An Iterated Three-Person Prisoner's Dilemma

1 Introduction

The Iterated Prisoner's Dilemma is an extremely well-researched area of game theory and has been used to model all kinds of evolutionary behaviour. In it, players choose between cooperating with each other, or not (Defecting), risking their reward for a potentially greater one. Conventionally the Prisoner's dilemma is a two-player game. However, it is possible to extend its ideas and qualities to one involving more players. Extending the experiment allows us to explore the effect an extra player has on previously established, successful strategies as well as further evaluate how cooperation, trust and rationality evolve in a larger group.

2 Extending The Game To Three Players

The two-player game is defined with the following standardised payoff matrices taken from here [5]:

$$Player\ 1. = \begin{pmatrix} R & S \\ T & P \end{pmatrix}, \quad Player\ 2. = \begin{pmatrix} R & T \\ S & P \end{pmatrix}. \quad Where, \quad 1. \quad T > R > P > S \quad and \quad 2. \quad 2R > T + S.$$

The given constraints ensure that that: a). defecting is always the dominant choice, and b). the sum of the utilities for both players is greatest when they cooperate (This is necessary for the iterated form as it prevents alternating cooperation and defection giving a greater reward than mutual cooperation). Combined, these two qualities instigate the dilemma that makes the repeated game so interesting to study, together they create a scenario where either player can choose to risk a slight loss of utility for the benefit of the collective.

Moving from two players to three, it is important to maintain these characteristics. But before we analyse the three-player game further, we must first introduce the notation used for representing the payoffs. There are four variations seen in the set of possible outcomes (Figure 1.): All players cooperate, all players defect, a majority cooperate and a majority defect. Since the game is symmetric, this leads to six different obtainable scores:

- Q and P - The scores given to all players for mutual cooperation/defection respectively.
- S and T - The scores given when a majority of players cooperate to the cooperators/defector respectively.
- U and V - The scores given when a majority of players defect to the cooperator/defectors respectively.

Hence payoff matrix can be derived as shown in Figure 2. (Where (S, S, T) implies a payoff of S for players 1 and 2, and a payoff of T for player 3 after players 1 and 2 cooperate and 3 defects).

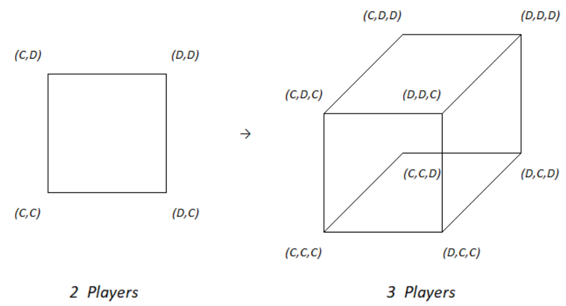

Figure 1: Representations of the two and three-player Prisoner's Dilemma. Where each vertex represents a different possible outcome (i.e. set of potential player actions).

		Player 2.			
		C		D	
		Player 3.		Player 3.	
		C	D	C	D
Player 1.	C	(Q, Q, Q)	(S, S, T)	(S, T, S)	(U, V, V)
	D	(T, S, S)	(V, V, U)	(V, U, V)	(P, P, P)

* Where C and D denote a player cooperating or defecting respectively.

Figure 2: The general pay-off matrix for a three player prisoner's dilemma (The exact values used later in this report are taken from this educational resource [3]).

Deciding on the specific values of Q,P,S,T,U and V simply requires us to modify the constraints mentioned earlier:

a). Defection should remain the dominant choice for each player. In other words, it should always better for a player to defect, regardless what their opponents do. This rule gives these constraints:

  - $T > Q$ (Utilities given when both opposing players cooperate)
  - $P > U$ (Utilities given when both opposing players defect)
  - $V > S$ (Utilities given when one opposing player defects and one cooperates)

b). The sum of utilities should still be maximised when all players cooperate. This gives:

  - $3Q > 2S + T, \quad 3Q > 2U + V \quad \text{and} \quad 3Q > 3P.$

At this point, it is important to note that there is no standard definition for this version of the prisoner's dilemma. These constraints only represent one possible translation of the two-player game.

