Real Financing Costs Over the Business Cycle: Emerging versus Advanced Economies*

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Abstract

I examine the differences in the response of developed and developing economies to shocks to their real interest rate. By employing empirical methods that exploit the panel structure of the data, I show that real interest rate shocks have more adverse effects on a set of macroeconomic variables in developing economies than in developed ones. To analyze the structural reasons behind these findings, I propose a theoretical model featuring multiple shocks and frictions that is estimated using data from Brazil and Canada. The results suggest that the interest rate faced by the Brazilian economy in international markets is much more sensitive to deviations in the debt level from its long-run value relative to that of Canada. The spread shock is found to be more volatile in Brazil, and it can explain a substantially higher share of its business cycle. A counterfactual exercise reveals that the volatility of the real interest rate shock is crucial in determining the correlations between real interest rates and macroeconomic aggregates. A final empirical exercise reveals that macroeconomic aggregates in Brazil are driven by US rate shocks to a larger extent relative to domestic (or spread) real interest rate shocks.

Keywords: Real Interest Rates; Developing economies; Real business cycles; Small Open Economies; Bayesian estimation of DSGE models.

JEL classification: C11; E32; E40; F41; F44

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1 Introduction

Business cycle fluctuations in developing economies are strongly associated with movements in the interest rates they face in international markets. As shown in Figure 1, periods of low (high) real borrowing costs, proxied by a measure of real interest rates,[1] are generally accompanied by pronounced peaks (drops) in economic activity for a set of developing economies.

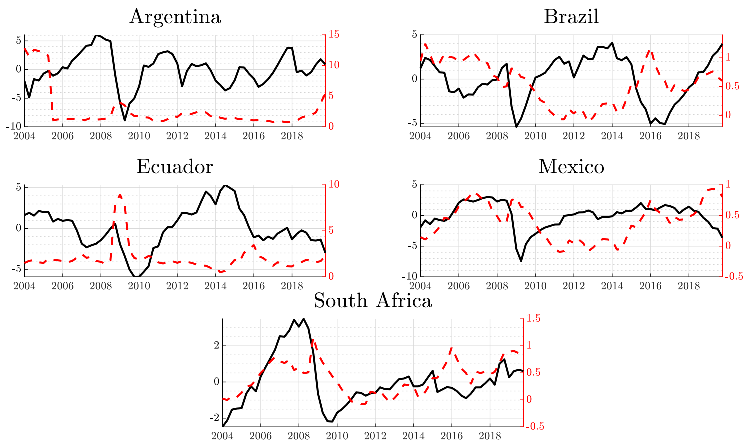


Figure 1: **Real GDP and real interest rates for a set of developing economies**. Black, solid line: Real Gross Domestic Product (log deviations from a quadratic trend, in %); red, solid line: Real interest rates (in %).

This observation does not appear to apply to developed economies, where real interest rates are either less acyclical or even procyclical in some cases (see Figure 2).

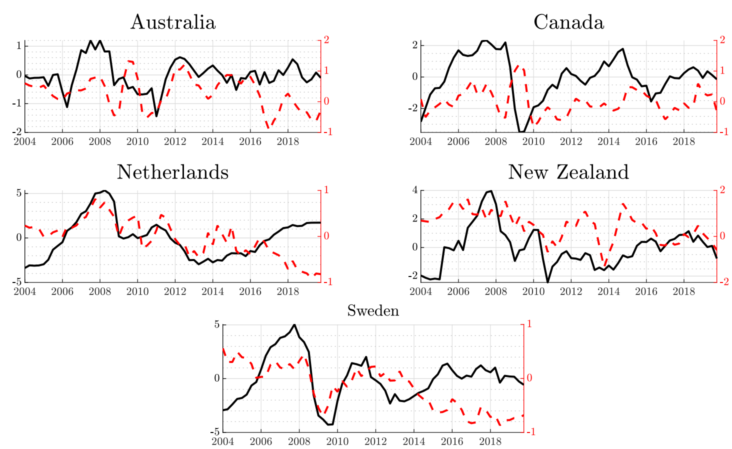


Figure 2: **Real GDP and real interest rates for a set of developed economies**. Black, solid line: Gross Domestic Product (log deviations from a quadratic trend, in %); red, solid line: Real interest rates (in %).

[1] Real interest rates are obtained by taking a measure of nominal interest rates and subtracting a measure of expected inflation. For more details on the variable description and construction, refer to the appendix.

Similarly, as shown in Figure 3, the correlation between detrended real GDP and lagged values of the real interest rate is negative across all lag horizons considered for developing economies, while it turns positive for developed ones.[2]

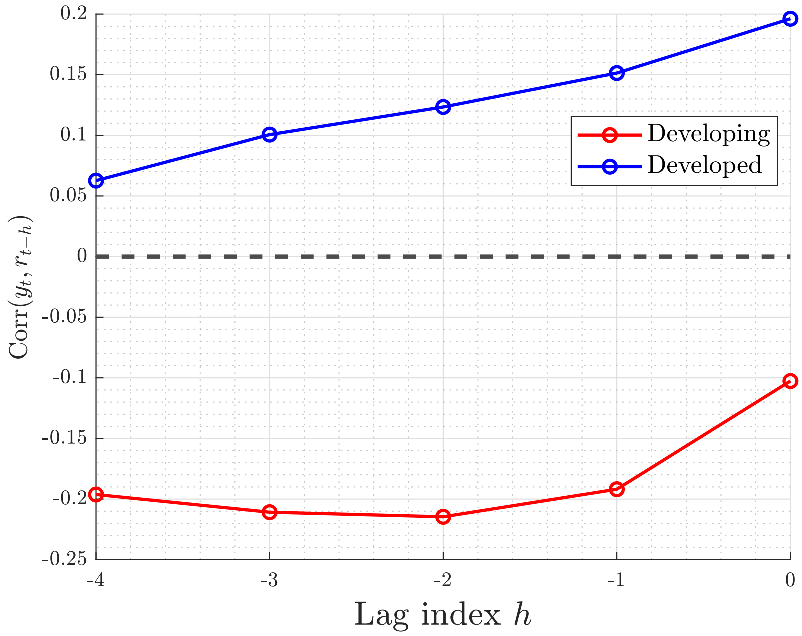


Figure 3: **Correlation between real GDP and lags of the real interest rate for developing and developed economies.** Real GDP is expressed as log deviations from a quadratic trend (in %).

Understanding the causes and magnitude of this relationship emerges as an important research question, both from the academic and policy dimensions. The fact that emerging economies are more sensitive to developments in international rates poses a threat to their economic and financial stability, which should trigger the application of appropriate policies aimed at cushioning the effect of interest rate hikes on their business cycles.

The purpose of this paper is to examine the differences in the response of developed and developing economies to shocks to their real interest rate. The empirical strategy proposed allows for exploiting the panel dimension of the data and aims to understand how this type of shock propagates into the two economy types. I then employ an augmented small open economy real-business-cycle model (SOE-RBC) featuring multiple shocks and frictions commonly considered relevant in explaining business cycle fluctuations in emerging economies. To examine the structural reasons behind these findings, the model is estimated using data from one developed (Canada) and one emerging (Brazil) economy in a Bayesian fashion.

The focus of this paper on real (instead of nominal) rates is justified by multiple reasons. First, real interest rates emerge as a fundamental variable in the decision-making process of agents. In particular, both consumers and firms take inflation into account when making consumption and investment decisions to assess the real purchasing power of money, which enables them to accurately determine the true cost of borrowing or the return on investment over time.

[2] A similar graph, including the country-specific correlations, can be found in the appendix.

Related to the latter point, Central Banks, therefore, target real interest rates when pursuing monetary policy with the purpose of influencing economic activity to achieve specific objectives within their mandates. By considering measures of expected inflation, Central Banks can adjust nominal interest rates to achieve the desired impact on real interest rates, which, in turn, directly influences borrowing and investment decisions. Failing to monitor expected inflation can lead to the under or overshooting of nominal interest rates, potentially triggering unintended consequences in the economy that do not align with the initial policy goals.

A final point relates to the nature of the countries considered. Emerging economies are found in the literature to have a high fraction of their global trade invoiced in international currencies (in particular the dollar and the euro; see, e.g., Goldberg & Tille 2016, Gopinath 2015, Kamps 2006), which makes them susceptible to inflationary pressures arising from a higher import price and trade volume sensitivity to fluctuations in their exchange rates (Boz et al., 2022).

Related literature. The literature that examines the drivers of economic fluctuations in emerging economies within the SOE-RBC framework is extensive. In terms of methodology, Mendoza (1991) emerged as the starting point for the use of RBC models to characterize business cycles. These models however are not able to reproduce a key finding that arises from the data: the fact that consumption volatility is higher than that of output in emerging economies. For this purpose, the literature explored modifications of the canonical RBC that would generate predictions in line with this empirical evidence. Aguiar & Gopinath (2007) extend the standard RBC model by allowing for two different types of productivity shocks, transitory and permanent, and find that shocks affecting output *trend* are more relevant than transitory shocks in explaining business cycles in emerging economies. In a recent application, Noh & Baek (2021) support the dominance of trend productivity shocks even when the standard RBC model is augmented with financial frictions, which, together with stationary technology shocks, are found to be relatively less relevant. Naoussi & Tripier (2013) find that the impact of trend shocks on the business cycle is inversely related to the overall levels of country development, at both the economic and institutional levels. Long-run technology shocks have also been shown to generate sudden stops in SOE models in the presence of collateral constraints (Seoane & Yurdagul, 2019). Boz et al. (2011) find that the presence of consumer informational frictions with regard to learning about the two types of technology shocks is more critical for emerging economies than for developed ones.

The extent to which trend productivity shocks can explain business cycles in emerging economies has been nonetheless subject to debate in the literature. In a seminal contribution, Garcia-Cicco et al. (2010) show that trend shocks have a negligible impact on the Argentinian business cycle by employing an SOE-RBC model featuring both types of productivity shocks and adding two building blocks to the model: financial frictions and a working capital constraint. The financial friction is based on the observation that the interest rates faced by a country in international markets are a positive function of its indebtedness. The working capital constraint emerges when firms are directly affected by interest rates because they need to pay a fraction of the total wage bill in advance, which must be raised by borrowing funds from the financial sector. In this way, interest rate shocks affect the intertemporal substitution of consumption and alter the firm's factor allocation and production. Chang & Fernández (2013) reinforce the role of financial imperfections in amplifying the effect of shocks and attribute a negligible role to trend TFP shocks. In a recent contribution, S. Hwang & Kim (2022) examine the extent to which Asian business cycles differ from Latin American ones by estimating the model in Garcia-Cicco et al. (2010) in a Bayesian fashion.

Neumeyer & Perri (2005) divide the interest rate shock into an "international" and a "country risk" components and show that the working capital constraint indeed amplifies the effect of fundamental shocks on the economy. Similarly, Uribe & Yue (2006) find feedback effects between the business cycle and country spreads, which are simultaneously sensitive to international rates. In a purely empirical application, Monacelli et al. (2023) show that capital inflows shocks (proxied by a drop in the real interest rate) generate TFP booms (drops) in emerging (developing) economies. Arellano (2008) develops an SOE model featuring incomplete markets that can generate an inverse relationship between cyclical output and interest rates, which, together with the probability of sovereign default, are endogenous to the endowment shocks to which the economy is subject.

Subsequent contributions reinforce the importance of financial frictions over trend productivity shocks in explaining business cycles in emerging economies (Chang & Fernández, 2013, Miyamoto & Nguyen, 2017, Y.-N. Hwang, 2012, Alvarez-Parra et al., 2013). Mendoza (1991) and Calvo (1998) employ theoretical models to understand how financial frictions can explain episodes of sudden slowdowns in capital inflows into the economy. Other relevant contributions focus on the importance of different types of labor market frictions in explaining the relatively high consumption volatility (Boz et al., 2015).

Overall, the literature identifies specific sets of shocks and frictions that are considered crucial for analyzing the main factors driving the business cycle in emerging economies: productivity shocks (both stationary and trend shocks), which account for the variation in output due to total factor productivity; financial frictions, which generate a positive relationship between deviations of the debt level from its long-run value and the interest rate faced by the agents; working capital constraints, which make firms' optimal labor choices (and hence production) to depend directly on interest rates; and interest rate shocks, which act as spread shocks that aim to capture how risky the financial sector deems the economy.

I follow the strand of the literature that tries to unveil the effect of interest rate shocks on the business cycle of emerging economies (see e.g. Uribe & Yue, 2006, Neumeyer & Perri, 2005) but I extend the analysis in several ways. First, the availability of new data at a quarterly frequency allows for the inclusion of periods in the analysis that have been of special economic relevance, such as the Great Recession. This particular contribution thus lies in examining the extent to which recent economic developments were able to shape the effect of interest rate shocks in both developed and emerging economies. Second, and more importantly, I analyze whether this effect is due to the structural aspects of the economy or the exposure to a particular path of shocks by estimating a theoretical model using data from Canada and Brazil. The estimation of the model allows examining in a rigorous and statistically founded manner the particularities of each economy that drive the differences in the response of the economy to interest rate shocks.

