Statistics 516
Homework 01

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Date: 2017/02/14

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2017/02/14

Fruit Flies_Backgroung

- **Code:**

```
library(faraway)
library(ggplot2)

fruitfly$activity <- factor(fruitfly$activity, levels = c("isolated", "one", "many",
"low", "high")) #reorder the levels of activity
p<-ggplot(fruitfly, aes(x=activity, y=longevity))+geom_boxplot()
p<-p+geom_jitter(height=0, width=0.1, alpha=0.5)
p<-p+xlab("Sexual Activity Condition") + ylab("Longevity (days)")
plot(p)

p<-ggplot(fruitfly, aes(x=thorax, y= longevity))
p<-p+xlab("Thorax Length (mm)") + ylab("Longevity (days)")
p<-p+facet_wrap(~activity, nrow=1) + geom_point(alpha=0.5)
plot(p)
```

- **Output:**

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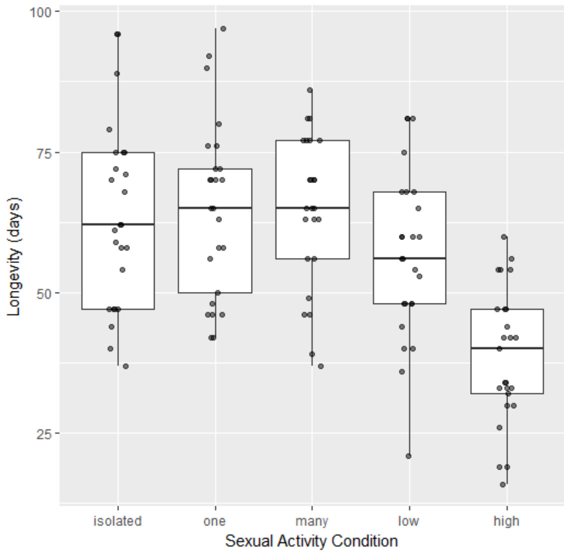

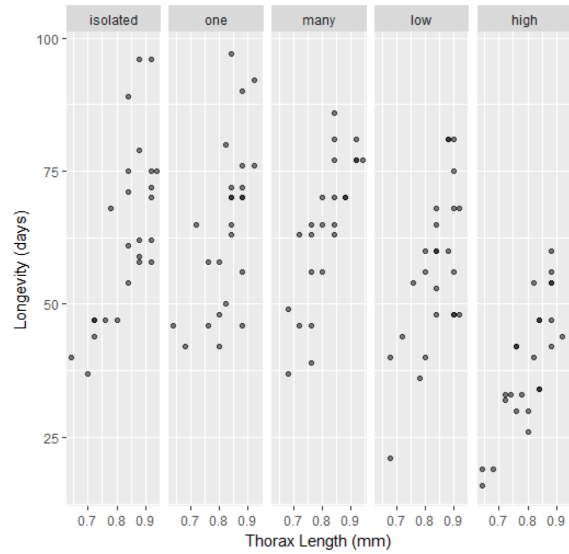

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Fruit Flies_1

- **Code:**

```
library(faraway)
m<-lm(longevity ~ activity, data = fruitfly)
options(digits=4) #adjust the outpput decimal points
cbind(summary(m)$coefficient,confint(m))
```

- **Output:**

```
             Estimate Std. Error t value  Pr(>|t|)   2.5 %  97.5 %
(Intercept)   63.5600      2.926 21.7202 3.213e-43  57.766  69.354
activityone    1.2400      4.138  0.2996 7.650e-01  -6.954   9.434
activitymany   0.9817      4.181  0.2348 8.148e-01  -7.298   9.261
activitylow   -6.8000      4.138 -1.6431 1.030e-01 -14.994   1.394
activityhigh -24.8400      4.138 -6.0023 2.161e-08 -33.034 -16.646
```

- **Discussion:**

$\hat{\beta}_0 \approx 63.56\,; Standard\ error = 2.926\,; Confident\ level\,(\alpha = 0.05) : 57.766 \text{¿} 69.354$

$\hat{\beta}_1 \approx 1.24\,; Standard\ error = 4.138\,; Confident\ level\,(\alpha = 0.05) : -6.954 \text{¿} 9.434$

$\hat{\beta}_2 \approx 0.98\,; Standard\ error = 4.181\,; Confident\ level\,(\alpha = 0.05) : -7.298 \text{¿} 9.261$

$\hat{\beta}_3 \approx -6.80\,; Standard\ error = 4.138\,; Confident\ level\,(\alpha = 0.05) : -14.994 \text{¿} 1.394$

$\hat{\beta}_4 \approx -24.84\,; Standard\ error = 4.138\,; Confident\ level\,(\alpha = 0.05) : -33.034 \text{¿} -16.646$

Fruit Flies_2

$E(Y_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4}$

$x_{i1} = 1\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ one\ \ group\,; x_{1i} = 0\, other\ wise$

$x_{i2} = 1\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ many\ \ group\,; x_{2i} = 0\, other\ wise$

$x_{i3} = 1\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ low\ \ group\,; x_{3i} = 0\, other\ wise$

$x_{i4} = 1\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ high\ \ group\,; x_{4i} = 0\, other\ wise$

$E(Y_i) = \beta_0\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{isolated}$

$E(Y_i) = \beta_0 + \beta_1\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{one}$

$E(Y_i) = \beta_0 + \beta_2\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{many}$

$E(Y_i) = \beta_0 + \beta_3\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{low}$

$E(Y_i) = \beta_0 + \beta_4\, if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{high}$

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Fruit Flies_3

- **Code:**

```
library(faraway)
library(contrast)
m<-lm(longevity ~ activity, data = fruitfly)
contrast(m,
        a = list(activity = c("isolated", "one", "many", "low", "high")),
        cnames=c("isolated", "one", "many", "low", "high"))
```

- **Output:**

```
lm model parameter contrast

         Contrast  S.E. Lower Upper     t  df Pr(>|t|)
isolated    63.56 2.926 57.77 69.35 21.72 119        0
one         64.80 2.926 59.01 70.59 22.14 119        0
many        64.54 2.987 58.63 70.46 21.61 119        0
low         56.76 2.926 50.97 62.55 19.40 119        0
high        38.72 2.926 32.93 44.51 13.23 119        0
```

