Statistics 516 Homework 03

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a. Code and output:

#building saturated model

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

options(digits=4)

Impact on Pesticides on Skylark Reproductivity

1.

```
skylark$batch<-factor(1:nrow(skylark))
         m.sat <- glm(count ~ batch, family = "poisson", data = skylark)
         m.pos<- glm(count ~ spray +field + year ,family = "poisson", data = skylark)
         summary(m.pos)
Call:
glm(formula = count ~ spray + field + year, family = "poisson",
  data = skylark
Deviance Residuals:
         10 Median
                        3Q
                              Max
-1.8682 -0.7351 0.0244 0.6697 1.8797
Coefficients:
      Estimate Std. Error z value Pr(>|z|)
(Intercept) 3.43094
                    0.13262 25.87 < 2e-16 ***
sprayyes -0.45613 0.09385 -4.86 1.2e-06 ***
fieldKr
         0.04909 0.12672 0.39 0.6985
fieldKu
         0.00496 0.12800 0.04 0.9691
fieldRd
        -0.17905 0.13417 -1.33 0.1820
year1993
           0.46262
                     0.13064 3.54 0.0004 ***
           0.06002
                     0.14149 0.42 0.6714
year1994
year1995
           0.32728
                    0.13411 2.44 0.0147 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for poisson family taken to be 1)
  Null deviance: 69.960 on 15 degrees of freedom
Residual deviance: 18.984 on 8 degrees of freedom
AIC: 118.3
Number of Fisher Scoring iterations: 4
         anova(m.pos,m.sat,test = "LRT")
Analysis of Deviance Table
Model 1: count ~ spray + field + year
Model 2: count ~ batch
Resid. Df Resid. Dev Df Deviance Pr(>Chi)
      8
            19
      0
             0 8
                    19 0.015 *
```

plot(predict(m.pos),rstudent(m.pos))

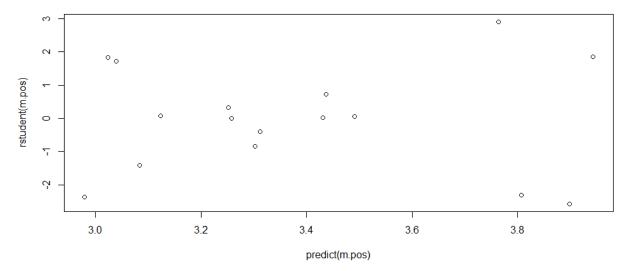


Figure 1 The residuals with different prediction value of the poisson model. m.quasipos <- glm(count \sim spray + field + year, family = quasipoisson, data = skylark)

plot(predict(m.quasipos),rstudent(m.quasipos))

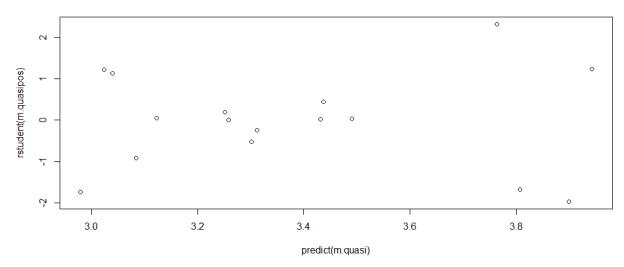


Figure 2 The residuals with different prediction value of the quasi-poisson model. summary(m.quasi)\$coefficients

```
Estimate Std. Error t value Pr(>|t|)
                       0.2029 16.91025 1.517e-07
(Intercept) 3.430943
sprayyes
          -0.456126
                       0.1436 -3.17687 1.306e-02
fieldKr
         0.049089
                     0.1939 0.25322 8.065e-01
fieldKu
          0.004964
                     0.1958 0.02535 9.804e-01
fieldRd
         -0.179048
                      0.2053 -0.87233 4.084e-01
year1993
            0.462623
                       0.1999 2.31467 4.933e-02
year1994
           0.060018
                       0.2164 0.27728 7.886e-01
                       0.2052 1.59520 1.493e-01
year1995
           0.327281
```

b. Discussion:

