Obtaining K and D meson properties from lattice QCD

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QCD Recap

We begin with a recap of QCD[1, 2].

$$\mathcal{L} = \sum_f \psi_f^\dagger (i\not{D} - m_f)\psi_f - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

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$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c$$

Lattice QCD Recap

Now we move on to a recap of lattice QCD[3, 2].

$$
\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, \mathcal{O} \exp\left(i \int \mathrm{d}^4 x \, \mathcal{L}(\psi, \partial\psi) \right) \\
\mathcal{Z} &= \int \mathcal{D}\psi \quad \exp\left(i \int \mathrm{d}^4 x \, \mathcal{L}(\psi, \partial\psi) \right)
\end{aligned}
$$

Lattice QCD Recap

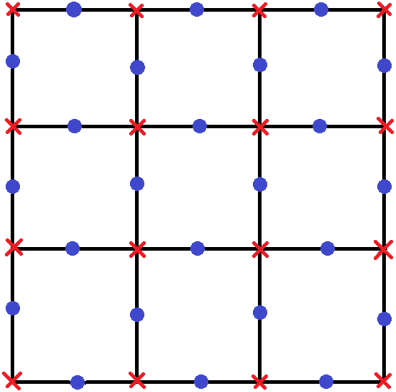

Figure: An example of a discretisation. Red crosses indicate quark fields, and blue dots indicate gauge fields.

Highly Improved Staggered Quark

- Fermions placed on a finite lattice give "doubled" fermions[4].

$$M^{-1} = \frac{-i\gamma_\mu \sin(ap_\mu)/a + m}{\sum_{\mu=0}^{3} \sin^2(ap_\mu)/a^2 + m^2} \tag{1}$$

- These are the reflections of the fermion mass within the Brillouin zone in each of the 4 spacetime dimensions, which gives $16 = 2^4$ extra modes, called "tastes".
- To remove this problem, Wilson[5] used a construction to give these tastes infinite mass, such that they decouple in the continuum limit.
- A procedure called "staggering" transformation[6] allows us to bring the system down to only 4 doublers.
- With some further optimizations this gives the Highly Improved Staggered Quark (HISQ) action, which has very small discretisation errors.

Data

- Data was provided by Dr. B. Chakraborty.
- This data has been produced by using relativistic HISQ action for both valuence and sea quarks on publicly available gauge configurations generated by MILC collaboration. The configurations have sea quark effects from up/down (light), strange, and charm quarks.[7].

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Very Coarse	0.1509	48	0.036
Coarse	0.124 04	64	0.2
Fine	0.090 23	96	0.2

Table: Parameters of the data used in the main part of this project. Corresponding to sets 1, 5, 6 from table I in [7] respectively.

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- Finite lattice leads to discretisation errors: must take continuum limit.
- Computations requirements mean we must use unphysical light quark masses: must take chiral limit.

Statistical Analysis: Jacknife averaging

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- A more complex variant of this procedure can be used to find an unbiased estimator for any parameter of F[8].

Statistical Analysis: Bayesian analysis

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Statistical Analysis: Bayesian analysis

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- Perform least-squares regression on 2-point correlator data.

$$G(t) = \sum_n a_n^2 (e^{-E_n t} + e^{-E_n (T-t)}) + (-1)^t \sum_n a_{o\,n}^2 (e^{-E_{o\,n} t} + e^{-E_{o\,n} (T-t)})$$

Preliminary Study: effective mass

$$G(t) = \sum_n a_n^2(e^{-E_n t} + e^{-E_n(T-t)}) + (-1)^t \sum_n {a_o}_n^2(e^{-{E_o}_n t} + e^{-{E_o}_n(T-t)})$$