- **Discussion:**

Expected longevity of fruit flies with sexual condition “isolated”: 63.56
Expected longevity of fruit flies with sexual condition “one”: 64.80
Expected longevity of fruit flies with sexual condition “many”: 64.54
Expected longevity of fruit flies with sexual condition “low”: 56.76
Expected longevity of fruit flies with sexual condition “high”: 38.72

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Fruit Flies_4

- **Code:**

```
library(faraway)
library(contrast)
m<-lm(longevity ~ activity, data = fruitfly)
contrast(m,
         a=list(activity=c("one","many","low","high")),
         b=list(activity="isolated"),
         cnames=c("one", "many", "low", "high"))
cbind(summary(m)$coefficient,confint(m))
```

- **Output:**

```
lm model parameter contrast

     Contrast  S.E.   Lower   Upper     t  df Pr(>|t|)
one    1.2400 4.138  -6.954   9.434  0.30 119   0.7650
many   0.9817 4.181  -7.298   9.261  0.23 119   0.8148
low   -6.8000 4.138 -14.994   1.394 -1.64 119   0.1030
high -24.8400 4.138 -33.034 -16.646 -6.00 119   0.0000

             Estimate Std. Error t value  Pr(>|t|)   2.5 %  97.5 %
(Intercept)   63.5600      2.926 21.7202 3.213e-43  57.766  69.354
activityone    1.2400      4.138  0.2996 7.650e-01  -6.954   9.434
activitymany   0.9817      4.181  0.2348 8.148e-01  -7.298   9.261
activitylow   -6.8000      4.138 -1.6431 1.030e-01 -14.994   1.394
activityhigh -24.8400      4.138 -6.0023 2.161e-08 -33.034 -16.646
```

- **Discussion:**

The estimated longevity, standard error, t-value, and confident interval difference between the fruit flies with “isolated” sexual condition with other four sexual conditions from “contrast” function were:

Table 1 The estimated longevity, standard error, t-value, and confident interval of the fruit flies with four sexual condition compare with the “isolated” sexual condition from “contrast” function

Sexual condition	Estimated Longevity	Standard Error	t-value	Confident interval
One	1.2400	4.138	0.2996	-6.954 to 9.434
Many	0.9817	4.181	0.2348	-7.298 to 9.261
Low	-6.8000	4.138	-1.6431	-14.994 to 1.394
High	-24.8400	4.138	-6.0023	-33.034 to -16.646

From the “summary” function, β_1=1.2400; β_2=0.9817; β_3=-6.8000; β_4=-24.8400. These β_j parameters equal to the estimated value from the “contrast” function. (β_1=estimated longevity of the fruit flies with sexual condition: one; β_2=estimated longevity of the fruit flies with sexual condition: many; β_3=estimated longevity of the fruit flies with sexual condition: low; β_4=estimated longevity of the fruit flies with sexual condition: high)

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Fruit Flies_5

- **Code:**

```
library(faraway)
library(contrast)
m<-lm(longevity ~ activity, data = fruitfly)
#Estimate the difference in expected longevity between the two conditions with one female fruit fly
contrast(m,
        a=list(activity="one"),
        b=list(activity="low"),
        cnames="one, pregnant - virgin")
#Estimate the difference in expected longevity between the two conditions with eight female fruit flies
contrast(m,
        a=list(activity="many"),
        b=list(activity="high"),
        cnames="eight, pregnant - virgin")
```

- **Output:**

```
                         Contrast  S.E.   Lower Upper    t  df Pr(>|t|)
one, pregnant - virgin     8.04 4.138 -0.1545 16.23 1.94 119   0.0544

                          Contrast  S.E. Lower Upper    t  df Pr(>|t|)
eight, pregnant - virgin    25.82 4.181 17.54  34.1 6.18 119        0
```

- **Discussion:**

The p-value of the test on one female fruit fly was 0.0544, which was higher than the default α value (0.05). This means the test fail to reject the null hypothesis that the pregnant status did not affects the longevity.

The p-value of the test on eight female fruit fly was 0, which was lower than the default α value (0.05). This means the test reject the null hypothesis that the pregnant status did not affects the longevity.

The confident interval of both test agreed with the results. The confident interval (while α = 0.05) for the first test was -0.1545 to 16.23, which included the 0 point. This meant there was no significant difference between the two tested group. The confident interval (while α = 0.05) of the second test was 17.54 to 34.1, which did not include the 0 point. This shows the two tested group in the second test had significance difference.

The tests show that the sexual activity does not has effect on longevity of the male fruit fly while there was only one female fly. However, the sexual activity has effect on longevity of the male fruit fly while there were eight female flies.

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Fruit Flies_6

- **Code:**

```
library(faraway)
m<-lm(longevity ~ activity + thorax, data = fruitfly)
cbind(summary(m)$coefficient,confint(m))
```

- **Output:**

```
             Estimate Std. Error t value  Pr(>|t|)   2.5 %  97.5 %
(Intercept)   -48.749     10.850 -4.4930 1.649e-05 -70.235 -27.263
activityone     2.637      2.984  0.8838 3.786e-01  -3.272   8.546
activitymany    4.139      3.027  1.3674 1.741e-01  -1.855  10.132
activitylow    -7.015      2.981 -2.3532 2.027e-02 -12.918  -1.112
activityhigh  -20.004      3.016 -6.6325 1.048e-09 -25.976 -14.031
thorax        134.341     12.731 10.5522 9.773e-19 109.130 159.553
```

- **Discussion:**

$\hat{\beta}_0 \approx -48.749\,;\, Standard\ error = 10.850\,;\, Confident\ level\ (\alpha = 0.05): -70.235 ¿ -27.263$

$\hat{\beta}_1 \approx 2.637\,;\, Standard\ error = 2.984\,;\, Confident\ level\ (\alpha = 0.05): -3.272 ¿ 8.546$

$\hat{\beta}_2 \approx 4.139\,;\, Standard\ error = 3.027\,;\, Confident\ level\ (\alpha = 0.05): -1.855 ¿ 10.132$

$\hat{\beta}_3 \approx -7.015\,;\, Standard\ error = 2.981\,;\, Confident\ level\ (\alpha = 0.05): -12.918 ¿ -1.112$