The results reveal that the Brazilian economy is subject to interest rate shocks that are substantially more volatile than those of the Canadian economy. In addition, the sensitivity of the interest rate to deviations in the debt level from its steady-state value is estimated to be higher for Brazil. Contrary to what is found for Canada, the Brazilian business cycle is substantially driven by shocks to the trend than by stationary technology shocks, and the rate spread shock can explain a substantially higher fraction of the cyclical component of investment growth and the trade balance-to-output ratio. A counterfactual exercise reveals that the volatility of the real rate shock matters substantially in determining the sign of the correlations between macroeconomic aggregates and the real interest rate in the Brazilian economy. Nevertheless, the results from a welfare analysis show that only reductions in the sensitivity of the real interest rate to deviations in the debt level from its long-run value are shown to be welfare-improving for Brazil, although at the expense of higher economic volatility. Offsetting the real interest rate shock completely would reduce investment volatility by a considerable amount, this way cushioning strong drops in this variable.

The remainder of this paper is organized as follows. Section 2 describes the empirical methodology and presents the macroeconomic responses to shocks to the real interest rate. In Section 3, the SOE-RBC model employed is outlined. Section 4 illustrates the calibration and estimation procedures, and Section 5 presents the estimation results and the counterfactual analysis. Finally, Section 6 concludes.

2 Empirical Evidence

I make use of a panel structural VAR (sVAR) model to quantify the macroeconomic effects of real interest rate shocks for both country types. The VAR is given by the standard representation

$$y_t = c + \sum_{k=1}^{p} A_p y_{t-k} + C x_t + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}\left(0, \Sigma\right), \tag{1}$$

where c denotes a constant term, A_p denotes the matrix of coefficients (assumed to be common across units), C denotes the coefficient matrix mapping the exogenous variables x_t to the endogenous variables, and Σ denotes the unit-variance-covariance matrix of the error term. The panel dimension of this model relies on the fact that the data comes from multiple units, each defined by whether countries fall into the developing or developed category (i.e., a "pooled" estimator). The model is estimated in a Bayesian framework by imposing a standard Normal-Wishart prior. In this vein, the prior for the coefficients of the system is assumed to follow a multivariate Normal distribution[3]

$$\beta \sim \mathcal{N}\left(\beta_0, \Sigma_c \otimes \Phi_0\right), \tag{2}$$

where Σ_c denotes the VAR residual variance-covariance matrix, and $\Sigma_c \otimes \Phi_0$ denotes the full covariance matrix. The prior for Σ_c is assumed to follow an Inverse-Wishart distribution

$$\Sigma_c \sim \mathcal{IW}\left(S_0, \alpha_0\right), \tag{3}$$

where S_0 denotes the prior scale matrix, and α_0 denotes the prior degrees of freedom, both defined as in Karlsson (2013).

Similarly to the Minnesota prior, Σ_c is assumed to be a diagonal matrix, while Φ_0 is constructed so as to mimic the variance matrix of the Minnesota prior (Doan et al., 1984): λ_1 denotes the hyperparameter

[3] See Canova & Ciccarelli (2013) and Dieppe et al. (2016) for an excellent review of panel VAR models.

governing the overall prior tightness, λ_2 governs the standard deviation of the prior on lags of variables other than the dependent variable, λ_3 governs the decay over lags of the own variable, and λ_4 relates to the variance of the constant and exogenous variables. The chosen values for the hyperparameters are commonly found in the literature and are given by:[4]

$$\lambda_1 = 0.1, \quad \lambda_2 = 1, \quad \lambda_3 = 1, \quad \lambda_4 = 100 \tag{4}$$

Given that the data enters the model in stationary form, the choice for the prior on β (i.e., β_0) imposes the absence of a unit root in the autoregressive coefficients by setting the prior equal to 0.8.

With this, I estimate a VAR model with five variables denoted respectively by GDP (gdp_t), private consumption (c_t), investment (gross fixed capital formation) (i_t), net exports (nx_t), and the real interest rate (r_t). Real interest rates are obtained by considering measures of nominal interest rates and subtracting a measure of expected inflation.[5]

All the variables except for net exports and the domestic interest rate are log-quadratically detrended. Given the potential negative values for net exports, I adjust them by the exponential of trend output and then remove a quadratic trend. I divide the countries into two groups: developed economies (Australia, Canada, the Netherlands, New Zealand, and Sweden) and developing economies (Argentina, Brazil, Ecuador, Mexico, and South Africa). The identification scheme adopted relies on a standard Cholesky decomposition under the assumption that the shock to the real interest rate does not affect the macroeconomic variables on impact. In other words, the ordering of the variables is such that $y_t = \{gdp_t, c_t, i_t, nx_t, r_t\}$.

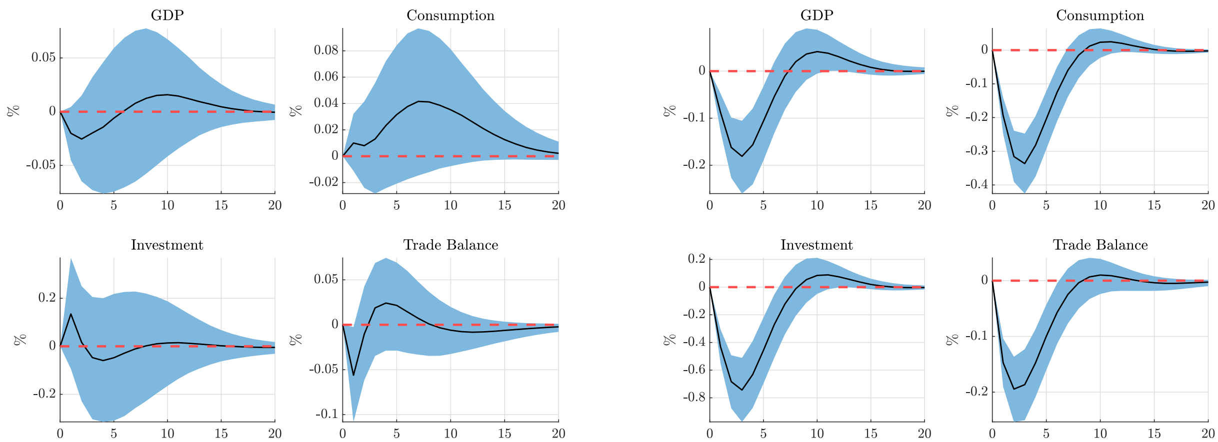
The total number of lags is set to $p = 4$ for all variables. In addition, I include the European Brent Spot Price as an exogenous variable of the system (sourced from the U.S. Energy Information Administration). The data consists of a balanced panel for both country types available for the period 2004q1 to 2019q4. The number of total iterations is set to 15,000 after a burn-in phase of 10,000 iterations.

Figure 4 depicts the estimated macroeconomic responses to a one standard deviation shock to the real interest rate for both developed and developing economies.[6] The median responses show a significant decline in all variables for developing economies, although no significant effect is found for developed economies. Overall, the empirical evidence from the panel sVAR points to the observation that real rate shocks have macroeconomic effects that are more adverse in developing economies than in developed ones. Taking this into consideration, the following section proposes a theoretical framework for understanding the fundamental reasons behind this finding.

[4] As shown in Dieppe et al. (2016), the variance-covariance matrix under a Normal-Wishart prior is a special case of that under a Minnesota prior in which Σ_c is assumed to be diagonal and $\lambda_2 = 1$ (see Karlsson, 2013, for details).

[5] For more information on the data sources and variable construction, see the appendix.

[6] See the appendix for the results obtained by employing an empirical model that allows for country-specific responses based on Jarociński 2010. Additionally, I also show in the appendix the results obtained by taking into account US unconventional monetary policy by employing the shadow rate by Wu & Xia (2016).



(a) Developed Economies

(b) Developing Economies

Figure 4: **Impulse responses to a one standard deviation shock to the real rate for developed and developing economies, in %.** The figure depicts the cross-country median effect (solid, black line) together with the 68% (light blue shaded area) credible bands.

3 The Model

3.1 Households

The model builds on the work of Garcia-Cicco et al. (2010). The economy is populated by an infinite number of infinitely lived agents with GHH preferences (Greenwood et al., 1988). The representative agent maximizes the discounted expected utility, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left[C_t - \omega^{-1} X_{t-1} H_t^{\omega}\right]^{1-\gamma} - 1}{1-\gamma} \right) Z_t, \tag{5}$$

where C_t denotes consumption, H_t denotes hours worked, $\beta \in (0,1)$ is a discount factor, γ denotes the parameter governing consumer risk aversion, ω denotes the parameter governing the disutility of labor, and X_t denotes a stochastic trend. The growth rate of the trend shock, denoted by g_t, is given by

$$g_t = \frac{X_t}{X_{t-1}} \tag{6}$$

and follows a (stationary) AR(1) process such that

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g; \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2). \tag{7}$$

In other words, X_t denotes a shock to the trend, whereas its growth rate follows a stationary process. The variable Z_t describes a preference shock following the process

$$\ln Z_t = \rho_Z Z_{t-1} + \varepsilon_t^Z; \qquad \varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2). \tag{8}$$

This shock may be thought of as an exogenous driver of consumers' (im)patience, that is, a shock that shifts the discount factor. Consumers face the following budget constraint

$$\frac{D_{t+1}^h}{1+r_t} = D_t^h - W_t H_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g\right)^2 K_t - \Pi_t, \tag{9}$$

where D_t^h denotes consumers' bond holdings acquired in $t-1$ and maturing in t, K_t denotes physical capital, I_t denotes investment in capital goods, W_t denotes the wage rate, u_t denotes the marginal product of capital, Π_t denotes total distributed profits (which consumers regard as exogenous), S_t denotes a government spending shock, and ϕ denotes the parameter governing the capital adjustment costs. The detrended version of the government spending shock $s_t = \frac{S_t}{X_{t-1}}$ is assumed to follow the process

$$\ln\left(\frac{s_t}{\bar{s}}\right) = \rho_s \ln\left(\frac{s_{t-1}}{\bar{s}}\right) + \varepsilon_t^s; \qquad \varepsilon_t^s \sim \mathcal{N}(0, \sigma_s^2), \tag{10}$$

where $\bar{s}$ denotes the steady-state level of the detrended government shock. The (gross) real interest rate faced by households is denoted by r_t and follows the process

$$r_t = r^* + \psi\left(e^{D_{t+1}-\bar{D}}\right) + \left(e^{\mu_t - 1} - 1\right), \tag{11}$$

where r^* denotes the steady-state interest rate, $\left(e^{D_{t+1}-\bar{D}}\right)$ denotes the external debt-elastic interest rate (EDEIR, Uribe & Yue, 2006),[7] D_t denotes the aggregate level of debt and $\bar{D}$ denotes its steady-state level, ψ denotes the parameter governing the sensitivity of the interest rate to deviations of debt from its steady-state value, $\left(e^{\mu_t-1} - 1\right)$ denotes the country spread, and μ_t denotes the spread shock, which follows the process

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_t^\mu; \qquad \varepsilon_t^\mu \sim \mathcal{N}(0, \sigma_\mu^2). \tag{12}$$

Capital is assumed to evolve according to the following process:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{13}$$

With this, consumers choose processes $\{C_t, H_t, K_{t+1}, D_{t+1}^h\}_{t=0}^\infty$ so as to maximize their expected discounted utility in Equation (5) subject to the budget constraint in Equation (9), the capital law of motion in Equation (13), and a standard no-Ponzi-game constraint of the form

$$\lim_{j\to\infty} \mathbb{E}_t \frac{D_{t+j+1}^h}{\Pi_{s=0}^j (1 + r_{t+s})} \leq 0. \tag{14}$$

3.2 Firms

A representative firm produces output using a standard Cobb-Douglas production function:[8]

$$Y_t = A_t K_t^\alpha (X_t H_t)^{1-\alpha}, \tag{15}$$

where α denotes the elasticity of output with respect to capital and A_t denotes a stationary technology shock following the process

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A; \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_A^2). \tag{16}$$

[7]The EDEIR is included so to induce stationarity in the model, that is, to ensure that the steady-state levels of the model variables are independent of the initial conditions. This also implies that the interest rate faced by the economy increases as debt rises above its steady-state value, which is an empirically plausible assumption.

[8]Note that the trend shock X_t appears in the firm's production function, which means that it can also be interpreted as a non-stationary TFP shock.

Firms are assumed to face a working capital constraint of the form:

$$M_t \geq \eta W_t H_t. \tag{17}$$

This expression states that the firm must hold an amount of working capital equal to M_t (which can be thought of as an asset that does not bear any interest) that has to be at least equal to a fraction η of the total labor costs, $W_t H_t$, in a form of precautionary behavior. In other words, η is defined as the fraction of the total wage bill that the firm is forced to pay prior to production. For the firm to pay this quantity in advance, it must raise funds by borrowing from the financial sector. The representative firm is assumed to borrow at the same rate as consumers, and the evolution of its debt is given by

$$\frac{D^f_{t+1}}{1+r_t} = D^f_t + \Delta M_t + u_t K_t + W_t H_t - Y_t + \Pi^f_t, \tag{18}$$

where D^f_t denotes the firm's debt, and Π^f_t denotes the firm's profits. The firm chooses processes $\{K_t, H_t, M_t, D^f_t\}_{t=0}^{\infty}$ that maximize their expected stream of profits[9]

$$\begin{aligned} &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t X_{t-1}^{-\gamma} \Pi^f_t \\ &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t X_{t-1}^{-\gamma} \left[\frac{D^f_{t+1}}{1+r_t} - D^f_t - \Delta M_t - u_t K_t - W_t H_t + Y_t \right] \end{aligned} \tag{19}$$

subject to the working capital constraint in Equation (17), and a no-Ponzi-game constraint of the form

$$\lim_{j\to\infty} \mathbb{E}_t \frac{D^f_{t+j+1} - M_{t+j+1}}{\Pi_{s=0}^{j}(1+r_{t+s})} \leq 0. \tag{20}$$

By combining the first-order conditions of the firm's problem, I am left with the firm's budget constraint as in Equation (18), and the returns for capital and labor, respectively:

$$u_t = \alpha \left(\frac{H_t}{K_t} \right)^{1-\alpha} \tag{21}$$

$$W_t = (1-\alpha) A_t \left(\frac{K_t}{H_t} \right)^{\alpha} \left[1 + \eta \frac{r_t}{1+r_t} \right]^{-1} \tag{22}$$

The expression above states that, as long as $\eta \neq 0$, the working capital constraint creates a wedge between the marginal product of labor and the wage earned by households. The presence of a working capital constraint implies that interest rate shocks can affect the production side of the economy, in addition to their effect on the intertemporal substitution of consumption.

