

Academic year 2020/2021

1. A *population counter* is a digital circuit which counts the number of bits of a word that have the value one. Assume that words are 32 bits long. Many solutions are possible. You should seek for solutions that trade a balance between hardware complexity and processing time in two different situations. In the first, the word is available in parallel; in the second, the word is provided bit by bit. For each case, do the following:

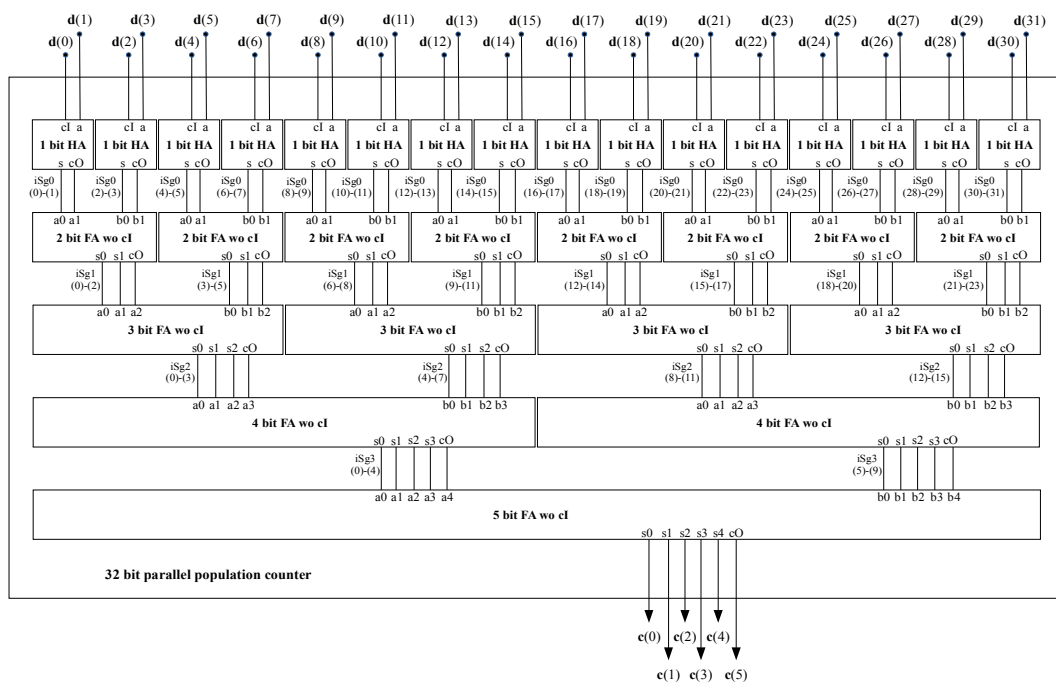
Parallel solution

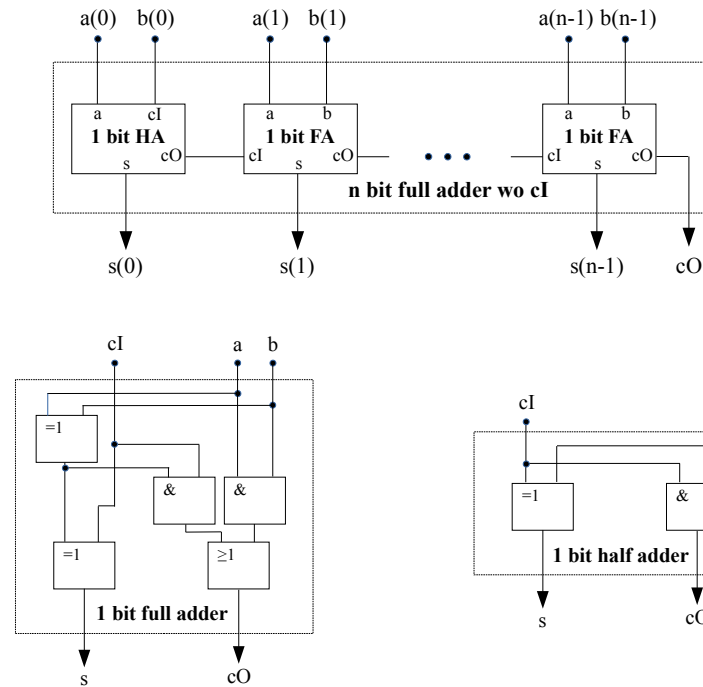
- 1.1. Specify the circuit interface and draw a schematics of its internal organization.