$\hat{\beta}_4 \approx -20.004\,;\, Standard\ error = 3.016\,;\, Confident\ level\ (\alpha = 0.05): -25.976 ¿ -14.031$

$\hat{\beta}_5 \approx 134.341\,;\, Standard\ error = 12.731\,;\, Confident\ level\ (\alpha = 0.05): 109.130 ¿ 159.553$

Fruit Flies_7

$E(Y_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5}$

$x_{i1} = 1\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ one\ \ group\,;\, x_{1i} = 0\ other\ wise$

$x_{i2} = 1\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ many\ \ group\,;\, x_{2i} = 0\ other\ wise$

$x_{i3} = 1\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ low\ \ group\,;\, x_{3i} = 0\ other\ wise$

$x_{i4} = 1\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was \in\ high\ \ group\,;\, x_{4i} = 0\ other\ wise$

$x_{i5} = The\ length\ of\ the\ thorax$

$E(Y_i) = \beta_0 + \beta_5 x_{i5}\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{isolated}$

$E(Y_i) = \beta_0 + \beta_1 + \beta_5 x_{i5}\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{one}$

$E(Y_i) = \beta_0 + \beta_2 + \beta_5 x_{i5}\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{many}$

$E(Y_i) = \beta_0 + \beta_3 + \beta_5 x_{i5}\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{low}$

$E(Y_i) = \beta_0 + \beta_4 + \beta_5 x_{i5}\ if\ the\ sexual\ activity\ of\ the\ i-th\ fruitfly\ was\ \text{high}$

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Fruit Flies_8

- **Code:**

```
library(faraway)
library(contrast)
library(ggplot2)
m<-lm(longevity ~ activity + thorax, data = fruitfly)
contrast(m,
        a = list(activity = c("isolated", "one", "many", "low", "high"),thorax=0.82),
        cnames=c("isolated", "one", "many", "low", "high"))

m<-lm(longevity ~ activity + thorax, data = fruitfly)
d<-expand.grid(activity=c("isolated","one","many","low","high"),
thorax=seq(0.5,1,0.1))
pred<-predict(m, d, interval="confidence")
d<-cbind(d,pred)
p<-ggplot(fruitfly, aes(x=thorax, y= longevity, color = activity))
p<-p+xlab("Thorax Length (mm)") + ylab("Longevity (days)") + geom_point()
+geom_line(aes(y=fit), data=d)
plot(p)
```

- **Output:**

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
isolated	61.41	2.118	57.22	65.60	29.00	118	0
one	64.05	2.109	59.87	68.22	30.37	118	0
many	65.55	2.153	61.28	69.81	30.44	118	0
low	54.40	2.120	50.20	58.59	25.66	118	0
high	41.41	2.123	37.20	45.61	19.50	118	0

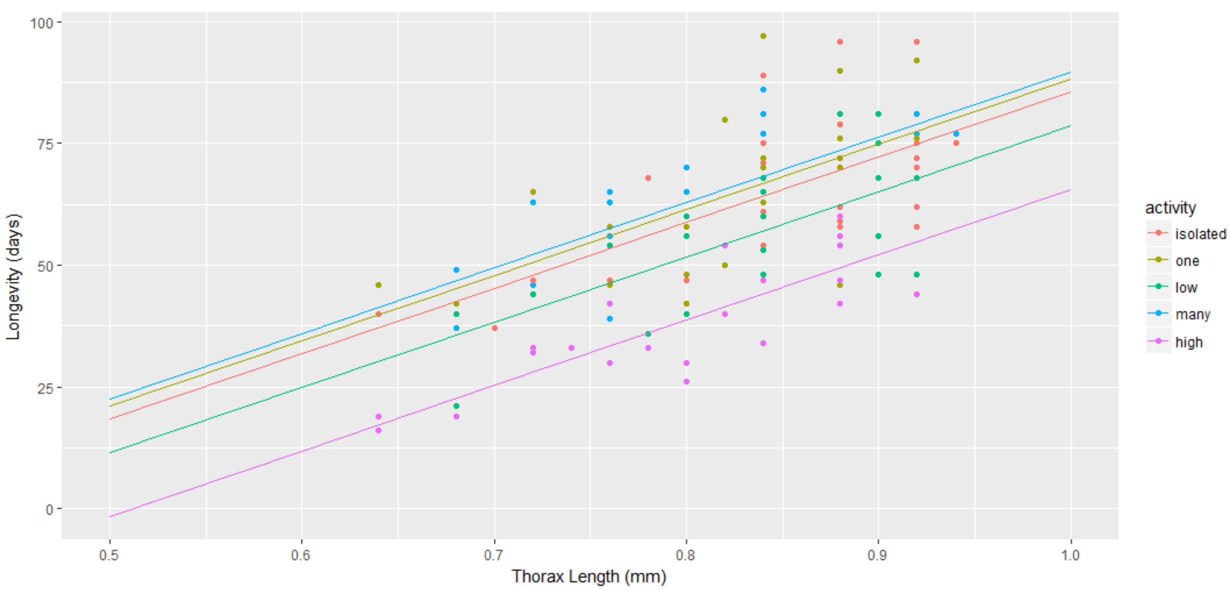

Figure 1 The expected longevity of the fruit flies with different sexual activity and the thorax length.

- **Discussion:**

Expected longevity of fruit flies with sexual condition “isolated” with 0.82 mm thorax: 61.41
Expected longevity of fruit flies with sexual condition “one” with 0.82 mm thorax: 64.05
Expected longevity of fruit flies with sexual condition “many” with 0.82 mm thorax: 65.55
Expected longevity of fruit flies with sexual condition “low” with 0.82 mm thorax: 54.40
Expected longevity of fruit flies with sexual condition “high” with 0.82 mm thorax: 41.41
These expected value agreed with the Figure 1.

