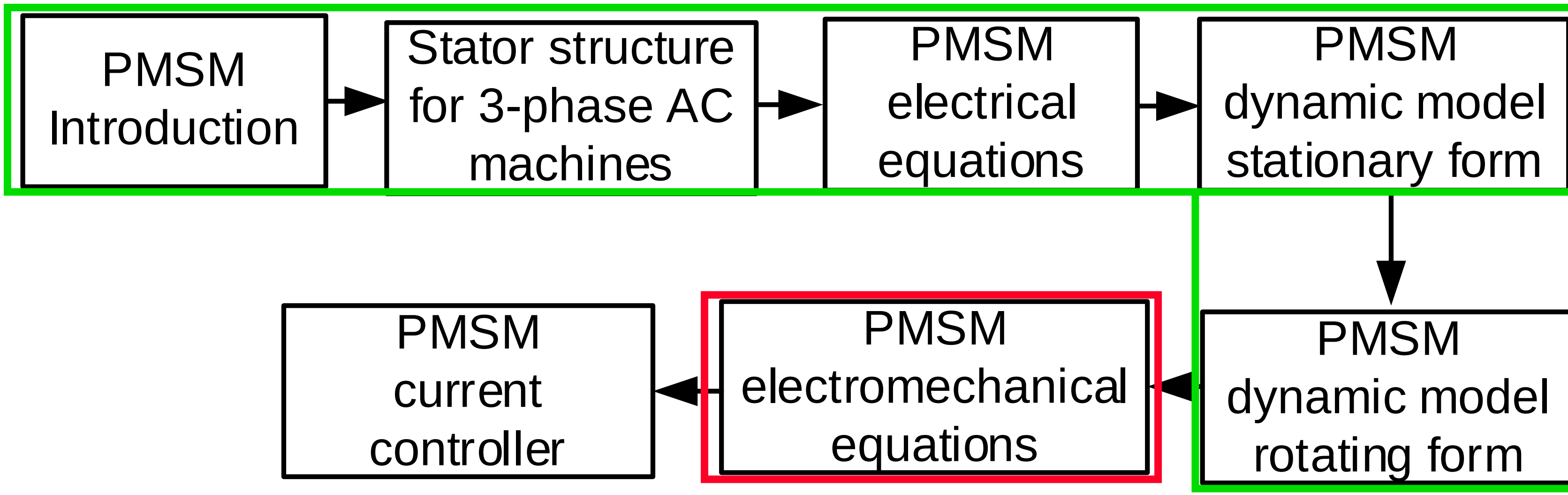
Into the stator

$$P_\mathrm{e} = \frac{3}{2K^2}\mathrm{Re}\left\{j\omega_r \underline{\psi}_s \underline{i}_s{}^*\right\}$$

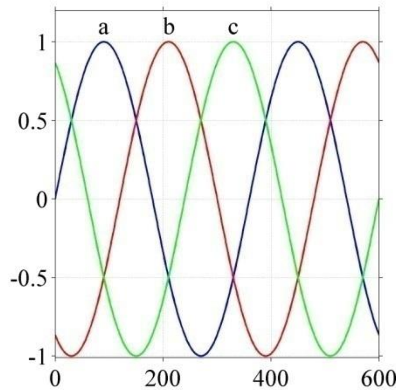
Shaft power

Stator copper loss $P_{cu1} = \frac{3}{2K^2} R_s \left|\underline{i}_s\right|^2$

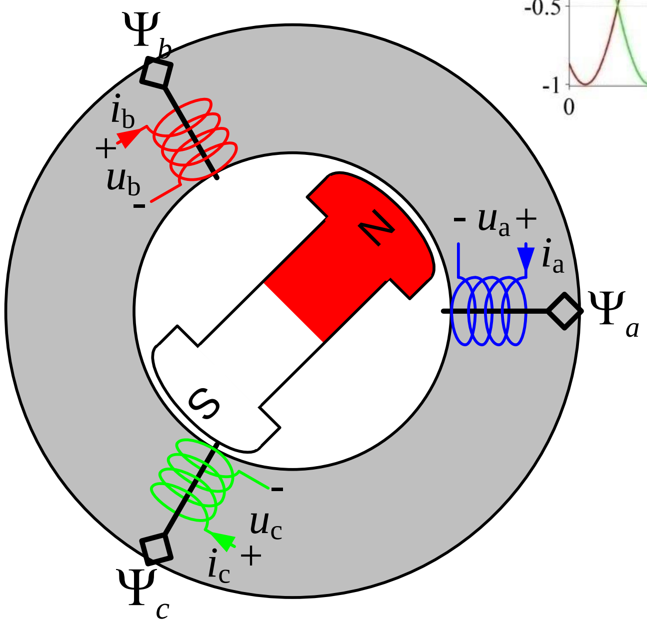
Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system



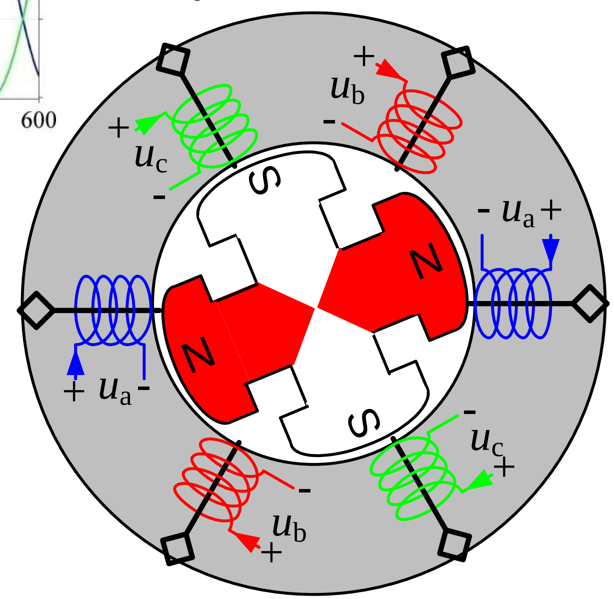
Number of Pole-pairs, n_p



n_p=1
2-pole machine



n_p=2
4-pole machine



Synchronous mechanical speed of the generator (stator field)

$$n_s = \frac{60\, f_s}{n_p}$$

n_p	n_s [RPM] (f_s 50 Hz)
1	3000
2	1500
3	1000
4	750

- n_p line periods required to make one mechanical turn
- Electrical equations require electrical speed, ω_r.
- Mechanical speed $\Omega_r = \omega_r / n_p$

Derivation of the torque produced by the PMSM

Shaft power:

$$P_e = \frac{3}{2K^2}\mathrm{Re}\left\{j\omega_r \underline{\psi}_s \underline{i}_s^{\,*}\right\} = \left[\mathrm{Re}\{j\underline{z}\} = -\mathrm{Im}\{\underline{z}\}\right] = -\frac{3\omega_r}{2K^2}\mathrm{Im}\left\{\underline{\psi}_s \underline{i}_s^{\,*}\right\} = \frac{3\omega_r}{2K^2}\mathrm{Im}\left\{\underline{\psi}_s^{\,*} \underline{i}_s\right\}$$

