

# Real Financing Costs Over The Business Cycle: Emerging Versus Advanced Economies\*

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## Abstract

I examine the differences in the response of developed and developing economies to real interest rate shocks. By employing empirical methods that exploit the panel structure of the data, I show that real interest rate shocks have more adverse effects on a set of macroeconomic variables in developing economies than in developed ones. To analyze the structural reasons behind these findings, I propose a theoretical model featuring multiple shocks and frictions that is estimated using data from Brazil and Canada. The results suggest that the interest rate faced by the Brazilian economy in international markets is much more sensitive to deviations in the debt level from its long-run value relative to that of Canada. The spread shock is found to be more volatile in Brazil, and it can explain a substantially higher share of its business cycle. A counterfactual exercise reveals that the volatility of the real interest rate shock is crucial in determining the relationship between real interest rates and macroeconomic aggregates. Reductions in the sensitivity of the real rate to deviations of the debt level relative to its steady-state value are shown to be welfare-improving for the Brazilian economy, although at the expense of higher economic volatility.

**Keywords:** Interest Rates; Developing economies; Real business cycles; Bayesian estimation of DSGE models;

**JEL classification:** C11; E32; E40; F41; F44

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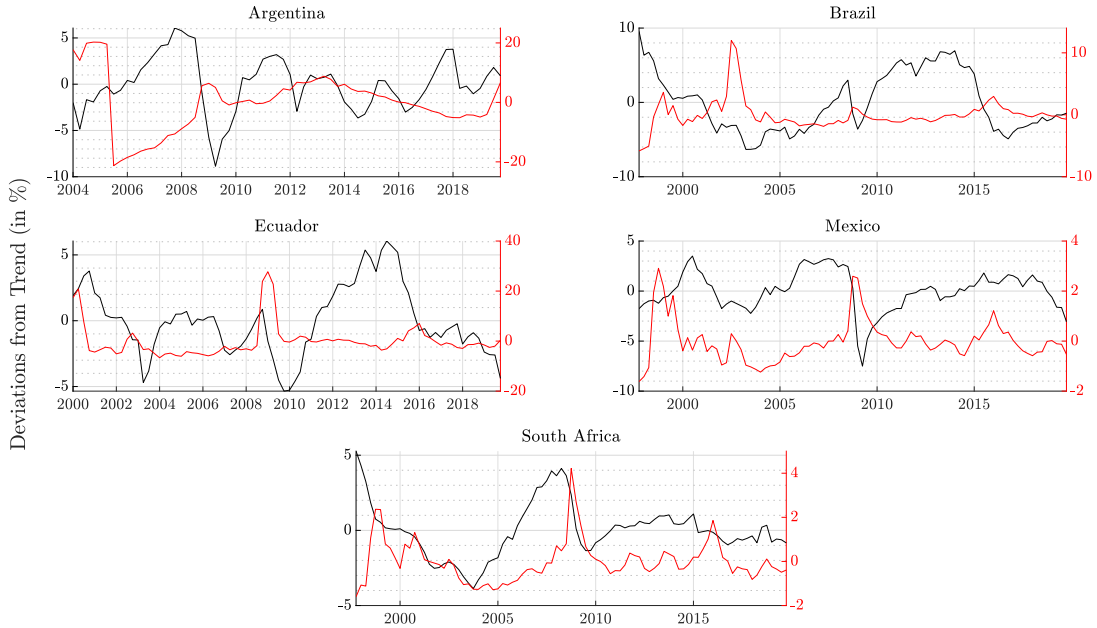
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# 1 Introduction

Business cycle fluctuations in developing economies are strongly associated with movements in the interest rates they face in international markets. As shown in Figure 1, periods of low (high) borrowing costs are generally accompanied by pronounced peaks (drops) in economic activity. This finding seems not to hold for developed economies, for which interest rates are found to be much more procyclical (Figure 2).

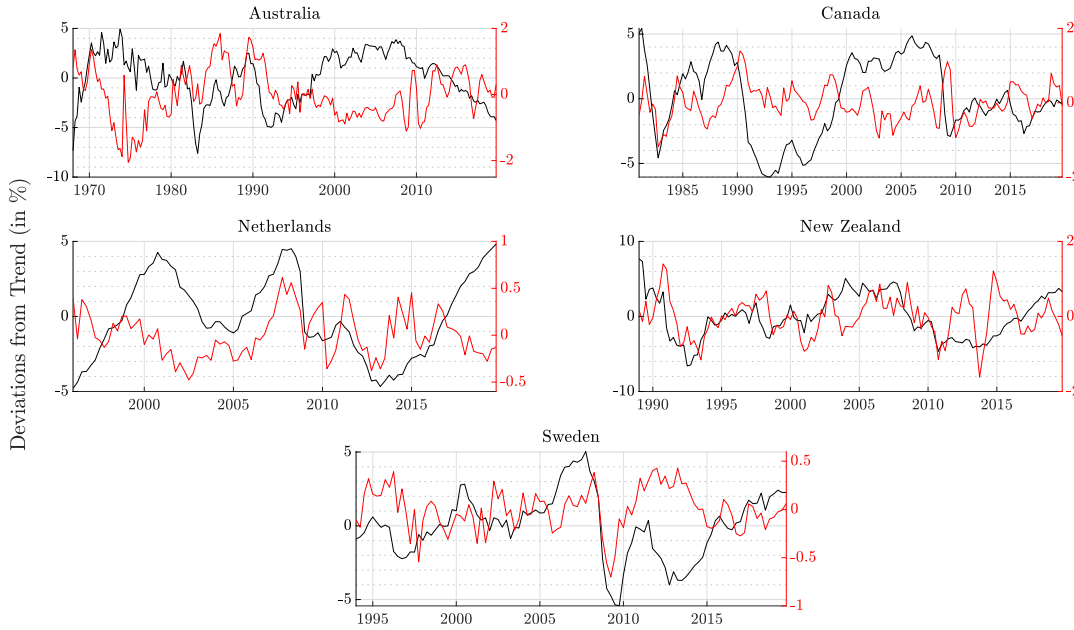
Figure 1: **GDP and real interest rates for a set of developing economies.** Black, solid line: Gross Domestic Product (log deviations from a quadratic trend, in %); red, solid line: Real interest rates (deviations from a quadratic trend, in %).



Understanding the causes and magnitude of this relationship emerges as an important research question, both from the academic and policy dimensions. The fact that emerging economies are more sensitive to developments in international rates poses a threat to their economic and financial stability, which should trigger the application of appropriate policies aimed at cushioning the effect of interest rate hikes on their business cycles.

The purpose of this paper is to examine the differences in the response of developed and developing economies to real interest rate shocks. The empirical strategy proposed allows exploiting the panel dimension of the data and aims to understand how this type of shock propagates into the economy. I then employ an augmented small open economy real-business-cycle model (SOE-RBC) featuring multiple shocks and frictions commonly considered relevant in explaining business cycle fluctuations in emerging economies. To examine the structural reasons behind these findings, the model is estimated using data from one developed (Canada) and one emerging (Brazil) economy in a Bayesian fashion.

Figure 2: **GDP and real interest rates for a set of developed economies.** Black, solid line: Gross Domestic Product (log deviations from a quadratic trend, in %); red, solid line: Real interest rates (deviations from a quadratic trend).



The focus of this paper on real (instead of nominal) rates is justified by multiple reasons. First, real interest rates emerge as a fundamental variable when it comes to the decision-making process of agents. In particular, both consumers and firms take inflation into account when making consumption and investment decisions in order to account for the real purchasing power of money, which allows them to evaluate the true cost of borrowing or the return on investment over time.

Related to the latter point, Central Banks, therefore, target real interest rates when pursuing monetary policy with the purpose of influencing economic activity to achieve specific objectives within their mandates. By considering measures of expected inflation, Central Banks can adjust nominal interest rates to achieve the desired impact on real interest rates, which has in turn a direct influence on borrowing and investment decisions. Failing to monitor expected inflation can lead to the under or overshooting of nominal interest rates, potentially triggering unintended consequences in the economy that do not align with the initial policy goals.

A final point relates to the nature of the countries considered. Emerging economies are found in the literature to have a high fraction of their global trade invoiced in international currencies (in particular the Dollar and the Euro, see e.g. [Goldberg & Tille 2016](#), [Gopinath 2015](#), [Kamps 2006](#)), which makes them susceptible to inflationary pressures arising from a higher import price and trade volume sensitivity to fluctuations in their exchange rates ([Boz et al., 2022](#)).

**Related literature.** The literature that examines the drivers of economic fluctuations in emerging economies

within the SOE-RBC framework is extensive. In terms of methodology, [Mendoza \(1991\)](#) emerged as the starting point for the use of RBC models to characterize business cycles. These models however are not able to reproduce a key finding that arises from the data: the fact that consumption volatility is higher than that of output in emerging economies. For this purpose, the literature explored modifications of the canonical RBC that would generate predictions in line with this empirical evidence. [Aguilar & Gopinath \(2007\)](#) extend the standard RBC model by allowing for two different types of productivity shocks, transitory and permanent, and find that shocks affecting output *trend* are more relevant than transitory shocks in explaining business cycles in emerging economies. In a recent application, [Noh & Baek \(2021\)](#) support the dominance of trend productivity shocks even when the standard RBC model is augmented with financial frictions, which, together with stationary technology shocks, are found to be relatively less relevant. [Naoussi & Tripier \(2013\)](#) find that the impact of trend shocks on the business cycle is inversely related to the overall levels of country development, at both the economic and institutional levels. Long-run technology shocks have also been shown to generate sudden stops in SOE models in the presence of collateral constraints ([Seoane & Yurdagul, 2019](#)). [Boz et al. \(2011\)](#) find that the presence of consumer informational frictions with regard to learning about the two types of technology shocks is more critical for emerging economies than for developed ones.

The extent to which trend productivity shocks can explain business cycles in emerging economies has been nonetheless subject to debate in the literature. In a seminal contribution, [Garcia-Cicco et al. \(2010\)](#) show that trend shocks have a negligible impact on the Argentinian business cycle by employing an SOE-RBC model featuring both types of productivity shocks and adding two building blocks to the model: financial frictions and a working capital constraint. The financial friction is based on the observation that the interest rates faced by a country in international markets are a positive function of its indebtedness. The working capital constraint emerges when firms are directly affected by interest rates because they need to pay a fraction of the total wage bill in advance, which must be raised by borrowing funds from the financial sector. In this way, interest rate shocks affect the intertemporal substitution of consumption and alter the firm’s factor allocation and production. [Chang & Fernández \(2013\)](#) reinforce the role of financial imperfections in amplifying the effect of shocks, and attribute a negligible role to trend TFP shocks. In a recent contribution, [S. Hwang & Kim \(2022\)](#) examine the extent to which Asian business cycles differ from Latin American ones by estimating the model in [Garcia-Cicco et al. \(2010\)](#) in a Bayesian fashion.

[Neumeyer & Perri \(2005\)](#) divide the interest rate shock into an “international” and a “country risk” components and show that the working capital constraint indeed amplifies the effect of fundamental shocks on the economy. Similarly, [Uribe & Yue \(2006\)](#) find feedback effects between the business cycle and country spreads, that are simultaneously sensitive to international rates. In a purely empirical application, [Monacelli et al. \(2023\)](#) show that capital inflows shocks (proxied by a drop in the real interest rate) generate TFP booms (drops) in emerging (developing) economies. [Arellano \(2008\)](#) develops an SOE model featuring incomplete markets that can generate an inverse relationship between cyclical output and interest rates, which, together with the sovereign default probabilities, are endogenous to the endowment shocks the economy is subject to.

Subsequent contributions reinforce the importance of financial frictions over trend productivity shocks in

explaining business cycles in emerging economies (Chang & Fernández, 2013, Miyamoto & Nguyen, 2017, Y.-N. Hwang, 2012, Alvarez-Parra et al., 2013). Mendoza (1991) and Calvo (1998) employ theoretical models to understand how financial frictions can explain episodes of sudden slowdowns in capital inflows into the economy. Other relevant contributions focus on the importance of different types of labor market frictions in explaining the relatively high consumption volatility (Boz et al., 2015).