Following Uribe & Yue (2006), I show that the firm's profits are equal to zero at all periods. I denote firms' net liabilities at t by Ξ_t, which are given by

$$\Xi_t = (1+r_t) D^f_t - M_t. \tag{23}$$

[9] Given that consumers are assumed to be the owners of the firms, the Lagrange multiplier associated with the sequential budget constraint is equivalent to the one associated with the sequential firm profits, $\beta^t \lambda_t X_{t-1}^{-\gamma}$ (Uribe & Schmitt-Grohé, 2017).

The last expression shows that net liabilities are equal to the debt that the firm needs to repay to the bank minus the wage bill that the firm sets aside and deposits in the bank. Under the assumption that $r_t \geq 0 \, \forall t$, it holds that

$$M_t = \eta W_t N_t. \tag{24}$$

I can therefore write the firm's liabilities at time t as follows

$$\Xi_t = (1 + r_t) D_t^f - \eta W_t N_t. \tag{25}$$

From Equation (18) I can rewrite the firm's profits

$$\Pi_t = Y_t - u_t K_t - W_t H_t + \frac{\Xi_t}{1 + r_t} - \Xi_{t-1} - \left(\frac{r_t}{1 + r_t} \right) \eta_t W_t N_t. \tag{26}$$

Under the assumption that firms do not possess any net liability at the beginning of the period $t = 0$, it implies that an optimal path for the firm would be to hold zero liabilities at each period: $\Xi_t = 0 \, \forall t$. With this, the firm's profits can be rewritten as follows

$$\Pi_t = Y_t - u_t K_t - W_t H_t \left(1 + \eta \frac{r_t}{1 + r_t} \right). \tag{27}$$

This shows that $\Pi_t = 0 \, \forall t$ under a particular path for firm liabilities.

3.2.1 Financial Sector

I assume a continuum mass of identical and perfectly competitive banks whose main activities are lending funds borrowed from international markets to domestic households and firms, and keeping deposits from firms (i.e., keeping the asset that bears no interest M_t). Their balance sheet is expressed as follows

$$\frac{D_{t+1}^h + D_{t+1}^f}{1 + r_t} = \frac{D_{t+1}}{1 + r_t} + M_t. \tag{28}$$

The left side of Equation (28) denotes the bank's assets, which are given by the (discounted) sum of the representative consumer and firm debt. The right-hand side of Equation (28) denotes the firm's liabilities, which are given by the sum of the debt acquired by the bank in international markets, D_{t+1} (acquired at t, maturing at $t + 1$; discounted), and the firm's deposits M_t. The bank's profits are assumed to be redistributed to consumers in a lump-sum manner and are given by

$$\Pi_t^b = D_t^h + D_t^f - D_t - M_{t-1}. \tag{29}$$

For more details on the model derivations, please refer to the appendix.

4 Model Calibration And Estimation

4.1 Calibration

The calibration of structural parameters relies on values commonly used in similar studies that employ RBC models. The consumer risk aversion coefficient is set to $\gamma = 2$, which aligns with the behavioral literature. I set the parameter depicting labor disutility to $\omega = 1.6$, generating a labor supply elasticity of $1/(1-\omega) \approx 1.67$ (Garcia-Cicco et al., 2010, Mendoza, 1991, Schmitt-Grohé & Uribe, 2003). The steady-state government spending-to-output ratio is set to the observed values of 15.5% and 14.9% for Brazil and Canada, respectively.[10] Similarly, the steady-state trade balance-to-output ratio is set to the observed value of 4.39% and -0.49% for Brazil and Canada, respectively.

4.2 Estimation

The remaining parameters, that is, those that are not calibrated, are estimated in a Bayesian fashion. Bayesian estimation of DSGE models allows us to obtain posterior distributions for the structural parameters of the model by combining three elements: the priors (defined as the researcher's prior knowledge and beliefs about certain parameters), the observed data, and the likelihood function (i.e., the DSGE model itself). Thus, the DSGE model can be translated into a state-space representation that allows the creation of a mapping between the model and the observed data:

$$X_t = A(\Theta)X_{t-1} + B(\Theta)u_t \tag{30}$$

$$\mathbf{Y}_t = C(\Theta) + D(\Theta)X_t + v_t, \tag{31}$$

where Θ denotes the vector of parameters to estimate, $\mathbf{Y}_t$ denotes the vector of endogenous (observed) variables, $C(\Theta)$ denotes a vector with data means,[11] $D(\Theta)$ denotes a matrix mapping the unobserved states to the data, and u_t, v_t denote vectors of structural shocks and measurement errors, respectively. With this, the Kalman filter is used to form an estimate of the unobservable states and to evaluate the likelihood function associated with the state-space model. Using Bayes' theorem, the posterior distribution of the parameters can be expressed as a function of the likelihood and the priors:

$$p(\Theta|\mathbf{Y}) = \frac{p(\mathbf{Y}|\Theta)p(\Theta)}{p(\mathbf{Y})} \propto p(\mathbf{Y}|\Theta)p(\Theta), \tag{32}$$

where $p(\mathbf{Y}|\Theta)$ denotes the likelihood function, $p(\Theta)$ denotes the prior density of the vector of parameters, and $p(\mathbf{Y}) = \int p(\mathbf{Y}|\Omega)p(\Omega)d\Omega$ denotes the (unconditional) data density function, which can be disregarded because it simply acts as a scaling factor. With this, the posterior distribution in Equation (32) can be computed by combining the prior distributions and likelihood function by applying the Kalman filter to the state-space representation in Equation (30), given the data $\mathbf{Y}$. The computation is typically performed considering the log of the posterior distribution, that is, (under the assumption of a priori independence)

$$\ln\left(p(\Theta|\mathbf{Y})\right) = \ln\left(p(\mathbf{Y}|\Theta)\right) + \sum_{j=1}^{J} \ln\left(p(\Theta_j)\right), \tag{33}$$

where J denotes the total number of parameters to estimate. Therefore, our vector of parameters to estimate is

$$\Theta = \{\rho_A, \rho_g, \rho_Z, \rho_s, \rho_\mu, \sigma_A, \sigma_g, \sigma_Z, \sigma_s, \sigma_\mu, \phi, \psi, \beta, \delta, \alpha, \eta, \sigma_{\Delta y}, \sigma_{\Delta c}, \sigma_{\Delta i}, \sigma_{tb/y}\}. \tag{34}$$

[10] For both countries, the (yearly) data employed in the model estimation belongs to the period 1951:2019 and are sourced from the Penn World Tables, version 10.1 (Feenstra et al., 2015).

[11] Given that the model is detrended prior to estimation, the growth rate of the trend g is also a parameter to estimate, and hence the data counterpart is an element of $C(\Theta)$.

The last four parameters denote the standard deviations of the measurement errors of the observed variables, which are included for robustness. The theoretical model is written in detrended form (see the appendix for further details) prior to estimation. The matrix of observed variables includes the growth rate of output, the growth rate of consumption, the growth rate of investment,[12] and the trade balance-to-output ratio. The posterior distributions for our parameters of interest are obtained using the slice sampler algorithm developed by Planas et al. (2015).[13] As depicted in Table 1, I assume uniform prior distributions for all the estimated parameters following the specification in S. Hwang & Kim (2022).

5 Results

5.1 Estimation Results

The estimation results are based on 15,000 draws[14] of the slice-sampling algorithm after a burn-in phase of 7,500 and five (sequentially drawn) parallel chains. The posterior values for all estimated parameters are depicted in Table 1. The results show that, except for the trend shock, Canada is found to experience shocks that are systematically more persistent than Brazil. Nonetheless, the Brazilian economy suffers from more volatile shocks, with the exception of the stationary TFP shock. This is particularly true for the volatility of the rate spread shock, which is estimated to be approximately nine times more volatile in Brazil than in Canada.

The posterior values for the structural parameters reveal that the Canadian economy is characterized by slightly higher capital adjustment costs, lower depreciation rates, and a relatively stronger working capital constraint. The latter is consistent with Canadian engaging in relatively stronger precautionary behavior, as the fraction of the total wage bill that they must hold is higher than that of Brazil. The estimated discount factor is relatively higher for Canada, which (given similar values for the trend growth rate g) indicates that the Brazilian economy is subject to higher real interest rates on average. Finally, I find that the sensitivity of the real rate to deviations in aggregate debt from its steady-state value is much higher in Brazil than in Canada. This implies that the Brazilian economy is much more penalized in international financial markets whenever its aggregate debt level diverges from its long-run (steady-state) values.

To understand the contribution of each shock to the business cycles in both economies, Table 2 shows the posterior variance decomposition at the posterior median of the parameters. The Brazilian economy is found to be relatively much more driven by shocks to its trend than by stationary TFP shocks, which goes in line with the argument *the cycle is the trend* motivated by Aguiar & Gopinath

[12]Note: all variables are transformed into per capita values.

[13]See Neal (2003) for a first introduction to slice sampling methods. Applications of the slice posterior sampling algorithm for the estimation of DSGE models in a Bayesian fashion can be found in e.g. Giovannini et al. (2019), Hohberger et al. (2019), Croitorov et al. (2020), Calés et al. (2017).

[14]The number of draws might seem a priori insufficient. However, while one M-H iteration requires exactly one likelihood evaluation, one slice iteration requires approximately $\#parameters \times 7$ likelihood evaluations. Therefore, a general rule of thumb states that if one wants to run N million draws using M-H, this is equivalent to running $N/(\#parameters \times 7)$ draws using the slice sampling algorithm.

Table 1: **Prior and posterior distributions for the estimated parameters.**

		Prior distribution		Posterior distribution			
				Brazil		Canada	
Parameter	Distribution	Min	Max	Median	Sd	Median	Sd
Shock persistence							
ρ_A	Uniform	0	0.99	0.6520	0.2779	0.8845	0.0258
ρ_g	Uniform	0	0.99	0.7736	0.0505	0.5907	0.2170
ρ_Z	Uniform	0	0.99	0.7488	0.1643	0.9811	0.0139
ρ_s	Uniform	0	0.99	0.5141	0.2831	0.5587	0.2844
ρ_μ	Uniform	0	0.99	0.8971	0.0567	0.9570	0.1050
Shock s.d.							
σ_A	Uniform	0	0.4	0.0021	0.0023	0.0144	0.0017
σ_g	Uniform	0	0.4	0.0157	0.0016	0.0028	0.0018
σ_Z	Uniform	0	0.4	0.1826	0.0632	0.1412	0.0510
σ_s	Uniform	0	0.4	0.0043	0.0037	0.0024	0.0022
σ_μ	Uniform	0	0.4	0.0088	0.0033	0.0010	0.0004
Structural parameters							
g	Uniform	1	1.05	1.0299	0.0072	1.0168	0.0023
ϕ	Uniform	0	8	2.0708	0.7450	2.8332	0.9047
ψ	Uniform	0	5	0.0534	0.0596	0.0015	0.5070
β	Uniform	0.9	0.999	0.9661	0.0179	0.9942	0.0053
δ	Uniform	0.01	0.15	0.1017	0.0213	0.0129	0.0042
α	Uniform	0.3	0.5	0.3206	0.0230	0.3674	0.0428
η	Uniform	0	1	0.0769	0.1537	0.8268	0.1970
Measurement errors							
	Distribution	Mean	Sd	Median	Sd	Median	Sd
$\sigma_{\Delta y}$	Gamma	0.02	0.01	0.0024	0.0008	0.0018	0.0005
$\sigma_{\Delta c}$	Gamma	0.02	0.01	0.0024	0.0009	0.0018	0.0007
$\sigma_{\Delta i}$	Gamma	0.02	0.01	0.0105	0.0033	0.0077	0.0024
$\sigma_{tb/y}$	Gamma	0.02	0.01	0.0016	0.0005	0.0007	0.0003

Note: $\rho_A, \rho_g \rho_Z, \rho_s \rho_\mu$ denote the autoregressive coefficients of the (stationary) technology, trend, preference, government spending, and interest rate shock processes, respectively. $\sigma_A, \sigma_g, \sigma_Z, \sigma_s, \sigma_\mu$ denote the standard deviations of the (stationary) technology, trend, preference, government spending, and interest rate shock processes, respectively. g denotes the growth rate of the trend shock, ϕ denotes the strength of the capital adjustment costs, ψ denotes the sensitivity of the real rate to deviations in the debt level, β denotes the discount factor, δ denotes the depreciation rate, α denotes the elasticity of output with respect to capital, and η denotes the strength of the working capital constraint. $\sigma_{\Delta y}, \sigma_{\Delta c}, \sigma_{\Delta i}, \sigma_{tb/y}$ denote the measurement errors of observed output growth, consumption growth, investment growth, and the trade balance-to-output, respectively.