A straightforward solution to the problem arises from the following set of observations

- a 32 bit population counter can be built using two 16 bit population counters plus a 5 bit full adder with *carry in* set to zero
- a 16 bit population counter can be built using two 8 bit population counters plus a 4 bit full adder with *carry in* set to zero
- a 8 bit population counter can be built using two 4 bit population counters plus a 3 bit full adder with *carry in* set to zero
- a 4 bit population counter can be built using two 2 bit population counters plus a 2 bit full adder with *carry in* set to zero
- a 2 bit population counter can be built using a 1 bit half adder.

Hence, resulting in





Implementation cost

16 1-bit half adders + 8 2-bit full adders wo cI + 4 3-bit full adders wo cI + 2 4-bit full adders wo cI + 1 5-bit full adder wo cI

31 1-bit half adders + 26 1 bit full adders

83 *and* gates + 26 *or* gates + 83 *xor* gates

Worst case propagation delay

(it arises from the propagation of the carry out signal at each stage but the first)

stage 0: 1 *xor* gate

stage 1: 1 *xor* gate + 1 *and* gate + 1 *or* gate

stage 2: 1 *xor* gate + 2 *and* gates + 2 *or* gates

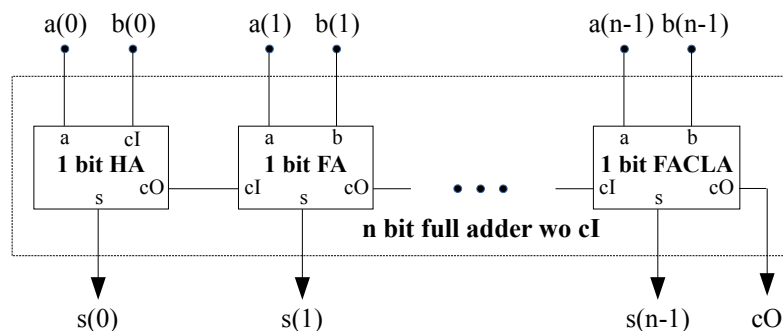
stage 3: 1 *xor* gate + 3 *and* gates + 3 *or* gates

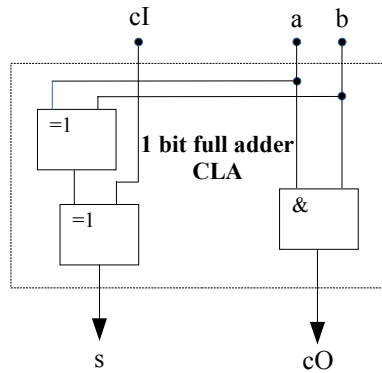
stage 4: 1 *xor* gate + 4 *and* gates + 4 *or* gates

total: 5 *xor* gates + 10 *and* gates + 10 *or* gates

A further observation leads to more simplification: *not all operand values can appear at stages 1 through 5*. In fact, at stage 1, 2 (10, in binary) is the maximum value; at stage 2, 4 (100, in binary) is the maximum value; at stage 3, 8 (1000, in binary) is the maximum value; at stage 4, 16 (10000, in binary) is the maximum value.

Thus, the n-bit full adder wo cI can be modified as





Implementation cost

16 1-bit half adders + 8 2-bit full adders wo cI + 4 3-bit full adders wo cI + 2 4-bit full adders wo cI + 1 5-bit full adder wo cI

31 1-bit half adders + 11 1 bit full adders + 15 1 bit full adders CLA

68 *and* gates + 11 *or* gates + 83 *xor* gates

Worst case propagation delay

(it arises from the propagation of the most significant *s* signal at each stage)

stage 0: 1 *xor* gate

stage 1: 2 *xor* gates

stage 2: 2 *xor* gates + 1 *and* gate + 1 *or* gate

stage 3: 3 *xor* gates + 2 *and* gates + 2 *or* gates

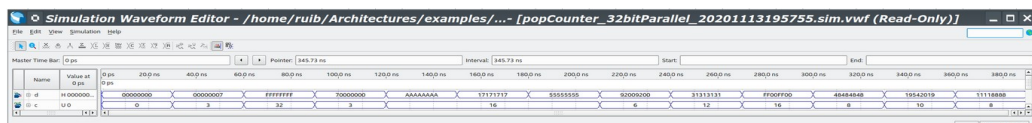
stage 4: 4 *xor* gates + 3 *and* gates + 3 *or* gates