When using the deviance of the residuals to test the Poisson regression model, the p-value was 0.015, which was lower than the α value we usually use (0.05). This means the deviance is not likely to happen under the null hypothesis ($D\!=\!2\big\lceil\log L_s\!-\!logL\big\rceil\!=\!0)$ is true. Where L and Ls are the likelihoods of the Poisson regression model and a saturated model. When plotting the Figure 1, the residuals with different prediction value of the Poisson model, there are some residuals higher than 2 or lower than -2. This also shows the evident of overdispersion

When I apply the quasi-Poisson regression model and plot the Figure 2, The residuals with different prediction value of the quasi-Poisson model, almost all the residuals are within -2 and 2, which means the model has no overdispersion.

```
\begin{split} E\big(Y_i\big) &= \exp\big(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \beta_6 x_{i6} + \beta_7 x_{i7}\big) \\ x_{i1} &= 1 \text{ if the pesticide is applied at } i - \text{th sample } ; x_{i1} = 0 \text{ other wise} \\ x_{i2} &= 1 \text{ if the } i - \text{th sample is} \in \text{field Kr}; x_{i2} = 0 \text{ other wise} \\ x_{i3} &= 1 \text{ if the } i - \text{th sample is} \in \text{field Ku}; x_{i3} = 0 \text{ other wise} \\ x_{i4} &= 1 \text{ if the } i - \text{th sample is} \in \text{field Rd}; x_{i4} = 0 \text{ other wise} \\ x_{i5} &= 1 \text{ if the } i - \text{th sample is at year 1993}; x_{i5} = 0 \text{ other wise} \\ x_{i6} &= 1 \text{ if the } i - \text{th sample is at year 1994}; x_{i6} = 0 \text{ other wise} \\ x_{i7} &= 1 \text{ if the } i - \text{th sample is at year 1995}; x_{i7} = 0 \text{ other wise} \end{split}
```

If the pesticide was not applied, the expected number of the skylark fledglings will be $\exp\left(\beta_0+\beta_2x_{i2}+\beta_3x_{i3}+\beta_4x_{i4}+\beta_5x_{i5}+\beta_6x_{i6}+\beta_7x_{i7}\right)$, and if the pesticide was applied, the expected number of the skylark fledglings will be $\exp\left(\beta_0-0.456126+\beta_2x_{i2}+\beta_3x_{i3}+\beta_4x_{i4}+\beta_5x_{i5}+\beta_6x_{i6}+\beta_7x_{i7}\right)$. Which means when the pesticide was applied, the expected number of the skylark fledglings will decrease by a factor of $\exp(-0.456126)=0.6337$, which also means the number will decrease 36.63%.

Moth Coloration and Natural Selection

1.

```
estimate
      2.5222 0.2823 1.4505 4.386 3.277 Inf 0.001048
Dark
Light 0.6286 0.2894 0.3564 1.108 -1.604 Inf 0.108629
            #Plot
            mydata dark <- data.frame(Distance = seq(0, 50, by = 0.5), Morph = "dark")
            mydata dark$logo<- predict(m, newdata = mydata dark)
            mydata dark$odds <- exp(mydata dark$logo)
            mydata light <- data.frame(Distance = seq(0, 50, by = 0.5), Morph = "light")
            mydata light$logo<- predict(m, newdata = mydata light)
            mydata light$odds <- exp(mydata light$logo)
            p<-ggplot(case2102, aes(x=Distance,y=Removed/(Placed-Removed)))
            p<-p+geom point(aes(fill=Morph),shape=21)
            p<-p+scale fill manual(values=c("black","white"))</pre>
            p<-p+ylab("Odds of Removed Moths")
            p<-p+xlab("Distance from Liverpool (km)")
            p < -p + geom line(aes(y = odds), data = mydata dark)
            p < -p + geom line(aes(y = odds), data = mydata light, linetype=2)
            plot(p)
```