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Fruit Flies_9

- **Code:**

```
library(faraway)
library(contrast)
m<-lm(longevity ~ activity+thorax, data = fruitfly)

contrast(m,
        a=list(activity="one", thorax=0.82),
        b=list(activity="low", thorax=0.82),
        cnames="one, pregnant - virgin")
contrast(m,
        a=list(activity="many",thorax=0.82),
        b=list(activity="high",thorax=0.82),
        cnames="eight, pregnant - virgin")
```

- **Output:**

```
                      Contrast  S.E. Lower Upper    t  df Pr(>|t|)
one, pregnant - virgin    9.652 2.985 3.741 15.56 3.23 118   0.0016

                        Contrast  S.E. Lower Upper t  df Pr(>|t|)
eight, pregnant - virgin    24.14 3.016 18.17 30.12 8 118        0
```

- **Discussion:**

The p-value of the test on one female fruit fly when thorax length equal to 0.82 was 0.0016, and the p-value of the test on eight female fruit flies when thorax length equal to 0.82 was 0. Both of the p-value were smaller than α = 0.05. This meant that the sexual activity of the fruit flies with 0.82 mm thorax length did affect the longevity of the male fruit fly.

The confidence interval of the one female and eight female tests were 3.741 to 15.56 and 18.17 to 30.12. Both of the confidence interval did not covered the 0 point. This also shows the sexual activity of the fruit flies with 0.82 thorax length did affect the longevity of the male fruit fly.

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Fruit Flies_10

- **Code:**

```
library(faraway)
library(contrast)
library(ggplot2)
#build up two model: m: model without covariate (thorax); m_c: model with
covariate (thorax)
m<-lm(longevity ~ activity, data = fruitfly)
m_c<-lm(longevity ~ activity + thorax, data = fruitfly)

#plot
d_nc<-expand.grid(activity=c("isolated","one","low","many","high"),
thorax=seq(0.6,1,0.1))
d_c<-expand.grid(activity=c("isolated","one","low","many","high"),
thorax=seq(0.6,1,0.1))
pred_nc<-predict(m, d_nc, interval="confidence")
pred_c<-predict(m_c,d_c,interval="confidence")
d_nc<-cbind(d_nc,pred_nc)
d_c<-cbind(d_c,pred_c)
p_nc<-ggplot(fruitfly, aes(x=thorax, y= longevity, color = activity))
p_c<-ggplot(fruitfly, aes(x=thorax, y= longevity, color = activity))
p_nc<-p_nc+xlab("Thorax Length (mm)") + ylab("Longevity (days)") +
geom_point()+geom_line(aes(y=fit), data=d_nc)
p_c<-p_c+xlab("Thorax Length (mm)") + ylab("Longevity (days)") + geom_point()
+geom_line(aes(y=fit),data=d_c)
plot(p_nc)
plot(p_c)

#Compare how the two models affect the result for question 8.
contrast(m,
        a = list(activity = c("isolated", "one", "many", "low", "high")),
        cnames=c("isolated", "one", "many", "low", "high"))
contrast(m_c,
        a = list(activity = c("isolated", "one", "many", "low", "high"),thorax=0.82),
        cnames=c("isolated", "one", "many", "low", "high"))

#Compare how the two models affect the result for question 9.
contrast(m,
        a=list(activity="one"),
        b=list(activity="low"),
        cnames="one, pregnant - virgin_nc")
contrast(m_c,
        a=list(activity="one", thorax=0.82),
        b=list(activity="low", thorax=0.82),
        cnames="one, pregnant - virgin_c")
```

```
contrast(m,
        a=list(activity="many"),
        b=list(activity="high"),
        cnames="eight, pregnant - virgin__nc")
contrast(m_c,
        a=list(activity="many",thorax=0.82),
        b=list(activity="high",thorax=0.82),
        cnames="eight, pregnant - virgin_c")
```

- **Output:**

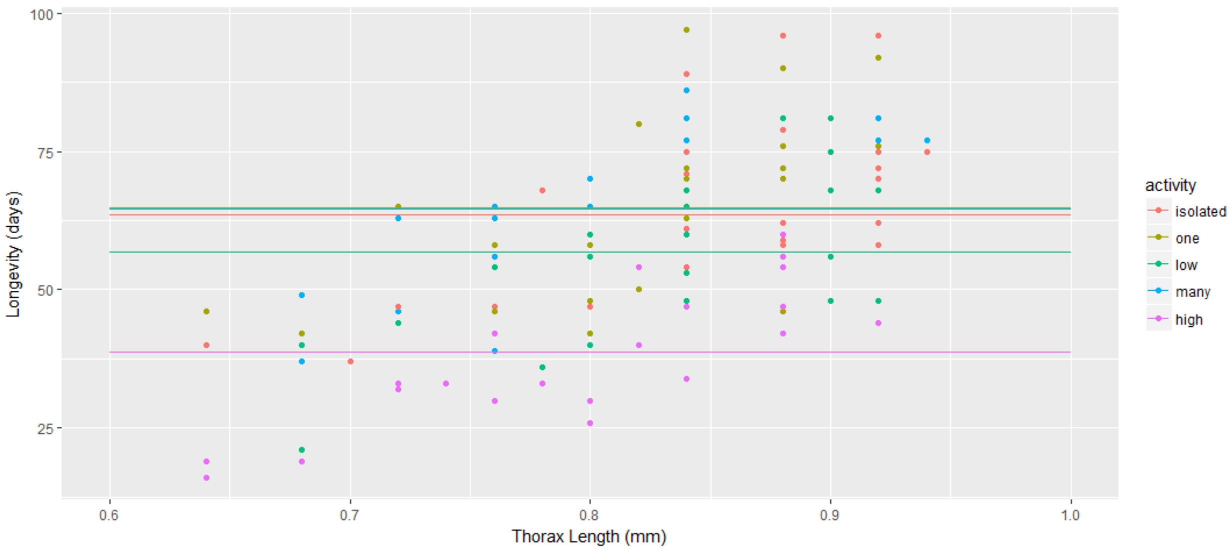

Figure 2 The expected longevity of the fruit flies with different sexual activity and the thorax length. The model was built without covariate (thorax)