Assuming only the first term dominates, and there is little contribution from the oscillatory terms, then we have:

$$m_{\text{eff}}(t) = \log\left(\frac{G(t)}{G(t+1)}\right).$$

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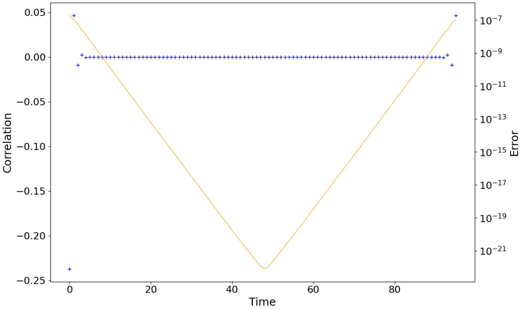

Figure: Jacknife averaged correlation data for D^0 meson on a fine lattice. This is for a slightly different set of parameters than otherwise discussed here, however it is sufficient for this discussion.

Preliminary Study: effective mass

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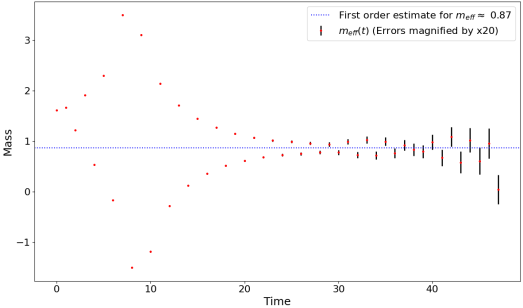

Figure: Estimate for effective mass of D^0 meson.

Data Analysis: fine lattice

- Fine dataset has $a \approx 0.09\,\mathrm{fm}$ and $m_l \approx 0.2 m_s$.
- K meson clearly converges for $n \geq 4$.

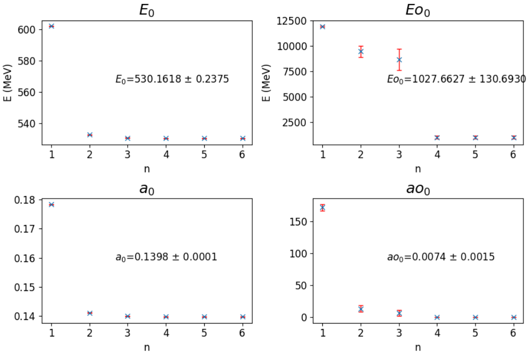

Figure: Results of fitting K meson dataset on a fine lattice.

Data Analysis: fine lattice

Combined fit quality plot shows that the fitting converged for all $t_{\min}$ and $n \geq 4$.

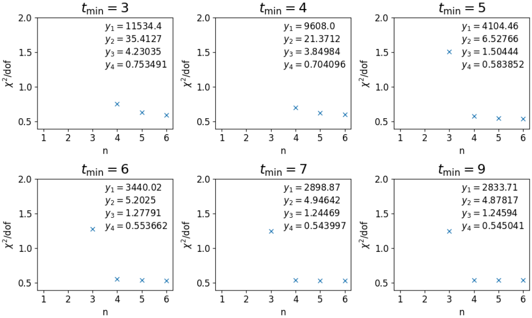

Figure: Fitting quality for fine lattice.

Data Analysis: coarse lattice

- Coarse dataset has $a \approx 0.12\,\text{fm}$ and $m_l \approx 0.2 m_s$.
- K meson converges for $n \geq 3$.

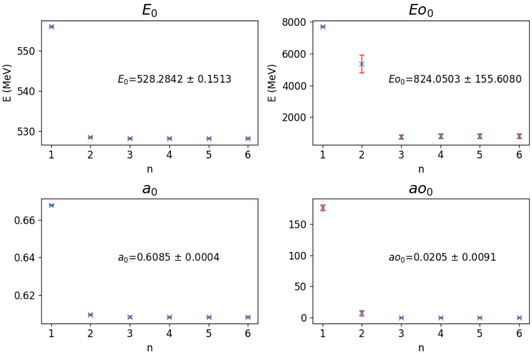

Figure: Results of fitting K meson dataset on a coarse lattice.

Data Analysis: coarse lattice

Combined fit quality plot shows that the fitting converged for all $t_{\min}$ and varying n.