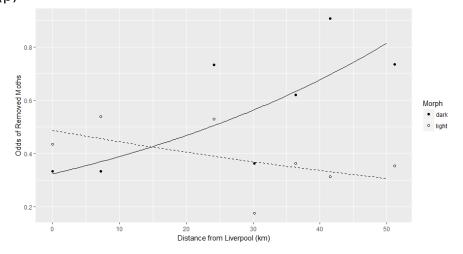


Figure 3 The odds of moths being removed with the different distance from Liverpool. The solid line represents the prediction curve of the dark moths, and the dash line indicates the prediction curve of the light moths.

b. Discussion:

Odds ratio for dark moth: 2.5222; Odds ratio for light moth: 0.6286

When the distance increased by 50 km, the odds that a dark moth will be removed changed by a factor of 2.5222, and the odds that a light moth will be removed changed by a factor of 0.6286.

These numbers agree with the Figure 3. The odds of dark moths being removed at 0 and 50 km are about 0.32 and 0.81, and the odds ratio is 0.81/0.32 =2.53 \approx 2.5222. The odds of light moths being removed at 0 and 50 km are about 0.48 and 0.30, and the odds ratio is 0.30/0.48 =0.625 \approx 0.6286.

2.

a. Code and output:

#margeff 50-0

```
margeff(m,
    a = list(Morph = c("dark","light"), Distance = 50),
    b = list(Morph = c("dark","light"), Distance = 0),
    cnames = c("Dark","Light"))
```

```
estimate se lower upper tvalue df pvalue
Dark 0.20486 0.05927 0.08869 0.32103 3.456 Inf 0.0005479
Light -0.09321 0.05867 -0.20820 0.02177 -1.589 Inf 0.1120949
```

#margeff 50-0 percent

```
margeff(m,
    a = list(Morph = c("dark","light"), Distance = 50),
    b = list(Morph = c("dark","light"), Distance = 0),
    cnames = c("Dark","Light"),type="percent")
```

```
estimate se lower upper tvalue df pvalue
Dark 83.84 35.29 14.68 153.0026 2.376 Inf 0.01751
Light -28.43 14.80 -57.43 0.5781 -1.921 Inf 0.05474
```

#margeff at 25

```
margeff(m, delta = 0.001,
    a = list(Distance = 25 + 0.001, Morph = c("dark","light")),
    b = list(Distance = 25, Morph = c("dark","light")),
    cnames = c("Dark","Light"))
```

```
estimate se lower upper tvalue df pvalue
Dark 0.004148 0.001229 0.001738 0.0065575 3.374 Inf 0.0007421
Light -0.001868 0.001180 -0.004181 0.0004452 -1.583 Inf 0.1134827
```

#margeff at dark - light at 0 and 50

```
margeff(m,
a = list(Morph = "dark", Distance = c(0,50)),
b = list(Morph = "light", Distance = c(0,50)),
```

```
cnames = c("0","50")
```

```
estimate se lower upper tvalue df pvalue
0 -0.08354 0.05561 -0.1925 0.02545 -1.502 Inf 1.330e-01
50 0.21453 0.04654 0.1233 0.30575 4.609 Inf 4.038e-06
```

#plot

```
mydata_dark <- data.frame(Distance = seq(0, 50, by = 0.1),Morph = "dark")
mydata_light <- data.frame(Distance = seq(0, 50, by = 0.1),Morph = "light")
mydata_dark$prob <- predict(m, newdata = mydata_dark,type="response")
mydata_light$prob <- predict(m, newdata = mydata_light,type = "response")

p<-ggplot(case2102, aes(x=Distance,y=Removed/Placed))
p<-p+geom_point(aes(fill=Morph),shape=21)
p<-p+scale_fill_manual(values=c("black","white"))
p<-p+ylab("Probability of Removed Moths")
p<-p+xlab("Distance from Liverpool (km)")
p<-p + geom_line(aes(y = prob), data = mydata_dark)
p<-p + geom_line(aes(y = prob), data = mydata_light, linetype = 2)
plot(p)
```

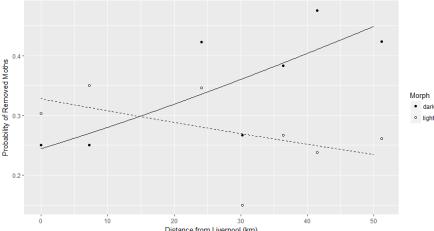


Figure 4 The probability of moths being removed with the different distance from Liverpool. The solid line represents the prediction curve of the dark moths, and the dash line indicates the prediction curve of the light moths.

b. Discussion:

The discrete marginal effects of increasing distance from 0 to 50 km on the probability of dark and light moth removal are 0.2048 and -0.0932, which means when the distance increased from 0 to 50 km, the probability of the dark and light moth being removed will increased 0.2048 (increase 83.84%) and decreased 0.0932 (decrease 28.43%).