3 Analysis of the Three-Player Game

Similar to the two-player game, the decision to defect provides a pure strategy Nash equilibrium despite resulting in a worse payoff, this can be proved programmatically or via induction. Essentially, this means that if all players chose to defect no one player can gain from changing their strategy, which is as a result of the constraints defining defection as the dominant strategy.

Interest in the iterated prisoner's dilemma was fueled by Robert Axelrod and his book The Evolution of Cooperation linked here [1]. His book details a tournament examining the iterated prisoner's dilemma between various strategies submitted by his colleagues. The winning strategy from his first tournament was named 'Tit For Tat', this strategy simply cooperated on the first turn and then copied whatever its opponent did on the previous turn. Emulating this algorithm with 3 players does pose some questions, for example, would it need both its opposing players to defect before it stopped cooperating? The method chosen for this report defines three-player Tit For Tat as the strategy of cooperating until *either* of its opponents defects. However, in the three-player environment tit for tat did not do quite so well (Figure 3.).

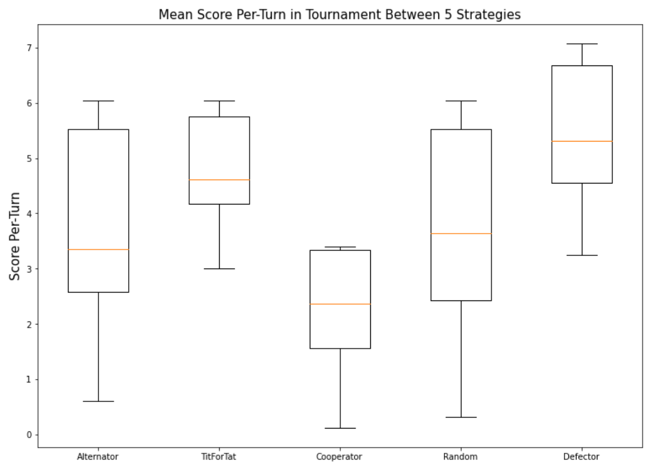

Figure 3: Results from an iterated 3-player Prisoner's Dilemma Tournament. Further details of the tournament format and strategies used can be found in the relevant repository here [4]. The code used is based on material from the Axelrod python library, shown here [2].

There could be many reasons for this, immediately it can be seen that the setting itself is notably different. For one, the round-robin tournament style is not strictly possible with more than two players. Instead, the tournament utilised a format where every suitable permutation of the strategy set played each other in an N-round match. Secondly, it did not contain as many strategies as Axelrod's original study and the ones that were included were not as sophisticated as many of his. In this particular environment, this may have allowed constant defection to take advantage of potentially weaker opponents. There are also the inherent differences in the three-player variant, in this game there is a greater opportunity to gain a larger score when defecting and less reliance on trusting a singular opponent.

In any case, this report offers a starting point to explore the properties of a three-player Prisoner's Dilemma more extensively. There is potential to modify other strategies featured in the Axelrod library and evaluate their effectiveness in a more extensive tournament. Beyond that, there is also the possibility of investigating other features such as noise, evolving strategies, or experimenting with Moran processes.

References

[1] Robert Axelrod. *The Evolution of Cooperation*. Basic Books, 1984. ISBN: 0-465-00564-0.
[2] Multiple Authors. *Axelrod*. URL: https://github.com/Axelrod-Python/Axelrod.
[3] Michael J. O'Donnell. *The Three-Player Prisoner's Dilemma*. URL: https://www.classes.cs.uchicago.edu/archive/1998/fall/CS105/Project/node6.html.
[4] Oscar Rose. *Game Theory*. URL: https://github.com/oscarhrose/Game-Theory.
[5] Wikipedia. *Prisoner's dilemma*. URL: https://en.wikipedia.org/wiki/Prisoner%27s_dilemma.