Task from lesson 3 **Óscar Iglesias González**
Statistical Mechanics and Applications on Simulations Universidade de Vigo

$$S = k_B \ln Q + k_B T \left(\frac{\partial \ln Q}{\partial N!}\right)_{N,V}$$
$$\alpha = \frac{2\pi m k_B T}{h^2}$$
$$Q = \frac{q^N}{N!} \quad q = (\alpha T)^{\frac{3}{2}} V g_{e,0}$$

$$\begin{aligned}
S &= k_B \ln \frac{\left[(\alpha T)^{\frac{3}{2}} V g_{e,0}\right]^N}{N!} + k_B T \left[\frac{1}{\left(\alpha^{\frac{3}{2}} T^{\frac{3}{2}} V g_{e,0}\right)^N} \cdot N \left(\alpha^{\frac{3}{2}} T^{\frac{3}{2}} V g_{e,0}\right)^{N-1} \cdot \frac{3}{2} \left(\alpha^{\frac{3}{2}} T^{\frac{1}{2}} V g_{e,0}\right)\right] \\
&= k_B \ln \left[(\alpha T)^{\frac{3}{2}} V g_{e,0}\right]^N - k_B \ln N! + k_B T \frac{3}{2} \frac{N}{T} \\
&= k_B N \ln (\alpha T)^{\frac{3}{2}} V g_{e,0} - k_B (N \ln N - N) + \frac{3}{2} k_B N \quad \text{(using Stirling's Approximation)} \\
&= k_B N \left[\ln (\alpha T)^{\frac{3}{2}} V g_{e,0} - \ln N + 1\right] + \frac{3}{2} k_B N \\
&= k_B N \left[\ln (\alpha T)^{\frac{3}{2}} \frac{Ve}{N}\right] + \frac{3}{2} k_B N + k_B N \ln g_{e,0} \\
&= \frac{3}{2} k_B N + k_B N \left[\ln \left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}} \frac{Ve}{N}\right] + k_B N \ln g_{e,0}
\end{aligned}$$