The estimated instantaneous rate of change in the probability of dark and light moth removal at 25 km are 0.004148 and -0.001868, which means at 25 km the slope

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of the prediction curve of dark and light moths being removed are 0.4148% per km and 0.1868% per km.

The discrete marginal effects for the difference in the probability of moth removal at 0 and 50 km are -0.08354 and 0.21453 (dark moth – light moth). Which means the dark moth has 8.354% lower probability to be removed at 0km, and 21.45% higher probability to be removed at 50 km. All the numbers obtained agree with Figure 3.

Cancer Death Rate of Atomic Bomb Survivors

1.

a. Code and output:

data\$batch<-factor(1:nrow(data))
m.sat <- glm(Deaths ~ batch, family = "poisson", data = data)
m<-glm(Deaths ~ YearsAfter+Exposure,offset = log(AtRisk), family = poisson, data
= data)
plot(predict(m),rstudent(m))</pre>

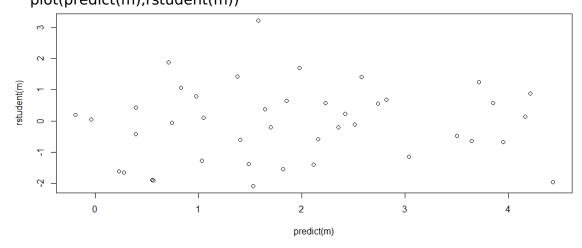


Figure 5 The residuals with different prediction value of the poisson model. anova(m,m.sat,test = "LRT")

```
Model 1: Deaths ~ YearsAfter + Exposure

Model 2: Deaths ~ batch

Resid. Df Resid. Dev Df Deviance Pr(>Chi)

1 34 50.1

2 0 0.0 34 50.1 0.037 *
```

m.quasi<-glm(Deaths ~ YearsAfter+Exposure, offset=log(AtRisk), family =
quasipoisson, data = data)
plot(predict(m.quasi),rstudent(m.quasi))</pre>

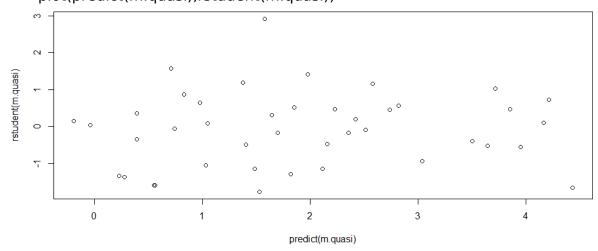


Figure 6 The residuals with different prediction value of the quasi-poisson model.

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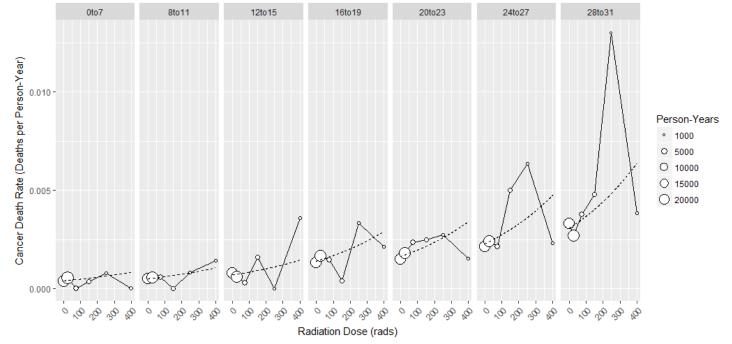


Figure 7 The cancer death rate with the different radiation dose and years after exposure. The dash line indicates the prediction curve of the quasi-poisson model.