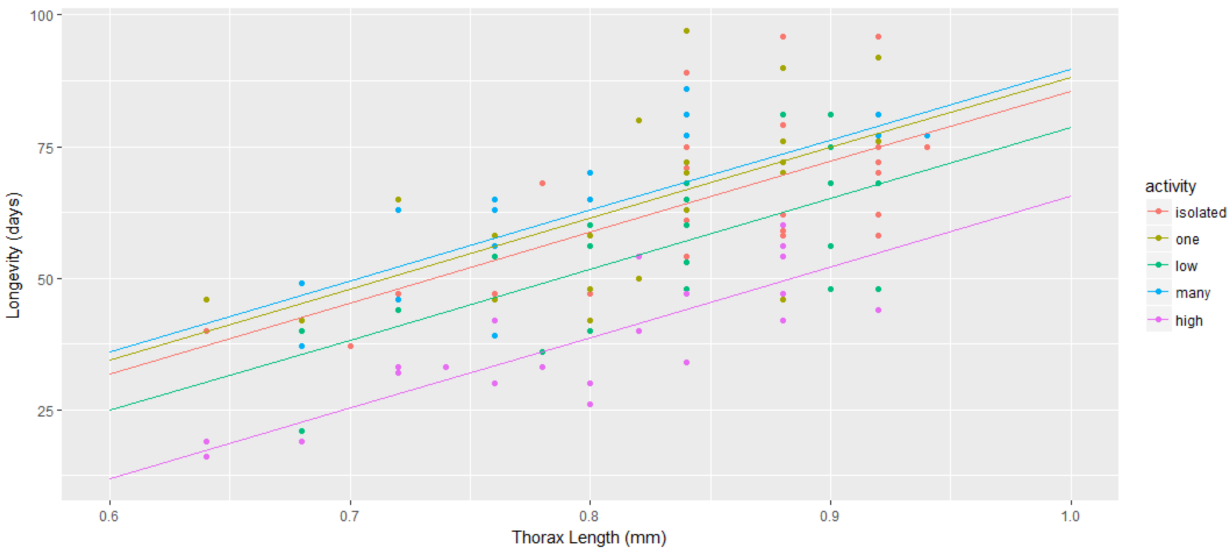

Figure 3 The expected longevity of the fruit flies with different sexual activity and the thorax length. The model was built with covariate (thorax)

#Compare how the two models affect the result for question 8.

#Without covariate

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
isolated	63.56	2.926	57.77	69.35	21.72	119	0
one	64.80	2.926	59.01	70.59	22.14	119	0
many	64.54	2.987	58.63	70.46	21.61	119	0
low	56.76	2.926	50.97	62.55	19.40	119	0
high	38.72	2.926	32.93	44.51	13.23	119	0

#With covariate

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
isolated	61.41	2.118	57.22	65.60	29.00	118	0
one	64.05	2.109	59.87	68.22	30.37	118	0
many	65.55	2.153	61.28	69.81	30.44	118	0
low	54.40	2.120	50.20	58.59	25.66	118	0
high	41.41	2.123	37.20	45.61	19.50	118	0

#Compare how the two models affect the result for question 9.

#One female fruit fly_without covariate

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
one, pregnant - virgin_nc	8.04	4.138	-0.1545	16.23	1.94	119	0.0544

#One female fruit fly_with covariate

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
one, pregnant - virgin_c	9.652	2.985	3.741	15.56	3.23	118	0.0016

#Eight female fruit fly_without covariate

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
eight, pregnant - virgin__nc	25.82	4.181	17.54	34.1	6.18	119	0

#Eight female fruit fly_with covariate

	Contrast	S.E.	Lower	Upper	t	df	Pr(>|t|)
eight, pregnant - virgin_c	24.14	3.016	18.17	30.12	8	118	0

- **Discussion:**

When using the models with and without covariate to predict the longevity of the fruit flies with 0.82 mm thorax length with different sexual activity, the model with covariate provide lower standard errors (isolated: 2.118; one: 2.109; many: 2.153; low: 2.120; high: 2.123) compared with the model without covariate (isolated: 2.926; one: 2.926; many: 2.987; low: 2.926; high: 2.926). The lower standard error means the linear model was more fit with the data (Figure 2 and Figure 3). The confidence intervals were narrower when apply the thorax length as covariate (isolated: 57.22 to 65.60; one: 59.87 to 68.22; many: 61.28 to 69.81; low: 50.20 to 58.59; high: 37.20 to 45.61) compared with the model without using thorax length as covariate (isolated: 57.77 to 69.35; one: 59.01 to 70.59; many: 58.63 to 70.46; low: 50.97 to 62.55; high: 32.93 to 44.51). The narrow confidence interval means the null hypothesis will easier to be rejected while it was not true. This will lead to the increasing of the accuracy.

When using the two models to predict the difference in expected longevity between the two sexual activity conditions (pregnant and virgin female flight), the standard errors decreased when applied the thorax length as covariate (one female: 2.985; eight female: 3.016) compared with the model without using thorax length as covariate (one female: 4.138; eight female: 4.181). The confidence intervals were also narrower (one female: 3.741 to 15.56; eight female: 18.17 to 30.12) compared with the model without covariate (one female: -0.1545 to 16.23; eight female: 17.54 to 34.1). The results were agreed with the conclusion made from comparing question 8 and question 3 that including the covariate (thorax length) in the model will increase the accuracy of the model.

