Prescriptive project

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1 Context

As a football fan, I have always been fascinated by the industry surrounding this sport. With the upcoming Euro 2024, many businesses will face the challenge of producing various football-related items while optimizing their production processes. In light of this, I have undertaken a project to optimize the production of football equipment such as jerseys, football shoes, balls, shin pads, and goalkeeper gloves.

I am now the owner of a football equipment company and aim to optimize production for the upcoming month, either by maximizing profit, minimizing shortages or a compromise. Each equipment item can be produced in two different ways (Method 1 or Method 2) where each method requires different components. Additionally, there are three types of workers capable of producing specific equipment items. Each type of worker has varying productivity levels and salaries. Therefore, I need to hire the necessary number of workers for production. To ensure efficient production, I face several constraints, including demand constraints, supply constraints and worker constraints. These constraints must be carefully managed to achieve optimal production outcomes.

2 One real company

Nike could be a company facing a similar problem. As a leading manufacturer of athletic footwear, Nike must optimize its production processes to meet the high demand for football-related products, especially with major events like the Euro 2024. To maximize profit and efficiency, Nike needs to carefully manage its production resources, including labor and materials. Like in my scenario, Nike could face constraints related to demand fluctuations, supply chain limitations and workforce availability. Therefore, Nike's operational challenges align closely with my problem, making it a relevant example in this context. However, this challenge is not unique to Nike but is shared by many other sportswear companies such as Adidas, Decathlon, Under Armour, and others, who also face similar demands and constraints in optimizing their production processes and resource management.

3 Realistic data

In this project I have 3 objective functions:

- maximize the profit
- minimize the shortage
- a compromise between the two

My decision variables are:

- the quantity to produce for each product and through each method: X_{ij}

    where i represents the product and j the method.

- The quantity of workers to hire by type and their allocation: W_{ij}

  where i represents the Type of worker and j the product.

I have 5 products (football shirts, football shoes, football balls, shin pads and GK(goalkeeper) gloaves) that can be done in 2 ways (method 1 and 2). In Figure 1, you can see the components needed for each method of each product (in KG).

	Quantity per product									
	Football shirts		Football shoes		Football balls		Shin pads		GK gloaves	
	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
Polyester	0.2		0.12				0.05	0.05	0.03	
Polyurethane	0.02		0.015	0.015	0.07					0.03
Polyethylene							0.05			
Cotton		0.25								
Leather						0.07				
Resin				0.12				0.05		
Latex									0.1	0.1

Figure 1: This is the components needed for each method for each product *Note: Empty cases indicate that the component is not utilized*

These products are manufactured by workers. There are three types of workers, each specializing in the production of specific items. Consequently, each type of worker is only capable of producing the products they have been trained for:

- Workers 1 are able to produce X_a, X_b, X_d, X_e
- Workers 2 are able to produce X_a, X_b, X_c
- Workers 3 are able to produce X_d

They all works 8 hours per day and 20 days a month. However, each type of workers don't have the same productivity:

- Workers 1 are able to product 5 X_a or 2 X_b or 6 X_d or 6 X_e by day
- Workers 2: 4 X_a or 3 X_b or 4 X_c by day
- Workers 3: 8 X_d by day

Their productivity per month is represented in Figure 2.

	Productivity per month		
	Workers 1	Workers 2	Workers 3
Football shirts	100	80	
Football shoes	40	60	
Football balls		80	
Shin pads	120		160
GK gloaves	120		

Figure 2: Productivity per worker type per product each month. *Note: Empty cases result from workers being unable to produce this product*

Therefore, they do not receive the same salary:

- Workers 1 earn 40 CHF per hour so 6400 CHF per month
- Workers 2 earn 37 CHF per hour so 5920 CHF per month

- Workers 3 earn 34 CHF per hour so 5440 CHF per month

On the market, there is only 100 workers 1, 50 workers 2 and 40 workers 3. To ensure that we do not exceed the available workforce, we can impose a constraint for each type of worker i:

$$\sum_{j \in J_i} W_{ij} \leq \text{Availability}_i$$

where:

- $J_1 = \{a, b, d, e\}$
- $J_2 = \{a, b, c\}$
- $J_3 = \{d\}$

We also have to mention: $W_{ij} \geq 0$ and are integer as W are workers.

