Computation of neutral K and D meson mass using non-relativistic lattice QCD.

Oscar Simpson
Supervisor: Dr. B. Chakraborty

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QCD Recap

We begin with a recap of QCD[1, 2].

$$\mathcal{L} = \sum_f \psi_f^\dagger (i\not{D} - m_f)\psi_f - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}$$

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$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

Lattice QCD Recap

Now we move on to a recap of lattice QCD[3, 2].

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, \mathcal{O} \exp\left(i \int \mathrm{d}^4 x \, \mathcal{L}(\psi, \partial\psi) \right)$$

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Lattice QCD Recap

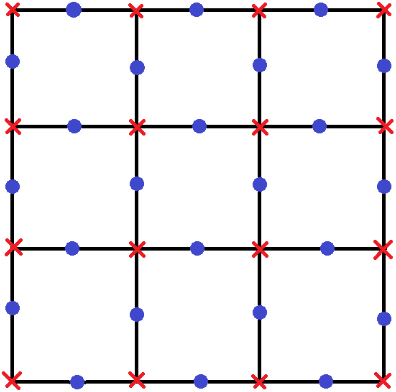

Figure: An example of a discretisation. Red crosses indicate quark fields, and blue dots indicate gauge fields.

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- This is computationally expensive if computed fully relativistically.
- Expand action in $\frac{1}{m_b}$.
- Also expanding in $\frac{v^2}{c^2}$ gives the Schrödinger equation[2],

$$\mathcal{L}_0 = \psi^\dagger(x)\left(iD_t + \frac{\mathbf{D}^2}{2m_b}\right)\psi(x).$$

- This action is only first order in the above expansions. The datasets used for this project are computed using an equivalent third order expansion.

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Coarse	0.124 04	64	0.2
Fine	0.090 23	96	0.2

Table: Parameters of the data used in the main part of this project. Corresponding to sets 1, 5, 6 from table I in [4] respectively.

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- Computations requirements mean we must use unphysical light quark masses: must take chiral limit.

Statistical Analysis: Jacknife averaging

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  - $\overline{x} = \sum_i \frac{\overline{x}_i}{n}$ is in fact an unbiased estimator for the mean of F.
- A more complex variant of this procedure can be used to find an unbiased estimator for any parameter of F[5].

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- Perform least-squares regression on 2-point correlator data.

$$G(t) = \sum_n a_n^2(e^{-E_n t} + e^{-E_n(T-t)}) + (-1)^t \sum_n {a_o}_n^2(e^{-E_{on} t} + e^{-E_{on}(T-t)})$$

Preliminary Study: effective mass

$$G(t) = \sum_n a_n^2 (e^{-E_n t} + e^{-E_n (T-t)}) + (-1)^t \sum_n {a_o}_n^2 (e^{-{E_o}_n t} + e^{-{E_o}_n (T-t)})$$

Assuming only the first term dominates, and there is little contribution from the oscillatory terms, then we have:

$$m_{\text{eff}}(t) = \log\left(\frac{G(t)}{G(t+1)}\right).$$

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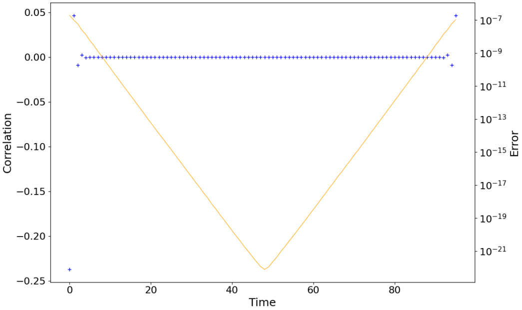

Figure: Jacknife averaged correlation data for D^0 meson on a fine lattice.

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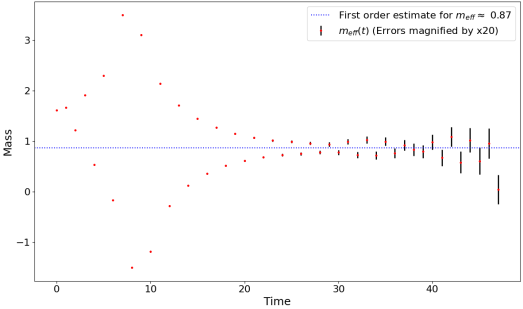

Figure: Estimate for effective mass of D^0 meson.

Data Analysis: fine lattice

- Fine dataset has $a \approx 0.09\,\text{fm}$ and $m_l \approx 0.2 m_s$.
- K meson clearly converges for $n \geq 4$.

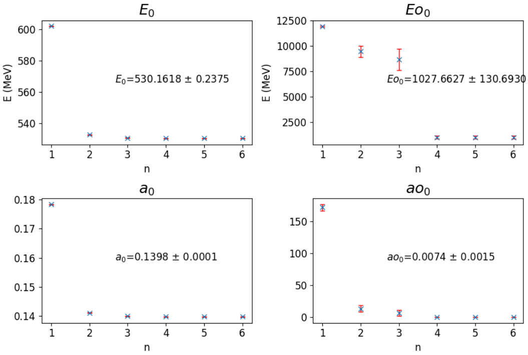

Figure: Results of fitting K meson dataset on a fine lattice.

Combined fit quality plot shows that the fitting converged for all $t_{\min}$ and $n \geq 4$.

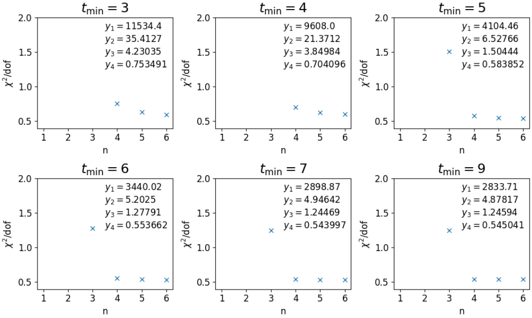

Figure: Fitting quality for fine lattice.