total: 10 *xor* gates + 6 *and* gates + 6 *or* gates

1.2. Write the VHDL code that describes it.

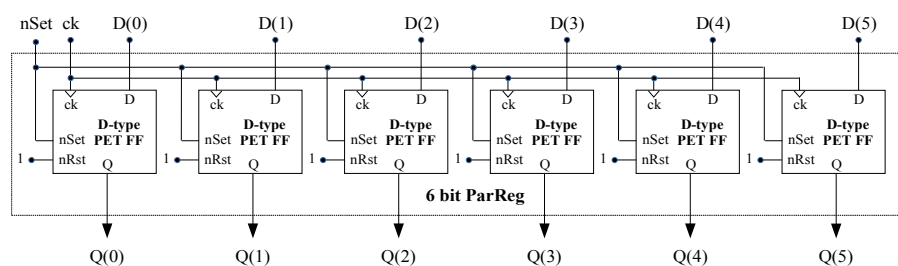
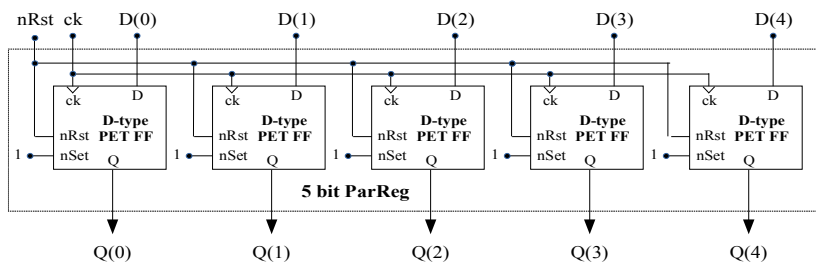
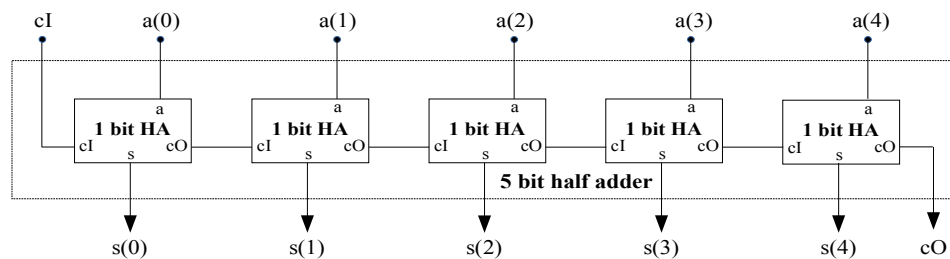
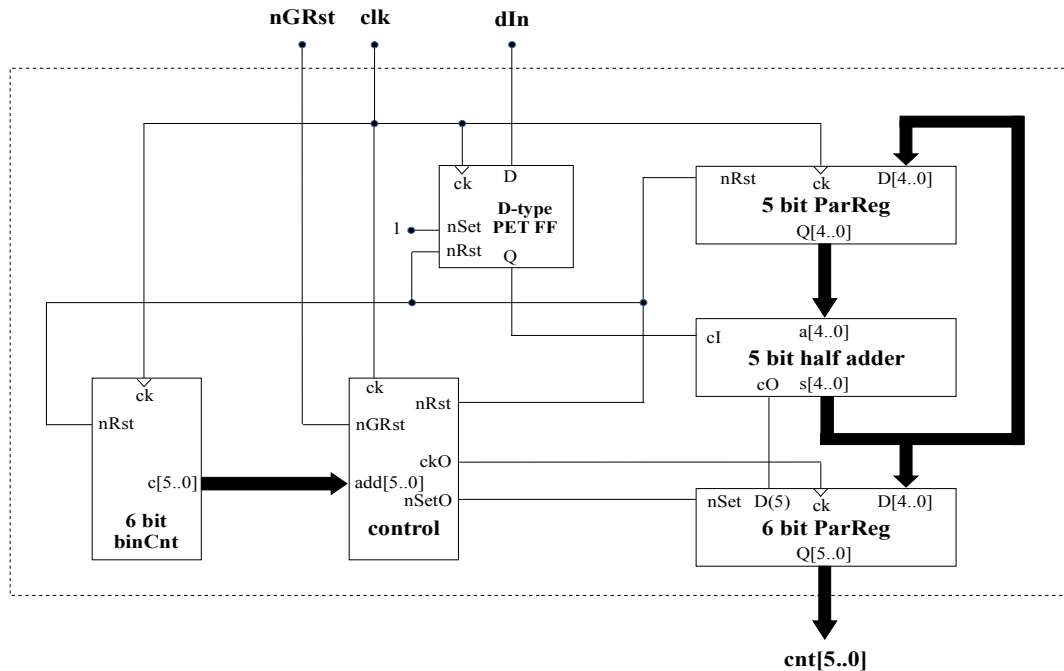
In a separate file.