The demand for each product (i) can be represented as followed:

$$\sum_{j=1}^{2} X_{ij} \leq \text{Demand}$$

Where i represents the product (denoted by a, b, c, d, or e), and j represents the method.

We also have to mention: $X_{ij} \geq 0$ and are integer as X are products. The demand limit for each product is as follows: $a \rightarrow 5000$, $b \rightarrow 4000$, $c \rightarrow 3000$, $d \rightarrow 1000$, and $e \rightarrow 1500$. We also ensure that the quantity of product we produce does not exceed the productivity of the workers for that product. This constraint links the quantity of production to the workforce productivity. For each product i, we enforce the following condition:

$$\sum_{j=1}^{2} X_{ij} \leq \sum_{k=1}^{3} p_{ki} \times W_{ki}$$

where p_{ki} represents the productivity of worker k for producing product i (Figure 2).

We should also ensure that the quantity of materials needed are not exceeding the supply.Therefore we have an equation for each material (i=polyester, polyurethane...).The quantity needed of component i can be represented by:

$$\sum_{j=a}^{e} \sum_{m=1}^{2} q_{jmi} \times X_{jm} \leq \text{Quantity available}_i$$

where i represents the components, j the product and m the method.q_{jmi} represents the quantity of component i required for method m in the production of product j (Figure 1).

The profit function is defines by the revenue minus the costs.

$$\sum \text{Price product} \times \text{Quantity product}$$
$$-\sum \text{Quantity needed of each component} \times \text{Price component} - \sum \text{Number of workers per type} \times \text{Salary}$$

$$= \sum_{i=a}^{e} P_i \times (X_{i1} + X_{i2}) - \sum_{i=polyester}^{latex} \sum_{j=a}^{e} \sum_{m=1}^{2} q_{jmi} \times X_{jm} \times C_i - \sum_{j=a}^{e} \sum_{i=1}^{3} S_i \times W_{ij}$$

where P are the price of the products (Figure 3) , C the costs of the components (Figure 4) and S the salary of the workers (see end page 2)

	Price per product				
	Football shirts	Football shoes	Football balls	Shin pads	GK gloaves
Price/unit	70	200	40	15	35

Figure 3: Price of the products

	Supplier						
	Polyester	Polyurethane	Polyethylene	Cotton	Leather	Resin	Latex
Price/KG	10	7	3	8	7	5	4

Figure 4: Price of the components per KG

In other words, it's the revenue from selling products minus the cost of components required for production and the labor expenses for the required workforce.

4 Excel solver results

4.1 Maximize profit

After solving the problem, I have a profit of 358'830 CHF. This success could be attributed to the high pricing of football components, despite their relatively low production costs. Examining the decision variables reveals a preference for producing primarily football shirts and football shoes (Figure 5). Specifically, they favor method 2 for football shoes production. Consequently, these preferences contribute to a notable shortage of 6'000 units.

	Production									
	Football shirts		Football shoes		Football balls		Shin pads		GK gloaves	
	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
Production/method	2500	2000	0	4000	0	0	0	0	0	0

Figure 5: Production while maximizing profit

Hence, the allocation of workers is divided between the production of football shirts and football shoes. While all workers from group 2 are employed, a significant portion of workers from group 1 are also engaged (Figure 6). I produce this specific number of products due to limitations in both workforce availability and the quantity of supplies. These constraints may hinder me from increasing production further.

	Allocation of workers		
	Workers 1	Workers 2	Workers 3
Football shirts	45	0	0
Football shoes	25	50	0
Football balls	0	0	0
Shin pads	0	0	0
GK gloaves	0	0	0

Figure 6: Allocation of workers while maximizing profit

4.2 Minimize shortage

After solving the problem, the profit has decreased to 3'598.9 CHF. This is a large decrease. Nevertheless, the shortage has also decreased substantially, aligning with our objective. The shortage now stands at only 2'250 units. To minimize this shortage, production includes all types of products (Figure 7). However, the remaining shortage can be attributed to the limited availability of components (supply) and workers.