Data Analysis: coarse lattice

- Coarse dataset has $a \approx 0.12\,\mathrm{fm}$ and $m_l \approx 0.2 m_s$.
- K meson converges for $n \geq 3$.

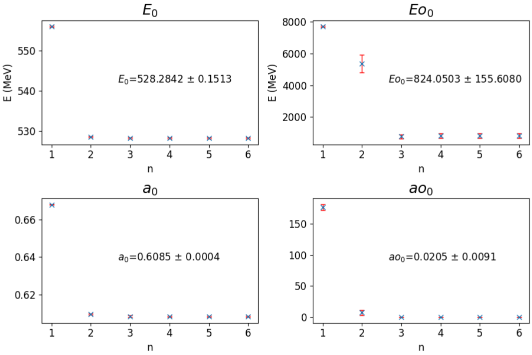

Figure: Results of fitting K meson dataset on a coarse lattice.

Data Analysis: coarse lattice

Combined fit quality plot shows that the fitting converged for all $t_{\min}$ and varying n.

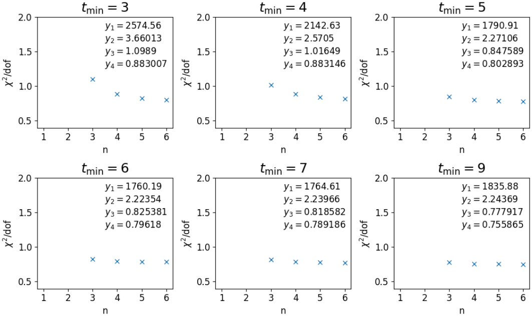

Figure: Fitting quality for coarse lattice.

Data Analysis: very coarse lattice

- Fine dataset has $a \approx 0.15\,\mathrm{fm}$ and $m_l \approx 0.036 m_s$.
- K meson appears to converg for $n \geq 2$.

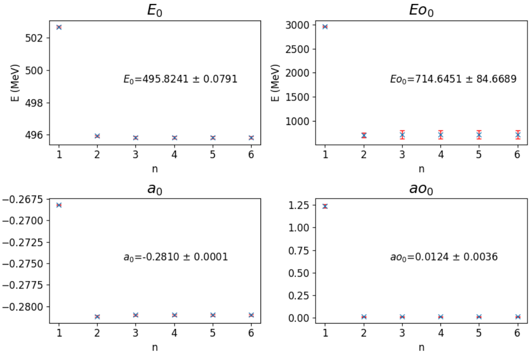

Figure: Results of fitting K meson dataset on a very coarse lattice.

Data Analysis: very coarse lattice

Combined fit quality plot shows that the fitting converged for a very limited selection of t_{min} and n.

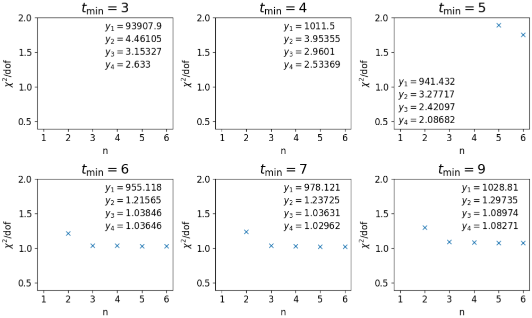

Figure: Fitting quality for very coarse lattice.

Results

For consistency between plots, $t_{\text{min}} = 6$ for D mesons, and $n = 4$ for all results, have been selected.

			E_0 (MeV)		a_0	
Label	a (fm)	m_l / m_s	K	D	K	D
Very Coarse	0.15	0.036	495.92(08)	1887.3(51)	−0.281 01(11)	−0.2286(65)
Coarse	0.12	0.2	528.28(15)	1893.0(15)	+0.608 53(37)	+0.1880(12)
Fine	0.09	0.2	530.16(24)	1889.3(11)	+0.139 78(12)	+0.1242(07)

Table: Results from fitting all datasets. Uncertainties given in parentheses are statistical.

These are reasonable mass values as literature[7] quotes $m_K \approx 498\,\text{MeV}$ and $m_D \approx 1865\,\text{MeV}$.

Chiral/Continuum Extrapolation

- Full extrapolation[8] requires 7 pairs of parameters.
- As we have very few datasets, we restrict to first order in each parameter.

$$m = m_{\mathrm{phys}} \left(1 + c_\delta \frac{m_l}{m_s}\right) \left(1 + c_{a^2} a^2\right)$$

Chiral/Continuum Extrapolation

The fit has $\chi^2/\mathrm{dof} = 0.23$ so we have a good quality fit, and importantly not an overfit.

	E_0 (MeV)		Predicted E_0 (MeV)	
Label	K	D	K	D
Very Coarse	495.92(08)	1887.3(51)	495.92(08)	1887.3(51)
Coarse	528.28(15)	1893.0(15)	528.31(25)	1892.8(15)
Fine	530.16(24)	1889.3(11)	530.13(24)	1889.4(11)
Chiral/Continuum limit	-	-	494.5(11)	1874.6(88)

Table: Comparison of observed and predicted mass values, including the extrapolated chiral/continuum limit values.

The extrapolated values are consistent with results from the literature[7], $m_{K^0} = 497.611(13)\,\mathrm{MeV}$ and $m_{D^0} = 1864.83(5)\,\mathrm{MeV}$ to within 3σ of our statistical uncertainty.

Remaining Work

- Compute partial decay widths, using amplitudes a_0.

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