# Optimal Harvesting Modelling

Final Report

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Abstract

Contents

# 1 Preliminary Concept

## 1.1 Constrained minimization in Banach spaces and Lagrange multipliers

Definition 1. Lower semi-continuous A functional $F$ is lower-semicontinuous if

$$F\left(\lim_{n\to\infty} x_n\right) \leq \liminf_{n\to\infty} F(x_n) \tag{1.1}$$

Definition 2. Derivative. The functional $F$ on a Banach space i

Let $X$, $Y$, $U$ be Hilbert spaces and Z be a Hilbert lattice. Consider the constrained minimization problem:

$$\min_{x\in C} J(x)$$

subject to

$$E(x) = 0$$
$$\text{and}$$
$$G(x) \leq 0$$

Where $C$ is a closed and convex set in $X$, $J : X \to \mathbb{R}$, $E : X \to Y$, and $G : X \to Z$ are continuously differentiable

## 1.2 Control Problem

# 3 Mathematical Models.

## 3.1 Exponential biological growth.

$$x = F(x) \tag{3.1}$$

## 3.2 Logistic Equation.

## 3.3 Wiener Process and noise.

# 4 Fishing Strategies and Optimizing Population

## 4.1 Open Loop Strategies.

### 4.1.1 Constant Harvesting Analysis.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1-\frac{x}{M}\right)-u \tag{4.1}$$

$$\beta = \frac{uM}{r} \tag{4.2}$$

$$\begin{aligned}
\frac{\mathrm{d}x}{rx\left(1-\frac{x}{M}\right)-u} &= \mathrm{d}t \\
\int_{x_0}^{x}\frac{\mathrm{d}\chi}{r\chi\left(1-\frac{\chi}{M}\right)-u} &= \int_0^t \mathrm{d}\tau \\
\frac{M}{r}\int_{x_0}^{x}\frac{\mathrm{d}\chi}{\chi\left(M-\chi\right)-\frac{Mu}{r}} &= t \\
-\frac{M}{r}\int_{x_0}^{x}\frac{\mathrm{d}\chi}{\chi^2-M\chi+\beta} &= t
\end{aligned}$$

$$-\frac{M}{r}\int_{x_0}^{x}\frac{\mathrm{d}\chi}{\left(\chi-\frac{M}{2}\right)^2-\frac{M^2}{4}+\beta} = t \tag{4.3}$$

$$\alpha = \beta-\frac{M^2}{4} = rM\left(u-\frac{rM}{4}\right) \tag{4.4}$$

If $u > rM/4$ implies $\alpha > 0$

$$\begin{aligned}
\int_{x_0}^{x}\frac{\mathrm{d}\chi}{\left(\chi-\frac{M}{2}\right)^2+\alpha} &= -\frac{r}{M}t \\
\frac{1}{\sqrt{\beta-\frac{M^2}{4}}}\left(\arctan\left(\frac{x-M/2}{\sqrt{\beta-M^2/4}}\right)-\arctan\left(\frac{x_0-M/2}{\sqrt{\beta-M^2/4}}\right)\right) &= -\frac{r}{M}t
\end{aligned}$$

$$x = \frac{M}{2}+\sqrt{\beta-\frac{M^2}{4}}\tan\left(\arctan\left(\frac{x_0-M/2}{\sqrt{\beta-M^2/4}}\right)-\frac{r\sqrt{\beta-M^2/4}}{M}t\right) \tag{4.5}$$

If $u < rM/4$ implies $\alpha < 0$,

$$\int_{x_0}^{x} \frac{\mathrm{d}\chi}{\left(\chi - \frac{M}{2}\right)^2 - \alpha} = -\frac{r}{M} t$$

$$\begin{aligned} \lambda &= \tfrac{M}{2} + \sqrt{\tfrac{M^2}{4} - \beta} \\ \overline{\lambda} &= \tfrac{M}{2} - \sqrt{\tfrac{M^2}{4} - \beta} \end{aligned} \tag{4.6}$$

$$\begin{aligned} \int_{x_0}^{x} \left( \frac{1}{\chi - \lambda} - \frac{1}{\chi - \overline{\lambda}} \right) \mathrm{d}\chi &= -\frac{2r\sqrt{M^2/4 - \beta}}{M} t \\ \ln \left| \frac{x - \lambda}{x - \overline{\lambda}} \right| &= \ln \left| \frac{x_0 - \lambda}{x_0 - \overline{\lambda}} \right| - \frac{2r\sqrt{M^2/4 - \beta}}{M} t \end{aligned}$$

$\gamma = \frac{2r\sqrt{M^2/4-\beta}}{M}$

$$\frac{x - \lambda}{x - \overline{\lambda}} = \frac{x_0 - \lambda}{x_0 - \overline{\lambda}} \mathrm{e}^{-\gamma t} \tag{4.7}$$

$$x - \lambda = \left( x - \overline{\lambda} \right) \left( \frac{x_0 - \lambda}{x_0 - \overline{\lambda}} \right) \mathrm{e}^{-\gamma t} \tag{4.8}$$

$\xi = \frac{x_0 - \lambda}{x_0 - \overline{\lambda}} \mathrm{e}^{-\gamma t}$

$$\begin{aligned} x\,(1 - \xi) &= \lambda - \overline{\lambda}\xi \\ x &= \frac{\lambda - \overline{\lambda}\xi}{1 - \xi} \\ x &= \frac{\frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} - \left( \frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta} \right) \xi}{1 - \xi} \\ x &= \frac{\frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} - \left( \frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta} \right) \xi}{1 - \xi} \\ x &= \frac{\frac{M}{2}(1 - \xi) + \sqrt{\frac{M^2}{4} - \beta}\,(1 + \xi)}{1 - \xi} \\ x &= \frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta}\,\frac{1 + \xi}{1 - \xi} \end{aligned}$$

$$x = \frac{M}{2} + \left( \sqrt{\frac{M^2}{4} - \beta} \right) \frac{(x_0 - M/2)(1 + \mathrm{e}^{-\gamma t}) - \sqrt{M^2/4 - \beta}\,(1 - \mathrm{e}^{-\gamma t})}{(x_0 - M/2)(1 - \mathrm{e}^{-\gamma t}) + \sqrt{M^2/4 - \beta}\,(1 + \mathrm{e}^{-\gamma t})} \tag{4.9}$$

### 4.1.2 Time Varying Harvesting.

### 4.1.3 Optimal Harvesting. Optimal Control Problem.

## 4.2 Closed Loop Strategies.

### 4.2.1 Constant Proportional Harvesting.

### 4.2.2 Optimal Proportional Harvesting.

# 5 Economical Profit

## 5.1 Linear Costs.

## 5.2 Quadratic Costs.

## 5.3 Stochastic Analysis.