Overall, the literature identifies specific sets of shocks and frictions that are considered crucial for analyzing the main factors driving the business cycle in emerging economies: productivity shocks (both stationary and trend shocks), which account for the variation in output due to total factor productivity; financial frictions, which generate a positive relationship between deviations of the debt level from its long-run value and the interest rate faced by the agents; working capital constraints, which make firms' optimal labor choices (and hence production) to depend directly on interest rates; and interest rate shocks, which act as spread shocks that aim to capture how risky the financial sector deems the economy.

I follow the strand of the literature that tries to unveil the effect of interest rate shocks on the business cycle of emerging economies (see e.g. Uribe & Yue, 2006, Neumeyer & Perri, 2005) but I extend the analysis in several ways. First, the availability of new data at a quarterly frequency allows for the inclusion of periods in the analysis that have been of special economic relevance, such as the Great Recession. The contribution thus lies in examining the extent to which recent economic developments were able to shape the effect of interest rate shocks in both developed and emerging economies. Second, and more importantly, I analyze whether this effect is due to the structural aspects of the economy or the exposure to a particular path of shocks by estimating a theoretical model using data from Canada and Brazil. The estimation of the model allows examining in a rigorous and statistically founded manner the particularities of each economy that drive the differences in the response of the economy to interest rate shocks.

The results reveal that the Brazilian economy is subject to interest rate shocks that are substantially more volatile than those of the Canadian economy. In addition, the sensitivity of the interest rate to deviations in the debt level from its steady-state value is estimated to be higher for Brazil. Contrary to what is found for Canada, the Brazilian business cycle is substantially driven by shocks to the trend than by stationary technology shocks, and the rate spread shock can explain a substantially higher fraction of the cyclical component of investment growth and the trade balance-to-output ratio. A counterfactual exercise reveals that the volatility of the real rate shock matters substantially in determining the sign of the relationship between macroeconomic aggregates and the real interest rate in the Brazilian economy. Nevertheless, the results from a welfare analysis show that only reductions in the sensitivity of the real interest rate to deviations in the debt level from its long-run value are shown to be welfare-improving for Brazil.

The remainder of this paper is organized as follows. Section 2 describes the empirical methodology and presents the macroeconomic responses to real rate shocks. In Section 3, the SOE-RBC model employed is outlined. Section 4 illustrates the calibration and estimation procedures, and Section 5 presents the estimation results and the counterfactual analysis. Finally, Section 6 concludes.

## 2 Empirical Evidence

I make use of a panel structural VAR (sVAR) model to quantify the macroeconomic effects of real interest rate shocks for both country types. The panel feature in this model is that the data comes from multiple units, each defined by whether countries fall into the developing or developed category. The estimator employed here is therefore a pooled estimator, and the VAR is given by the standard representation

$$y_t = c + \sum_{k=1}^p A_p y_{t-k} + Cx_t + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma), \quad (1)$$

where  $c$  denotes a constant term,  $A_p$  denote the matrices of coefficients (assumed to be common across units),  $C$  denotes the coefficient matrix mapping the exogenous variables  $x_t$  to the endogenous variables, and  $\Sigma$  denotes the unit-variance-covariance matrix of the error term. The model is estimated in a Bayesian framework by imposing the standard Normal-Wishart prior.<sup>1</sup> In this vein, the prior for the coefficients of the system is assumed to follow a multivariate Normal distribution<sup>2</sup>

$$\beta \sim \mathcal{N}(\beta_0, \Sigma_c \otimes \Phi_0), \quad (2)$$

where  $\Sigma_c$  denotes the VAR residual variance-covariance matrix, and  $\Sigma_c \otimes \Phi_0$  denotes the full covariance matrix. The prior for  $\Sigma_c$  is assumed to follow an Inverse-Wishart distribution

$$\Sigma_c \sim \mathcal{IW}(S_0, \alpha_0), \quad (3)$$

where  $S_0$  denotes the prior scale matrix, and  $\alpha_0$  denotes the prior degrees of freedom, both defined as in [Karlsson \(2013\)](#).

Similarly to the Minnesota prior,  $\Sigma_c$  is assumed to be a diagonal matrix, while  $\Phi_0$  is constructed so as to mimick the variance matrix of the Minnesota prior ([Doan et al., 1984](#)):  $\lambda_1$  denotes the hyperparameter governing the overall prior tightness,  $\lambda_2$  governs the standard deviation of the prior on lags of variables other than the dependent variable,  $\lambda_3$  governs the decay over lags of the own variable, and  $\lambda_4$  relates to the variance of the constant and exogenous variables. The chosen values for the hyperparameters are commonly found in the literature and are given by:<sup>3</sup>

$$\lambda_1 = 0.1, \quad \lambda_2 = 1, \quad \lambda_3 = 1, \quad \lambda_4 = 100 \quad (4)$$

Given that the data enters the model in stationary form,  $\beta_0$  imposes the absence of a unit root in the autoregressive coefficients.

With this, I estimate a VAR model with five variables denoted respectively by GDP ( $gdp_t$ ), private consumption ( $c_t$ ), investment (gross fixed capital formation) ( $i_t$ ), net exports ( $nx_t$ ), and the real interest rate ( $r_t$ ).<sup>4</sup>

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<sup>1</sup>Contrary to the Minnesota prior, under the Normal-Wishart prior both  $\beta$ ,  $\Sigma_c$  are assumed to be unknown and hence they can be estimated.

<sup>2</sup>See [Canova & Ciccarelli \(2013\)](#) and [Dieppe et al. \(2016\)](#) for an excellent review of panel VAR models.

<sup>3</sup>As shown in [Dieppe et al. \(2016\)](#), the variance-covariance matrix under a Normal-Wishart prior is a special case of that under a Minnesota prior in which  $\Sigma_c$  is assumed to be diagonal and  $\lambda_2 = 1$ .

<sup>4</sup>For more information on the data sources, see Appendix [A.1](#).

All the variables except for net exports and the domestic interest rate are log-quadratically detrended. Given the potential negative values for net exports, I adjust them by the exponential of trend output and then remove a quadratic trend. I divide the countries into two groups: developed economies (Australia, Canada, the Netherlands, New Zealand, and Sweden) and developing economies (Argentina, Brazil, Ecuador, Mexico, and South Africa). The identification scheme adopted relies on the sign restrictions approach following [Arias et al. \(2018\)](#) by assuming the following:<sup>5</sup>

$$\begin{array}{c}
 \text{GDP} \\
 \text{Consumption} \\
 \text{Investment} \\
 \text{NX} \\
 \text{Real int.rate}
 \end{array}
 \begin{pmatrix}
 \text{GDP} & \text{Consumption} & \text{Investment} & \text{NX} & \text{Real int.rate} \\
 * & * & * & * & - \\
 * & * & * & * & - \\
 * & * & * & * & - \\
 * & * & * & * & * \\
 * & * & * & * & +
 \end{pmatrix}
 . \quad (5)$$

In words, it is assumed that the real financing cost shock increases the real interest rate, while it has a negative impact on GDP, consumption, and investment (and the effect on net exports is not restricted to any sign). All restrictions are set on the quarter of impact.

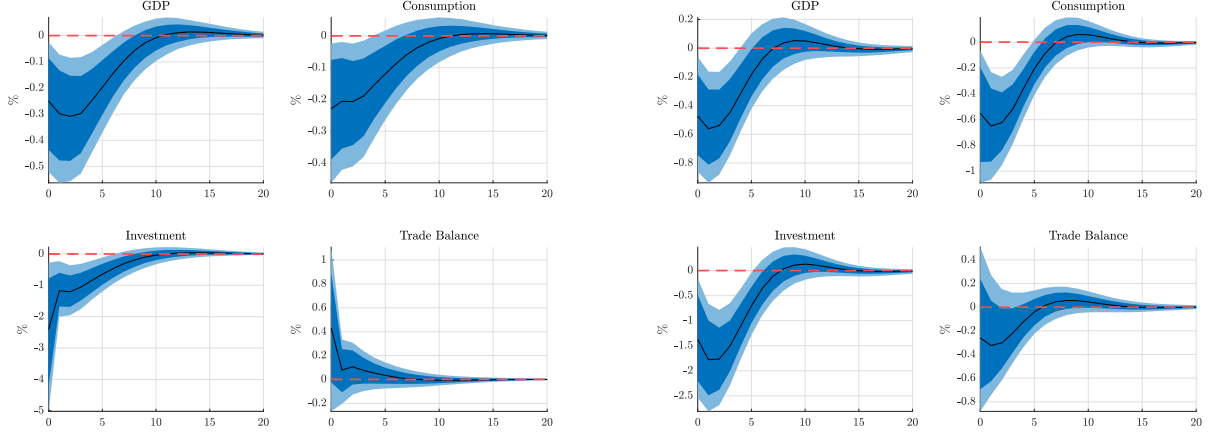
The total number of lags is set to  $p = 4$  for all variables. In addition, I include the European Brent Spot Price as an exogenous variable of the system (sourced from the U.S. Energy Information Administration). The data consists of a balanced panel for both country types available for the period 2004q1:2019q4. The number of total iterations is set to 15,000 after a burn-in phase of 10,000 iterations.

Figure 3 depicts the estimated macroeconomic responses to a one standard deviation shock to the real interest rate for both developed and developing economies. The mean responses show a decline in GDP for both groups of countries, although the effect on developing economies is relatively more adverse. The same finding holds when considering the responses of consumption, which go back to zero substantially faster for developed economies. Despite the (negative) reaction of investment being stronger for developed economies on impact, the path back to zero is relatively more adverse for developing economies. I document a decline (increase) on net exports for developed (developing) economies to the shock, although the effect seems not to be significant. Overall, the empirical evidence from the panel sVAR points to the observation that real rate shocks have macroeconomic effects that are more adverse in developing economies than in developed ones. Taking this into consideration, the following section proposes a theoretical framework for understanding the fundamental reasons behind this finding.

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<sup>5</sup>Note: column denotes shock, row denotes response; the asterisk denotes that no restriction on the response to the shock is imposed.

Figure 3: **Impulse responses to a one standard deviation shock to the real rate for developed and developing economies, in %.** The figure depicts the cross-country median effect (solid, black line) together with the 68% (dark blue shaded area) and 90% (light blue shaded area) credible bands.



(a) Developed Economies

(b) Developing Economies

### 3 The Model

#### 3.1 Households

The model builds on the work of [Garcia-Cicco et al. \(2010\)](#). The economy is populated by an infinite number of infinitely lived agents with GHH preferences ([Greenwood et al., 1988](#)). The representative agent maximizes the discounted expected utility, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{[C_t - \omega^{-1} X_{t-1} H_t^{\omega}]^{1-\gamma} - 1}{1-\gamma} \right) Z_t, \quad (6)$$

where  $C_t$  denotes consumption,  $H_t$  denotes hours worked,  $\beta \in (0, 1)$  is a discount factor,  $\gamma$  denotes the parameter governing consumer risk aversion,  $\omega$  denotes the parameter governing the disutility of labor, and  $X_t$  denotes a stochastic trend. The growth rate of the trend shock, denoted by  $g_t$ , is given by

$$g_t = \frac{X_t}{X_{t-1}} \quad (7)$$

and follows a (stationary) AR(1) process such that

$$\ln \left( \frac{g_t}{g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{g} \right) + \varepsilon_t^g; \quad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2). \quad (8)$$

In other words,  $X_t$  denotes a shock to the trend, whereas its growth rate follows a stationary process. The variable  $Z_t$  describes a preference shock following the process

$$\ln Z_t = \rho_Z Z_{t-1} + \varepsilon_t^Z; \quad \varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2). \quad (9)$$