Torque:

$T_e = \frac{P_e}{\Omega_r}$ Torque is mechanical power divided with the mechanical speed

$$T_e = \frac{n_p P_e}{\omega_r} = \frac{3\omega_r n_p}{2\omega_r K^2} Im\left\{\underline{\psi}_s^{s\,*} \underline{i}_s^s\right\} = \frac{3n_p}{2K^2} Im\left\{\underline{\psi}_s^{s\,*} \underline{i}_s^s\right\} =$$

$$= \frac{3n_p}{2K^2} Im\left\{\left(L_{sd}i_{sd} + jL_{sq}i_{sq} + \psi_m\right)^* \underline{i}_s\right\} =$$

$$= \frac{3n_p}{2K^2} Im\left\{\left(L_{sd}i_{sd} - jL_{sq}i_{sq}\right)\left(i_{sd} + ji_{sq}\right) + \psi_m \underline{i}_s\right\} =$$

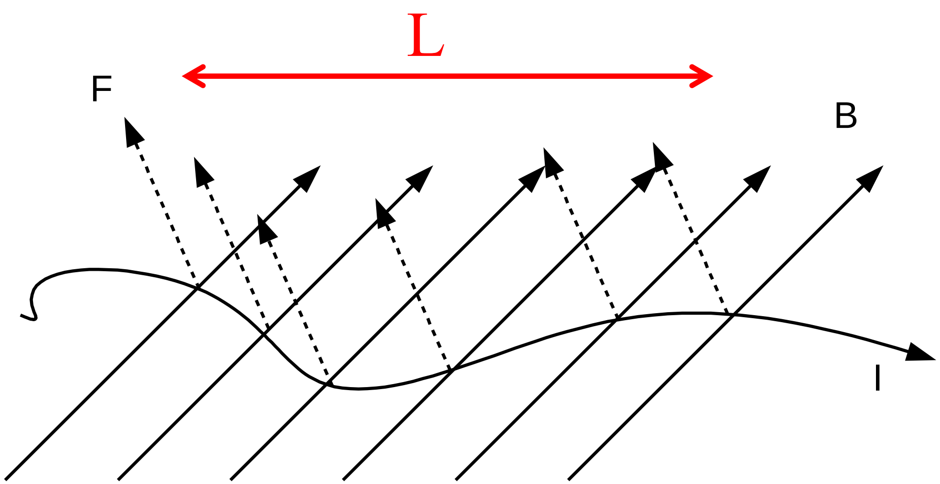
$$= \frac{3n_p}{2K^2}\left(L_{sd}i_{sd}i_{sq} - L_{sq}i_{sq}i_{sd} + \psi_m i_{sq}\right) = \frac{3n_p}{2K^2}\left(\psi_m i_{sq} + \left(L_{sd} - L_{sq}\right)i_{sd}i_{sq}\right)$$

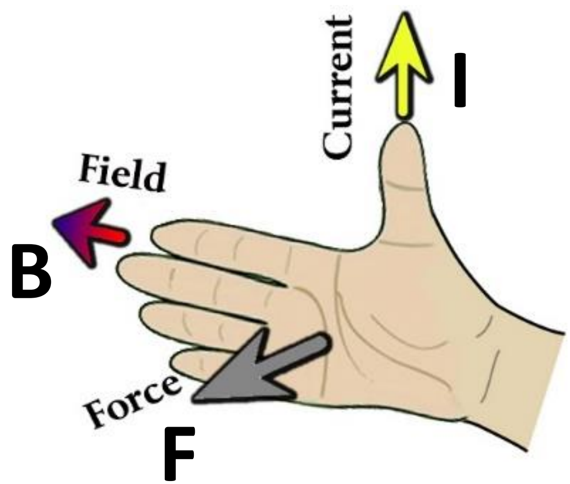
Remember, we use amplitude invariant scaling, K=1

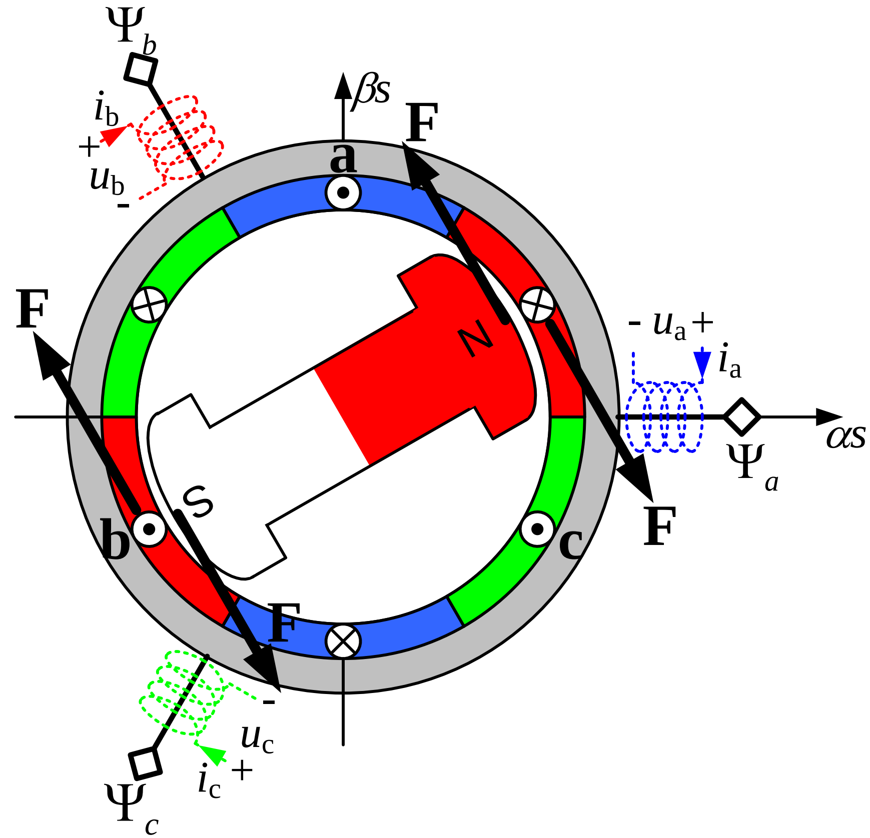
As can be noticed the torque is composed of two terms

- one term from the flux from the magnets in the rotor, the magnet torque
- one term from the difference in the inductances, this is called the reluctance torque. The flux created by the stator coils wants to minimize the reluctance it has in the flux path, so it wants to align the rotor so it gives the lowest reluctance (smallest air-gap towards it)