(2007). The rate spread shock can explain a substantially higher fraction of both investment growth and the trade balance-to-output ratio in the Brazilian economy than in the Canadian economy.

Table 3 presents a comparison between the data moments and those generated by the model when all parameters are set to their corresponding estimated values. The model correctly predicts that consumption volatility is higher than GDP volatility for Brazil (whereas the opposite applies to Canada) and the fact that investment growth is the most volatile series. While it correctly generates a negative correlation between the trade balance-to-output ratio for the Brazilian economy, it is unable to capture

Table 2: **Posterior median variance decomposition results (in %).** Measurement errors are not reported given their negligible contribution in driving the observed variables. Standard errors are depicted in parentheses.

	Brazil				Canada			
Shock	Δy	Δc	Δi	tb/y	Δy	Δc	Δi	tb/y
Technology (stat.)	1.38	0.56	0.13	0.16	93.10	63.89	57.44	7.86
	(4.93)	(3.12)	(2.37)	(1.21)	(8.24)	(8.56)	(10.14)	(5.35)
Trend	93.61	63.09	49.39	21.44	5.43	7.15	5.04	2.99
	(5.07)	(6.76)	(9.38)	(14.02)	(8.25)	(8.41)	(8.54)	(10.27)
Preference	0.49	33.40	1.98	21.94	0.50	25.42	3.81	64.59
	(1.15)	(5.81)	(3.23)	(11.14)	(0.32)	(5.02)	(2.68)	(19.23)
Interest rate	2.94	1.58	45.60	51.73	0.32	1.39	30.46	21.19
	(1.35)	(0.82)	(7.89)	(15.26)	(0.15)	(0.65)	(8.14)	(14.23)
Government spending	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.01
	(0.00)	(0.03)	(0.01)	(0.08)	(0.00)	(0.04)	(0.03)	(0.04)

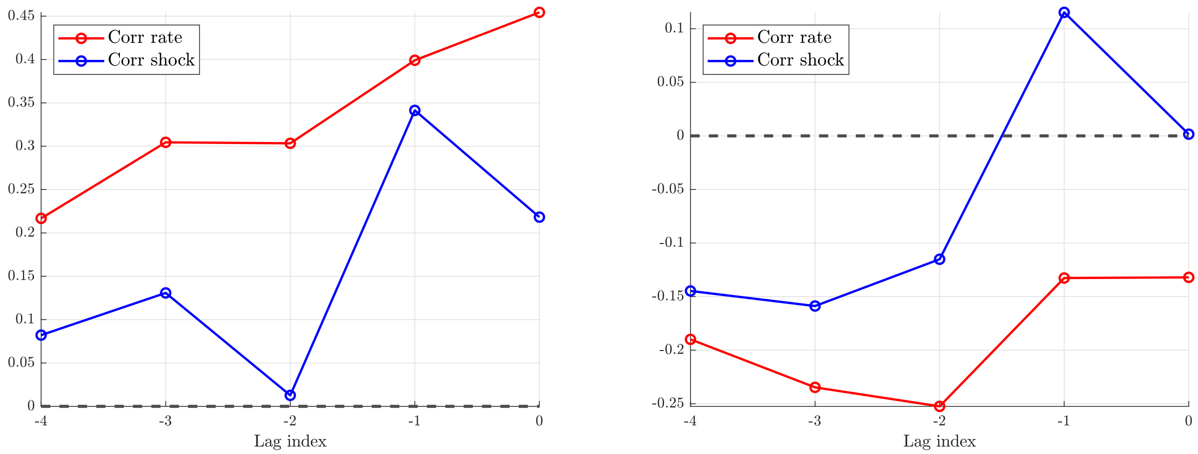
Note: $\Delta y, \Delta c, \Delta i, tb/y$ denote the growth rates of output, consumption, and investment, and the trade balance-to-output ratio, respectively.

the positive empirical correlation between these two series for Canada.

Table 3: **Comparison between data and model moments**. The model moments are obtained by setting the model parameter values at the posterior median of the estimated parameters.

	Brazil							
	Data				Model			
	Δy	Δc	Δi	tb/y	Δy	Δc	Δi	tb/y
$\sigma(x)$	3.6	4.01	10.79	3.4	3.58	4.72	10.28	3.67
$\rho(x, \Delta y)$	1	0.81	0.71	-0.3	1	0.81	0.64	-0.27
$\rho(x_t, x_{t-1})$	0.4	0.16	0.01	0.82	0.14	-0.01	-0.08	0.75
	Canada							
	Data				Model			
	Δy	Δc	Δi	tb/y	Δy	Δc	Δi	tb/y
$\sigma(x)$	2.21	1.5	8.91	3.17	2.5	2.31	9.12	4.16
$\rho(x, \Delta y)$	1	0.78	0.82	0.21	1	0.85	0.77	-0.06
$\rho(x_t, x_{t-1})$	0.19	0.45	-0.15	0.91	-0.04	-0.02	-0.07	0.94

To further understand the model predictions of certain business cycle characteristics, Figure 5 depicts the theoretical correlations between smoothed GDP and real interest rates and the smoothed spread shock ε_t^{μ}.[15] The Brazilian business cycle is found to be negatively correlated with both the theoretical real interest rate and its corresponding spread shocks at almost all lags, although the latter is not true for the Canadian economy.



(a) Canada (b) Brazil

Figure 5: **Theoretical correlations between GDP growth and real interest rate and real interest rate shock**. The blue lines depict the theoretical correlations between smoothed GDP growth and the real interest rate; the blue lines depict the theoretical correlations between smoothed GDP growth and the spread shock ε_t^{μ}.

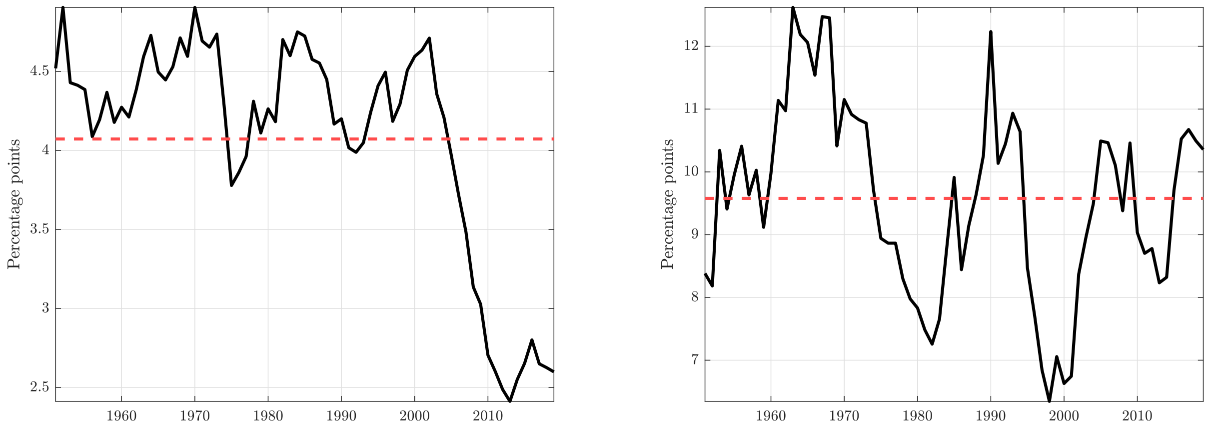
In a similar vein, Figure 6 shows the smoothed real interest rate (in annualized terms) over the estimation period, which is substantially higher for Brazil than for Canada (9.58% and 4.07%, respectively).

5.2 Counterfactual Analysis

5.2.1 Theoretical IRFs And Correlations

Structural estimation of theoretical models allows producing counterfactual analysis that sheds light on certain characteristics of the economy under different parameter values. With this purpose, I study how the response of the Brazilian economy to shocks to the real rate is influenced by two key parameters: the elasticity of the real interest rate to deviations of the debt level relative to its steady state value (i.e., ψ), and the volatility of the real rate shock, σ_{μ}. I start by setting all the parameters to their estimated posterior median and modify the value of ψ by creating a linearly spaced vector of length three between the estimated value for the Brazilian economy and the Canadian. Figure 7 shows that the Brazilian growth rate of output, consumption, and investment would suffer a stronger downturn on impact when facing a shock to its real rate, although the effect seems to be compensated at higher time periods. The picture looks the opposite for the trade balance-to-output ratio, which experiences a

[15] Note: Smoothed shocks are computed by employing the Kalman smoother and represent the "best guess" for the structural shocks given all observations



(a) Canada

(b) Brazil

Figure 6: **Smoothed real interest rates for Canada and Brazil (annualized terms, in %).** Dashed, red horizontal lines depict the average real interest rate over the observed period.

more positive shock on impact when ψ is set to the estimated value of Canada (this seems nonetheless to be compensated by a downturn at higher time periods).

Figure 8 shows the counterfactual IRFs to a real rate shock under different shock volatility values. The evidence points out the fact that the (negative) reaction of macroeconomic aggregates (output, consumption, and investment growth) to real rate shocks is substantially smaller once the volatility of the real rate shock of Brazil tends to that of Canada. The trade balance-to-output ratio, however, reacts substantially less not only on impact but also at higher time values.

I also explore the extent to which the parameters $\{\psi, \sigma_\mu\}$ alter the relationship between the growth rate of output and the real interest rate. Similarly to the previous exercise, I construct a fine grid of length a hundred for both parameters ranging from the estimated posterior value for Brazil to that of Canada, and compute the model-based correlations between output growth and the real interest rate for each point in the grid. Figure 9 reveal the fact that the correlation between the real rate and GDP and consumption growth and the trade balance-to-output ratio decreases monotonically as ψ^{BR} approaches ψ^{CA}. However, the latter is not true for the growth rate of investment, which appears to have a higher correlation (i.e., less negative) as ψ^{BR} tends to ψ^{CA}.

The interpretation changes once I examine the correlation between real rates and macroeconomic aggregates as a function of the volatility of the real rate shock itself, σ_μ (Figure 10). I show that, as σ_μ^{BR} tends to σ_μ^{CA}, these correlations decrease (i.e., they become less negative) for the growth rate of output, consumption, and investment, while this relationship appears to be weak when considering the trade balance. Under the assumption that these correlations being negative implies that the Brazilian economy is very vulnerable to real rate hikes, this result shows that the volatility of the real rate shock matters substantially in determining the relationship between macroeconomic aggregates and financing costs.

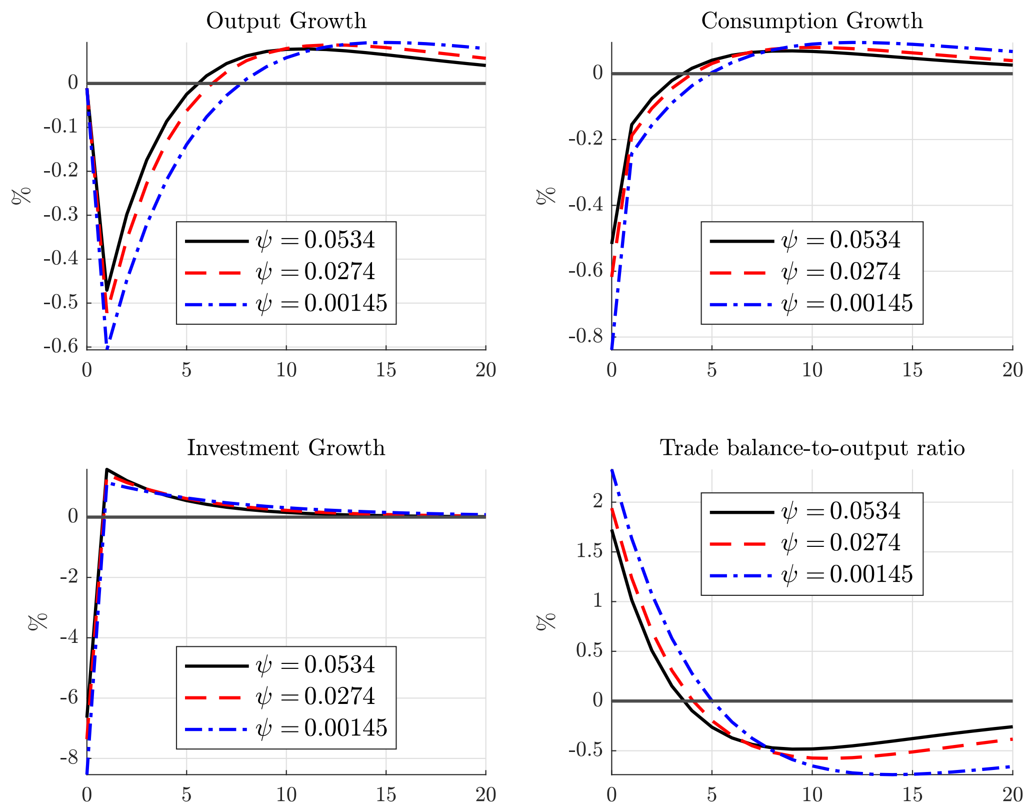


Figure 7: **Model-based IRFs to a (one standard deviation) shock to the real interest rate for different values of** ψ. Dark, solid lines: ψ set to the posterior median for Brazil. Blue, dashed lines: ψ set to the posterior median for Canada. Red, dashed lines: ψ set at an intermediate value.