This shock may be thought of as an exogenous driver of consumers' (im)patience, that is, a shock that shifts the discount factor. Consumers face the following budget constraint

$$\frac{D_{t+1}^h}{1+r_t} = D_t^h - W_t h_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t - \Pi_t, \quad (10)$$

where  $D_t^h$  denotes consumers' bond holdings acquired in  $t-1$  and maturing in  $t$ ,  $K_t$  denotes physical capital,  $I_t$  denotes investment in capital goods,  $W_t$  denotes the wage rate,  $u_t$  denotes the marginal product of capital,  $\Pi_t$  denotes total distributed profits (which consumers regard as exogenous),  $S_t$  denotes a government spending shock, and  $\phi$  denotes the parameter governing the capital adjustment costs. The detrended version of the government spending shock  $s_t = \frac{S_t}{\bar{X}_{t-1}}$  is assumed to follow the process

$$\ln \left( \frac{s_t}{\bar{s}} \right) = \rho_s \ln \left( \frac{s_{t-1}}{\bar{s}} \right) + \varepsilon_t^s; \quad \varepsilon_t^s \sim \mathcal{N}(0, \sigma_s^2), \quad (11)$$

where  $\bar{s}$  denotes the steady-state level of the detrended government shock. The gross interest rate faced by households is denoted by  $r_t$  and follows the process

$$r_t = r^* + \psi \left( e^{D_{t+1} - \bar{D}} \right) + (e^{\mu_t - 1} - 1), \quad (12)$$

where  $r^*$  denotes the steady-state interest rate,  $(e^{D_{t+1} - \bar{D}})$  denotes the external debt-elastic interest rate (EDEIR, [Uribe & Yue, 2006](#)),<sup>6</sup>  $D_t$  denotes the aggregate level of debt and  $\bar{D}$  denotes its steady-state level,  $\psi$  denotes the parameter governing the sensitivity of the interest rate to deviations of debt from its steady-state value,  $(e^{\mu_t - 1} - 1)$  denotes the country spread, and  $\mu_t$  denotes the spread shock, which follows the process

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_t^\mu; \quad \varepsilon_t^\mu \sim \mathcal{N}(0, \sigma_\mu^2). \quad (13)$$

Capital is assumed to evolve according to the following process:

$$K_{t+1} = (1 - \delta)K_t + I_t. \quad (14)$$

With this, consumers choose processes  $\{C_t, H_t, K_{t+1}, D_{t+1}^h\}_{t=0}^\infty$  so as to maximize their expected discounted utility in Equation (6) subject to the budget constraint in Equation (10), the capital law of motion in Equation (14), and a standard no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} \mathbb{E}_t \frac{D_{t+j+1}^h}{\prod_{s=0}^j (1 + r_{t+s})} \leq 0. \quad (15)$$

## 3.2 Firms

A representative firm produces output using a standard Cobb-Douglas production function:<sup>7</sup>

$$Y_t = A_t K_t^\alpha (X_t H_t)^{1-\alpha}, \quad (16)$$

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<sup>6</sup>The EDEIR is included so to induce stationarity in the model, that is, to ensure that the steady-state levels of the model variables are independent of the initial conditions. This also implies that the interest rate faced by the economy increases as debt rises above its steady-state value, which is an empirically plausible assumption.

<sup>7</sup>Note that the trend shock  $X_t$  appears in the firm's production function, which means that it can also be interpreted as a non-stationary TFP shock.

where  $\alpha$  denotes the elasticity of output with respect to capital and  $A_t$  denotes a stationary technology shock following the process

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A; \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_A^2). \quad (17)$$

Firms are assumed to face a working capital constraint of the form:

$$M_t \geq \eta W_t H_t. \quad (18)$$

This expression states that the firm must hold an amount of working capital equal to  $M_t$  (which can be thought of as an asset that does not bear any interest) that has to be at least equal to a fraction  $\eta$  of the total labor costs,  $W_t H_t$ . In other words,  $\eta$  is defined as the fraction of the total wage bill that the firm is forced to pay prior to production. For the firm to pay this quantity in advance, it must raise funds by borrowing from the financial sector. The representative firm is assumed to borrow at the same rate as consumers, and the evolution of its debt is given by

$$\frac{D_{t+1}^f}{1+r_t} = D_t^f + \Delta M_t + u_t K_t + W_t h_t - Y_t + \Pi_t^f, \quad (19)$$

where  $D_t^f$  denotes the firm's debt, and  $\Pi_t^f$  denotes the firm's profits. The firm chooses processes  $\{K_t, H_t, M_t, D_t^f\}_{t=0}^{\infty}$  that maximize their expected stream of profits<sup>8</sup>

$$\begin{aligned} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t X_{t-1}^{-\gamma} \Pi_t^f \\ & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t X_{t-1}^{-\gamma} \left[ \frac{D_{t+1}^f}{1+r_t} - D_t^f - \Delta M_t - u_t K_t - W_t h_t + Y_t \right] \end{aligned} \quad (20)$$

subject to the working capital constraint in Equation (18), and a no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} \mathbb{E}_t \frac{D_{t+j+1}^f - M_{t+j+1}}{\prod_{s=0}^j (1+r_{t+s})} \leq 0. \quad (21)$$

By combining the first-order conditions of the firm's problem, I am left with the firm's budget constraint as in Equation (19), and the returns for capital and labor, respectively:

$$u_t = \alpha \left( \frac{H_t}{K_t} \right)^{1-\alpha} \quad (22)$$

$$W_t = (1-\alpha) A_t \left( \frac{K_t}{H_t} \right)^{\alpha} \left[ 1 + \eta \frac{r_t}{1+r_t} \right]^{-1} \quad (23)$$

The expression above states that, as long as  $\eta \neq 0$ , the working capital constraint creates a wedge between the marginal product of labor and the wage earned by households. The presence of a working capital constraint implies that interest rate shocks can affect the production side of the economy, in addition to their effect on the intertemporal substitution of consumption.

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<sup>8</sup>Given that consumers are assumed to be the owners of the firms, the Lagrange multiplier associated with the sequential budget constraint is equivalent to the one associated with the sequential firm profits,  $\beta^t \lambda_t X_{t-1}^{-\gamma}$  (Uribe & Schmitt-Grohé, 2017).

Following [Uribe & Yue \(2006\)](#), I show that the firm's profits are equal to zero at all periods. I denote firms' net liabilities at  $t$  by  $\Xi_t$ , which are given by

$$\Xi_t = (1 + r_t)D_t^f - M_t. \quad (24)$$

The last expression shows that net liabilities are equal to the debt that the firm needs to repay to the bank minus the wage bill that the firm sets aside and deposits in the bank. Under the assumption that  $r_t \geq 0 \forall t$ , it holds that

$$M_t = \eta W_t N_t. \quad (25)$$

I can therefore write the firm's liabilities at time  $t$  as follows

$$\Xi_t = (1 + r_t)D_t^f - \eta W_t N_t. \quad (26)$$

From Equation (19) I can rewrite the firm's profits

$$\Pi_t = Y_t - u_t K_t - W_t H_t + \frac{\Xi_t}{1 + r_t} - \Xi_{t-1} - \left( \frac{r_t}{1 + r_t} \right) \eta W_t N_t. \quad (27)$$

Under the assumption that firms do not possess any net liability at the beginning of the period  $t = 0$ , it implies that an optimal path for the firm would be to hold zero liabilities at each period:  $\Xi_t = 0 \forall t$ . With this, the firm's profits can be rewritten as follows

$$\Pi_t = Y_t - u_t K_t - W_t H_t \left( 1 + \eta \frac{r_t}{1 + r_t} \right). \quad (28)$$

This shows that  $\Pi_t = 0 \forall t$  under a particular path for firm liabilities.

### 3.2.1 Financial Sector

I assume a continuum mass of identical and perfectly competitive banks whose main activities are lending funds borrowed from international markets to domestic households and firms, and keeping deposits from firms (i.e., keeping the asset that bears no interest  $M_t$ ). Their balance sheet is expressed as follows

$$\frac{D_{t+1}^h + D_{t+1}^f}{1 + r_t} = \frac{D_{t+1}}{1 + r_t} + M_t. \quad (29)$$

The left side of Equation (29) denotes the bank's assets, which are given by the (discounted) sum of the representative consumer and firm debt. The right-hand side of Equation (29) denotes the firm's liabilities, which are given by the sum of the debt acquired by the bank in international markets,  $D_{t+1}$  (acquired at  $t$ , maturing at  $t + 1$ ; discounted), and the firm's deposits  $M_t$ . The bank's profits are assumed to be redistributed to consumers in a lump-sum manner and are given by

$$\Pi_t^b = D_t^h + D_t^f - D_t - M_{t-1}. \quad (30)$$

For more details on the model derivations, see Appendix [A.2](#).

## 4 Model Calibration And Estimation

### 4.1 Calibration

The calibration of structural parameters relies on values commonly used in similar studies that employ RBC models. The consumer risk aversion coefficient is set to  $\gamma = 2$ , which aligns with the behavioral literature. I set the parameter depicting labor disutility to  $\omega = 1.6$ , generating a labor supply elasticity of  $1/(1 - \omega) \approx 1.67$  (Garcia-Cicco et al., 2010, Mendoza, 1991, Schmitt-Grohé & Uribe, 2003). The steady-state government spending-to-output ratio is set to the observed values of 15.5% and 14.9% for Brazil and Canada, respectively.<sup>9</sup> Similarly, the steady-state trade balance-to-output ratio is set to the observed value of 4.39% and -0.49% for Brazil and Canada, respectively.

### 4.2 Estimation

The remaining parameters, that is, those that are not calibrated, are estimated in a Bayesian fashion. Bayesian estimation of DSGE models allows us to obtain posterior distributions for the structural parameters of the model by combining three elements: the priors (defined as the researcher's prior knowledge and beliefs about certain parameters), the observed data, and the likelihood function (i.e., the DSGE model itself). Thus, the DSGE model can be translated into a state-space representation that allows the creation of a mapping between the model and the observed data:

$$X_t = A(\Theta)X_{t-1} + B(\Theta)u_t \quad (31)$$

$$\mathbf{Y}_t = C(\Theta) + D(\Theta)X_t + v_t, \quad (32)$$

where  $\Theta$  denotes the vector of parameters to estimate,  $\mathbf{Y}_t$  denotes the vector of endogenous (observed) variables,  $C(\Theta)$  denotes a vector with data means,<sup>10</sup>  $D(\Theta)$  denotes a matrix mapping the unobserved states to the data, and  $u_t, v_t$  denote vectors of structural shocks and measurement errors, respectively. With this, the Kalman filter is used to form an estimate of the unobservable states and to evaluate the likelihood function associated with the state-space model. Using Bayes' theorem, the posterior distribution of the parameters can be expressed as a function of the likelihood and the priors:

$$p(\Theta|\mathbf{Y}) = \frac{p(\mathbf{Y}|\Theta)p(\Theta)}{p(\mathbf{Y})} \propto p(\mathbf{Y}|\Theta)p(\Theta), \quad (33)$$

where  $p(\mathbf{Y}|\Theta)$  denotes the likelihood function,  $p(\Theta)$  denotes the prior density of the vector of parameters, and  $p(\mathbf{Y}) = \int p(\mathbf{Y}|\Omega)p(\Omega)d\Omega$  denotes the (unconditional) data density function, which can be disregarded because it simply acts as a scaling factor. With this, the posterior distribution in Equation (33) can be computed by combining the prior distributions and likelihood function by applying the Kalman filter to the state-space

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<sup>9</sup>For both countries, the (yearly) data employed in the model estimation belongs to the period 1951:2019 and are sourced from the Penn World Tables, version 10.1 (Feenstra et al., 2015).

<sup>10</sup>Given that the model is detrended prior to estimation, the growth rate of the trend  $g$  is also a parameter to estimate, and hence the data counterpart is an element of  $C(\Theta)$ .

representation in Equation (31), given the data  $\mathbf{Y}$ . The computation is typically performed considering the log of the posterior distribution, that is, (under the assumption of a priori independence)

$$\ln(p(\Theta|\mathbf{Y})) = \ln(p(\mathbf{Y}|\Theta)) + \sum_{j=1}^J \ln(p(\Theta_j)), \quad (34)$$

where  $J$  denotes the total number of parameters to estimate. Therefore, our vector of parameters to estimate is

$$\Theta = \{\rho_A, \rho_g, \rho_Z, \rho_s, \rho_\mu, \sigma_A, \sigma_g, \sigma_Z, \sigma_s, \sigma_\mu, \phi, \psi, \beta, \delta, \alpha, \eta, \sigma_{\Delta y}, \sigma_{\Delta c}, \sigma_{\Delta i}, \sigma_{tb/y}\}. \quad (35)$$

The last four parameters denote the standard deviations of the measurement errors of the observed variables, which are included for robustness. The theoretical model is written in detrended form (see Appendix A.2 for further details) prior to estimation. The matrix of observed variables includes the growth rate of output, the growth rate of consumption, the growth rate of investment,<sup>11</sup> and the trade balance-to-output ratio. The model is solved and estimated using the *Dynare* (Adjemian et al., 2022), and the posterior distributions for our parameters of interest are obtained using the slice sampler algorithm developed by Planas et al. (2015).<sup>12</sup> I refer to the Online Appendix for details on the convergence diagnostics. As depicted in Table 1, I assume uniform prior distributions for all the estimated parameters following the specification in S. Hwang & Kim (2022).