Force on a current-conducting conductor in a magnetic field

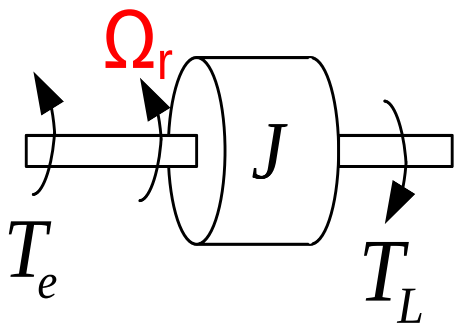






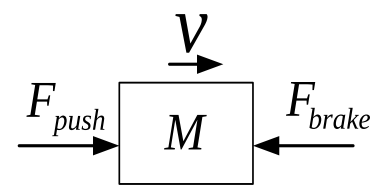
If B and I are perpendicular to each other then F is perpendicular and can be calculated as :
$F = BIL$ where L is the length

Mechanical equations



This resembles Newton's 2nd law

$$M\frac{dv}{dt} = F_{push} - F_{brake}$$



Speed equations

$$J\frac{d\Omega_r}{dt} = T_e - T_L$$

Relations

$$\Omega_r = \frac{\omega_r}{n_p}$$

Parameters

n_p = pole pair number

Ω_r = mechanical rotor speed

ω_r = electrical rotor speed

Φ_r = mechanical rotor position

φ_r = electrical rotor position

T_e = electrodynamical torque

T_L = load torque

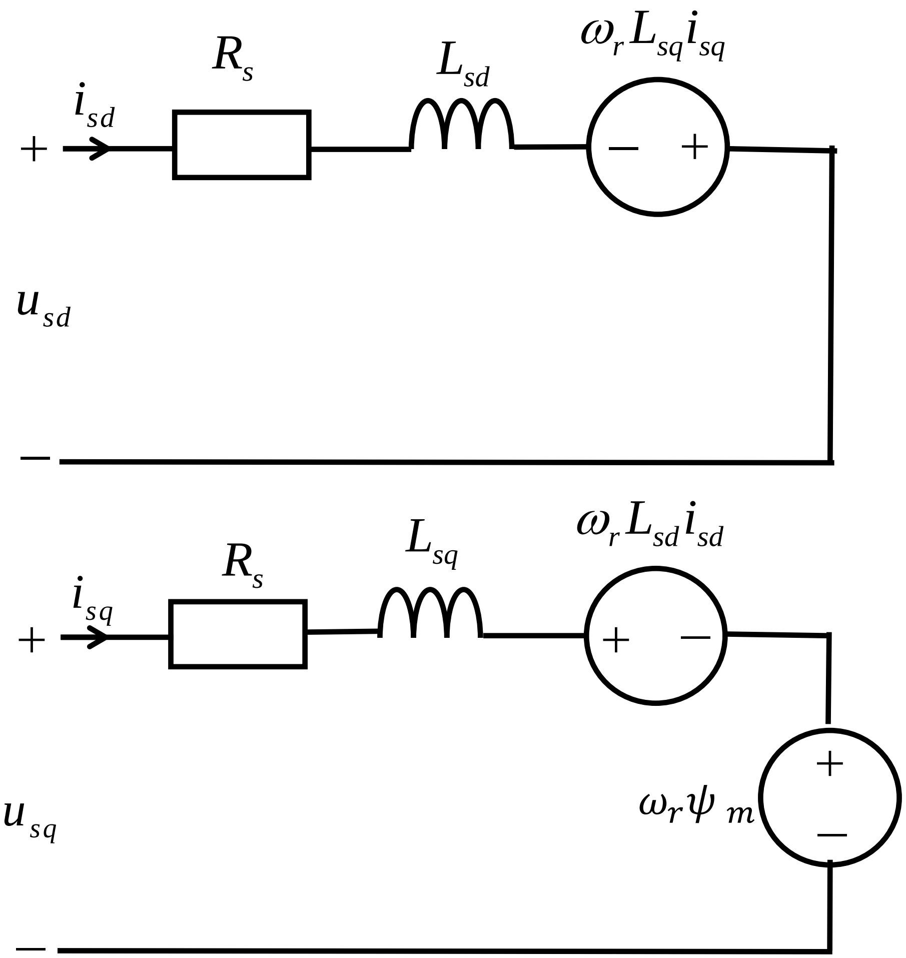
J = inertia of machine

$$T_e = \frac{3n_p}{2}\mathrm{Im}\left\{\underline{\psi}_s^{*}\,\underline{i}_s\right\} = \frac{3n_p}{2}\left(\Psi_m i_{sq} + \left(L_{sd} - L_{sq}\right) i_{sd} i_{sq}\right)$$

$$\frac{J}{n_p}\frac{d\omega_r}{dt} = T_e - T_L$$

$$\frac{d\phi_r}{dt} = \omega_r \qquad \Phi_r = \frac{\phi_r}{n_p}$$

PMSM modeling, dq-system, amplitude invariant transformation



Component form:

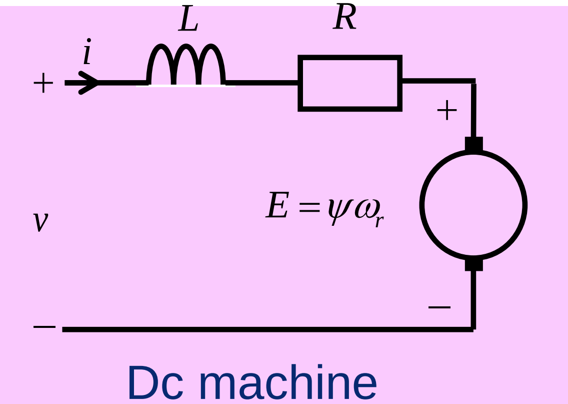
$$u_{sd} = R_s i_{sd} + L_{sd}\frac{di_{sd}}{dt} - \omega_r L_{sq} i_{sq}$$

$$u_{sq} = R_s i_{sq} + L_{sq}\frac{di_{sq}}{dt} + \omega_r L_{sd} i_{sd} + \omega_r \Psi_m$$

$$T_e = \frac{3n_p}{2}\left(\Psi_m i_{sq} + \left(L_{sd} - L_{sq}\right) i_{sd} i_{sq}\right)$$

$$\frac{J}{n_p}\frac{d\omega_r}{dt} = T_e - T_L$$

$$\frac{d\phi_r}{dt} = \omega_r$$



PMSM model implementation in Simulink

Model in Component form:

$$u_{sd} = R_s i_{sd} + L_{sd}\frac{di_{sd}}{dt} - \omega_r L_{sq} i_{sq}$$

$$u_{sq} = R_s i_{sq} + L_{sq}\frac{di_{sq}}{dt} + \omega_r L_{sd} i_{sd} + \omega_r \Psi_m$$

$$T_e = \frac{3n_p}{2}\left(\Psi_m i_{sq} + \left(L_{sd} - L_{sq}\right) i_{sd} i_{sq}\right)$$

$$\frac{J}{n_\mathrm{p}}\frac{d\omega_r}{dt} = T_\mathrm{e} - T_\mathrm{L} = T_\mathrm{e} - \frac{B}{n_p}\omega_r - T_{\mathrm{L},extra}$$

$$\frac{d\phi_r}{dt} = \omega_r$$

$$\omega_r = n_p \Omega_r$$

$$\phi_r = n_p \Theta_r$$

To implement the model into Simulink we first write the model on state-space form.