5.2.2 Welfare Analysis

To finalize the counterfactual analysis of the estimated theoretical model, I examine the potential changes in welfare under alternative parameter values. Considering the utility function of the representative agent in Equation (5), I can define a simple welfare criterion as follows:

$$\mathcal{W}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left[C_t - \omega^{-1} X_{t-1} H_t^{\omega}\right]^{1-\gamma} - 1}{1-\gamma} \right) Z_t, \tag{35}$$

which can be rewritten as

$$\mathcal{W}_t = \left(\frac{\left[C_t - \omega^{-1} X_{t-1} H_t^{\omega}\right]^{1-\gamma} - 1}{1-\gamma} \right) Z_t + \beta\, \mathbb{E}_t\, \mathcal{W}_{t+1}. \tag{36}$$

With this, I examine how values of ψ and σ_μ affect welfare in the Brazilian economy. I proceed in a similar way to the previous subsection by creating equally spaced (joint) grids of ψ and σ_μ and analyzing their impact on welfare for each possible parameter combination by solving the model using a second-order approximation around the deterministic steady state.

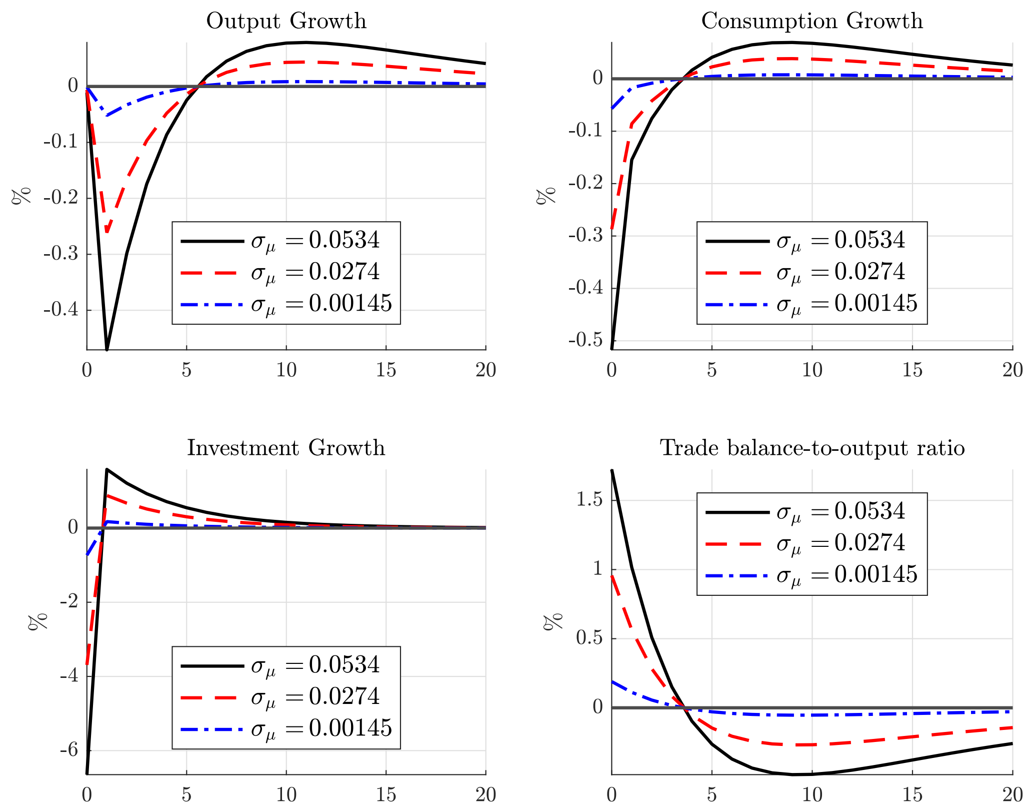


Figure 8: **Model-based IRFs to a (one standard deviation) shock to the real interest rate for different values of σ_μ**. Dark, solid lines: σ_μ set to the posterior median for Brazil. Blue, dashed lines: σ_μ set to the posterior median for Canada. Red, dashed lines: σ_μ set at an intermediate value.

Figure 11 depicts the results of the counterfactual welfare analysis. Conditional on ψ being set to its estimated value for Brazil, reductions in the spread shock volatility imply monotonic reductions in welfare. On the other hand, reductions in the sensitivity of the real rate to deviations of the aggregate debt level relative to its steady-state value (i.e., reductions in ψ conditional on σ_μ) are shown to be welfare-improving. Taken together, these results show that it is not the spread shock volatility the parameter that matters when analyzing welfare, but the extent to which the real interest rate faced by the Brazilian economy reacts to movements in the debt level. Despite the observation that lower values of ψ imply lower (i.e., more negative) correlations between the growth rates of GDP and consumption, these are found to induce higher welfare values (those implied by the simple welfare rule in Equation (35)) in Brazil.

5.2.3 Counterfactual Macroeconomic Volatility

Since the role of ψ has been identified as significant for welfare, the next step is to examine how different levels of this parameter influence the overall volatility of the economy. For this purpose, I first obtain the smoothed shocks (i.e., the best guesses of the structural shocks given the data observed), which

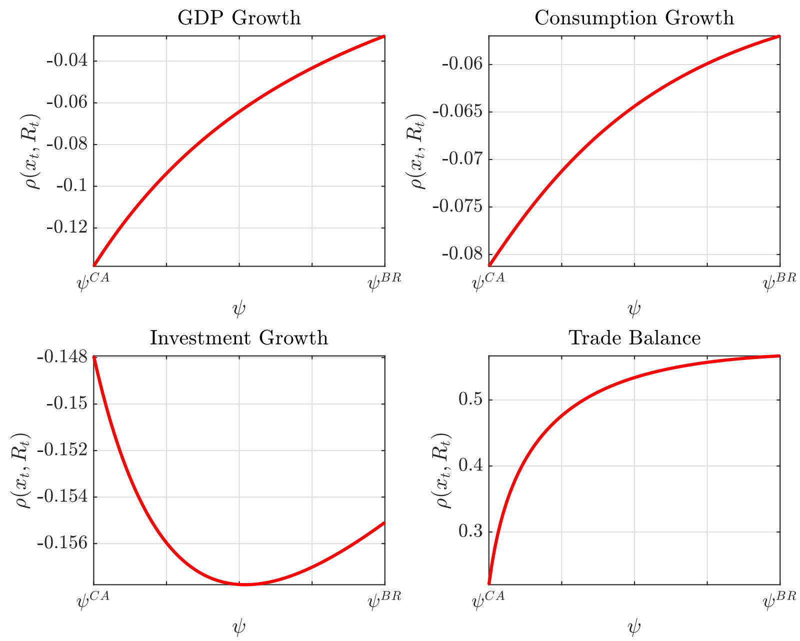


Figure 9: **Theoretical correlation between output growth and real interest rate as a function of the parameter ψ.** Note: I consider a grid of length a hundred that ranges from the posterior estimated value for Canada (ψ^{CA}) to that for Brazil (ψ^{BR}).

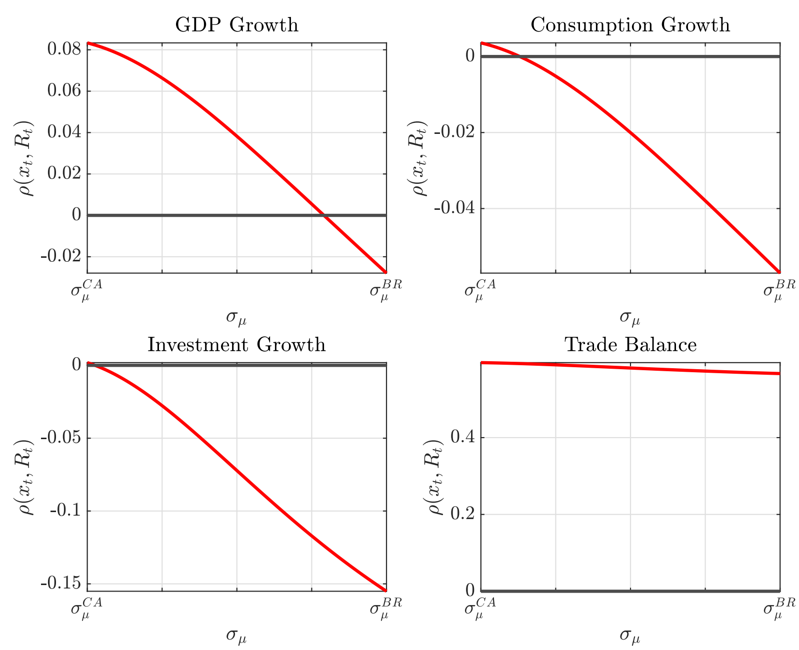


Figure 10: **Theoretical correlation between output growth and real interest rate as a function of the parameter σ_μ.** Note: I consider a grid of length a hundred that ranges from the posterior estimated value for Canada (σ_μ^{CA}) to that for Brazil (σ_μ^{BR}).

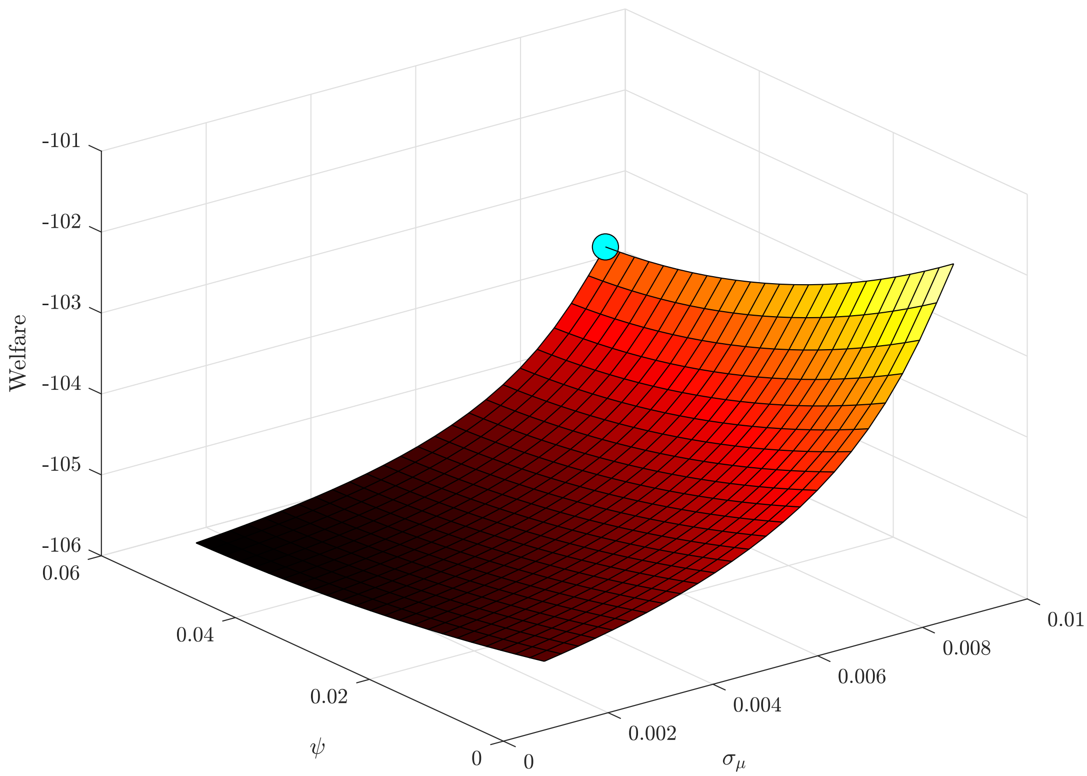


Figure 11: **Welfare in the Brazilian economy as a function of the parameters ψ and σ_μ.** The circle colored in cyan represents the welfare value at the estimated parameters $\{\psi, \sigma_\mu\}$ when using Brazilian data.

are obtained with the estimation of the model. With this, I input them into the theoretical model and vary ψ accordingly. In this sense, the shocks are not *deterministic* but *determined*, i.e., agents in the economy still form expectations because they do not know the future realization of the shocks but these shocks are in turn *determined* (i.e., they are not *simulated shocks*). On top of this, I analyze the extent to which macroeconomic volatility is shaped by the real interest rate shock by setting this shock to zero at all time periods (i.e., I set $\varepsilon_t^\mu = 0$).

Figure 12 reveals the results from these counterfactual experiments. The evidence shows that all three variables would experience higher volatility levels if the sensitivity of the real interest rate in Brazil were to be equal to that of Canada (second column in each of the three subplots of Figure 12). The reason for this is the fact that if ψ were to be reduced, firms and especially consumers would engage in debt paths that would be substantially more volatile so as to finance consumption levels that would be welfare-improving (as I show in Section 5.2.2 with the examination of a simple welfare rule), despite this coming at the cost of higher volatility. Therefore, varying the degree to which the real interest rate faced by Brazil responds to movements in aggregate debt levels generates a tension between higher *levels* of consumption (which generate higher *levels* of welfare) and higher consumption *volatility* as a consequence.

