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# Optimal Harvesting Modelling

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Report 1

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## Abstract

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## 1 Preliminary Concept

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### 1.1 Constrained minimization in Banach spaces and Lagrange multipliers

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Definition 1. Lower semi-continuous A functional  $F$  is lower-semicontinuous if

$$F\left(\lim_{n \rightarrow \infty} x_n\right) \leq \liminf_{n \rightarrow \infty} F(x_n) \quad (1.1)$$

Definition 2. Derivative. The functional  $F$  on a Banach space  $X$

Let  $X, Y, U$  be Hilbert spaces and  $Z$  be a Hilbert lattice. Consider the constrained minimization problem:

$$\min_{x \in C} J(x)$$

subject to

$$E(x) = 0$$

and

$$G(x) \leq 0$$

Where  $C$  is a closed and convex set in  $X$ ,  $J : X \rightarrow \mathbb{R}$ ,  $E : X \rightarrow Y$ , and  $G : X \rightarrow Z$  are continuously differentiable

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### 1.2 Control Problem

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## 2 Problem Framework

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### 3 Mathematical Models.

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#### 3.1 Exponential biological growth.

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#### 3.2 Logistic Equation.

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#### 3.3 Wiener Process and noise.

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## 4 Fishing Strategies and Optimizing Population

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### 4.1 Open Loop Strategies.

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#### 4.1.1 Constant Harvesting Analysis.

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#### 4.1.2 Time Varying Harvesting.

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#### 4.1.3 Optimal Harvesting. Optimal Control Problem.

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### 4.2 Closed Loop Strategies.

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#### 4.2.1 Constant Proportional Harvesting.

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#### 4.2.2 Optimal Proportional Harvesting.

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## 5 Economical Profit

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### 5.1 Linear Costs.

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### 5.2 Quadratic Costs.

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### 5.3 Stochastic Analysis.

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## 6 Further Research

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