## 5 Results

### 5.1 Estimation Results

The estimation results are based on 15,000 draws<sup>13</sup> of the slice-sampling algorithm after a burn-in phase of 7,500 and five (sequentially drawn) parallel chains. The posterior values for all estimated parameters are depicted in Table 1. The results show that, except for the trend shock, Canada is found to experience shocks that are systematically more persistent than Brazil. Nonetheless, the Brazilian economy suffers from more volatile shocks, with the exception of the stationary TFP shock. This is particularly true for the volatility of the rate spread shock, which is estimated to be approximately nine times more volatile in Brazil than in Canada.

The posterior values for the structural parameters reveal that the Canadian economy is characterized by slightly higher capital adjustment costs, lower depreciation rates, and a relatively stronger working capital constraint. This means that, for a given shock to the real rate, the wedge between the marginal product of labor and the wage is relatively larger in Canada.<sup>14</sup> The estimated discount factor is relatively higher for Canada,

<sup>11</sup>Note: all variables are transformed into per capita values.

<sup>12</sup>See Neal (2003) for a first introduction to slice sampling methods. Applications of the slice posterior sampling algorithm for the estimation of DSGE models in a Bayesian fashion can be found in e.g. Giovannini et al. (2019), Hohberger et al. (2019), Croitorov et al. (2020). Calés et al. (2017) showed that the slice sampling algorithm has better properties than the standard Metropolis-Hastings (M-H) sampling algorithm.

<sup>13</sup>The number of draws might seem a priori insufficient. However, note that while one M-H iteration requires exactly one likelihood evaluation, one slice iteration requires approximately  $\#parameters \times 7$  likelihood evaluations. Therefore, a general rule of thumb states that if one wants to run  $N$  million draws using M-H, this is equivalent to running  $N/(\#parameters \times 7)$  draws using the slice sampling algorithm.

<sup>14</sup>In other words, the effect through which changes in the real rate affect aggregate production is found to be larger for the Canadian economy.

which (given similar values for the trend growth rate  $g$ ) indicates that the Brazilian economy is subject to higher real interest rates. Finally, I find that the sensitivity of the real rate to deviations in aggregate debt from its steady-state value is much higher in Brazil than in Canada. This implies that the Brazilian economy is much more penalized in international financial markets whenever its aggregate debt level diverges from its long-run (steady-state) values.

Figure 4 depicts the response of the growth rate of output, consumption, and investment, and the trade balance-to-output ratio to a real interest rate shock. All variables (except for the trade balance) respond more negatively to the rate shock in the Brazilian economy than in the Canadian economy, especially output growth. Canada seems to return to its steady-state value substantially faster, which shows that shocks to interest rates do not percolate into the Canadian economy as strongly as they do into the Brazilian economy.

To understand the contribution of each shock to the business cycles in both economies, Table 2 shows the posterior variance decomposition at the posterior median of the parameters. The Brazilian economy is found to be relatively much more driven by shocks to its trend than by stationary TFP shocks, which goes in line with the argument *the cycle is the trend* motivated by Aguiar & Gopinath (2007). The rate spread shock can explain a substantially higher fraction of both investment growth and the trade balance-to-output ratio in the Brazilian economy than in the Canadian economy.

Table 3 presents a comparison between the data moments and those generated by the model when all parameters are set to their corresponding estimated values. The model correctly predicts that consumption volatility is higher than GDP volatility for Brazil (whereas the opposite applies to Canada) and the fact that investment growth is the most volatile series. While it correctly generates a negative correlation between the trade balance-to-output ratio for the Brazilian economy, it is unable to capture the positive empirical correlation between these two series for Canada.

The relationship between the smoothed rate shock and GDP growth is shown in Figure 5. The results reveal that the smoothed interest rate shock is negatively correlated with cyclical output in Brazil, whereas the correlation is negligible for Canada. The volatility of the smoothed rate shock is estimated to be seven times higher than that in Canada.

## 5.2 Counterfactual Analysis

### 5.2.1 Theoretical IRFs And Correlations

Structural estimation of theoretical models allows producing counterfactual analysis that sheds light on certain characteristics of the economy under different parameter values. With this purpose, I study how the response of the Brazilian economy to shocks to the real rate is influenced by two key parameters: the elasticity of the real interest rate to deviations of the debt level relative to its steady state value (i.e.,  $\psi$ ), and the volatility of the real rate shock,  $\sigma_\mu$ . I start by setting all the parameters to their estimated posterior median and modify the value of  $\psi$  by creating a linearly spaced vector of length three between the estimated value for the Brazilian economy and the Canadian. Figure 6 shows that the Brazilian growth rate of output, consumption, and investment

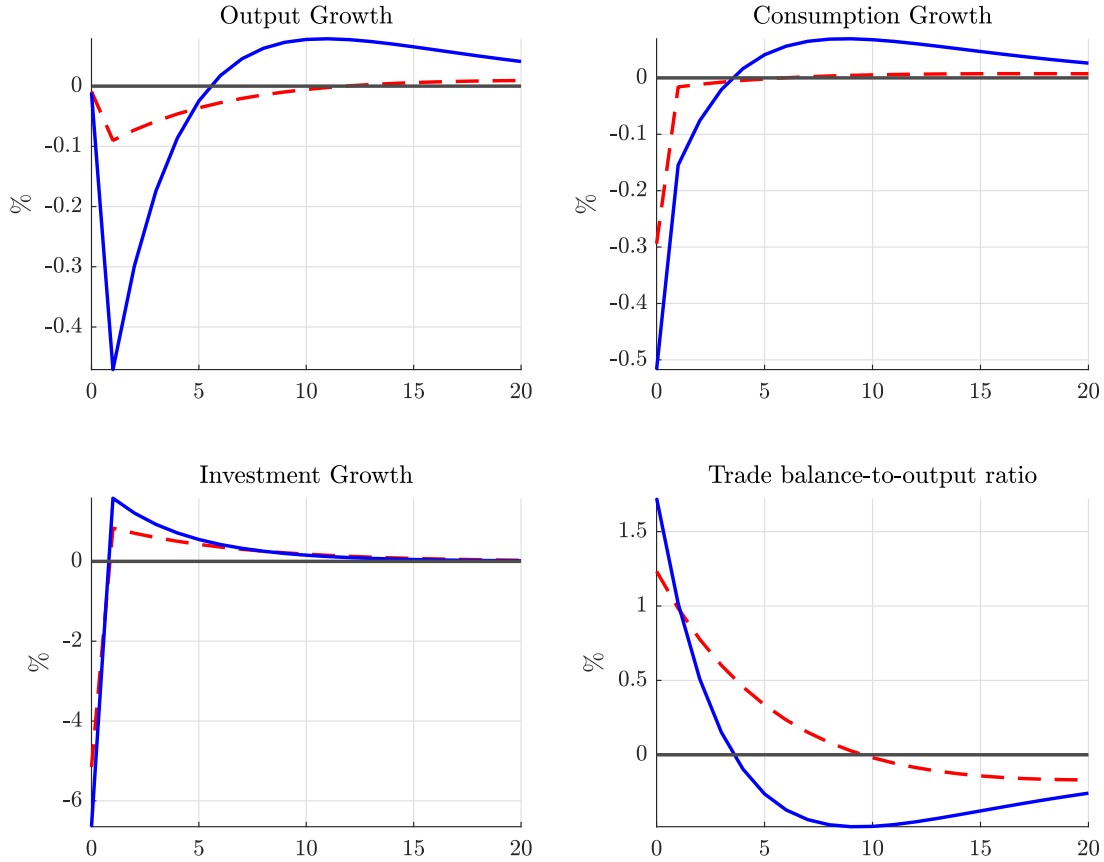
Table 1: Priors and posterior distributions for the estimated parameters.

		Prior distribution		Posterior distribution			
Parameter	Distribution	Min	Max	Brazil		Canada	
				Median	Sd	Median	Sd
<i>Shock persistence</i>							
$\rho_A$	Uniform	0	0.99	0.6520	0.2779	0.8845	0.0258
$\rho_g$	Uniform	0	0.99	0.7736	0.0505	0.5907	0.2170
$\rho_Z$	Uniform	0	0.99	0.7488	0.1643	0.9811	0.0139
$\rho_s$	Uniform	0	0.99	0.5141	0.2831	0.5587	0.2844
$\rho_\mu$	Uniform	0	0.99	0.8971	0.0567	0.9570	0.1050
<i>Shock s.d.</i>							
$\sigma_A$	Uniform	0	0.4	0.0021	0.0023	0.0144	0.0017
$\sigma_g$	Uniform	0	0.4	0.0157	0.0016	0.0028	0.0018
$\sigma_Z$	Uniform	0	0.4	0.1826	0.0632	0.1412	0.0510
$\sigma_s$	Uniform	0	0.4	0.0043	0.0037	0.0024	0.0022
$\sigma_\mu$	Uniform	0	0.4	0.0088	0.0033	0.0010	0.0004
<i>Structural parameters</i>							
$g$	Uniform	1	1.05	1.0299	0.0072	1.0168	0.0023
$\phi$	Uniform	0	8	2.0708	0.7450	2.8332	0.9047
$\psi$	Uniform	0	5	0.0534	0.0596	0.0015	0.5070
$\beta$	Uniform	0.9	0.999	0.9661	0.0179	0.9942	0.0053
$\delta$	Uniform	0.01	0.15	0.1017	0.0213	0.0129	0.0042
$\alpha$	Uniform	0.3	0.5	0.3206	0.0230	0.3674	0.0428
$\eta$	Uniform	0	1	0.0769	0.1537	0.8268	0.1970
<i>Measurement errors</i>							
	Distribution	Mean	Sd	Median	Sd	Median	Sd
$\sigma_{\Delta y}$	Gamma	0.02	0.01	0.0024	0.0008	0.0018	0.0005
$\sigma_{\Delta c}$	Gamma	0.02	0.01	0.0024	0.0009	0.0018	0.0007
$\sigma_{\Delta i}$	Gamma	0.02	0.01	0.0105	0.0033	0.0077	0.0024
$\sigma_{tb/y}$	Gamma	0.02	0.01	0.0016	0.0005	0.0007	0.0003

Note:  $\rho_A, \rho_g, \rho_Z, \rho_s, \rho_\mu$  denote the autoregressive coefficients of the (stationary) technology, trend, preference, government spending, and interest rate shock processes, respectively.  $\sigma_A, \sigma_g, \sigma_Z, \sigma_s, \sigma_\mu$  denote the standard deviations of the (stationary) technology, trend, preference, government spending, and interest rate shock processes, respectively.  $g$  denotes the growth rate of the trend shock,  $\phi$  denotes the strength of the capital adjustment costs,  $\psi$  denotes the sensitivity of the real rate to deviations in the debt level,  $\beta$  denotes the discount factor,  $\delta$  denotes the depreciation rate,  $\alpha$  denotes the elasticity of output with respect to capital, and  $\eta$  denotes the strength of the working capital constraint.  $\sigma_{\Delta y}, \sigma_{\Delta c}, \sigma_{\Delta i}, \sigma_{tb/y}$  denote the measurement errors of observed output growth, consumption growth, investment growth, and the trade balance-to-output, respectively.

would suffer a stronger downturn on impact when facing a shock to its real rate, although the effect seems to be compensated at higher time periods. The picture looks the opposite for the trade balance-to-output ratio, which experiences a more positive shock on impact when  $\psi$  is set to the estimated value of Canada (this seems nonetheless to be compensated by a downturn at higher time periods).