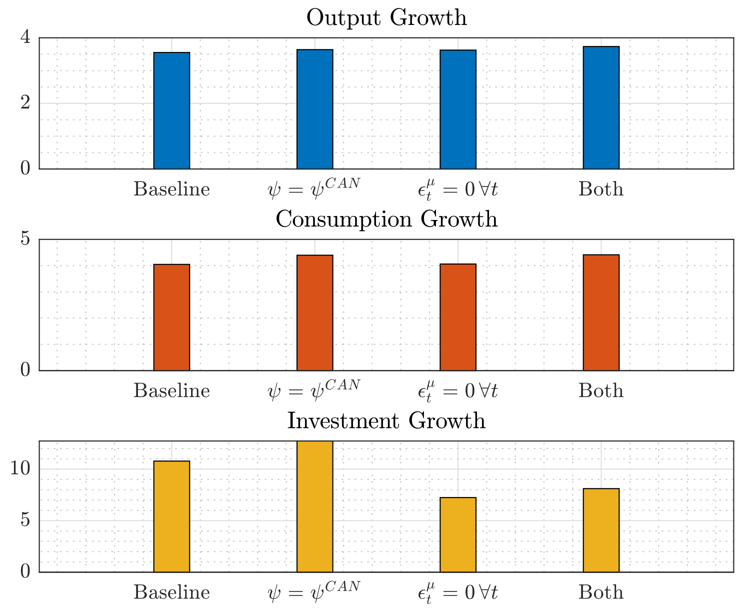


Figure 12: **Macroeconomic volatility under counterfactual scenarios.** The bars of the baseline scenario represent the standard deviation of each of the smoothed variables after estimation. For the other scenarios, the bars represent the standard deviation of the simulated variables under alternative specifications conditional on the (determined) smoothed shocks obtained from the model estimation. "Both" indicates that both conditions are imposed in the simulation, i.e. $\psi = \psi^{CAN}$ and $\varepsilon_t^\mu = 0\,\forall t$

In a similar vein, Figure 11 shows that decreasing the volatility of the real interest rate shock in the Brazilian economy induced lower levels of welfare, at least from the perspective of the simple welfare function in Equation (35). To examine the effect of the real interest rate shock on overall economic volatility, I repeat the same exercise but now set the smoothed rate shock to zero (third column in each of the three subplots of Figure 12). The results reveal that while consumption and output growth volatilities are barely affected, the volatility of investment growth decreases dramatically when the real interest rate shock is set to zero, relative to the baseline scenario. This suggests that a substantial part of investment volatility in the Brazilian economy is driven by the shocks to the real interest rate it faces in international markets, as previously shown in Table 2.

Figure 13 confirms the latter point by depicting the baseline smoothed investment growth and the counterfactual one when the real interest rate shock is set to zero. The evidence points out the fact that if Brazil were to completely offset the shocks to the real interest rates they face in international markets, investment growth volatility would decrease substantially, hence dampening the strong swings in this variable. The counterfactual investment series presents downturns that are not as deep as those in the baseline scenario across all time periods (with the exception of the considerable drop in the early 1980s).

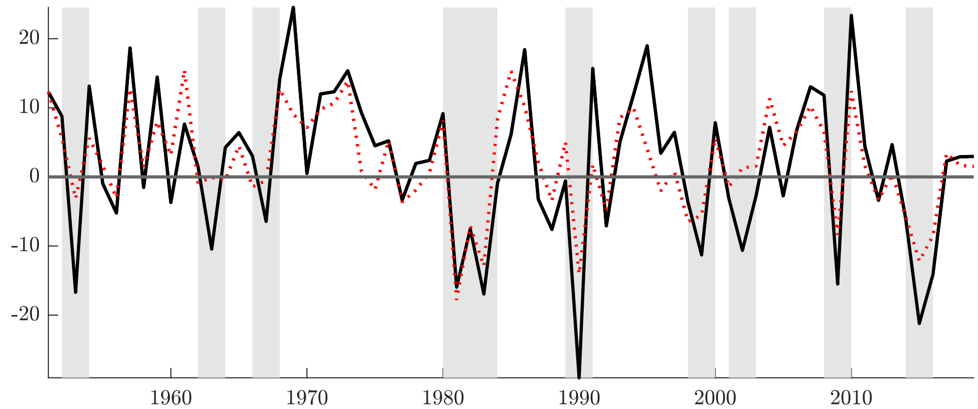


Figure 13: **Counterfactual investment growth series.** Black, solid lines: smoothed investment series (i.e., after estimation); red, dashed lines: smoothed investment series when real interest rate shock is set to zero ($\varepsilon_t^\mu = 0\,\forall t$). Grey bars denote periods when output growth was substantially low for ease of readability.

5.3 On the source of interest rate shocks

So far, the nature and sources of the shock to the real interest rate have not yet been discussed. For instance, the interest rate shock in Brazil and its corresponding effects on the economy could be divided into an international/external component and a country-specific spread component, depending on whether the shock is imported from international financial markets or originates domestically. For this purpose, I estimate an empirical model using only Brazilian data to disentangle the interest rate shock into a US and a country-specific real interest rate shock in the spirit of Uribe & Yue (2006).

The model is a standard structural VAR estimated in a Bayesian fashion by imposing a Minnesota prior. The data are exactly the same as those in Section 2 (in terms of sources and treatment), with the exception that now the interest rate is divided into a country-spread and an external component. More specifically, the variables included are:

$$y_t = [r_t^{US}, r_t, gdp_t, c_t, i_t, nx_t] \tag{37}$$

Where r_t^{US} denotes a proxy for the US real interest rate, r_t denotes the Brazil country-specific real interest rate,[16] and gdp_t, c_t, i_t, nx_t denote GDP, consumption, gross fixed capital formation, and net exports for the Brazilian economy, respectively. Except for the real interest rates, and as depicted in Section 2, all variables are log-quadratically detrended. The number of lags is set to 4, and the model is identified via Cholesky decomposition, following the variable order in Equation (37).

To ensure that Brazilian variables do not affect the US real interest rate, block exogeneity (also known as no-Granger causality) is imposed by setting tight priors around zero on the corresponding

[16] Similarly to the empirical model depicted in Section 2, the US real interest rate r_t^{US} is computed by taking the 3-month Treasury Bill and subtracting a measure of expected inflation, which is proxied by the average between current GDP deflator inflation and the previous 3 quarters. The Brazilian-specific real interest rate is computed by taking the US real interest rate r_t^{US} and adding the JP Morgan EMBI spread.

coefficients.[17] This implies that the US real interest rate simply follows an AR(4) process. Note further that, following closely Uribe & Yue (2006), the shock to the Brazilian real interest rate (denoted by r_t), can be interpreted as a country spread shock by construction.

In Figure 14 I depict the historical decomposition of Brazilian macroeconomic aggregates to both country-specific and US real interest rate shocks. In a similar vein, Table 4 shows the cumulative sum of the absolute value of the contribution of both shocks to Brazilian variables. The results reveal that Brazilian macroeconomic aggregates are relatively driven by US interest rate shocks to a larger degree, especially gross fixed capital formation.

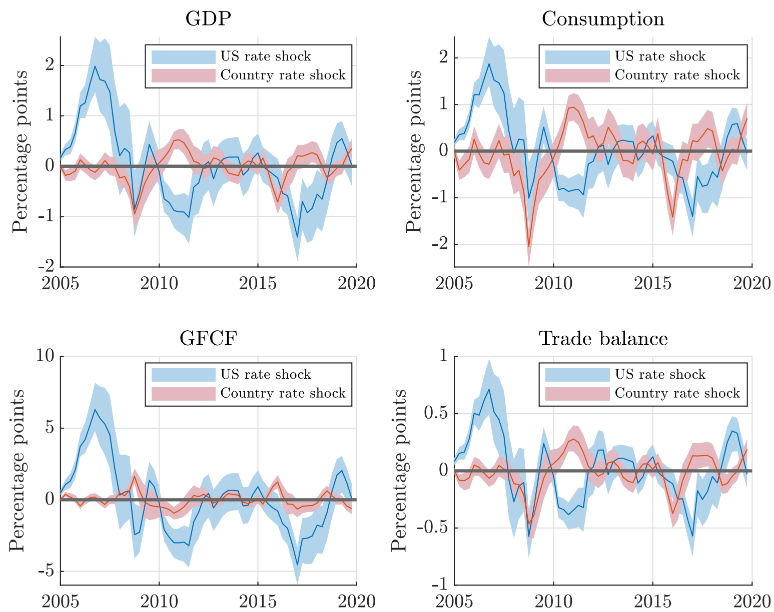


Figure 14: **Historical decomposition of Brazilian variables to domestic and US rate shocks.** Shaded blue and red areas denote the 68% confidence bands.

Table 4: **Cumulative sum of domestic and US rate shock contributions in absolute values to Brazilian variables (in %).**

	US rate shock	Country rate shock
Output	33.20	11.56
Consumption	31.93	23.11
GFCF	107.59	21.84
Trade Balance	13.13	6.33

This finding is further confirmed by the forecast error variance decomposition in Figure 15, with the exception of consumption, which seems to be driven by both rate shocks to an equal amount.

[17]The variable ordering is slightly different than that in Section 2 because in order to ensure no feedback from Brazilian variables to the US real interest rate, the no-Granger causality needs to be additionally combined with a zero restriction on impact from Brazilian to US variables, which is ensured by this particular variable ordering.

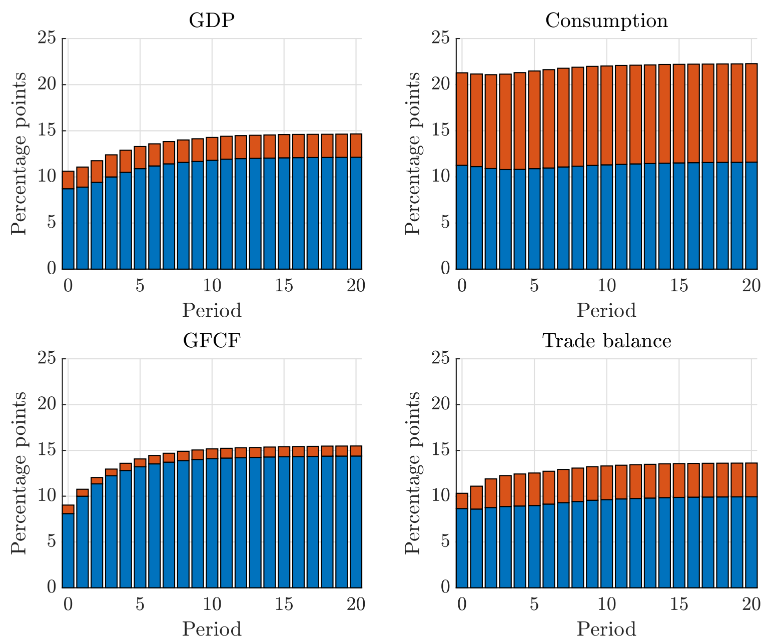


Figure 15: **Forecast error variance decomposition of Brazilian variables to domestic and US rate shocks.**

6 Conclusions

In this paper, I examine the differences in the response of developed and developing economies to real interest rate shocks. Using structural VAR models that exploit the panel structure of the data, the results show that real rate shocks have more adverse effects on a set of macroeconomic variables in developing economies than in developed ones.

To analyze the structural reasons behind these findings, I make use of an SOE-RBC model featuring multiple shocks and frictions that is estimated using data from Brazil and Canada. The estimation results suggest that the discrepancies between the two countries emerge mainly from the differences in the sensitivity of the interest rate to deviations in the debt level from its steady-state value and the observation that real rate shocks are more volatile in the Brazilian economy. The Brazilian business cycle is driven more by shocks to the trend than by stationary technology shocks, and the rate spread shock can explain a substantially higher fraction of both investment growth and the trade balance-to-output ratio in the Brazilian economy relative to the Canadian economy.

I perform a counterfactual exercise and show that reductions in the spread shock volatility can reverse the sign of the correlation between the real interest rate and the growth rates of macroeconomic aggregates. I further reveal that the Brazilian economy would attain higher welfare values if the sensitivity of the real interest rate to deviations in the debt level from its long-run value decreased, although this comes at the expense of higher economic volatility. The counterfactual scenario in which the real interest rate shock is set to zero shows that investment growth volatility would decrease substantially, hence dampening strong investment downturns observed in the data.

A final empirical exercise in which the Brazilian real interest rate is disentangled into a domestic and a US component reveals that macroeconomic aggregates in Brazil are driven by US rate shocks to a larger extent relative to domestic (or spread) real interest rate shocks.