This means we put the time derivatives on the left side and everything on the right side of the equation. The torque equation does not contain any derivative, so we leave it as it is for now. This gives

$$\frac{di_{sd}}{dt} = \frac{1}{L_{sd}}\left(u_{sd} - R_s i_{sd} + \omega_r L_{sq} i_{sq}\right)$$

$$\frac{di_{sq}}{dt} = \frac{1}{L_{sq}}\left(u_{sq} - R_s i_{sq} - \omega_r L_{sd} i_{sd} - \omega_r \Psi_m\right)$$

$$\frac{d\omega_r}{dt} = \frac{n_\mathrm{p}}{J}\left(T_\mathrm{e} - \frac{B}{n_p}\omega_r - T_{\mathrm{L},extra}\right)$$

$$\frac{d\phi_r}{dt} = \omega_r$$

Can we implement this in a normal state-space model using matrixes $\dot{x} = Ax + Bu$?

NO, the model is nonlinear, so this is not possible

Now we take the Laplace transform of this

PMSM model implementation in Simulink

Laplace transform of the model

$$si_{sd} = \frac{1}{L_{sd}}\left(u_{sd} - R_s i_{sd} + \omega_r L_{sq} i_{sq}\right)$$

$$si_{sq} = \frac{1}{L_{sq}}\left(u_{sq} - R_s i_{sq} - \omega_r L_{sd} i_{sd} - \omega_r \Psi_m\right)$$

$$s\omega_r = \frac{n_\mathrm{p}}{J}\left(T_\mathrm{e} - \frac{B}{n_p}\omega_r - T_{\mathrm{L},extra}\right)$$

$$s\phi_r = \omega_r$$

Now we divide with s

$$i_{sd} = \frac{1}{s}\frac{1}{L_{sd}}\left(u_{sd} - R_s i_{sd} + \omega_r L_{sq} i_{sq}\right)$$

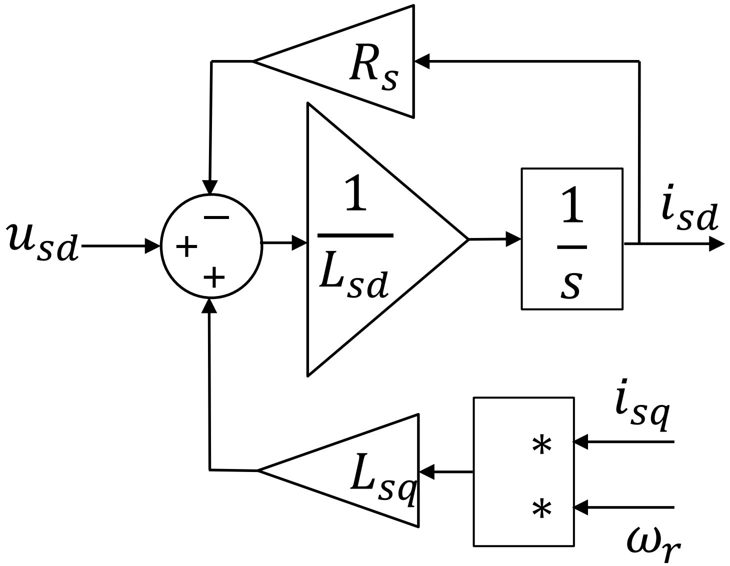
$$i_{sq} = \frac{1}{s}\frac{1}{L_{sq}}\left(u_{sq} - R_s i_{sq} - \omega_r L_{sd} i_{sd} - \omega_r \Psi_m\right)$$

$$\omega_r = \frac{1}{s}\frac{n_\mathrm{p}}{J}\left(T_\mathrm{e} - \frac{B}{n_p}\omega_r - T_{\mathrm{L},extra}\right)$$