Figure 4: **Model-based IRFs to a (one standard deviation) shock to the real interest rate.** Blue, solid lines: Brazil; dashed, red lines: Canada.



Note: IRFs are obtained by setting all parameters to their corresponding estimated posterior median values for each of the two countries.

Figure 7 shows the counterfactual IRFs to a real rate shock under different shock volatility values. The evidence points out the fact that the (negative) reaction of macroeconomic aggregates (output, consumption, and investment growth) to real rate shocks is substantially smaller once the volatility of the real rate shock of Brazil tends to that of Canada. The trade balance-to-output ratio, however, reacts substantially less not only on impact but also at higher time values.

I also explore the extent to which the parameters  $\{\psi, \sigma_\mu\}$  alter the relationship between the growth rate of output and the real interest rate. Similarly to the previous exercise, I construct a fine grid of length a hundred for both parameters ranging from the estimated posterior value for Brazil to that of Canada and compute the model-based correlations between output growth and the real interest rate for each point in the grid. Figure 8



Table 2: **Posterior median variance decomposition results (in %)**. Measurement errors are not reported given their negligible contribution in driving the observed variables. Standard errors are depicted in parentheses.

	Brazil				Canada			
Shock	$\Delta y$	$\Delta c$	$\Delta i$	$tb/y$	$\Delta y$	$\Delta c$	$\Delta i$	$tb/y$
Technology (stat.)	1.38 (4.93)	0.56 (3.12)	0.13 (2.37)	0.16 (1.21)	93.10 (8.24)	63.89 (8.56)	57.44 (10.14)	7.86 (5.35)
Trend	93.61 (5.07)	63.09 (6.76)	49.39 (9.38)	21.44 (14.02)	5.43 (8.25)	7.15 (8.41)	5.04 (8.54)	2.99 (10.27)
Preference	0.49 (1.15)	33.40 (5.81)	1.98 (3.23)	21.94 (11.14)	0.50 (0.32)	25.42 (5.02)	3.81 (2.68)	64.59 (19.23)
Interest rate	2.94 (1.35)	1.58 (0.82)	45.60 (7.89)	51.73 (15.26)	0.32 (0.15)	1.39 (0.65)	30.46 (8.14)	21.19 (14.23)
Government spending	0.00 (0.00)	0.00 (0.03)	0.00 (0.01)	0.02 (0.08)	0.00 (0.00)	0.00 (0.04)	0.00 (0.03)	0.01 (0.04)

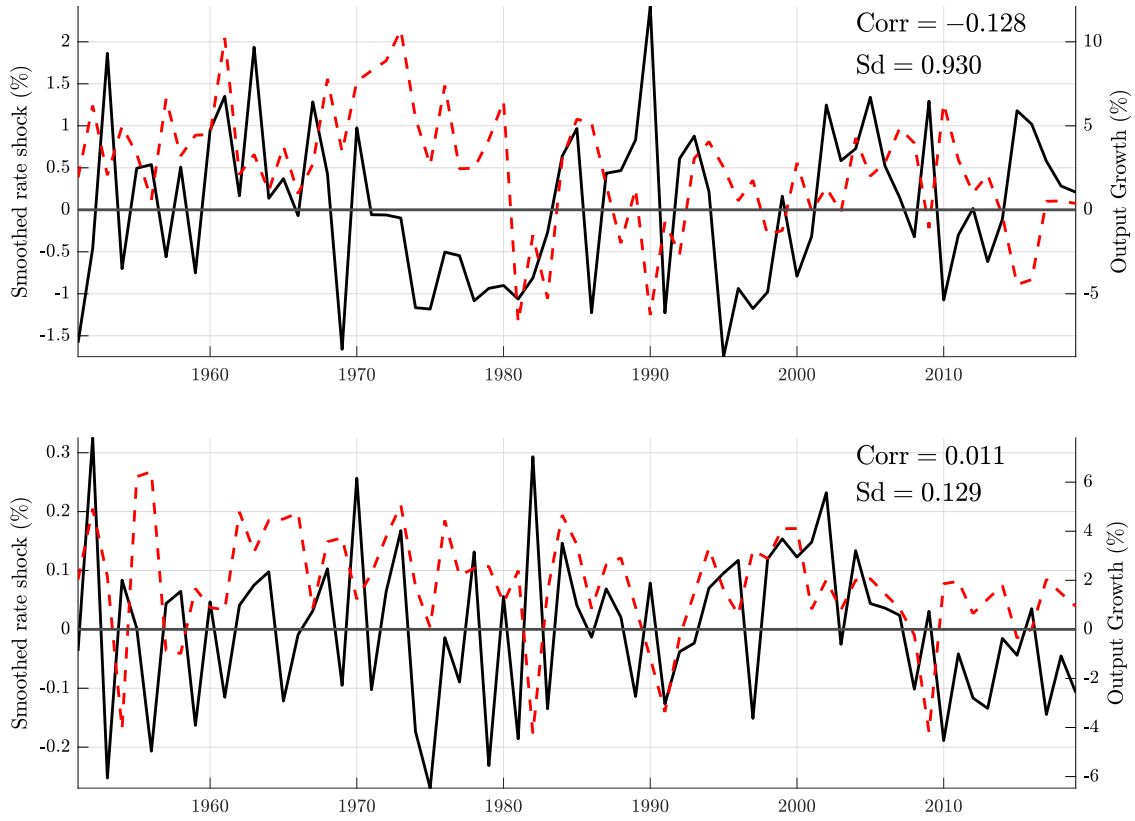
Note:  $\Delta y, \Delta c, \Delta i, tb/y$  denote the growth rates of output, consumption, and investment, and the trade balance-to-output ratio, respectively.

Table 3: **Comparison between data and model moments**. The model moments are obtained by setting the model parameter values at the posterior median of the estimated parameters.

	Brazil							
	Data				Model			
	$\Delta y$	$\Delta c$	$\Delta i$	$tb/y$	$\Delta y$	$\Delta c$	$\Delta i$	$tb/y$
$\sigma(x)$	3.6	4.01	10.79	3.4	3.58	4.72	10.28	3.67
$\rho(x, \Delta y)$	1	0.81	0.71	-0.3	1	0.81	0.64	-0.27
$\rho(x_t, x_{t-1})$	0.4	0.16	0.01	0.82	0.14	-0.01	-0.08	0.75
	Canada							
	Data				Model			
	$\Delta y$	$\Delta c$	$\Delta i$	$tb/y$	$\Delta y$	$\Delta c$	$\Delta i$	$tb/y$
$\sigma(x)$	2.21	1.5	8.91	3.17	2.5	2.31	9.12	4.16
$\rho(x, \Delta y)$	1	0.78	0.82	0.21	1	0.85	0.77	-0.06
$\rho(x_t, x_{t-1})$	0.19	0.45	-0.15	0.91	-0.04	-0.02	-0.07	0.94

reveal the fact that the correlation between the real rate and GDP and consumption growth and the trade balance-to-output ratio decreases monotonically as  $\psi^{BR}$  approaches  $\psi^{CA}$ . However, the latter is not true for the growth rate of investment, which appears to have a higher correlation (i.e., less negative) as  $\psi^{BR}$  tends to

Figure 5: **Smoothed real interest rate shocks and GDP growth for Brazil (top panel) and Canada (lower panel).** Solid, dark lines: GDP growth. Red, dashed lines: smoothed rate shocks.

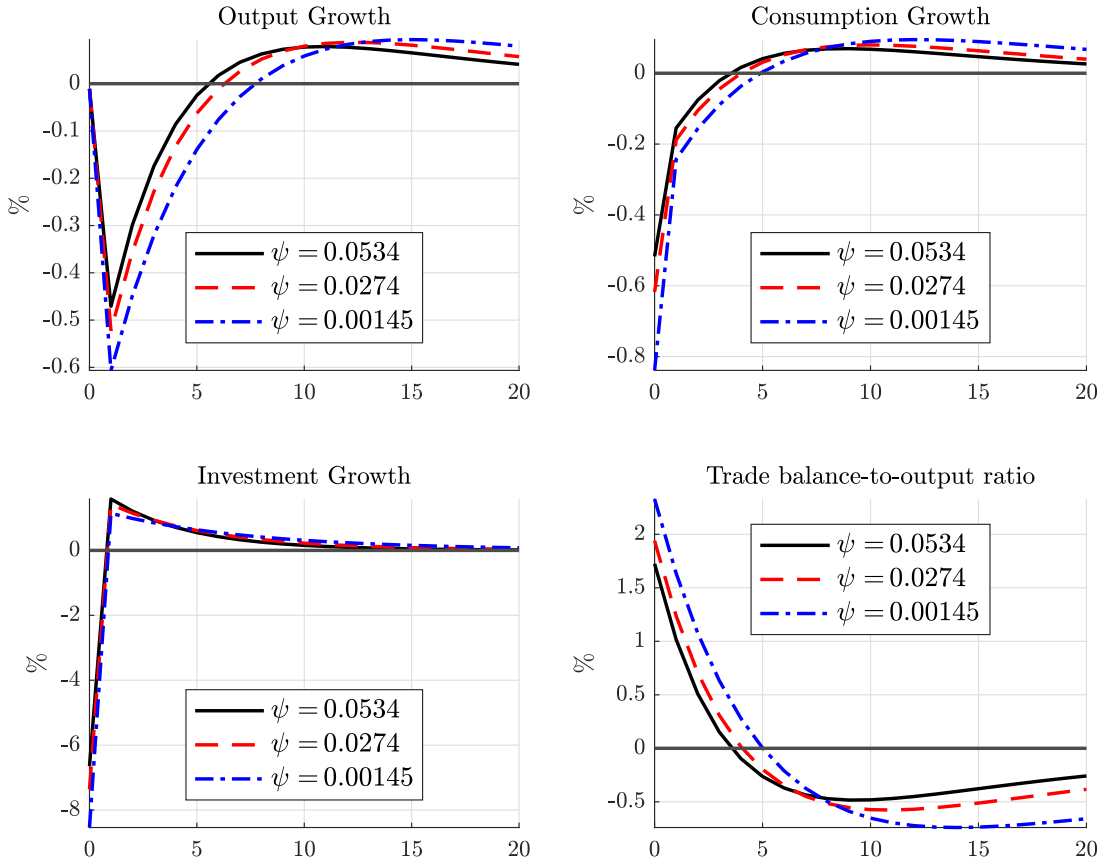


Note: Smoothed shocks are computed by employing the Kalman smoother and represent the “best guess” for the structural shocks given all observations. The empirical correlation (denoted by Corr) between the two variables and the standard deviation of the smoothed shock (denoted by Sd) are shown in both graphs.

$\psi^{CA}$ .

The interpretation changes once I examine the correlation between real rates and macroeconomic aggregates as a function of the volatility of the real rate shock itself,  $\sigma_\mu$  (Figure 9). I show that, as  $\sigma_\mu^{BR}$  tends to  $\sigma_\mu^{CA}$ , these correlations decrease (i.e., they become less negative) for the growth rate of output, consumption, and investment, while this relationship appears to be weak when considering the trade balance. Under the assumption that these correlations being negative implies that the Brazilian economy is very vulnerable to real rate hikes, this result shows that the volatility of the real rate shock matters substantially in determining the relationship between macroeconomic aggregates and financing costs.

Figure 6: **Model-based IRFs to a (one standard deviation) shock to the real interest rate for different values of  $\psi$ .** Dark, solid lines:  $\psi$  set to the posterior median for Brazil. Blue, dashed lines:  $\psi$  set to the posterior median for Canada. Red, dashed lines:  $\psi$  set at an intermediate value.