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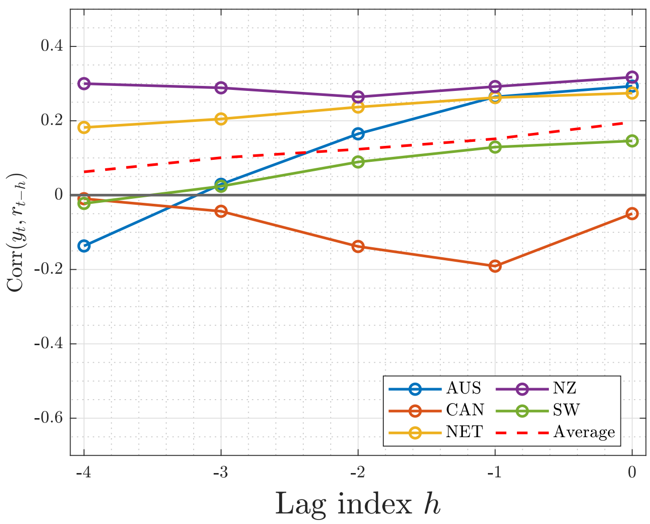
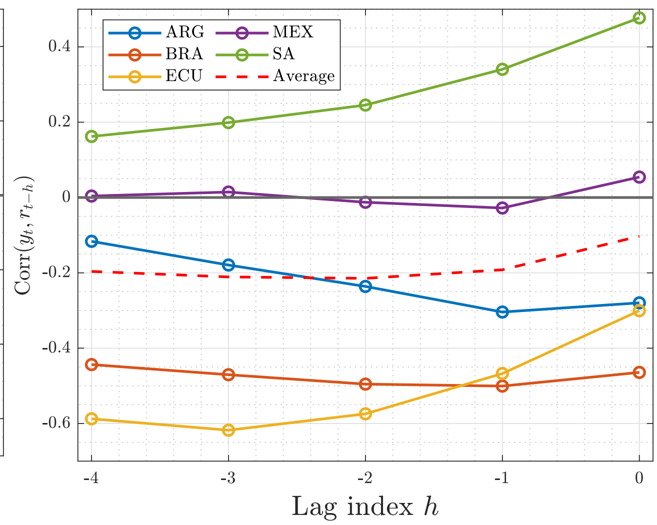
A Appendix



(a) Developed Economies



(b) Developing Economies

Figure A.1: **Correlations between GDP and lagged real interest rates for developed and developing economies (country-specific and cross-country averages).**

A.1 Data Sources

A.1.1 Developed Economies

Data on quarterly national accounts (real, seasonally adjusted) for developed economies were obtained from the OECD Quarterly National Accounts. Interest rate data were obtained from the Federal Reserve Economic Data (FRED). The following interest rates were selected for each country:

- Canada: 3-Month or 90-day Rates and Yields; Commercial/Corporate Paper
- Australia: 3-Month or 90-day Rates and Yields; Bank Bills
- New Zealand: 3-Month or 90-day Rates and Yields; Bank Bills
- Sweden: 3-Month or 90-day Rates and Yields; Treasury Securities
- Netherlands: 3-Month or 90-day Rates and Yields; Interbank Rates for the Netherlands

The real rate is computed by subtracting the expected GDP deflator inflation from the nominal rate. Expected GDP deflator inflation is computed by taking the average of the current GDP deflator inflation and the previous three lags:

$$r_t = i_t - \frac{1}{4}\sum_{j=0}^{3} \pi_{t-j}, \tag{A.1}$$

where r_t denotes the real rate, i_t denotes the nominal rate, and π_t denotes GDP deflator inflation.

A.1.2 Developing Economies

Data on quarterly national accounts (real, seasonally adjusted) for developing economies were obtained from the IMF International Financial Statistics (IMF IFS). The nominal interest rate for this group of countries is constructed as the sum of the J.P. Morgan Emerging Markets Bond Spread and the 3-Month US Treasury Bill rate. As in the case of developed economies, the real rate is constructed by taking the nominal interest rate and subtracting a measure of expected GDP deflator inflation.

A.2 Alternative real interest rate for developing economies

In this subsection, I depict the results of a structural VAR model with the exact same specifications as in Section 2 but using an alternative measure of the real interest rate for emerging economies that takes into account the zero lower bound period for US interest rates. Instead of using the 3-Month Treasury Bill rate as the nominal interest rate for the US to build the real rate for emerging economies, I consider now using the Federal Funds Rate (FFR). Following closely Boeck & Mori (2023), I replace the FFR with the shadow rate of Wu & Xia (2016) during the period 2008M12-2015M12 to account for conventional and unconventional US monetary policy (see Figure A.2).

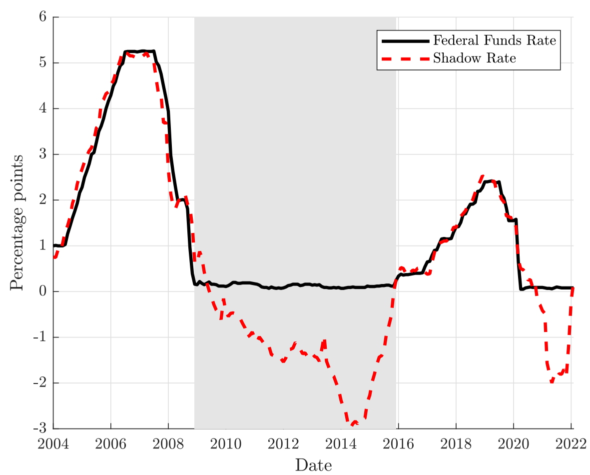


Figure A.2: **Federal Funds Rate and Shadow Rate (Wu & Xia, 2016)**

To understand the developments in the two different measures, Figure A.3 depicts both real interest rates for developing economies. Accounting for US unconventional monetary policy is related to lower real interest rates in developing economies.

The results of the empirical model with the same specification described in Section 2 but employing real interest rates adjusted for the shadow rate by Wu & Xia (2016) are shown in Figure A.4. The results yield qualitatively similar conclusions to those obtained by not accounting for the US shadow rate.

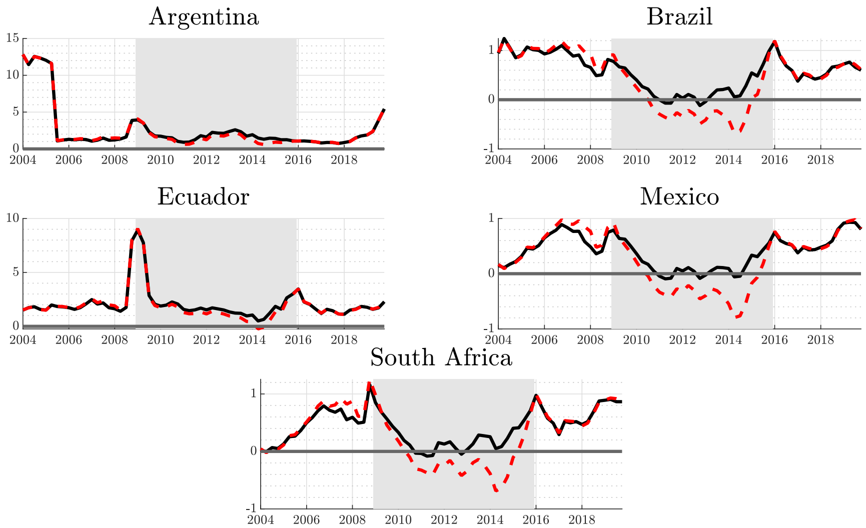


Figure A.3: **Real interest rates under different US nominal interest rates for developing economies**. Black, solid lines: real interest rates when using 3-month TBill rates as a measure of US nominal interest rates. Red, dashed lines: real interest rates when using the shadow rate (Wu & Xia, 2016) as a measure of US nominal interest rates. Grey-shaded areas indicate the period when the FFR is replaced by the shadow rate.

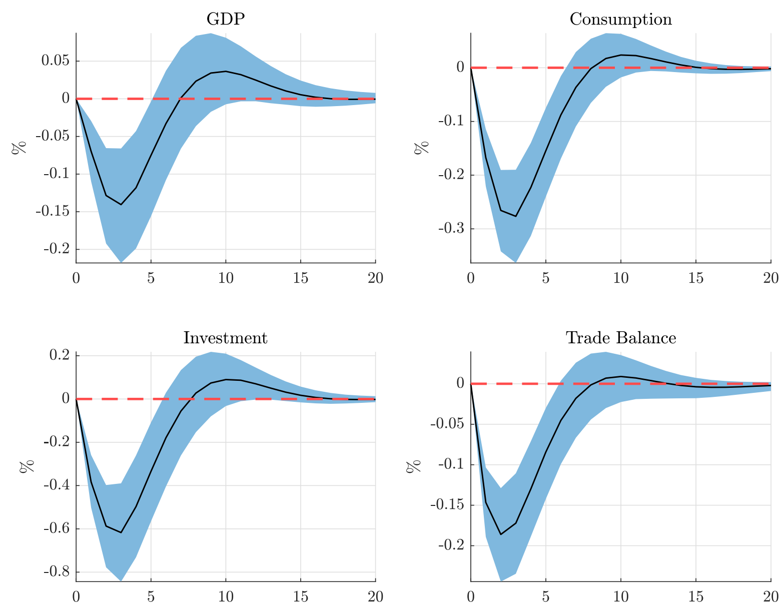


Figure A.4: **Impulse responses to a one standard deviation shock to the real rate (when using the shadow rate by Wu & Xia 2016) for developing economies, in %**. The figure depicts the cross-country median effect (solid, black line) together with the 68% (light blue shaded area) credible bands.

A.3 Alternative sVAR model

In this section, I depict the results of a model that allows estimating country-specific VAR matrices for the sake of robustness. For each country $i = \{1, ..., N\}$ the panel VAR is given by

$$y_{it} = c_i + \sum_{k=1}^{p} A_i^p y_{i,t-k} + C_i x_t + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim \mathcal{N}(0, \Sigma_i), \tag{A.2}$$

where c_i denotes a constant term, A_i^p denotes the (unit-specific) $N \times N$ coefficient matrix, C_i denotes the coefficient matrix mapping the exogenous variables x_t to the endogenous variables, and Σ_i denotes the unit-specific variance-covariance matrix. In this application, I adopt the hierarchical prior scheme as in Jarociński (2010). The model is estimated in a Bayesian framework in which the coefficients of the system are assumed to be drawn from a normal distribution[18]

$$\beta_i \sim \mathcal{N}(\mu, \Sigma_b), \tag{A.3}$$

where $\beta_i = \text{vec}(B_i)$ denotes the vectorized version of the stacked coefficients $B_i = [(c_i, (A_i^1)', ...(A_i^p)')]'$. I select an improper prior for the common mean such that

$$\pi(b) \propto 1. \tag{A.4}$$

The common variance-covariance matrix is defined as follows

$$\Sigma_b = (\lambda_1 \otimes I_q)\,\Omega_b, \tag{A.5}$$

where λ_1 denotes the hyperparameter governing the overall prior tightness (i.e., the differences in the coefficients across units), I_q denotes the $q \times q$ identity matrix and Ω_b denotes a $q \times q$ diagonal variance-covariance. The hyperparameters of Ω_b have the usual interpretation, as in the case of the Minnesota prior (Doan et al., 1984): λ_2 governs the standard deviation of the prior on lags of variables other than the dependent variable, λ_3 governs the decay over lags, and λ_4 relates to the variance of the constant and exogenous variables. The tightness hyperparameter is assumed to follow an inverse gamma distribution

$$\lambda_1 \sim \mathcal{IG}\left(\frac{s_0}{2}, \frac{v_0}{2}\right). \tag{A.6}$$

The chosen values for the hyperparameters are commonly found in the literature (see e.g. Dieppe et al., 2016, Jarociński, 2010, Gelman, 2006) and are listed in Table A.1.

The identification scheme adopted is based on a standard Cholesky decomposition, under the assumption that shocks to the real interest rate affect all the variables in the system on impact. Therefore, the variable ordering is such that $y_t = \{r_t, gdp_t, c_t, i_t, nx_t\}$. The total number of lags is set to $p = 4$ for all variables. Analogously to the previous empirical model, I include the European Brent Spot Price as an exogenous variable of the system (sourced from the U.S. Energy Information Administration). The number of total iterations is set to 15,000 after a burn-in phase of 5,000 iterations.

Figures A.5 and A.6 depict the estimated macroeconomic responses to a one standard deviation shock to the real interest rate for both developed and developing economies, respectively (obtained by computing the cross-country average IRFs within each group), together with the country-specific responses.

[18] See Canova & Ciccarelli (2013) and Dieppe et al. (2016) for an excellent review of panel VAR models.

Table A.1: **Selected hyperparameter values for the alternative panel VAR**

Hyperparameter	λ_2	λ_3	λ_4	s_0	v_0
Value	0.5	1	100	0.001	0.001

Note: λ_1 governs the overall tightness, λ_2 governs the standard deviation of the prior on lags of variables other than the dependent variable, λ_3 governs the decay over lags, λ_4 relates to the variance of the constant and exogenous variables; s_0, v_0 denote the shape and scale of the prior distribution of λ_1, respectively.

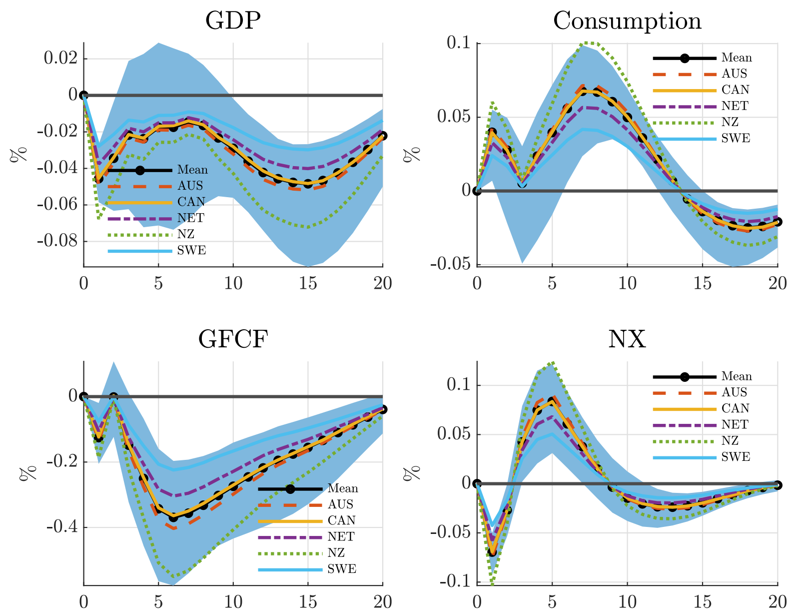


Figure A.5: **Impulse responses to a one standard deviation shock to the real rate for developed economies, in %**. The figure depicts the cross-country average effect (solid, black line) together with the 68% credible bands (dashed, red lines) and the country-specific average IRFs.

For completeness, I depict the results using the exact same empirical model defined in this subsection (based on Jarociński 2010) but employing the shadow rate by Wu & Xia (2016) as a measure of US nominal interest rate in Figure A.7.