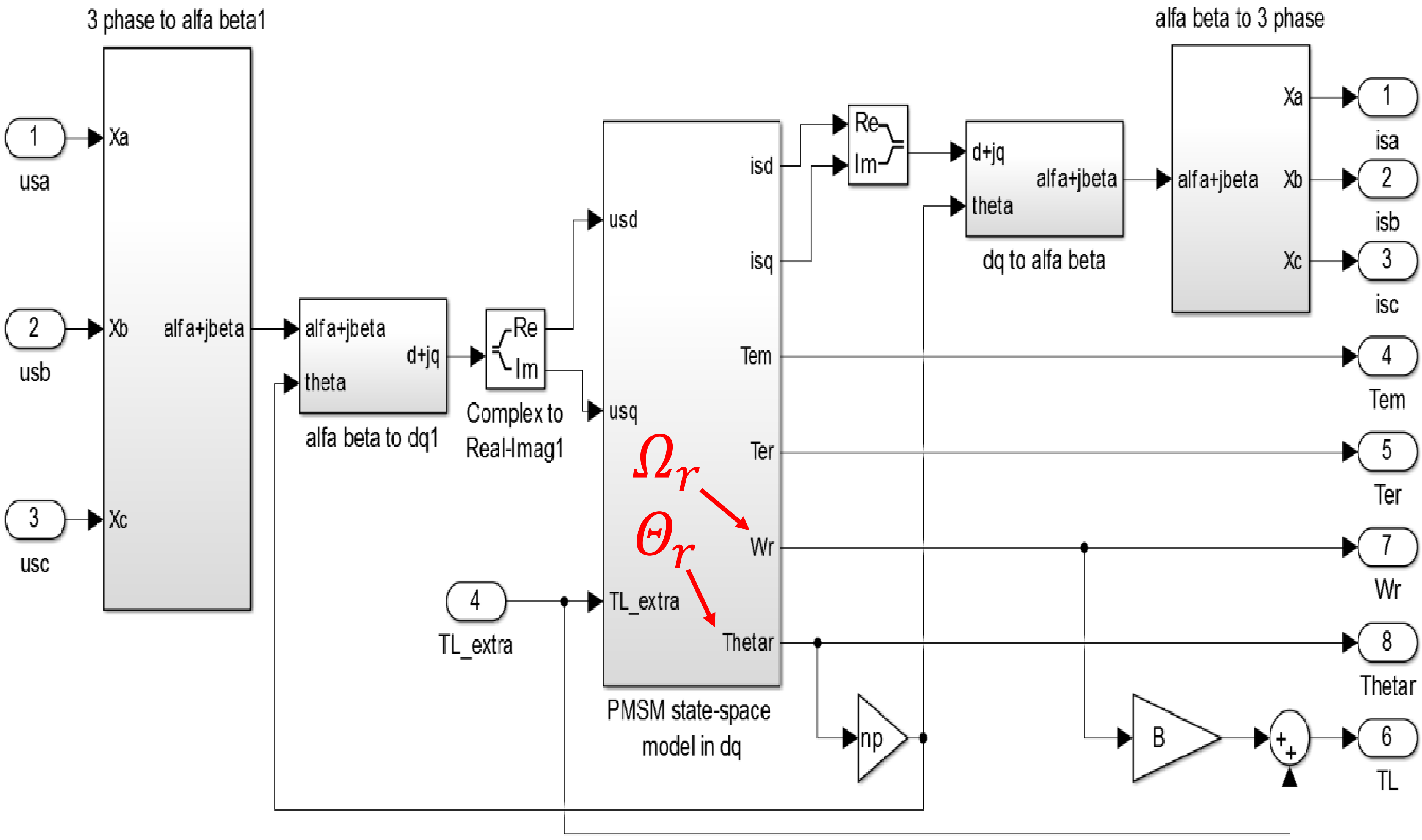
$$\phi_r = \frac{1}{s}\omega_r$$

This can be implemented in Simulink as

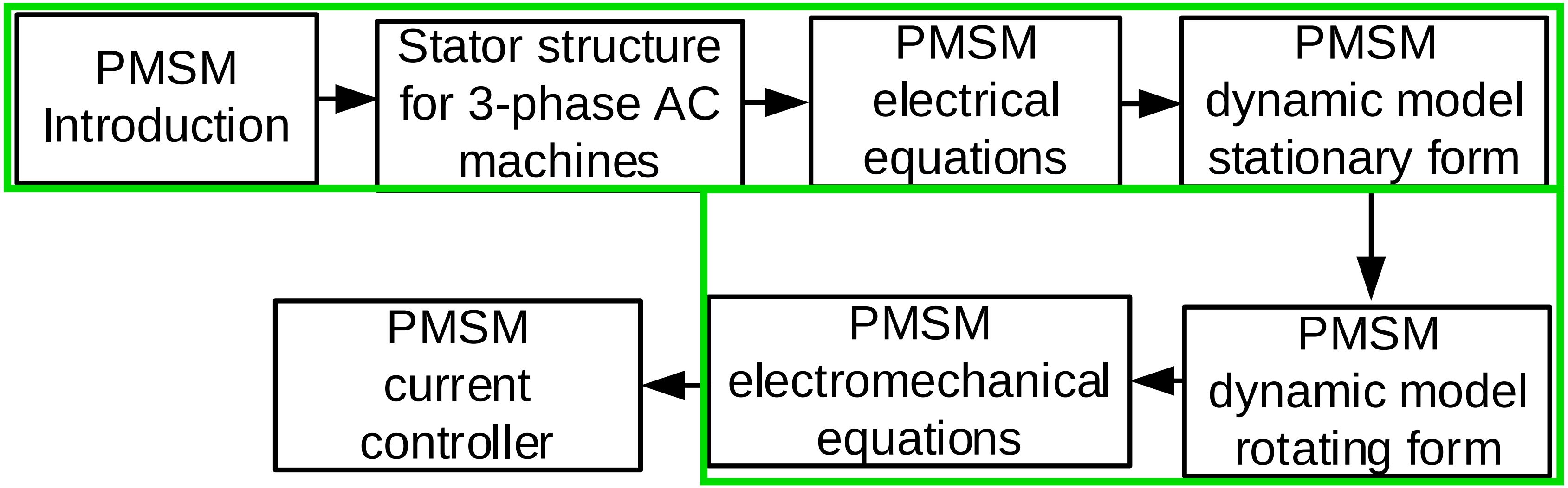
I only do this for the d-current



PMSM model implementation in Simulink

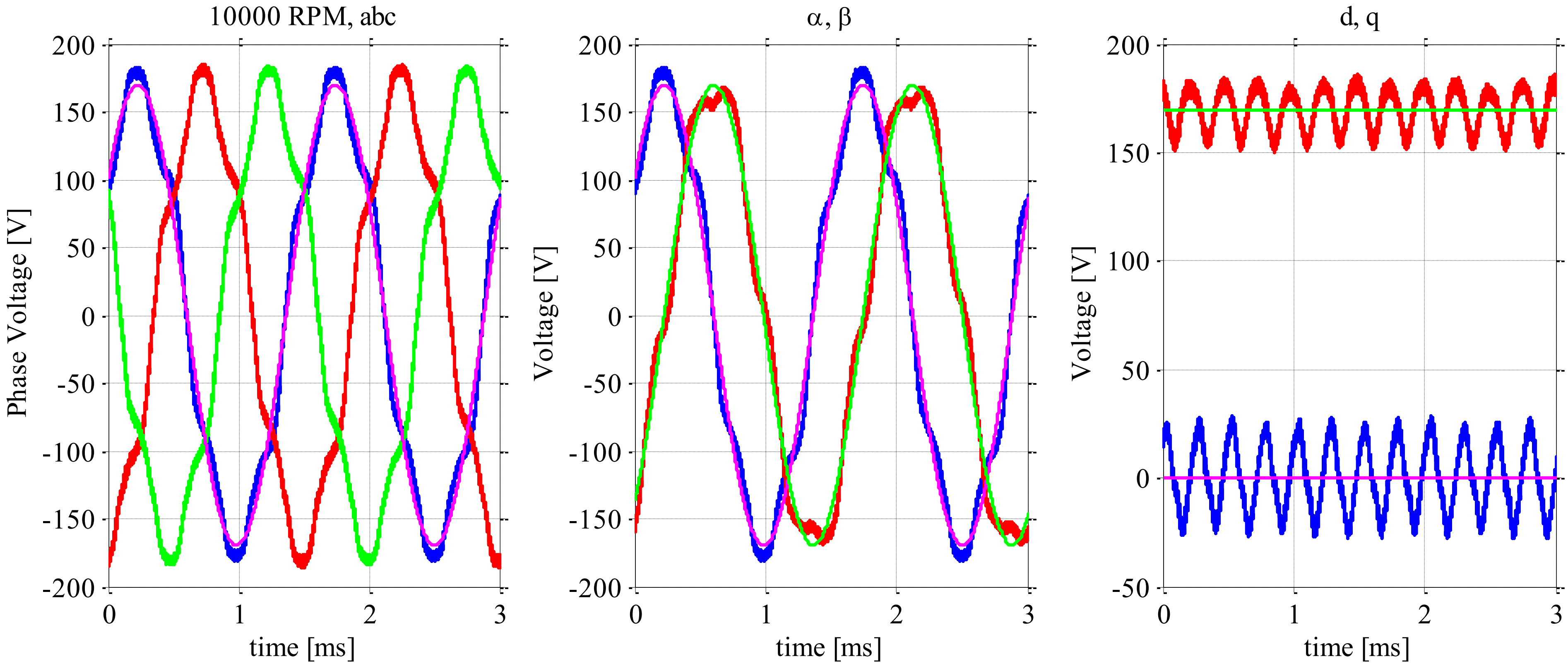


Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system