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Cherry Tree_Background

- **Code:**

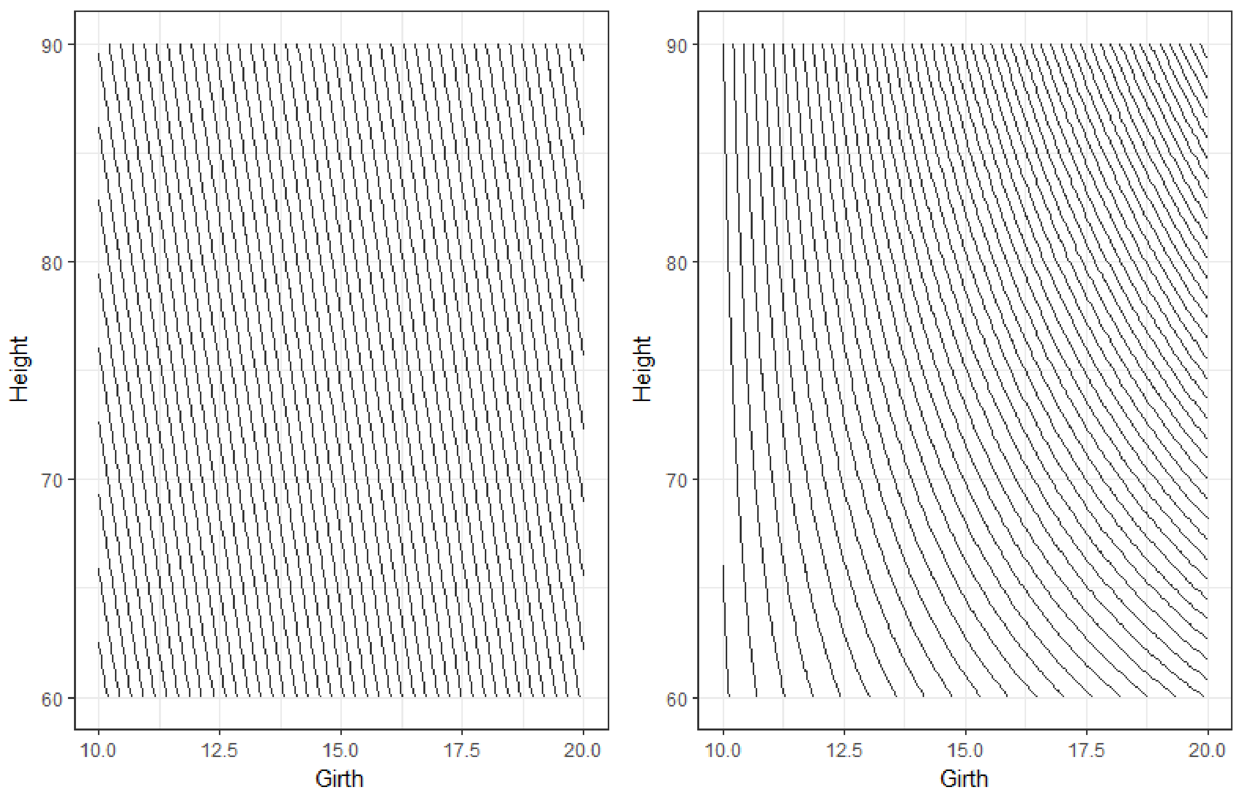
```
library(ggplot2)
library(contrast)

options(digits = 4)
m1<-lm(Volume~Girth + Height, data =trees)
m2<-lm(Volume ~ Girth + Height +Girth:Height, data = trees)

d<-expand.grid(Girth = seq(10,20, length = 100),
            Height = seq(60,90, length = 100))
d$yhat1<-predict(m1, newdata = d)
d$yhat2<-predict(m2, newdata = d)

p1<- ggplot(d, aes(x=Girth, y=Height, z=yhat1))
p1<- p1+ geom_contour(bins=50, color = grey(0.2))+theme_bw()
p2<- ggplot(d, aes(x=Girth, y=Height, z=yhat2))
p2<- p2+ geom_contour(bins=50, color = grey(0.2))+theme_bw()
gridExtra::grid.arrange(p1, p2, nrow=1)
 (p1, p2, nrow=1)
```

- **Output:**



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Cherry Tree_1

- **Code:**

```
library(contrast)
m2<-lm(Volume ~ Girth + Height +Girth:Height, data = trees)
contrast(m2,
        a=list(Height =c(60, 75, 90), Girth = 16),
        b=list(Height =c(60, 75, 90), Girth = 15),
        cnames=c("60","75","90")
        )
```

- **Output:**

```
   Contrast   S.E. Lower Upper     t df Pr(>|t|)
60    2.223 0.4862 1.226 3.221  4.57 27    1e-04
75    4.243 0.2027 3.827 4.659 20.93 27    0e+00
90    6.263 0.3365 5.573 6.954 18.61 27    0e+00
```

- **Discussion:**

The estimated rate of change of volume with height of 60, 75, and 90 feet when the girth was 15 to 16 inches was 2.223, 4.243 and 6.263 ft^3/inch.

Cherry Tree_2

- **Code:**

```
library(contrast)
m2<-lm(Volume ~ Girth + Height +Girth:Height, data = trees)

contrast(m2,
        a=list(Girth =c(10, 15, 20), Height = 71),
        b=list(Girth =c(10, 15, 20), Height = 70),
        cnames=c("10","15","20")
        )
```

- **Output:**

```
   Contrast   S.E.   Lower  Upper    t df Pr(>|t|)
10  0.04946 0.1049 -0.1657 0.2647 0.47 27    0.641
15  0.72273 0.1143  0.4882 0.9573 6.32 27    0.000
20  1.39600 0.2118  0.9615 1.8305 6.59 27    0.000
```

- **Discussion:**

The estimated rate of change of volume with girth of 10, 15, 20 inches when the height was 70 to 71 feet was 0.04946, 0.72273 and 1.39600 ft^3/foot.

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Cherry Tree_3

- **Code:**

```
library(contrast)
m1<-lm(Volume~Girth + Height, data =trees)
m2<-lm(Volume ~ Girth + Height +Girth:Height, data = trees)

contrast(m1,
        a=list(Height =c(60, 75, 90), Girth = 15),
        b=list(Height =c(60, 75, 90), Girth = 16),
        cnames=c("60","75","90")
        )

contrast(m1,
        a=list(Girth =c(10, 15, 20), Height = 70),
        b=list(Girth =c(10, 15, 20), Height = 71),
        cnames=c("10","15","20")
        )

summary(m1)$coefficient
summary(m2)$coefficient
```

- **Output:**

```
   Contrast   S.E. Lower Upper     t df Pr(>|t|)
60    4.708 0.2643 4.167 5.249 17.82 28        0
75    4.708 0.2643 4.167 5.249 17.82 28        0
90    4.708 0.2643 4.167 5.249 17.82 28        0

   Contrast   S.E.   Lower  Upper    t df Pr(>|t|)
10   0.3393 0.1302 0.07265 0.6059 2.61 28   0.0145
15   0.3393 0.1302 0.07265 0.6059 2.61 28   0.0145
20   0.3393 0.1302 0.07265 0.6059 2.61 28   0.0145

            Estimate Std. Error t value  Pr(>|t|)
(Intercept) -57.9877     8.6382  -6.713 2.750e-07
Girth         4.7082     0.2643  17.816 8.223e-17
Height        0.3393     0.1302   2.607 1.449e-02

             Estimate Std. Error t value  Pr(>|t|)
(Intercept)   69.3963   23.83575   2.911 7.131e-03
Girth         -5.8558    1.92134  -3.048 5.109e-03
Height        -1.2971    0.30984  -4.186 2.699e-04
Girth:Height   0.1347    0.02438   5.524 7.484e-06
```

- **Discussion:**

When using the model without using the term $\beta_3 g_i h_i$, the volume changed per inch of girth were the same (4.708 ft^3/inch) when the height were 60, 75, and 90 feet. The volume changed per foot of height were also the same (0.3393 ft^3/foot) when the girth were 10, 15, and 20 inches. The expected volume change with respect to height was independent on gird, and the expected volume change with respect to grid was independent on height. This result agreed with the output when I applied summary function.

