Optimal Harvesting Modelling

Final Report

Centre de Recerca Matemàtica
Universitat Autonònoma de Barcelona
Group number 4.

Abstract

Contents

1 Preliminary Concept

$$r = 0.8 \qquad (2.1)$$
$$M = 780500 \qquad (2.2)$$

3 Mathematical Models.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F(x, t) \tag{3.1}$$

3.1 Exponential biological growth.

$$F(x, t) = rx \tag{3.2}$$

3.2 Logistic Equation.

Logistic equation.

$$F(x, t) = rx\left(1 - \frac{x}{M}\right) \tag{3.3}$$

3.3 Wiener Process and noise.

$$dx = \left(rx\left(1 - \frac{x}{M}\right) - u\right)\mathrm{d}t + \sigma x \mathrm{d}W \tag{3.4}$$

A unique solution exists if both Itó conditions hold (Fleming and Rishel, 1975). The first is the linear growth condition.

$$\left|rx\left(1 - \frac{x}{M}\right) - u\right| \leq K(1 + |x|) \tag{3.5}$$

$$|\sigma x| \leq K(1 + |x|) \tag{3.6}$$

and the Lipschitz condition,

$$\left|rx_2\left(1 - \frac{x_2}{M}\right) - rx_1\left(1 - \frac{x_1}{M}\right)\right| \leq L|x_2 - x_1| \tag{3.7}$$

$$|\sigma(x_2 - x_1)| \leq L|x_2 - x_1| \tag{3.8}$$

For some independent constants L and K.

4 Fishing Strategies and Optimizing Population

4.1 Open Loop Strategies.

4.1.1 Constant Harvesting Analysis.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{M}\right) - u \tag{4.1}$$

$$\beta = \frac{uM}{r} \tag{4.2}$$

$$\frac{\mathrm{d}x}{rx\left(1 - \frac{x}{M}\right) - u} = \mathrm{d}t$$

$$\int_{x_0}^{x} \frac{\mathrm{d}\chi}{r\chi\left(1 - \frac{\chi}{M}\right) - u} = \int_{0}^{t} \mathrm{d}\tau$$

$$\frac{M}{r} \int_{x_0}^{x} \frac{\mathrm{d}\chi}{\chi\left(M - \chi\right) - \frac{Mu}{r}} = t$$

$$-\frac{M}{r} \int_{x_0}^{x} \frac{\mathrm{d}\chi}{\chi^2 - M\chi + \beta} = t$$

$$-\frac{M}{r} \int_{x_0}^{x} \frac{\mathrm{d}\chi}{\left(\chi - \frac{M}{2}\right)^2 - \frac{M^2}{4} + \beta} = t \tag{4.3}$$

$$\alpha = \beta - \frac{M^2}{4} = rM\left(u - \frac{rM}{4}\right) \tag{4.4}$$

If $u > rM/4$ implies $\alpha > 0$

$$\int_{x_0}^{x} \frac{\mathrm{d}\chi}{\left(\chi - \frac{M}{2}\right)^2 + \alpha} = -\frac{r}{M}t$$

$$\frac{1}{\sqrt{\beta - \frac{M^2}{4}}}\left(\arctan\left(\frac{x - M/2}{\sqrt{\beta - M^2/4}}\right) - \arctan\left(\frac{x_0 - M/2}{\sqrt{\beta - M^2/4}}\right)\right) = -\frac{r}{M}t$$

$$x = \frac{M}{2} + \sqrt{\beta - \frac{M^2}{4}}\tan\left(\arctan\left(\frac{x_0 - M/2}{\sqrt{\beta - M^2/4}}\right) - \frac{r\sqrt{\beta - M^2/4}}{M}t\right) \tag{4.5}$$