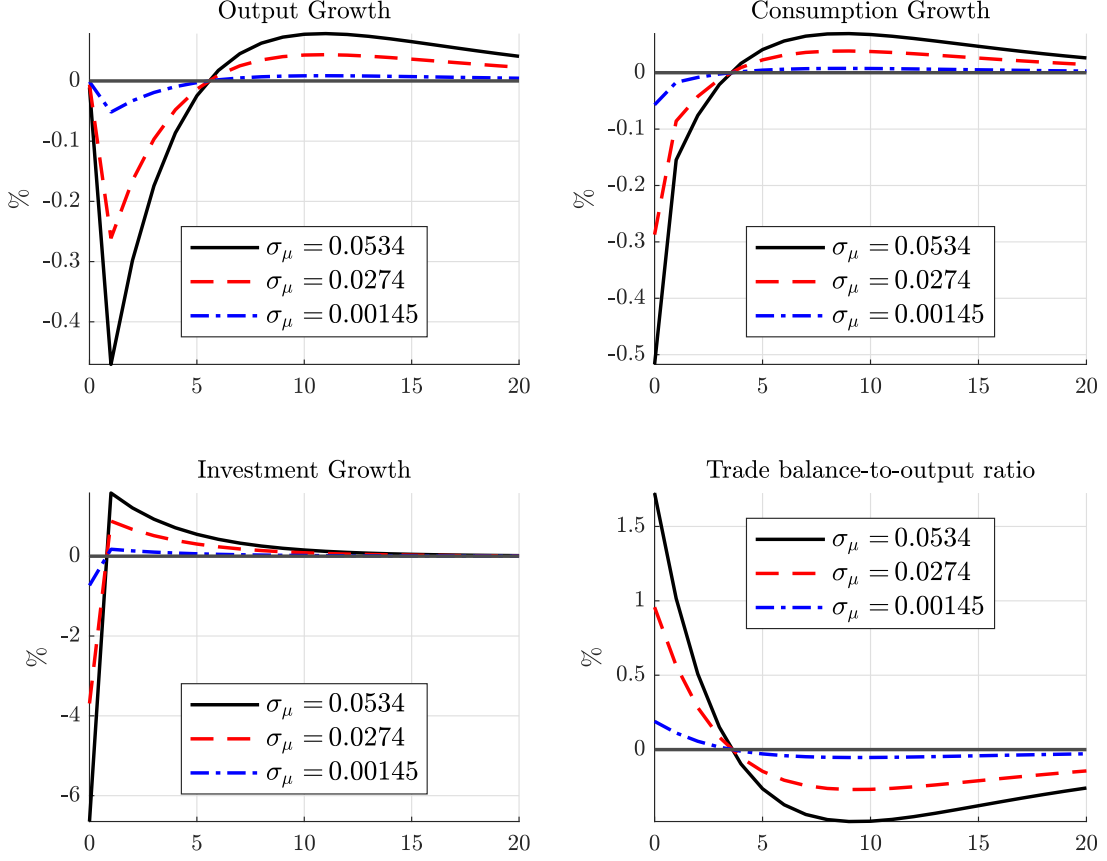
Note: IRFs are obtained by setting all parameters to their corresponding estimated posterior median values for Brazil, and varying the parameter  $\psi$ .

### 5.2.2 Welfare Analysis

To finalize the counterfactual analysis of the estimated DSGE model, I examine the potential changes in welfare under alternative parameter values. Considering the utility function of the representative agent in Equation (6), I can define a simple welfare criterion as follows:

$$\mathcal{W}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{[C_t - \omega^{-1} X_{t-1} H_t^\omega]^{1-\gamma} - 1}{1-\gamma} \right) Z_t, \quad (36)$$

Figure 7: **Model-based IRFs to a (one standard deviation) shock to the real interest rate for different values of  $\sigma_\mu$ .** Dark, solid lines:  $\sigma_\mu$  set to the posterior median for Brazil. Blue, dashed lines:  $\sigma_\mu$  set to the posterior median for Canada. Red, dashed lines:  $\sigma_\mu$  set at an intermediate value.



Note: IRFs are obtained by setting all parameters to their corresponding estimated posterior median values for Brazil, and varying the parameter  $\sigma_\mu$ .

which can be rewritten as

$$\mathcal{W}_t = \left( \frac{[C_t - \omega^{-1} X_{t-1} H_t^\omega]^{1-\gamma} - 1}{1-\gamma} \right) Z_t + \beta \mathbb{E}_t \mathcal{W}_{t+1}. \quad (37)$$

With this, I examine how values of  $\psi$  and  $\sigma_\mu$  affect welfare in the Brazilian economy. I proceed in a similar way to the previous subsection by creating equally spaced (joint) grids of  $\psi$  and  $\sigma_\mu$  and analyzing their impact on welfare for each possible parameter combination by solving the model using a second-order approximation around the deterministic steady state.

Figure 10 depicts the results of the counterfactual welfare analysis. Conditional on  $\psi$  being set to its estimated value for Brazil, reductions in the spread shock volatility imply monotonic reductions in welfare. On

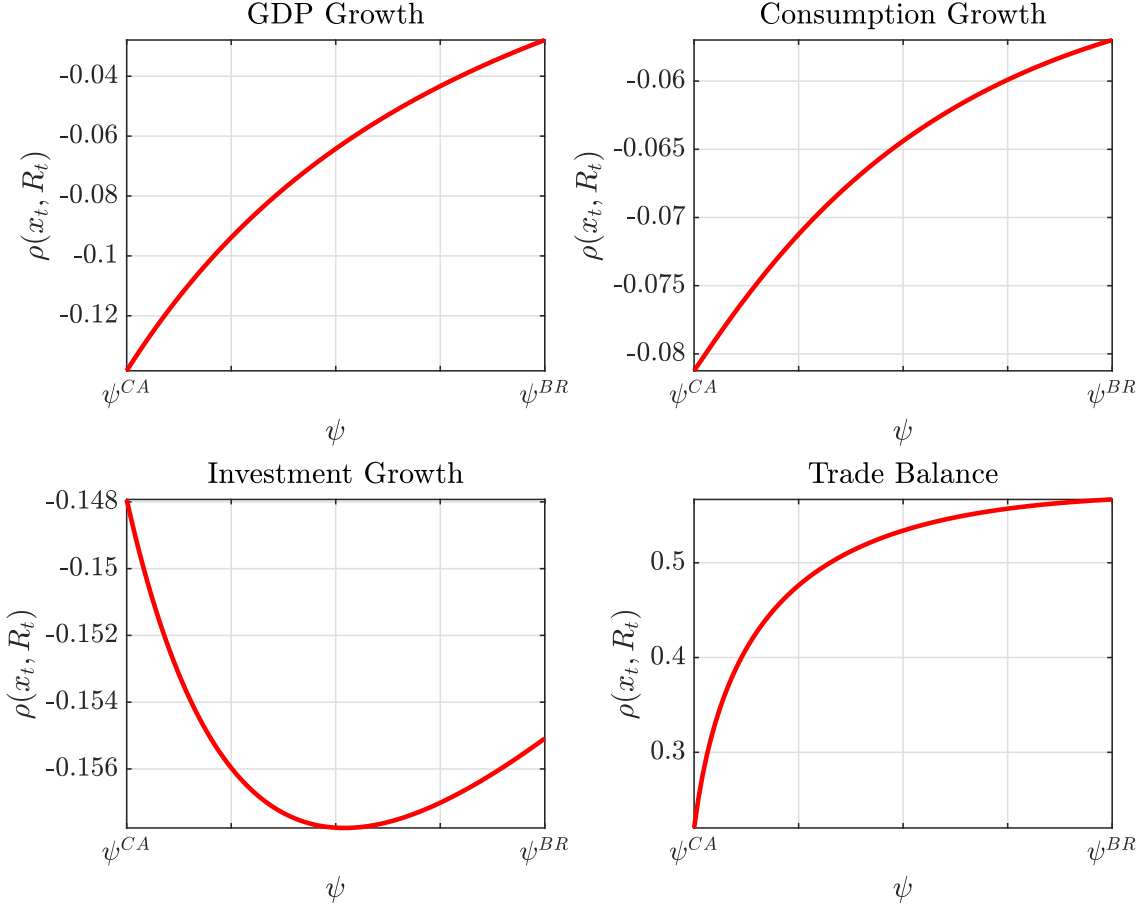


Figure 8: **Theoretical correlation between output growth and real interest rate as a function of the parameter  $\psi$ .** Note: I consider a grid of length a hundred that ranges from the posterior estimated value for Canada ( $\psi^{CA}$ ) to that for Brazil ( $\psi^{BR}$ ).

the other hand, reductions in the sensitivity of the real rate to deviations of the aggregate debt level relative to its steady-state value (i.e., reductions in  $\psi$  conditional on  $\sigma_\mu$ ) are shown to be welfare-improving. Taken together, these results show that it is not the spread shock volatility the parameter that matters when analyzing welfare, but the extent to which the real interest rate faced by the Brazilian economy reacts to movements in the debt level. Despite the observation that lower values of  $\psi$  imply lower (i.e., more negative) correlations between the growth rates of GDP and consumption, these are found to induce higher welfare values (those implied by the simple welfare rule in Equation (36)) in Brazil.

### 5.2.3 Counterfactual Macroeconomic Volatility

Since the role of  $\psi$  has been identified as significant for welfare, the next step is to examine how different levels of this parameter influence the overall volatility of the economy. For this purpose, I first obtain the smoothed shocks (i.e., the best guesses of the structural shocks given the data observed), which are obtained with the estimation of the model. With this, I input them into the theoretical model and vary  $\psi$  accordingly. In

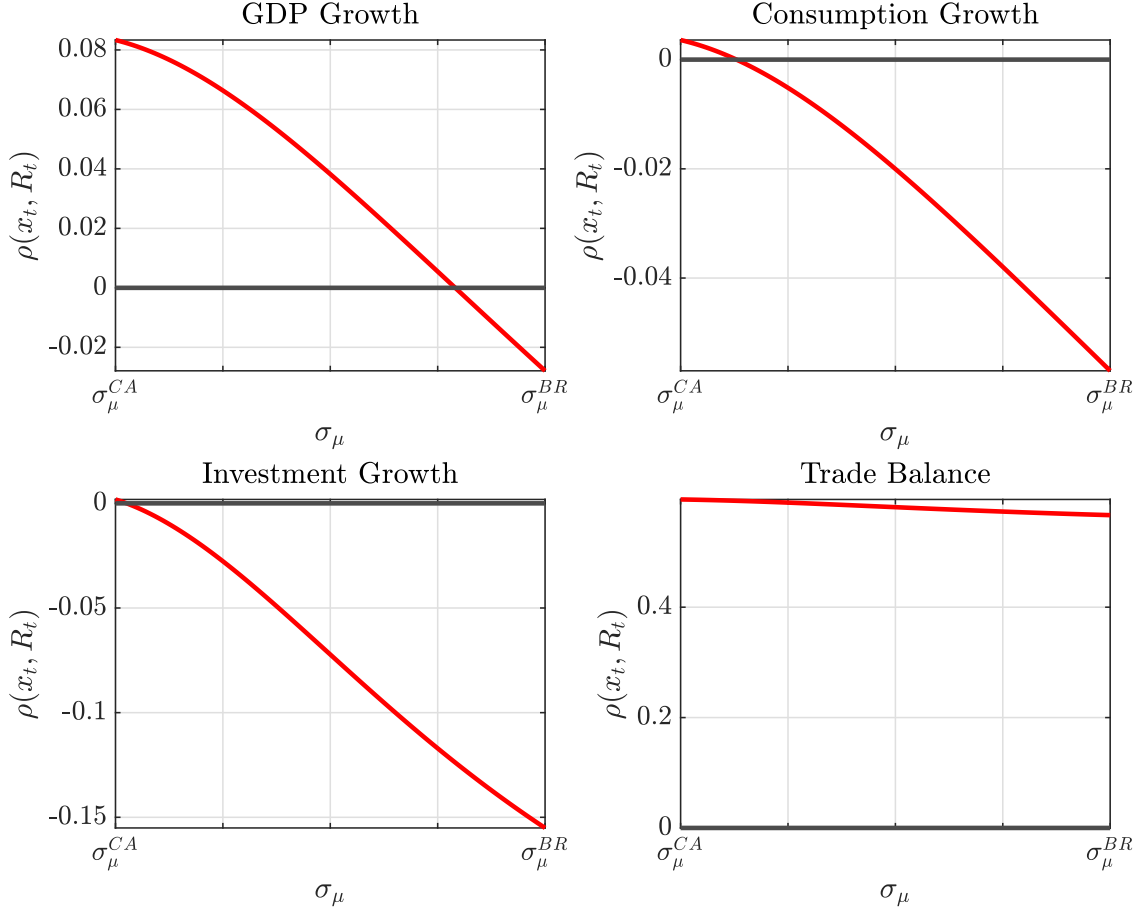


Figure 9: **Theoretical correlation between output growth and real interest rate as a function of the parameter  $\sigma_\mu$ .** Note: I consider a grid of length a hundred that ranges from the posterior estimated value for Canada ( $\sigma_\mu^{CA}$ ) to that for Brazil ( $\sigma_\mu^{BR}$ ).

this sense, the shocks are not *deterministic* but *determined*, i.e., agents in the economy still form expectations because they do not know the future realization of the shocks but these shocks are in turn *determined* (i.e., they are not *simulated shocks*). On top of this, I analyze the extent to which macroeconomic volatility is shaped by the real interest rate shock by setting this shock to zero at all time periods (i.e., I set  $\varepsilon_t^\mu = 0$ ).