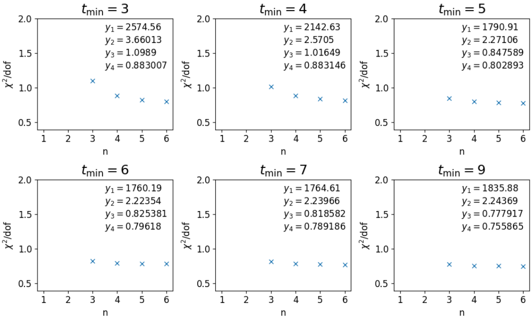

Figure: Fitting quality for coarse lattice.

Data Analysis: very coarse lattice

- Fine dataset has $a \approx 0.15\,\mathrm{fm}$ and $m_l \approx 0.036 m_s$.
- K meson appears to converge for $n \geq 2$.

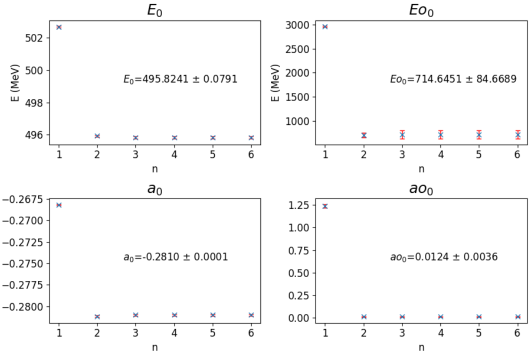

Figure: Results of fitting K meson dataset on a very coarse lattice.

Data Analysis: very coarse lattice

Combined fit quality plot shows that the fitting converged for a very limited selection of t_{min} and n.

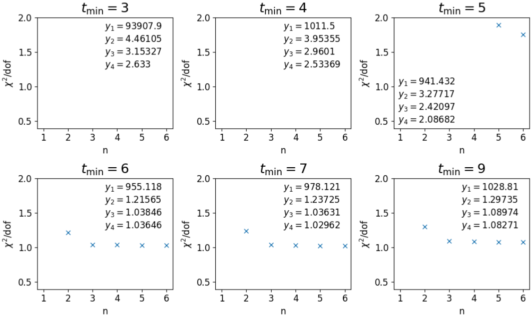

Figure: Fitting quality for very coarse lattice.

Results

For consistency between plots, $t_{\min} = 6$ for D mesons, and $n = 4$ for all results, have been selected.

			E_0 (MeV)		$\lvert a_0 \rvert$	
Label	a (fm)	m_l / m_s	K	D	K	D
Very Coarse	0.15	0.036	495.92(08)	1887.3(51)	0.281 01(11)	0.2286(65)
Coarse	0.12	0.2	528.28(15)	1893.0(15)	0.608 53(37)	0.1880(12)
Fine	0.09	0.2	530.16(24)	1889.3(11)	0.139 78(12)	0.1242(07)

Table: Results from fitting all datasets. Uncertainties given in parentheses are statistical.

These are reasonable mass values as literature[10] quotes $m_K \approx 498\,\text{MeV}$ and $m_D \approx 1865\,\text{MeV}$.

Chiral/Continuum Extrapolation

- Full extrapolation[11] requires 7 pairs of parameters.
- As we have very few datasets, we restrict to first order in each parameter.

$$m = m_{\mathrm{phys}} \left(1 + c_\delta \frac{m_l}{m_s}\right) \left(1 + c_{a^2} a^2\right)$$

Chiral/Continuum Extrapolation

The fit has $\chi^2/\mathrm{dof} = 0.23$ so we have a good quality fit, and importantly not an overfit.

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Table: Comparison of observed and predicted mass values, including the extrapolated chiral/continuum limit values.

The extrapolated values are consistent with results from the literature[10], $m_{K^0} = 497.611(13)\,\mathrm{MeV}$ and $m_{D^0} = 1864.83(5)\,\mathrm{MeV}$ to within 3σ of our statistical uncertainty.

Remaining Work

- Compute partial decay widths, using amplitudes a_0.

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