contrast(m.quasi,

```
a = list(YearsAfter = "0to7", Exposure = 100, AtRisk = 1),
b = list(YearsAfter = "0to7", Exposure = 0, AtRisk = 1), tf = exp)
```

```
estimate se lower upper tvalue df pvalue 1.201 0.05236 1.08 1.336 3.498 34 0.001328
```

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```
exp(summary(m.quasi)$coefficients)
```

```
Estimate Std. Error t value Pr(>|t|)
(Intercept)
            0.0004017
                          1.250 5.726e-16
                                          1.000
YearsAfter8to11 1.2627945
                            1.352 2.169e+00 1.559
YearsAfter12to15 1.7361996
                             1.327 7.040e+00
                                              1.061
YearsAfter16to19 3.4842188
                             1.289 1.356e+02
                                              1.000
YearsAfter20to23 4.0709552
                             1.284 2.724e+02
                                              1.000
YearsAfter24to27 5.6783267
                             1.275 1.272e+03
                                               1.000
YearsAfter28to31 7.6228003
                             1.269 5.066e+03
                                               1.000
Exposure
             1.0018333
                          1.001 3.305e+01
                                            1.001
            1.0018333^100
```

[1] 1.201

b. Discussion:

The p-value of the deviance of residuals test on the Poisson regression model was 0.37, which was close to α value we usually use (0.05), and the Figure 5 also shows most of the residuals are in the range of -2 and 2, which means the model has a little bit over dispersion. When applying the quasi-Poisson regression model, Figure 6 shows the range of the residuals decreased (no significant different from the Poisson model).

Since the year after exposing does not interact with the dose, I only used the data set with 0 to 7 years after exposure in the contrast function to predict the dose effects on the death rate. The estimated value from the contrast function was 1.201, which means when the dose increase 100 rad, the death rate will increase by a factor of 1.201 (increased 20.1%). We can also calculate this number from the exp(summary(m.quasi)\$coefficients) function. The estimated increasing of death rate while the exposure increased 1 rad was 1.0018333. the effect of increasing 100 rad will be $1.0018333^{100} = 1.201$, which agrees with the data from the contrast function.

Producing Embryogenic Anthers

1.

a. Code and output:

 $m2 <- glm(y/n \sim force * storage,data = anthers, family = tweedie(link.power = -3, var.power = 2))$ summary(m2)