Figure 11 reveals the results from these counterfactual experiments. The evidence shows that all three variables would experience higher volatility levels if the sensitivity of the real interest rate in Brazil were to be equal to that of Canada (second column in each of the three subplots of Figure 11). The reason for this is the fact that if  $\psi$  were to be reduced, firms and especially consumers would engage in debt paths that would be substantially more volatile so as to achieve consumption levels that would be welfare-improving (as I show in Section 5.2.2 with the introduction of a simple welfare rule), despite this coming at the cost of higher volatility. Therefore, varying the degree to which the real interest rate faced by Brazil responds to movements in aggregate debt levels generates a tension between higher *levels* of consumption (which generate higher *levels* of welfare) and higher consumption *volatility* as a consequence.

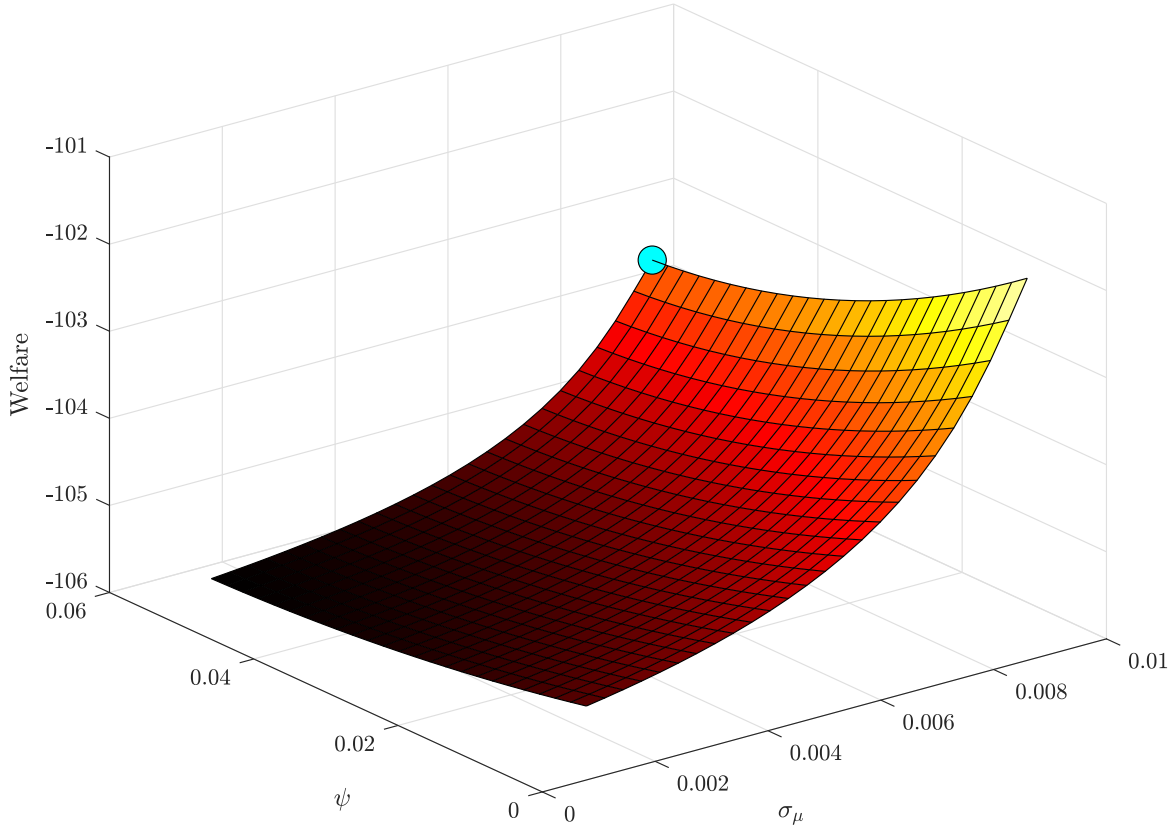
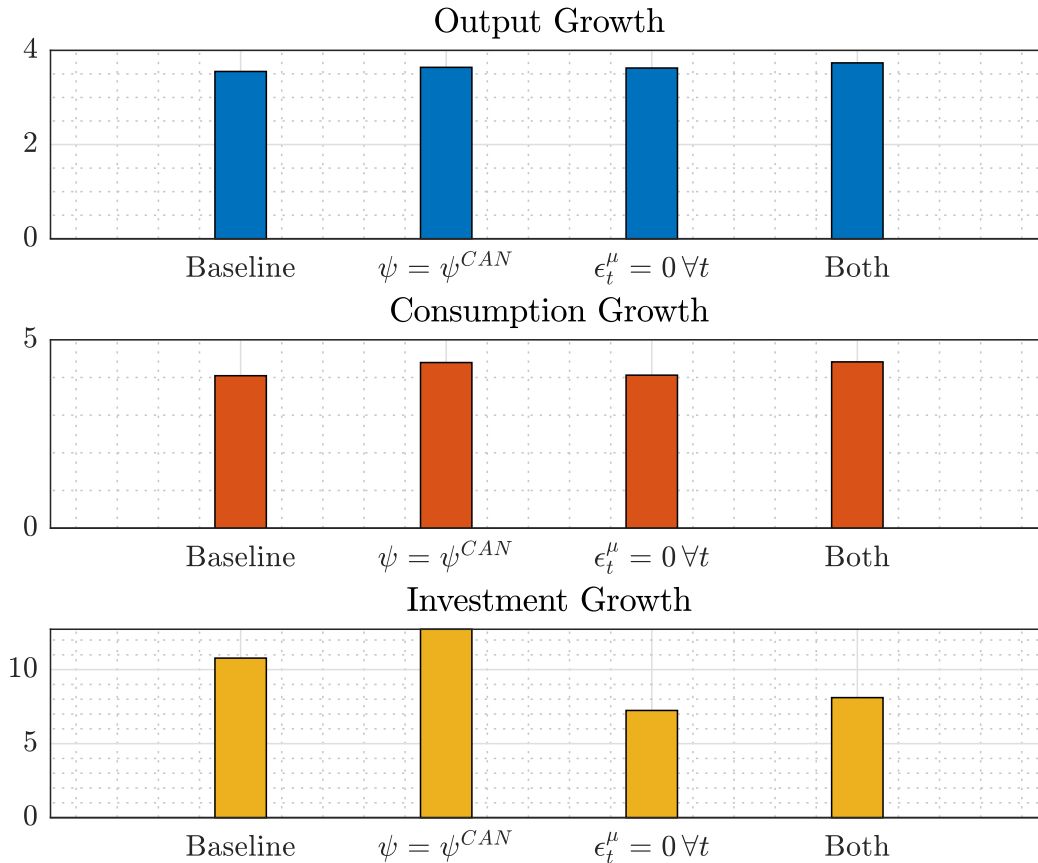


Figure 10: **Welfare in the Brazilian economy as a function of the parameters  $\psi$  and  $\sigma_\mu$ .** The circle colored in cyan represents the welfare value at the estimated parameters  $\{\psi, \sigma_\mu\}$  when using Brazilian data.

In a similar vein, Figure 10 shows that decreasing the volatility of the real interest rate shock in the Brazilian economy induced lower levels of welfare, at least from the perspective of the simple welfare function in Equation (36). To examine the effect of the real interest rate shock on overall economic volatility, I repeat the same exercise but now set the smoothed rate shock to zero (third column in each of the three subplots of Figure 11). The results reveal that while consumption and output growth volatilities are barely affected, the volatility of investment growth decreases dramatically when the real interest rate shock is set to zero, relative to the baseline scenario. This suggests that a substantial part of investment volatility in the Brazilian economy is driven by the shocks to the real interest rate it faces in international markets.

Figure 12 confirms the latter point by depicting the baseline smoothed investment growth and the counterfactual one when the real interest rate shock is set to zero. The evidence points out the fact that if Brazil were to completely offset the shocks to the real interest rates they face in international markets, investment growth volatility would decrease substantially, hence dampening the strong swings in this variable. The counterfactual investment series presents downturns that are not as deep as those in the baseline scenario across all time periods (with the exception of the considerable drop in the early 1980s).

Figure 11: **Macroeconomic volatility under counterfactual scenarios.** The bars of the baseline scenario represent the standard deviation of each of the smoothed variables after estimation. For the other scenarios, the bars represent the standard deviation of the simulated variables under alternative specifications conditional on the (determined) smoothed shocks obtained from the model estimation. “Both” indicates that both conditions are imposed in the simulation, i.e.  $\psi = \psi^{CAN}$  and  $\epsilon_t^\mu = 0 \forall t$



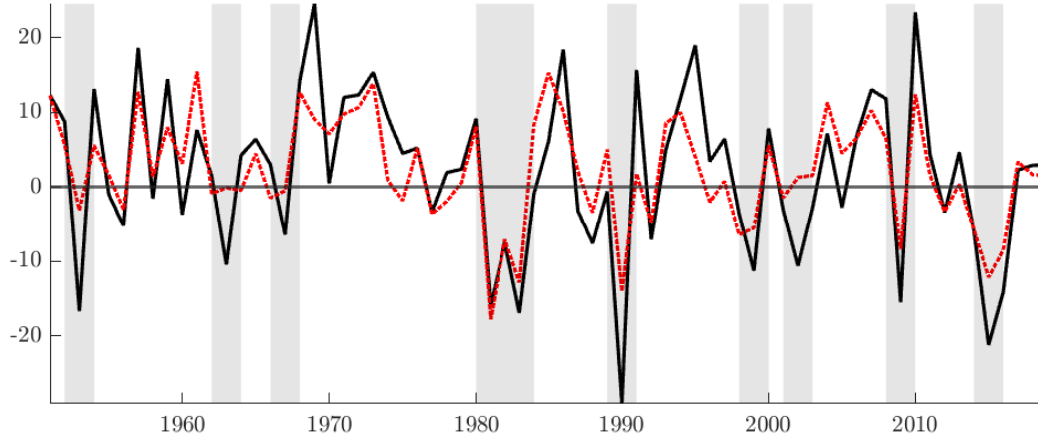
## 6 Conclusions

In this paper, I examine the differences in the response of developed and developing economies to real interest rate shocks. Using structural VAR models that exploit the panel structure of the data, the results show that real rate shocks have more adverse effects on a set of macroeconomic variables in developing economies than in developed ones.

To analyze the structural reasons behind these findings, I make use of an SOE-RBC model featuring multiple shocks and frictions that is estimated using data from Brazil and Canada. The estimation results suggest that the discrepancies between the two countries emerge mainly from the differences in the sensitivity of the interest rate to deviations in the debt level from its steady-state value and the observation that real rate shocks are



Figure 12: **Counterfactual investment growth series.** Blue, solid lines: smoothed investment series (i.e., after estimation); dark, dashed lines: smoothed investment series when real interest rate shock is set to zero ( $\varepsilon_t^\mu = 0 \forall t$ ). Grey bars denote periods when output growth was substantially low for ease of readability.



more volatile in the Brazilian economy. The Brazilian business cycle is driven more by shocks to the trend than by stationary technology shocks, and the rate spread shock can explain a substantially higher fraction of both investment growth and the trade balance-to-output ratio in the Brazilian economy relative to the Canadian economy.