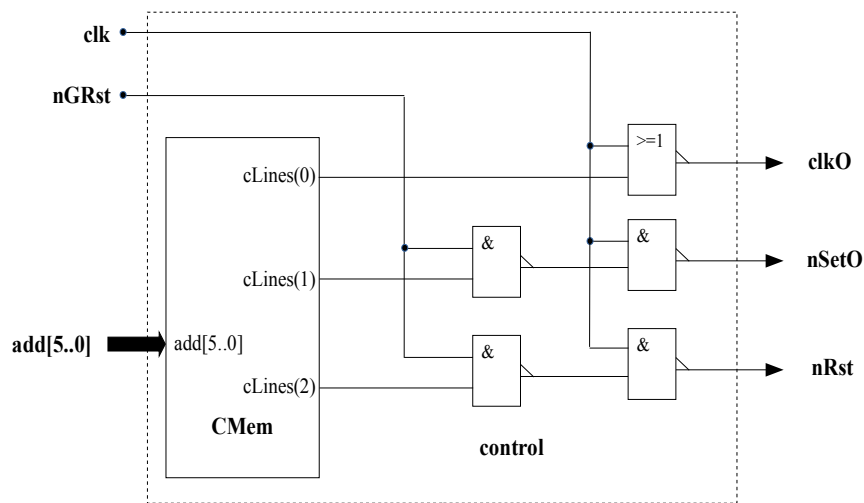
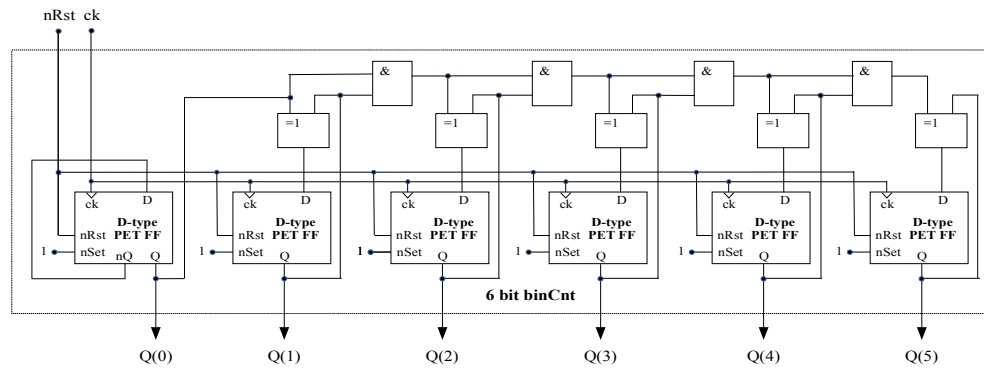
1.3. Create a Quartus project and simulate its operation.



Bit-serial solution

1.1. Specify the circuit interface and draw a schematics of its internal organization.





ROM contents

| add[5..0] | cLines[2..0] |
|-----------|--------------|
| 00 | 7 |
| 01 | 3 |
| 02 | 7 |
| ... | ... |
| 1F | 7 |
| 20 | 6 |
| 21 | 3 |
| 22 | 7 |
| ... | ... |
| 3F | 7 |

Implementation cost

1 D-type flip flop + 1 5-bit parallel register + 1 5-bit half adder + 1 6-bit parallel register + 1 6-bit binary counter + 1 control unit

18 D-type flip flops + 9 *and* gates + 10 *xor* gates + 4 *nand* gates + 1 *nor* gate

1.2. Write the VHDL code that describes it.

In a separate file.

1.3. Create a Quartus project and simulate its operation.