PMSM – A few details

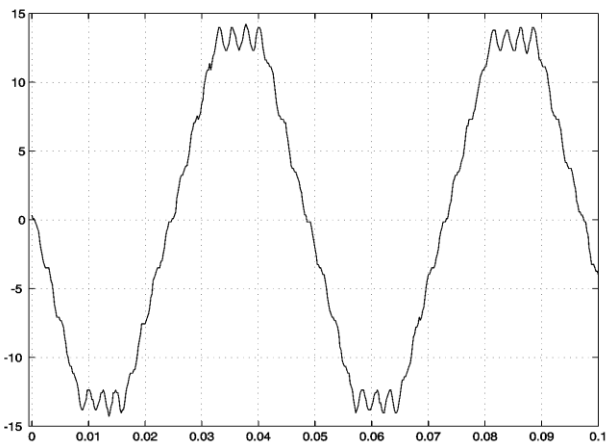
Back-emf with harmonics



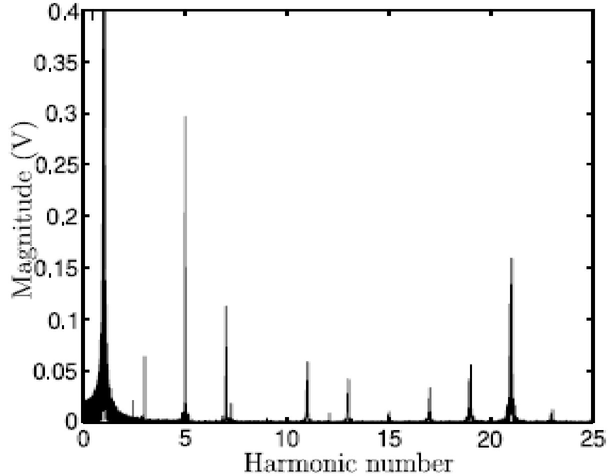
If there is no zero-sequence, no information is lost when transforming from three-phase to two-phase to dq!

PMSM – A few details

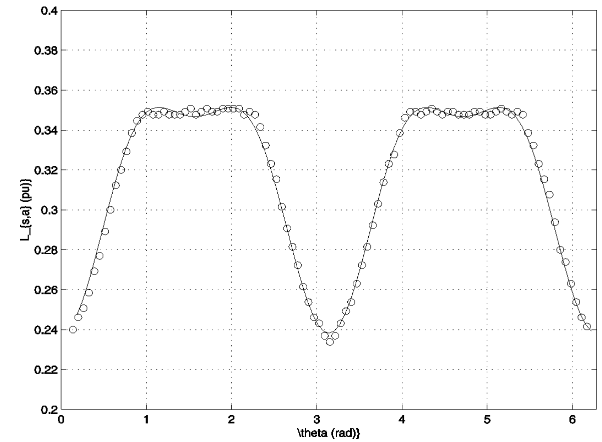
•Voltage not sinusoidal, inductances not constant