I perform a counterfactual exercise and show that reductions in the spread shock volatility can reverse the sign of the correlation between the real interest rate and the growth rates of macroeconomic aggregates. I further reveal that the Brazilian economy would attain higher welfare values if the sensitivity of the real interest rate to deviations in the debt level from its long-run value decreased, although this comes at the expense of higher economic volatility. A final counterfactual scenario in which the real interest rate shock is set to zero shows that investment growth volatility would decrease substantially, hence dampening strong investment downturns observed in the data.

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# A Appendix

## A.1 Data Sources

### A.1.1 Developed Economies

Data on quarterly national accounts (real, seasonally adjusted) for developed economies were obtained from the OECD Quarterly National Accounts. Interest rate data were obtained from the Federal Reserve Economic Data (FRED). The following interest rates were selected for each country:

- Canada: 3-Month or 90-day Rates and Yields; Commercial/Corporate Paper
- Australia: 3-Month or 90-day Rates and Yields; Bank Bills
- New Zealand: 3-Month or 90-day Rates and Yields; Bank Bills
- Sweden: 3-Month or 90-day Rates and Yields; Treasury Securities
- Netherlands: 3-Month or 90-day Rates and Yields; Interbank Rates for the Netherlands

The real rate is computed by subtracting the expected GDP deflator inflation from the nominal rate. Expected GDP deflator inflation is computed by taking the average of the current GDP deflator inflation and the previous three lags:

$$r_t = i_t - \frac{1}{4} \sum_{j=0}^3 \pi_{t-j}, \quad (\text{A.1})$$

where  $r_t$  denotes the real rate,  $i_t$  denotes the nominal rate, and  $\pi_t$  denotes GDP deflator inflation.

### A.1.2 Developing Economies

Data on quarterly national accounts (real, seasonally adjusted) for developing economies were obtained from the IMF International Financial Statistics (IMF IFS). The nominal interest rate for this group of countries is constructed as the sum of the J.P. Morgan Emerging Markets Bond Spread and the 3-Month US Treasury Bill rate. As in the case of developed economies, the real rate is constructed by taking the nominal interest rate and subtracting a measure of expected GDP deflator inflation.

## A.2 Model Derivations

### A.2.1 Households

Recall that households maximize expected discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{[C_t - \omega^{-1} X_{t-1} H_t^\omega]^{1-\gamma} - 1}{1-\gamma} \right) Z_t \quad (\text{A.2})$$

subject to the sequential budget constraint

$$\frac{D_{t+1}^h}{1+r_t} = D_t^h - W_t h_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t - \Pi_t. \quad (\text{A.3})$$

With this, the household problem boils down to maximizing the following Lagrangian function:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{[C_t - \omega^{-1} X_{t-1} h_t^\omega]^{1-\gamma}}{1-\gamma} \right) Z_t + \right. \quad (\text{A.4})$$

$$\left. X_{t-1}^{-\gamma} \lambda_t \left( \frac{D_{t+1}^h}{1+r_t} - D_t^h + W_t h_t + u_t K_t - C_t - S_t - I_t - \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t + \Pi_t \right) \right\}. \quad (\text{A.5})$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial C_t} : \quad Z_t [C_t / X_{t-1} - \omega^{-1} h_t^\omega] = \lambda_t \quad (\text{A.6})$$

$$\frac{\partial \mathcal{L}}{\partial h_t} : \quad Z_t [C_t / X_{t-1} - \omega^{-1} h_t^\omega] h_t^{\omega-1} = \lambda_t \frac{W_t}{X_{t-1}} \quad (\text{A.7})$$

$$\frac{\partial \mathcal{L}}{\partial D_{t+1}^h} : \quad \lambda_t = \beta(1+r_t) g_t^{-\gamma} \mathbb{E}_t \lambda_{t+1} \quad (\text{A.8})$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} : \quad \lambda_t \left[ 1 + \phi \left( \frac{K_{t+1}}{K_t} - g \right) \right] = \beta g_t^{-\gamma} \mathbb{E}_t \lambda_{t+1} \left[ 1 - \delta + u_{t+1} + \phi \left( \frac{K_{t+2}}{K_{t+1}} \right) \left( \frac{K_{t+2}}{K_{t+1}} - g \right) - \frac{\phi}{2} \left( \frac{K_{t+2}}{K_{t+1}} - g \right)^2 \right] \quad (\text{A.9})$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : \quad \frac{D_{t+1}^h}{1+r_t} = D_t^h - W_t h_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t - \Pi_t. \quad (\text{A.10})$$

### A.2.2 Firms

Recall that the firm's budget constraint is given by

$$\frac{D_{t+1}^f}{1+r_t} = D_t^f + \Delta M_t + u_t K_t + W_t h_t - Y_t + \Pi_t^f. \quad (\text{A.11})$$

With this, the firm chooses processes  $\{K_t, h_t, M_t, D_t^f\}_{t=0}^{\infty}$  that maximize its expected stream of profits

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t X_{t-1}^{-\gamma} \lambda_t \left[ Y_t - u_t K_t - W_t h_t - \Delta M_t + \frac{D_{t+1}^f}{1+r_t} - D_t^f + \zeta_t (M_t - \eta W_t h_t) \right]. \quad (\text{A.12})$$

The first-order conditions yield

$$\frac{\partial \mathcal{L}}{\partial K_t} : \quad u_t = \alpha A_t \left( \frac{X_t h_t}{K_t} \right)^{1-\alpha} \quad (\text{A.13})$$

$$\frac{\partial \mathcal{L}}{\partial h_t} : \quad (1-\alpha) A_t X_t \left( \frac{K_t}{X_t h_t} \right)^\alpha = W_t [1 + \eta \zeta_t] \quad (\text{A.14})$$

$$\frac{\partial \mathcal{L}}{\partial M_t} : \quad \lambda_t (1 - \zeta_t) = \beta g_t^{-\gamma} \mathbb{E}_t \lambda_{t+1} \quad (\text{A.15})$$

$$\frac{\partial \mathcal{L}}{\partial B_t^f} : \quad \lambda_t = \beta(1+r_t) g_t^{-\gamma} \mathbb{E}_t \lambda_{t+1}. \quad (\text{A.16})$$

Note that Equation (A.13) already provides the optimality condition for capital (the rental rate equals the marginal product of capital). Dividing Equation (A.15) by Equation (A.16) and solving for  $\zeta_t$ , I obtain

$$\zeta_t = \frac{r_t}{1+r_t}. \quad (\text{A.17})$$

Plugging this expression into (A.14), I get

$$\underbrace{(1-\alpha)A_tX_t\left(\frac{K_t}{X_th_t}\right)^\alpha}_{\equiv \text{MPL}} = W_t \left[1 + \eta \frac{r_t}{1+r_t}\right]. \quad (\text{A.18})$$

### A.2.3 Financial Sector

To close the model, the equilibrium in the financial sector must be specified. The bank's balance sheet is given by the expression

$$\frac{D_{t+1}^h + D_{t+1}^f}{1+r_t} = \frac{D_{t+1}}{1+r_t} + M_t. \quad (\text{A.19})$$

With this, the bank's profits are assumed to be

$$\Pi_t^b = D_t^h + D_t^f - D_t - M_{t-1}. \quad (\text{A.20})$$

Consumers' profits are given by:

$$\Pi_t = \Pi_t^f + \Pi_t^b. \quad (\text{A.21})$$

To close the equilibrium, the economy's resource constraint must be found. For this purpose, consider plugging the households' budget constraint in Equation (A.10) and the firms' budget constraint in Equation (A.11) into the LHS of Equation (29):

$$\frac{D_{t+1}}{1+r_t} + M_t = C_t + S_t + I_t + \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t - \Pi_t + \Delta M_t - Y_t + \Pi_t^f \quad (\text{A.22})$$

I use Equation (A.21) together with Equation (A.20) to express the total household profits as follows

$$\Pi_t = \Pi_t^f + D_t^h + D_t^f - D_t - M_{t-1}. \quad (\text{A.23})$$

Plugging this last expression in Equation (A.22), I obtain the resource constraint of the economy

$$\frac{D_{t+1}}{1+r_t} = D_t + C_t + S_t + I_t + \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t - Y_t. \quad (\text{A.24})$$

### A.2.4 Equilibrium

Denoting with lowercase letters the detrended versions of the variables  $c_t = C_t/X_{t-1}$ ,  $k_t = K_t/X_{t-1}$ ,  $w_t = w_t/X_{t-1}$ ,  $i_t = I_t/X_{t-1}$ ,  $d_t = D_t/X_{t-1}$ ,  $s_t = S_t/X_{t-1}$ , the resulting equilibrium conditions are as follows

$$\lambda_t = Z_t [c_t - \omega^{-1} h_t^\omega]^{-\gamma} \quad (\text{A.25})$$

$$h_t^{\omega-1} = (1-\alpha) A_t g_t^{1-\alpha} \left( \frac{k_t}{h_t} \right)^\alpha \left[ 1 + \eta \frac{r_t}{1+r_t} \right]^{-1} \quad (\text{A.26})$$

$$\lambda_t = \beta g_t^{-\gamma} (1+r_t) \mathbb{E}_t \lambda_{t+1} \quad (\text{A.27})$$

$$\begin{aligned} \lambda_t \left[ 1 + \phi \left( \frac{k_{t+1}}{k_t} g_t - g \right) \right] &= \beta g_t^{-\gamma} \mathbb{E}_t \lambda_{t+1} \left[ 1 - \delta + \alpha A_{t+1} \left( \frac{h_{t+1} g_{t+1}}{k_t} \right)^{1-\alpha} + \right. \\ &\quad \left. \phi \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} \right) \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) - \frac{\phi}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \right] \end{aligned} \quad (\text{A.28})$$

$$\frac{d_{t+1}}{1+r_t}g_t = d_t - y_t + c_t + s_t + i_t + \frac{\phi}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t \quad (\text{A.29})$$

$$r_t = r^* + \psi \left( e^{d_{t+1}-\bar{d}} \right) + e^{\mu_t-1} - 1 \quad (\text{A.30})$$

$$k_{t+1}g_t = (1-\delta)k_t + i_t \quad (\text{A.31})$$

$$y_t = A_t k_t^\alpha (g_t h_t)^{1-\alpha} \quad (\text{A.32})$$

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2) \quad (\text{A.33})$$

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_t^Z, \quad \varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2) \quad (\text{A.34})$$

$$\ln \left( \frac{g_t}{g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{g} \right) + \varepsilon_t^g, \quad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2) \quad (\text{A.35})$$

$$\ln \left( \frac{s_t}{\bar{s}} \right) = \rho_s \ln \left( \frac{s_{t-1}}{\bar{s}} \right) + \varepsilon_t^s, \quad \varepsilon_t^s \sim \mathcal{N}(0, \sigma_s^2) \quad (\text{A.36})$$

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_t^\mu, \quad \varepsilon_t^\mu \sim \mathcal{N}(0, \sigma_\mu^2) \quad (\text{A.37})$$

### A.2.5 Model And Observed Variables

Once the detrended version model is introduced, the mapping between the observed and the model variables can be defined as follows:

$$\underbrace{\begin{bmatrix} (\Delta Y_t)^{data} \\ (\Delta C_t)^{data} \\ (\Delta I_t)^{data} \\ (TB_t/Y_t)^{data} \end{bmatrix}}_{\text{Observed variables}} = \underbrace{\begin{bmatrix} \ln \left( \frac{y_t}{y_{t-1}} g_t \right) \\ \ln \left( \frac{c_t}{c_{t-1}} g_t \right) \\ \ln \left( \frac{i_t}{i_{t-1}} g_t \right) \\ \frac{d_{t-1} - \frac{d_t g_t}{1+r_t}}{y_t} \end{bmatrix}}_{\text{Model variables}} + \underbrace{\begin{bmatrix} \sigma_{\Delta y} \\ \sigma_{\Delta c} \\ \sigma_{\Delta i} \\ \sigma_{\Delta tb/y} \end{bmatrix}}_{\text{Measurement errors}} \quad (\text{A.38})$$