If $u < rM/4$ implies $\alpha < 0$,

$$\int_{x_0}^{x} \frac{\mathrm{d}\chi}{\left(\chi - \frac{M}{2}\right)^2 - \alpha} = -\frac{r}{M}t$$

$$\begin{aligned}
\lambda &= \frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} \\
\overline{\lambda} &= \frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta}
\end{aligned} \tag{4.6}$$

$$\int_{x_0}^{x} \left(\frac{1}{\chi - \lambda} - \frac{1}{\chi - \overline{\lambda}}\right)\mathrm{d}\chi = -\frac{2r\sqrt{M^2/4 - \beta}}{M}t$$

$$\ln\left|\frac{x - \lambda}{x - \overline{\lambda}}\right| = \ln\left|\frac{x_0 - \lambda}{x_0 - \overline{\lambda}}\right| - \frac{2r\sqrt{M^2/4 - \beta}}{M}t$$

$\gamma = \frac{2r\sqrt{M^2/4 - \beta}}{M}$

$$\frac{x - \lambda}{x - \overline{\lambda}} = \frac{x_0 - \lambda}{x_0 - \overline{\lambda}}\mathrm{e}^{-\gamma t} \tag{4.7}$$

$$x - \lambda = \left(x - \overline{\lambda}\right)\left(\frac{x_0 - \lambda}{x_0 - \overline{\lambda}}\right)\mathrm{e}^{-\gamma t} \tag{4.8}$$

$\xi = \frac{x_0 - \lambda}{x_0 - \overline{\lambda}}\mathrm{e}^{-\gamma t}$

$$\begin{aligned}
x\left(1 - \xi\right) &= \lambda - \overline{\lambda}\xi \\
x &= \frac{\lambda - \overline{\lambda}\xi}{1 - \xi} \\
x &= \frac{\frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} - \left(\frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta}\right)\xi}{1 - \xi} \\
x &= \frac{\frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta} - \left(\frac{M}{2} - \sqrt{\frac{M^2}{4} - \beta}\right)\xi}{1 - \xi} \\
x &= \frac{\frac{M}{2}\left(1 - \xi\right) + \sqrt{\frac{M^2}{4} - \beta}\left(1 + \xi\right)}{1 - \xi} \\
x &= \frac{M}{2} + \sqrt{\frac{M^2}{4} - \beta}\frac{1 + \xi}{1 - \xi}
\end{aligned}$$

$$x = \frac{M}{2} + \left(\sqrt{\frac{M^2}{4} - \beta}\right)\frac{(x_0 - M/2)(1 + \mathrm{e}^{-\gamma t}) - \sqrt{M^2/4 - \beta}(1 - \mathrm{e}^{-\gamma t})}{(x_0 - M/2)(1 - \mathrm{e}^{-\gamma t}) + \sqrt{M^2/4 - \beta}(1 + \mathrm{e}^{-\gamma t})} \tag{4.9}$$

4.1.2 Time Varying Harvesting.

4.1.3 Optimal Harvesting. Optimal Control Problem.

4.2 Closed Loop Strategies.

4.2.1 Constant Proportional Harvesting.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{M}\right) - px \tag{4.10}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{p}{r} - \frac{x}{M}\right) \tag{4.11}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = r\left(1 - \frac{p}{r}\right)\left(1 - \frac{x}{M\left(1 - \frac{p}{r}\right)}\right)x \tag{4.12}$$

$\gamma = r\left(1 - \frac{p}{r}\right)$, $K = M\left(1 - \frac{p}{r}\right)$. With $\frac{p}{r} < 1$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \gamma x\left(1 - \frac{x}{K}\right) \tag{4.13}$$

$$x = \frac{Kx_0}{x_0 + (K - x_0)\mathrm{e}^{-\gamma t}} \tag{4.14}$$

$$x = \frac{M\left(1 - \frac{p}{r}\right)x_0}{x_0 + \left(M - \frac{Mp}{r} - x_0\right)\mathrm{e}^{-\gamma t}} \tag{4.15}$$

4.2.2 Optimal Proportional Harvesting.

5 Economical Profit

5.1 Linear Costs.

5.2 Quadratic Costs.

5.3 Stochastic Analysis.

6 Further Research