Lab3

niedziela, 21 marca 2021 13:25

Oscar Teeninga

$$A = \begin{bmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{bmatrix}$$

Zadanie 1

$$\|A\|_F = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} |a_{ij}|^2\right)^{\frac{1}{2}} = \left(4^2+9^2+2^2+3^2+5^2+7^2+8^2+1^2+6^2\right)^{\frac{1}{2}} = \sqrt{285}$$

$$\|A\|_1 = \max_{j=1,2,\dots,n} \sum_{i=1}^{n} |a_{ij}| = \max\{4+3+8,\ 9+5+1,\ 2+7+6\} = 15$$

$$\|A\|_\infty = \max_{i=1,2,\dots,n} \sum_{j=1}^{n} |a_{ij}| = \max\{4+9+2,\ 3+5+7,\ 8+1+6\} = 15$$

$$|A| = \det(A) = 4\cdot5\cdot6 + 3\cdot1\cdot2 + 8\cdot9\cdot7 - 2\cdot5\cdot8 - 4\cdot7\cdot1 - 6\cdot9\cdot3 = 360$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A) = \frac{1}{\det(A)} \cdot \operatorname{cof}(A)^T$$

$$\operatorname{cof}(A) = \begin{bmatrix} \begin{vmatrix} 5 & 7 \\ 1 & 6 \end{vmatrix} & -\begin{vmatrix} 3 & 7 \\ 8 & 6 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 8 & 1 \end{vmatrix} \\ -\begin{vmatrix} 9 & 2 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 8 & 6 \end{vmatrix} & -\begin{vmatrix} 4 & 9 \\ 8 & 1 \end{vmatrix} \\ \begin{vmatrix} 9 & 2 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 4 & 2 \\ 3 & 7 \end{vmatrix} & \begin{vmatrix} 4 & 9 \\ 3 & 5 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 23 & 38 & -37 \\ -52 & 8 & 68 \\ 53 & -22 & -7 \end{bmatrix}$$

$$\operatorname{cof}(A)^T = \begin{bmatrix} 23 & -52 & 53 \\ 38 & 8 & -22 \\ -37 & 68 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{360}\begin{bmatrix} 23 & -52 & 53 \\ 38 & 8 & -22 \\ -37 & 68 & -7 \end{bmatrix}$$

$$\operatorname{cond}(A)_1 = \|A\|_1 \|A^{-1}\|_1$$

$$\|A\|_1 = 15$$

$$\|A^{-1}\|_1 = \max\left\{\frac{23+38+37}{360}, \frac{52+8+68}{360}, \frac{53+22+7}{360}\right\} = \frac{128}{360}$$

$$\operatorname{cond}(A)_1 = 15 \cdot \frac{128}{360} = \frac{16}{3}$$

$$\operatorname{cond}(A)_\infty = \|A\|_\infty \|A^{-1}\|_\infty$$

$$\|A\|_\infty = 15$$

$$\|A^{-1}\|_\infty = \max\left\{\frac{23+52+53}{360}, \frac{38+8+22}{360}, \frac{37+68+7}{360}\right\} = \frac{128}{360}$$

$$\operatorname{cond}(A)_\infty = \frac{16}{3}$$