No-load voltage

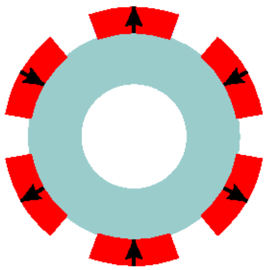


No-load voltage FFT



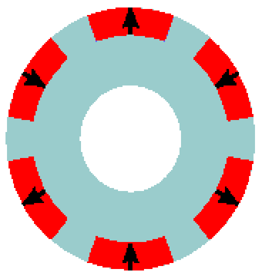
Variation of inductance

• Inductances in d and q direction often not equal -> salient machine



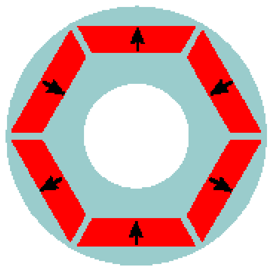
Surface mounted

$L_d \approx L_q$



Inset mounted

$L_d < L_q$



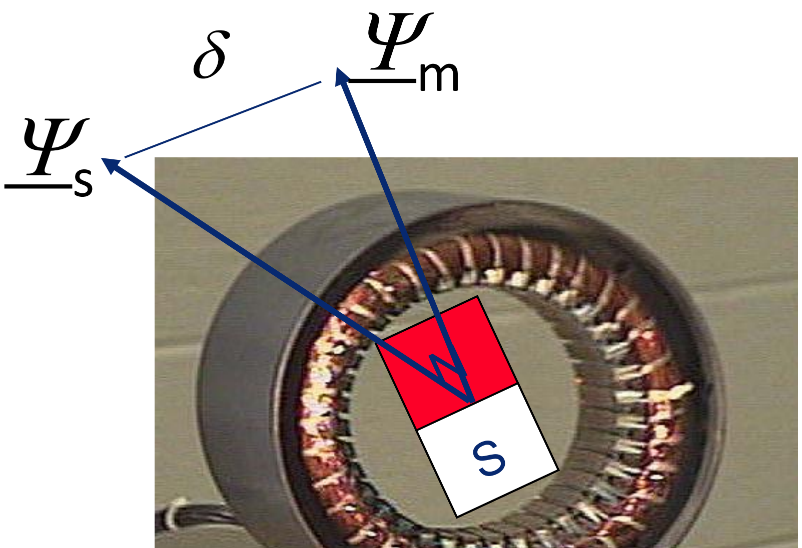
Interior mounting

$L_d < L_q$

$$L = \frac{\mu_0 A N^2}{\frac{l_j}{\mu_r} + l_g}$$

$$\mu_{r,magnet} \approx 1$$

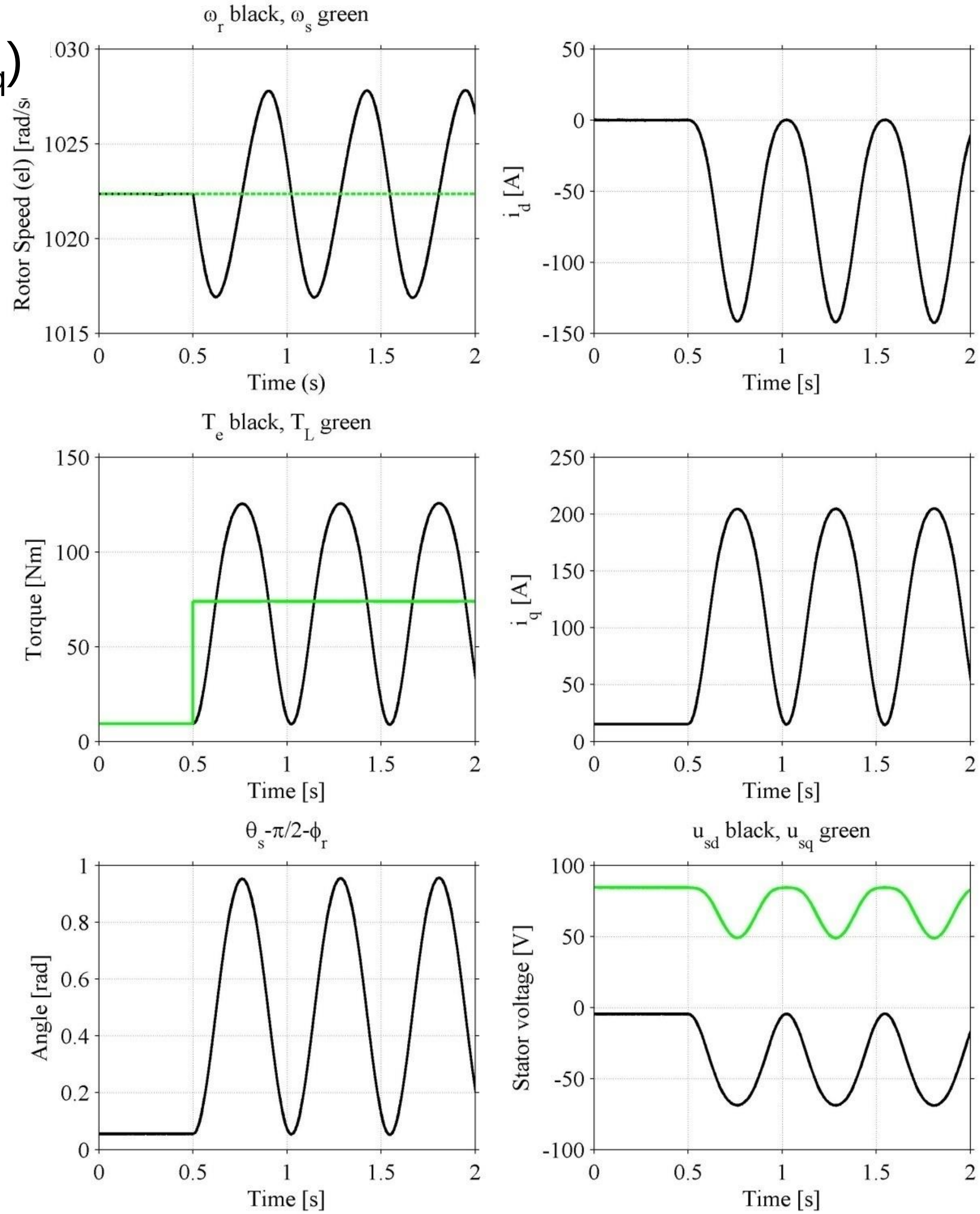
PMSM – A few details, open loop (grid connected) non-salient (L_d=L_q)



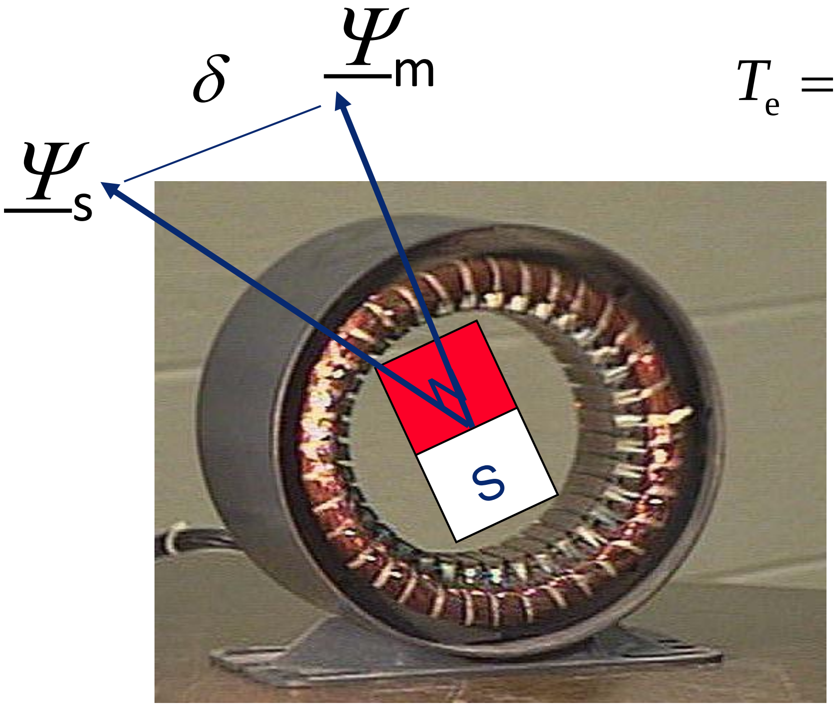
$$\underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt}$$

$$u_{sd} = R_s i_{sd} + L_s \frac{di_{sd}}{dt} - \omega_r L_s i_{sq}$$

$$u_{sq} = R_s i_{sq} + L_s \frac{di_{sq}}{dt} + \omega_r L_s i_{sd} + \omega_r \Psi_m$$



PMSM – Modeling, non-salient machine (Ld=Lq)



$$T_{\mathrm{e}} = \frac{3n_{\mathrm{p}}}{2}\,\mathrm{Im}\left\{\underline{\psi}_s^{s*}\,\underline{i}_s^{s}\right\} \qquad \underline{\psi}_s^{s} = \underbrace{L_{\mathrm{s}}}_{=L_{\mathrm{sd}}=L_{\mathrm{sq}}}\,\underline{i}_s^{s} + \Psi_{\mathrm{m}} e^{j\phi_{\mathrm{r}}}$$

$$\underline{i}_s^{s} = \frac{\underline{\psi}_s^{s} - \Psi_{\mathrm{m}} e^{j\phi_{\mathrm{r}}}}{L_{\mathrm{s}}}$$

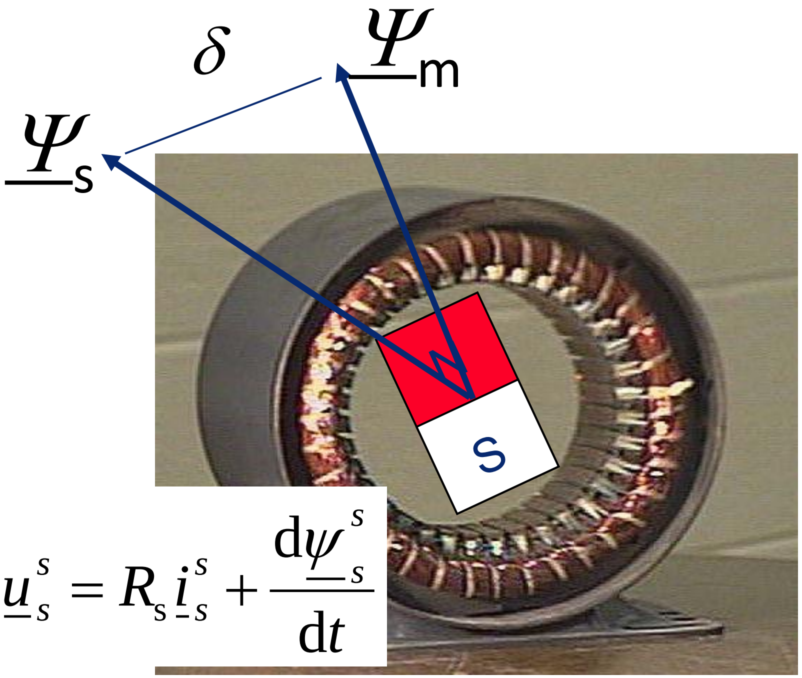
$$T_{\mathrm{e}} = \frac{3n_{\mathrm{p}}}{2}\,\mathrm{Im}\left\{\underline{\psi}_s^{s*}\,\frac{\underline{\psi}_s^{s} - \Psi_{\mathrm{m}} e^{j\phi_{\mathrm{r}}}}{L_{\mathrm{s}}}\right\} =$$