#For the model with $\beta_3 g_i h_i$ term:

$E(Y_i) = \beta_0 + \beta_1 g_i + \beta_2 h_i + \beta_3 g_i h_i$

$Y_i = The\ volume\ of\ the\ i-th\ tree\ (ft^3)$
$g_i = The\ diameter\ of\ the\ i-th\ tree\ (inches)$
$h_i = The\ height\ of\ the\ i-th\ tree\ (ft)$

$\hat{\beta}_0 \approx 69.3963;\ \hat{\beta}_1 \approx -5.8558;\ \hat{\beta}_2 \approx -1.2971;\ \hat{\beta}_3 \approx 0.1347;$

The rate of change in the expected volume (δ_i) with respect to girth was depends on height:

$\delta_i = (\beta_1 + \beta_3 h_i)$

The rate of change in the expected volume (γ_i) with respect to height was depends on girth:

$\gamma_i = (\beta_2 + \beta_3 g_i)$

#For the model without $\beta_3 g_i h_i$ term:

$E(Y_i) = \beta_0 + \beta_1 g_i + \beta_2 h_i$

$Y_i = The\ volume\ of\ the\ i-th\ tree\ (ft^3)$
$g_i = The\ diameter\ of\ the\ i-th\ tree\ (inches)$
$h_i = The\ height\ of\ the\ i-th\ tree\ (ft)$

$\hat{\beta}_0 \approx -57.9877;\ \hat{\beta}_1 \approx 4.7082;\ \hat{\beta}_2 \approx 0.3393$

The rate of change in the expected volume (δ_i) with respect to girth was independent from height:

$\delta_i = \beta_1 = 4.7082\left(\frac{ft^3}{inches}\right)(agreed\ with\ the\ result\ ¿\ constrast\ function)$

The rate of change in the expected volume (γ_i) with respect to height was independent on girth:

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$\gamma_i = \beta_2 = 0.3393\left(\frac{ft^3}{ft}\right)\left(agreed\ with\ the\ result\ ¿\ constrast\ function\right)$

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Anorexia

- **Code:**

```
library(MASS)
library(contrast)l

options(digits = 4)
anorexia$Treat<-relevel(anorexia$Treat, ref="Cont")

#1 Postwt~Treat
m<-lm(Postwt~Treat, data = anorexia)
summary(m)$coefficient

#2
m<-lm(Postwt~Treat-1, data = anorexia)
summary(m)$coefficient

#3
m<-lm(Postwt~Treat+Prewt, data = anorexia)
summary(m)$coefficient

#4
m<-lm(Postwt~Treat+Prewt-1, data = anorexia)
summary(m)$coefficient

#5
m<-lm(Postwt~Treat+Prewt+Treat:Prewt, data = anorexia)
summary(m)$coefficient

#6
m<-lm(Postwt~Treat+Treat:Prewt-1, data = anorexia)
summary(m)$coefficient
```

- **Output and discussion:**

#1 Postwt~Treat

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	81.108	1.429	56.746	1.221e-59
TreatCBT	4.589	1.968	2.331	2.267e-02
TreatFT	9.386	2.273	4.129	1.004e-04

$E(Y_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}$

$x_{i1} = 1\, if\ the\, i-th\, subject\, was\, received\, CBT\ ;\ x_{1i} = 0\, other\, wise$
$x_{i2} = 1\, if\ the\, i-th\, subject\, was\, received\, FT\ ;\ x_{2i} = 0\, other\, wise$

$\hat{\beta}_0 \approx 81.108\,;\,\hat{\beta}_1 \approx 4.589\,;\,\hat{\beta}_2 \approx 9.386\,;$

$E(Y_i) = \beta_0\, if\ the\ i-th\, subject\, was \in the\, control\, group$
$E(Y_i) = \beta_0 + \beta_1\, if\ the\, i-th\, subject\, received\, CBT$

$E(Y_i) = \beta_0 + \beta_2\ if\ the\ i-th\ subject\ received\ FT$

#2 Postwt~Treat-1

	Estimate	Std. Error	t value	Pr(>|t|)
TreatCont	81.11	1.429	56.75	1.221e-59
TreatCBT	85.70	1.353	63.32	7.286e-63
TreatFT	90.49	1.768	51.20	1.261e-56

$E(Y_i) = \beta_0 x_{i0} + \beta_1 x_{i1} + \beta_2 x_{i2}$

$x_{i0} = 1\ if\ the\ i-th\ subject\ was \in the\ control\ group\ ; x_{0i} = 0\ other\ wise$
$x_{i1} = 1\ if\ the\ i-th\ subject\ was\ received\ CBT\ ; x_{1i} = 0\ other\ wise$
$x_{i2} = 1\ if\ the\ i-th\ subject\ was\ received\ FT\ ; x_{2i} = 0\ other\ wise$

$\hat{\beta}_0 \approx 81.11\ ; \hat{\beta}_1 \approx 85.70\ ; \hat{\beta}_2 \approx 90.49\ ;$

$E(Y_i) = \beta_0\ if\ the\ i-th\ subject\ was \in the\ control\ group$
$E(Y_i) = \beta_1\ if\ the\ i-th\ subject\ received\ CBT$
$E(Y_i) = \beta_2\ if\ the\ i-th\ subject\ received\ FT$

#3 Postwt~Treat+Prewt

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	45.6740	13.2167	3.456	0.0009499
TreatCBT	4.0971	1.8935	2.164	0.0339993
TreatFT	8.6601	2.1931	3.949	0.0001890
Prewt	0.4345	0.1612	2.695	0.0088500