```
Call:
glm(formula = y/n \sim force * storage, family = tweedie(link.power = -3,
  var.power = 2), data = anthers)
Deviance Residuals:
0.00757 -0.01205 0.00440 0.01179 -0.02658 0.01444
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                2.30025 0.23309 9.87 0.010 *
(Intercept)
              force
                 4.17706 0.54622 7.65 0.017 *
storagecontrol
force:storagecontrol -0.00964 0.00278 -3.47 0.074.
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for Tweedie family taken to be 0.000633)
  Null deviance: 0.0818936 on 5 degrees of freedom
Residual deviance: 0.0012762 on 2 degrees of freedom
AIC: NA
Number of Fisher Scoring iterations: 3
```

plot(predict(m2),rstudent(m2))

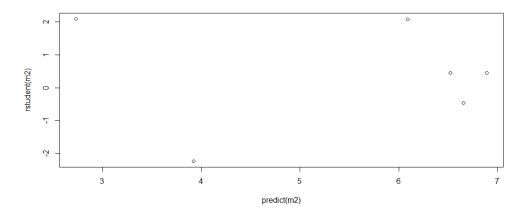


Figure 8 The residuals with different prediction value of the tweedie model. d <- expand.grid(force = seq(40,350, length = 100), storage = c("cold", "control"), n=1)

d\$yhat<-predict(m2,newdata = d, type = "response")
p<-ggplot(anthers, aes(x=force,y=y/n))
p<-p+geom_line(aes(y=yhat, group=storage), data =d)
p<-p+geom_point(aes(size = n, fill = storage), shape = 21)
p<-p+scale_fill_manual(values = c("white",grey(0.75)))
p<-p+scale_size(breaks=c(50,150,250,350))
p<-p+labs(y="Expected Proportion of \n Embryogenic Anthers", x="Force (g)", size = "Number of \nAnthers", fill = "Storage \n Condition")</pre>

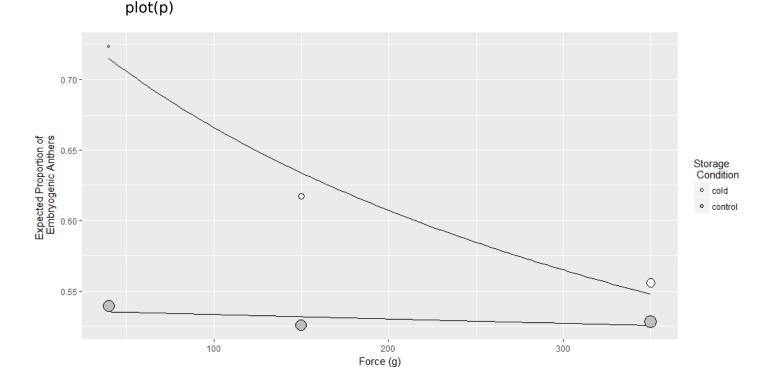


Figure 9 The expected proportion of embryogenic anthers with different storage condition, g force.

b. Discussion:

The tweedie model was used to fit the data, and Figure 8 shows there was no overdispersion in the model. The mean structure of the tweedie model is: $g[E(Y_i)] = \eta_i = E(Y_i)^{-3}$, hence $\eta_i^{-1/3} = E(Y_i)$.

$$E(Y_i) = (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2})^{-1/3}$$

$$x_{i1} = g \text{ force}$$

$$x_{i2} = 1 \text{ if the storage condition of the } i - th \text{ fruitfly was} \in \text{ con trol} \quad \text{group } ; x_{2i} = 0 \text{ other wise}$$

$$E(Y_i) = (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2})^{\frac{-1}{3}}$$
 if the storage condition of the i -th fruitfly was \in control group

$$E(Y_i) = (\beta_0 + \beta_1 x_{i1})^{\frac{-1}{3}}$$

if the storage condition of the i-th fruitfly was \in cold group

The effect of force on the expected proportion for "control" group at force = f was:

$$\frac{d}{df}(2.300+0.011*f+4.177+(-0.009)*f)^{\frac{-1}{3}} = \frac{-0.02(0.002f+6.477)^{\frac{-4}{3}}}{3}$$

The effect of force on the expected proportion for "cold" group at force = f was:

$$\frac{d}{df}(2.30+0.011*f)^{\frac{-1}{3}} = \frac{-0.011(0.011f+2.3)^{\frac{-4}{3}}}{3}$$

The storage condition effects on the sample at 40g was:

$$\frac{E_{cold}(Y_i)}{E_{control}(Y_i)} = \left(\frac{2.300 + 0.011 * 40}{2.300 + 0.011 * 40 + 4.177 + (-0.009) * 40}\right)^{\frac{-1}{3}} = 1.3376$$

When stored the sample at cold condition with 40 g, the expected proportion of embryogenic anthers will increase by a factor of 1.3376 (33.76% increasing)

The storage condition effects on the sample at 150g was:

$$\frac{E_{cold}(Y_i)}{E_{control}(Y_i)} = \left(\frac{2.300 + 0.011 * 150}{2.300 + 0.011 * 150 + 4.177 + (-0.009) * 150}\right)^{\frac{-1}{3}} = 1.1971$$

When stored the sample at cold condition with 150 g, the expected proportion of embryogenic anthers will increase by a factor of 1.1971 (19.71% increasing)

The storage condition effects on the sample at 350g was:

$$\frac{E_{cold}(Y_i)}{E_{control}(Y_i)} = \left(\frac{2.300 + 0.011 * 350}{2.300 + 0.011 * 350 + 4.177 + (-0.009) * 350}\right)^{\frac{-1}{3}} = 1.0528$$

When stored the sample at cold condition with 350 g, the expected proportion of embryogenic anthers will increase by a factor of 1.0528 (5.28% increasing)