$$= \frac{3n_{\mathrm{p}}}{2L_{\mathrm{s}}}\,\mathrm{Im}\left\{\underline{\psi}_s^{s*}\,\underline{\psi}_s^{s} - \underline{\psi}_s^{s*}\,\Psi_{\mathrm{m}} e^{j\phi_{\mathrm{r}}}\right\}$$

$$T_{\mathrm{e}} = \left[\text{put } \underline{\psi}_s^{s} = \Psi_{\mathrm{s}} e^{j\theta_{\mathrm{s}}}\right] = \frac{3n_{\mathrm{p}}}{2L_s}\,\mathrm{Im}\left\{-\Psi_{\mathrm{s}}\left(e^{-j\theta_s}\Psi_{\mathrm{m}} e^{j\phi_r}\right\} = -\frac{3n_p}{2L_s}\,\mathrm{Im}\left\{\psi_{\mathrm{m}}\psi_s e^{j(\phi_r-\theta_s)}\right\}$$

$$= \frac{3n_{\mathrm{p}}}{2L_s}\,\Psi_{\mathrm{m}}\Psi_{\mathrm{s}}\sin\left(\theta_s - \phi_r\right) = \frac{3n_{\mathrm{p}}}{2L_s}\,\Psi_{\mathrm{m}}\Psi_{\mathrm{s}}\sin(\delta)$$

PMSM – A few details, open loop (grid connected) non-salient (L_d=L_q)



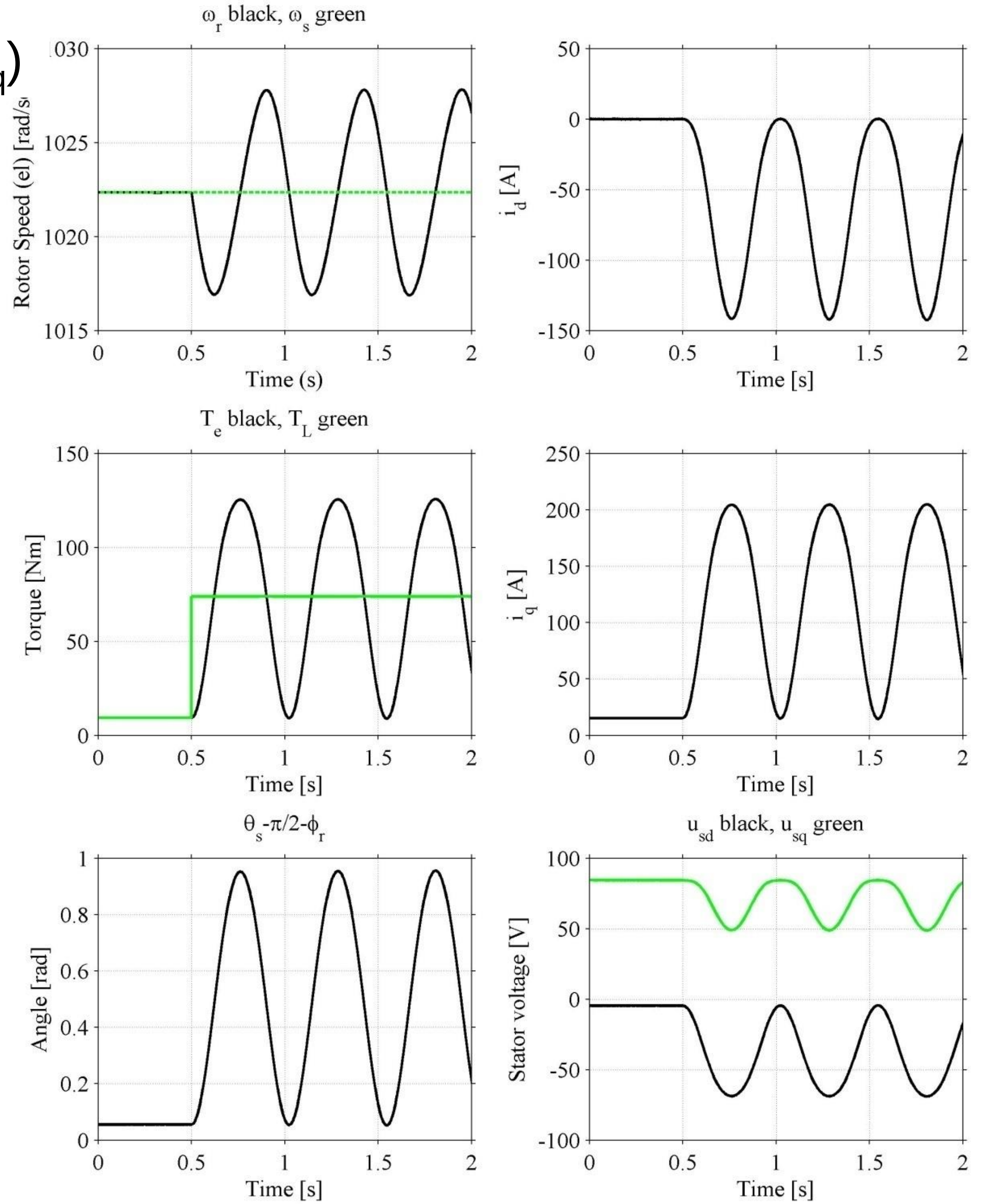
$$\underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt}$$

$$u_{sd} = R_s i_{sd} + L_s \frac{di_{sd}}{dt} - \omega_r L_s i_{sq}$$

$$u_{sq} = R_s i_{sq} + L_s \frac{di_{sq}}{dt} + \omega_r L_s i_{sd} + \omega_r \Psi_m$$

$$T_e = \frac{3n_p}{2L_s} \Psi_m \Psi_s \sin(\delta) = \frac{3n_p}{2} (\Psi_m i_{sq})$$

$$\frac{J}{n_p} \frac{d\omega_r}{dt} = T_e - T_L$$



Permanent Magnetized Synchronous Machine (PMSM) – derivation of dynamic model and control system