
Optimal Harvesting Modelling

Final Report

UAB
Universitat Autònoma
de Barcelona

Centre de Recerca Matemàtica
Universitat Autònoma de Barcelona
Group number 4.

Abstract

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1 Preliminary Concept

1.1 Constrained minimization in Banach spaces and Lagrange multipliers

Definition 1. Lower semi-continuous A functional F is lower-semicontinuous if

$$F\left(\lim_{n \rightarrow \infty} x_n\right) \leq \liminf_{n \rightarrow \infty} F(x_n) \quad (1.1)$$

Definition 2. Derivative. The functional F on a Banach space X

Let X, Y, U be Hilbert spaces and Z be a Hilbert lattice. Consider the constrained minimization problem:

$$\min_{x \in C} J(x)$$

subject to

$$\begin{aligned} E(x) &= 0 \\ \text{and} \\ G(x) &\leq 0 \end{aligned}$$

Where C is a closed and convex set in X , $J : X \rightarrow \mathbb{R}$, $E : X \rightarrow Y$, and $G : X \rightarrow Z$ are continuously differentiable

1.2 Control Problem



2 Problem Framework

3 Mathematical Models.

3.1 Exponential biological growth.

$$x = F(x) \tag{3.1}$$

3.2 Logistic Equation.

3.3 Wiener Process and noise.

4 Fishing Strategies and Optimizing Population

4.1 Open Loop Strategies.

4.1.1 Constant Harvesting Analysis.

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{M}\right) - u \quad (4.1)$$

$$\beta = \frac{uM}{r} \quad (4.2)$$

$$\begin{aligned} \frac{dx}{rx \left(1 - \frac{x}{M}\right) - u} &= dt \\ \int_{x_0}^x \frac{d\chi}{r\chi \left(1 - \frac{\chi}{M}\right) - u} &= \int_0^t d\tau \\ \frac{M}{r} \int_{x_0}^x \frac{d\chi}{\chi(M - \chi) - \frac{Mu}{r}} &= t \\ -\frac{M}{r} \int_{x_0}^x \frac{d\chi}{\chi^2 - M\chi + \beta} &= t \\ -\frac{M}{r} \int_{x_0}^x \frac{d\chi}{\left(\chi - \frac{M}{2}\right)^2 - \frac{M^2}{4} + \beta} &= t \end{aligned} \quad (4.3)$$

$$\alpha = \beta - \frac{M^2}{4} = rM \left(u - \frac{rM}{4}\right) \quad (4.4)$$

If $u > rM/4$ implies $\alpha > 0$

$$\begin{aligned} \int_{x_0}^x \frac{d\chi}{\left(\chi - \frac{M}{2}\right)^2 + \alpha} &= -\frac{r}{M} t \\ \frac{1}{\sqrt{\beta - \frac{M^2}{4}}} \left(\arctan\left(\frac{x - M/2}{\sqrt{\beta - M^2/4}}\right) - \arctan\left(\frac{x_0 - M/2}{\sqrt{\beta - M^2/4}}\right) \right) &= -\frac{r}{M} t \end{aligned}$$

$$x = \frac{M}{2} + \sqrt{\beta - \frac{M^2}{4}} \tan\left(\arctan\left(\frac{x_0 - M/2}{\sqrt{\beta - M^2/4}}\right) - \frac{r\sqrt{\beta - M^2/4}}{M} t\right) \quad (4.5)$$

If $u < rM/4$ implies $\alpha < 0$,

$$\int_{x_0}^x \frac{d\chi}{\left(\chi - \frac{M}{2}\right)^2 - \alpha} = -\frac{r}{M}t$$

$$\begin{aligned}\lambda &= \frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} \\ \bar{\lambda} &= \frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta}\end{aligned}\tag{4.6}$$

$$\begin{aligned}\int_{x_0}^x \left(\frac{1}{\chi - \lambda} - \frac{1}{\chi - \bar{\lambda}} \right) d\chi &= -\frac{2r\sqrt{M^2/4 - \beta}}{M}t \\ \ln \left| \frac{x - \lambda}{x - \bar{\lambda}} \right| &= \ln \left| \frac{x_0 - \lambda}{x_0 - \bar{\lambda}} \right| - \frac{2r\sqrt{M^2/4 - \beta}}{M}t\end{aligned}$$

$$\gamma = \frac{2r\sqrt{M^2/4 - \beta}}{M}$$

$$\frac{x - \lambda}{x - \bar{\lambda}} = \frac{x_0 - \lambda}{x_0 - \bar{\lambda}} e^{-\gamma t}\tag{4.7}$$

$$x - \lambda = (x - \bar{\lambda}) \left(\frac{x_0 - \lambda}{x_0 - \bar{\lambda}} \right) e^{-\gamma t}\tag{4.8}$$

$$\xi = \frac{x_0 - \lambda}{x_0 - \bar{\lambda}} e^{-\gamma t}$$

$$\begin{aligned}x(1 - \xi) &= \lambda - \bar{\lambda}\xi \\ x &= \frac{\lambda - \bar{\lambda}\xi}{1 - \xi} \\ x &= \frac{\frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} - \left(\frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta} \right) \xi}{1 - \xi} \\ x &= \frac{\frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} - \left(\frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta} \right) \xi}{1 - \xi} \\ x &= \frac{\frac{M}{2}(1 - \xi) + \sqrt{\frac{M^2}{4} - \beta}(1 + \xi)}{1 - \xi} \\ x &= \frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} \frac{1 + \xi}{1 - \xi}\end{aligned}$$

$$x = \frac{M}{2} + \left(\sqrt{\frac{M^2}{4} - \beta} \right) \frac{(x_0 - M/2)(1 + e^{-\gamma t}) - \sqrt{M^2/4 - \beta}(1 - e^{-\gamma t})}{(x_0 - M/2)(1 - e^{-\gamma t}) + \sqrt{M^2/4 - \beta}(1 + e^{-\gamma t})}\tag{4.9}$$

4.1.2 Time Varying Harvesting.

4.1.3 Optimal Harvesting. Optimal Control Problem.

4.2 Closed Loop Strategies.

4.2.1 Constant Proportional Harvesting.

4.2.2 Optimal Proportional Harvesting.

5 Economical Profit

5.1 Linear Costs.

5.2 Quadratic Costs.

5.3 Stochastic Analysis.

6 Further Research