$E(Y_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3}$

$x_{i1} = 1\ if\ the\ i-th\ subject\ was\ received\ CBT\ ; x_{1i} = 0\ other\ wise$
$x_{i2} = 1\ if\ the\ i-th\ subject\ was\ received\ FT\ ; x_{2i} = 0\ other\ wise$
$x_{i3} = The\ preweight\ of\ i-subject$

$\hat{\beta}_0 \approx 45.6740\ ; \hat{\beta}_1 \approx 4.0971\ ; \hat{\beta}_2 \approx 8.6601\ ; \hat{\beta}_3 \approx 0.4345$

$E(Y_i) = \beta_0 + \beta_3 * preweight\ , if\ the\ i-th\ subject\ was \in the\ control\ group$
$E(Y_i) = \beta_0 + \beta_1 + \beta_3 * preweight\ , if\ the\ i-th\ subject\ received\ CBT$
$E(Y_i) = \beta_0 + \beta_2 + \beta_3 * preweight\ , if\ the\ i-th\ subject\ received\ FT$

#4 Postwt~Treat+Prewt-1

	Estimate	Std. Error	t value	Pr(>|t|)
TreatCont	45.6740	13.2167	3.456	0.0009499

TreatCBT	49.7711	13.3910	3.717	0.0004101
TreatFT	54.3342	13.5215	4.018	0.0001491
Prewt	0.4345	0.1612	2.695	0.0088500

$$E(Y_i)=\beta_0 x_{i0}+\beta_1 x_{i1}+\beta_2 x_{i2}+\beta_3 x_{i3}$$

$x_{i0}=1$ *if the* $i-th$ *subject was* $\in$ *control group* ; $x_{0i}=0$ *other wise*
$x_{i1}=1$ *if the* $i-th$ *subject was received CBT* ; $x_{1i}=0$ *other wise*
$x_{i2}=1$ *if the* $i-th$ *subject was received FT* ; $x_{2i}=0$ *other wise*
$x_{i3}=$ *The preweight of* $i-$*subject*

$$\hat{\beta}_0\approx 45.6740 ; \hat{\beta}_1\approx 49.7711 ; \hat{\beta}_2\approx 54.3342 ; \hat{\beta}_3\approx 0.4345$$

$E(Y_i)=\beta_0+\beta_3 * preweight$, *if the* $i-th$ *subject was* $\in$ *the control group*
$E(Y_i)=\beta_1+\beta_3 * preweight$, *if the* $i-th$ *subject received CBT*
$E(Y_i)=\beta_2+\beta_3 * preweight$, *if the* $i-th$ *subject received FT*

#5 Postwt~Treat+Prewt+Treat:Prewt

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	92.0515	18.8085	4.8941	6.672e-06
TreatCBT	-76.4742	28.3470	-2.6978	8.852e-03
TreatFT	-77.2317	33.1328	-2.3310	2.282e-02
Prewt	-0.1342	0.2301	-0.5832	5.617e-01
TreatCBT:Prewt	0.9822	0.3442	2.8532	5.776e-03
TreatFT:Prewt	1.0434	0.4000	2.6087	1.123e-02

$$E(Y_i)=\beta_0+\beta_1 x_{i1}+\beta_2 x_{i2}+\beta_3 x_{i3}+\beta_4 x_{i1} x_{i3}+\beta_5 x_{i2} x_{i3}$$

$x_{i1}=1$ *if the* $i-th$ *subject was received CBT* ; $x_{1i}=0$ *other wise*
$x_{i2}=1$ *if the* $i-th$ *subject was received FT* ; $x_{2i}=0$ *other wise*
$x_{i3}=$ *The preweight of* $i-$*subject*

$$\hat{\beta}_0\approx 92.0515 ; \hat{\beta}_1\approx -76.4742 ; \hat{\beta}_2\approx -77.2317 ; \hat{\beta}_3\approx -0.1342 ; \hat{\beta}_4\approx 0.9822 ; \hat{\beta}_5\approx 1.0434$$

$E(Y_i)=\beta_0+\beta_3 * preweight$ *if the* $i-th$ *subject was* $\in$ *the control group*
$E(Y_i)=\beta_0+\beta_1+(\beta_3+\beta_4) * preweight$ *if the* $i-th$ *subject received CBT*
$E(Y_i)=\beta_0+\beta_2+(\beta_3+\beta_5) * preweight$ *if the* $i-th$ *subject received FT*

#6 Postwt~Treat+Treat:Prewt-1

	Estimate	Std. Error	t value	Pr(>|t|)
TreatCont	92.0515	18.8085	4.8941	6.672e-06
TreatCBT	15.5772	21.2083	0.7345	4.653e-01

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```
TreatFT          14.8198    27.2768  0.5433 5.887e-01
TreatCont:Prewt  -0.1342     0.2301 -0.5832 5.617e-01
TreatCBT:Prewt    0.8480     0.2561  3.3117 1.507e-03
TreatFT:Prewt     0.9092     0.3272  2.7791 7.094e-03
```

$E(Y_i) = \beta_0 x_{i0} + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i0} x_{i3} + \beta_4 x_{i1} x_{i3} + \beta_5 x_{i2} x_{i3}$

$x_{i0} = 1\ if\ the\ i-th\ subject\ was \in control\ group\ ; x_{0i} = 0\ other\ wise$
$x_{i1} = 1\ if\ the\ i-th\ subject\ was\ received\ CBT\ ; x_{1i} = 0\ other\ wise$
$x_{i2} = 1\ if\ the\ i-th\ subject\ was\ received\ FT\ ; x_{2i} = 0\ other\ wise$
$x_{i3} = The\ preweight\ of\ i-subject$

$\hat{\beta}_0 \approx 92.0515 ; \hat{\beta}_1 \approx 15.5772 ; \hat{\beta}_2 \approx 14.8198 ; \hat{\beta}_3 \approx -0.1342 ; \hat{\beta}_4 \approx 0.8480 \hat{\beta}_5 \approx 0.9092$

$E(Y_i) = \beta_0 + \beta_3 * preweight\ if\ the\ i-th\ subject\ was \in the\ control\ group$
$E(Y_i) = \beta_1 + \beta_4 * preweight\ if\ the\ i-th\ subject\ received\ CBT$
$E(Y_i) = \beta_2 + \beta_5 * preweight\ if\ the\ i-th\ subject\ received\ FT$