A.4 Model Derivations

A.4.1 Households

Recall that households maximize expected discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left[C_t - \omega^{-1} X_{t-1} H_t^{\omega}\right]^{1-\gamma} - 1}{1-\gamma} \right) Z_t \tag{A.7}$$

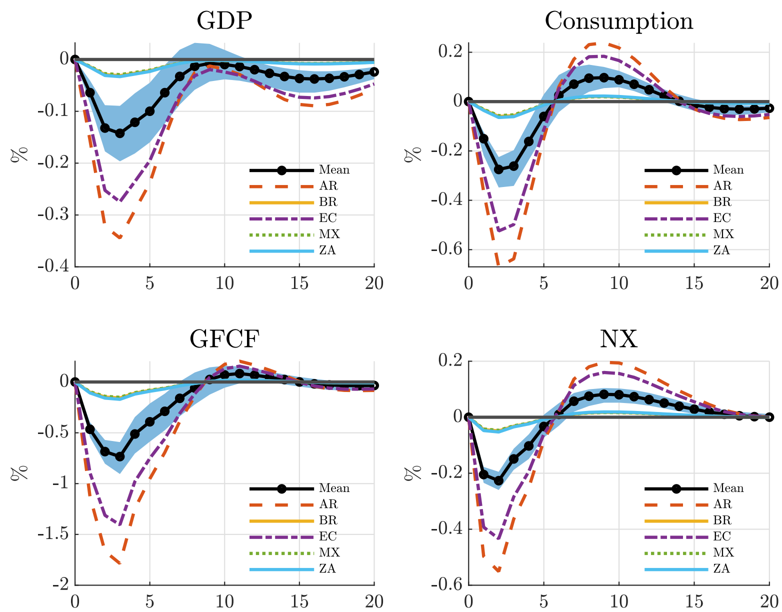


Figure A.6: **Impulse responses to a one standard deviation shock to the real rate for developing economies, in %.** The figure depicts the cross-country average effect (solid, black line) together with the 68% credible bands (dashed, red lines) and the country-specific average IRFs.

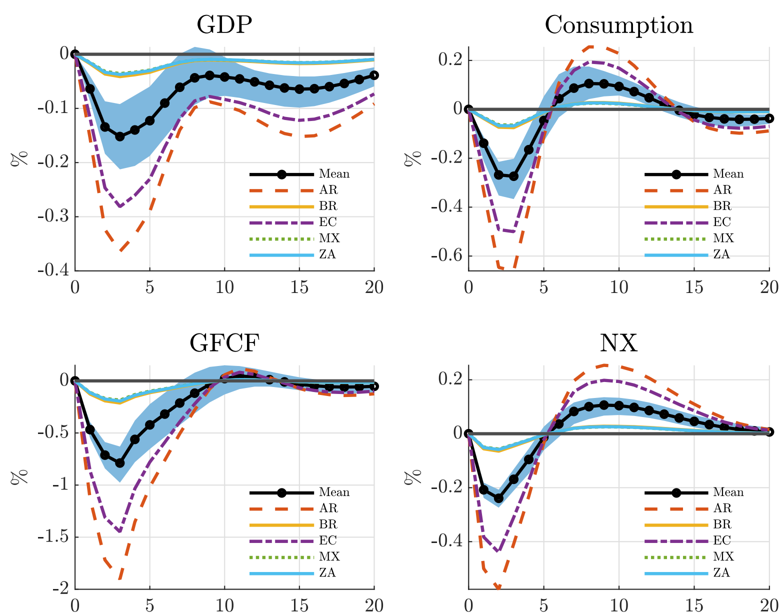


Figure A.7: **Impulse responses to a one standard deviation shock to the real rate (when employing the shadow rate by Wu & Xia 2016) for developing economies, in %.** The figure depicts the cross-country average effect (solid, black line) together with the 68% credible bands (dashed, red lines) and the country-specific average IRFs.

subject to the sequential budget constraint

$$\frac{D_{t+1}^h}{1+r_t} = D_t^h - W_t H_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g\right)^2 K_t - \Pi_t. \tag{A.8}$$

With this, the household problem boils down to maximizing the following Lagrangian function:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\frac{[C_t - \omega^{-1} X_{t-1} H_t^{\omega}]^{1-\gamma}}{1-\gamma} \right) Z_t + \tag{A.9}$$

$$X_{t-1}^{-\gamma} \lambda_t \left(\frac{D_{t+1}^h}{1+r_t} - D_t^h + W_t H_t + u_t K_t - C_t - S_t - I_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - g \right)^2 K_t + \Pi_t \right) \Bigg\}. \tag{A.10}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial C_t} : \quad Z_t [C_t / X_{t-1} - \omega^{-1} H_t^{\omega}] = \lambda_t \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} : \quad Z_t [C_t / X_{t-1} - \omega^{-1} H_t^{\omega}] H_t^{\omega - 1} = \lambda_t \frac{W_t}{X_{t-1}} \tag{A.12}$$

$$\frac{\partial \mathcal{L}}{\partial D_{t+1}^h} : \quad \lambda_t = \beta (1 + r_t) g_t^{-\gamma} \, \mathbb{E}_t \, \lambda_{t+1} \tag{A.13}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_{t+1}} : \quad & \lambda_t \left[1 + \phi \left(\frac{K_{t+1}}{K_t} - g \right) \right] = \\ & \beta g_t^{-\gamma} \, \mathbb{E}_t \, \lambda_{t+1} \left[1 - \delta + u_{t+1} + \phi \left(\frac{K_{t+2}}{K_{t+1}} \right) \left(\frac{K_{t+2}}{K_{t+1}} - g \right) - \frac{\phi}{2} \left(\frac{K_{t+2}}{K_{t+1}} - g \right)^2 \right] \end{aligned} \tag{A.14}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : \qquad \frac{D_{t+1}^h}{1+r_t} = D_t^h - W_t H_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - g \right)^2 K_t - \Pi_t. \tag{A.15}$$

A.4.2 Firms

Recall that the firm's budget constraint is given by

$$\frac{D_{t+1}^f}{1+r_t} = D_t^f + \Delta M_t + u_t K_t + W_t H_t - Y_t + \Pi_t^f. \tag{A.16}$$

With this, the firm chooses processes $\{K_t, H_t, M_t, D_t^f\}_{t=0}^{\infty}$ that maximize its expected stream of profits

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t X_{t-1}^{-\gamma} \lambda_t \left[Y_t - u_t K_t - W_t H_t - \Delta M_t + \frac{D_{t+1}^f}{1+r_t} - D_t^f + \zeta_t (M_t - \eta W_t H_t) \right]. \tag{A.17}$$

The first-order conditions yield

$$\frac{\partial \mathcal{L}}{\partial K_t} : \qquad u_t = \alpha A_t \left(\frac{X_t H_t}{K_t} \right)^{1-\alpha} \tag{A.18}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} : \qquad (1-\alpha) A_t X_t \left(\frac{K_t}{X_t H_t} \right)^{\alpha} = W_t \left[1 + \eta \zeta_t \right] \tag{A.19}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} : \qquad \lambda_t (1 - \zeta_t) = \beta g_t^{-\gamma} \, \mathbb{E}_t \, \lambda_{t+1} \tag{A.20}$$

$$\frac{\partial \mathcal{L}}{\partial B_t^f} : \qquad \lambda_t = \beta (1 + r_t) g_t^{-\gamma} \, \mathbb{E}_t \, \lambda_{t+1}. \tag{A.21}$$

Note that Equation (A.18) already provides the optimality condition for capital (the rental rate equals the marginal product of capital). Dividing Equation (A.20) by Equation (A.21) and solving for ζ_t, I obtain

$$\zeta_t = \frac{r_t}{1+r_t}. \tag{A.22}$$

Plugging this expression into Equation (A.19), I get

$$\underbrace{(1-\alpha)A_t X_t \left(\frac{K_t}{X_t H_t}\right)^{\alpha}}_{\equiv \text{MPL}} = W_t \left[1 + \eta \frac{r_t}{1+r_t}\right]. \tag{A.23}$$

A.4.3 Financial Sector

To close the model, the equilibrium in the financial sector must be specified. The bank's balance sheet is given by the expression

$$\frac{D^h_{t+1} + D^f_{t+1}}{1+r_t} = \frac{D_{t+1}}{1+r_t} + M_t. \tag{A.24}$$

With this, the bank's profits are assumed to be

$$\Pi^b_t = D^h_t + D^f_t - D_t - M_{t-1}. \tag{A.25}$$

Consumers' profits are given by:

$$\Pi_t = \Pi^f_t + \Pi^b_t. \tag{A.26}$$

To close the equilibrium, the economy's resource constraint must be found. For this purpose, consider plugging the households' budget constraint in Equation (A.15) and the firms' budget constraint in Equation (A.16) into the LHS of Equation (28):

$$\frac{D_{t+1}}{1+r_t} + M_t = C_t + S_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g\right)^2 K_t - \Pi_t + \Delta M_t - Y_t + \Pi^f_t \tag{A.27}$$

I use Equation (A.26) together with Equation (A.25) to express the total household profits as follows

$$\Pi_t = \Pi^f_t + D^h_t + D^f_t - D_t - M_{t-1}. \tag{A.28}$$

Plugging this last expression in Equation (A.27), I obtain the resource constraint of the economy

$$\frac{D_{t+1}}{1+r_t} = D_t + C_t + S_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g\right)^2 K_t - Y_t. \tag{A.29}$$

A.4.4 Equilibrium

Denoting with lowercase letters the detrended versions of the variables $c_t = C_t/X_{t-1}$, $k_t = K_t/X_{t-1}$, $W_t = w_t/X_{t-1}$, $i_t = I_t/X_{t-1}$, $d_t = D_t/X_{t-1}$, $s_t = S_t/X_{t-1}$, the resulting equilibrium conditions are as follows

$$\lambda_t = Z_t[c_t - \omega^{-1} H^{\omega}_t]^{-\gamma} \tag{A.30}$$

$$H^{\omega-1}_t = (1-\alpha)A_t g^{1-\alpha}_t \left(\frac{k_t}{H_t}\right)^{\alpha} \left[1 + \eta \frac{r_t}{1+r_t}\right]^{-1} \tag{A.31}$$

$$\lambda_t = \beta g_t^{-\gamma}(1+r_t)\,\mathbb{E}_t\,\lambda_{t+1} \tag{A.32}$$

$$\begin{aligned}\lambda_t\left[1+\phi\left(\frac{k_{t+1}}{k_t}g_t - g\right)\right] = \beta g_t^{-\gamma}\,\mathbb{E}_t\,\lambda_{t+1}\Bigg[1-\delta+\alpha A_{t+1}\left(\frac{h_{t+1}g_{t+1}}{k_t}\right)^{1-\alpha} + \\ \phi\left(\frac{k_{t+2}}{k_{t+1}}g_{t+1}\right)\left(\frac{k_{t+2}}{k_{t+1}}g_{t+1}-g\right) - \frac{\phi}{2}\left(\frac{k_{t+2}}{k_{t+1}}g_{t+1}-g\right)^2\Bigg]\end{aligned} \tag{A.33}$$

$$\frac{d_{t+1}}{1+r_t}g_t = d_t - y_t + c_t + s_t + i_t + \frac{\phi}{2}\left(\frac{k_{t+1}}{k_t}g_t - g\right)^2 k_t \tag{A.34}$$

$$r_t = r^* + \psi\left(e^{d_{t+1}-\bar{d}}\right) + e^{\mu_t - 1} - 1 \tag{A.35}$$

$$k_{t+1}g_t = (1-\delta)k_t + i_t \tag{A.36}$$

$$y_t = A_t k_t^{\alpha}(g_t H_t)^{1-\alpha} \tag{A.37}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim \mathcal{N}(0,\sigma_A^2) \tag{A.38}$$

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_t^Z, \quad \varepsilon_t^Z \sim \mathcal{N}(0,\sigma_Z^2) \tag{A.39}$$

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g, \quad \varepsilon_t^g \sim \mathcal{N}(0,\sigma_g^2) \tag{A.40}$$

$$\ln\left(\frac{s_t}{\bar{s}}\right) = \rho_s \ln\left(\frac{s_{t-1}}{\bar{s}}\right) + \varepsilon_t^s, \quad \varepsilon_t^s \sim \mathcal{N}(0,\sigma_s^2) \tag{A.41}$$

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_t^\mu, \quad \varepsilon_t^\mu \sim \mathcal{N}(0,\sigma_\mu^2) \tag{A.42}$$

A.4.5 Model And Observed Variables

Once the detrended version model is introduced, the mapping between the observed and the model variables can be defined as follows:

$$\underbrace{\begin{bmatrix}(\Delta Y_t)^{data} \\ (\Delta C_t)^{data} \\ (\Delta I_t)^{data} \\ (TB_t/Y_t)^{data}\end{bmatrix}}_{\text{Observed variables}} = \underbrace{\begin{bmatrix}\ln\left(\frac{y_t}{y_{t-1}}g_t\right) \\ \ln\left(\frac{c_t}{c_{t-1}}g_t\right) \\ \ln\left(\frac{i_t}{i_{t-1}}g_t\right) \\ \frac{d_{t-1}-\frac{d_t g_t}{1+r_t}}{y_t}\end{bmatrix}}_{\text{Model variables}} + \underbrace{\begin{bmatrix}\sigma_{\Delta y} \\ \sigma_{\Delta c} \\ \sigma_{\Delta i} \\ \sigma_{\Delta tb/y}\end{bmatrix}}_{\text{Measurement errors}} \tag{A.43}$$