	Production									
	Football shirts		Football shoes		Football balls		Shin pads		GK gloaves	
	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
Production/method	2250	2000	0	2540	0	2960	0	1000	0	1500

Figure 7: Production while minimizing shortage

All workers from groups 1 and 2 have been employed and assigned to produce the majority of available products. Additionally, it's noteworthy that workers from group 1 delegate the production of shin pads to workers from group 3, while workers from group 2 delegate the production of football shirts to workers from group 1, primarily due to productivity considerations (Figure 8).

	Allocation of workers		
	Workers 1	Workers 2	Workers 3
Football shirts	43	0	0
Football shoes	44	13	0
Football balls	0	37	0
Shin pads	0	0	7
GK gloaves	13	0	0

Figure 8: Allocation of workers while minimizing shortage

4.3 Compromise

It could be good to have a good compromise. Therefore, we decided to minimizing the shortage while having a profit not under 200'000 CHF. This result with a profit of 201'677.9 CHF and a shortage of 2'700. We have augmented a lot the profit (compared to minimize shortage) while not increase too much the shortage. Production include all type of products using approximately all the methods (Figure 9).

	Production									
	Football shirts		Football shoes		Football balls		Shin pads		GK gloaves	
	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
Production/method	2200	2000	80	3820	1200	0	168	832	13	1487

Figure 9: Production while doing a compromise

There is exactly the same number of workers as for the minimization of the shortage. However, they are not allocated the same way. To maximize the profit, workers 1 and 2 concentrate on football shoes (Figure 10).

	Allocation of workers		
	Workers 1	Workers 2	Workers 3
Football shirts	42	0	0
Football shoes	45	35	0
Football balls	0	15	0
Shin pads	0	0	7
GK gloaves	13	0	0

Figure 10: Allocation of workers while doing a compromise

5 Sensitivity analysis

For the sensitivity analysis, I envisaged a scenario wherein one of our suppliers, the supplier of polyurethane, encounters significant production disruptions. This disruption may arise from an environmental catastrophe, resulting in the destruction of a large part of the supplier's production plans, or from a significant issue with their delivery trucks, thereby impeding their ability to fulfill our orders. As a result, the amount of polyurethane supplied by my supplier has decreased by 90%. Beforehand, his production capacity was 300 KG, thus he now possesses only 0.1*300 = 30 KG.
In such a circumstance, where our reliance on this supplier is apparent, there exists a potential risk. Other suppliers, particularly those offering substitutable goods, may capitalize on our limited alternatives. Knowing

we rely on them, they might raise prices since they know we urgently need their materials. In my scenario, the cotton supplier will raise the price to 10 CHF/kg, recognizing cotton as a substitute material for polyurethane in football shirt production. Similarly, the leather supplier will adjust the price to 9 CHF/kg, acknowledging leather's role as a substitute material for football manufacturing.

5.1 Maximize profit scenario

In this scenario, the profit will decrease to 206'249.5 CHF, accompanied by a significant shortage of 10'520 units. My focus will remain on producing football shirts, primarily utilizing method 2 (cotton-based) due to insufficient polyurethane for method 1. The remaining polyurethane will be allocated to football shoe production. The workforce will consist of both type 1 and type 2 workers, with type 1 focusing on football shirt production and type 2 dedicated to football shoe manufacturing.

5.2 Minimize shortage scenario

In this scenario, I'll end up with a negative profit of -102'440 CHF and a shortage of 5'000 units. I'll opt to produce a mix of products, using method 2 for both football shirts (cotton-based) and football balls (leather-based). This decision might contribute to the significant deficit due to the increased prices of cotton and leather. Additionally, this approach will require a considerable workforce.
In this situation, finding a balance between maximizing profit and minimizing shortages could be advantageous. Striking a compromise ensures that customers aren't left disappointed by significant shortages while also preventing excessive financial losses.

6 Challenge extension

One extension could have been to enhance the production optimization process by incorporating seasonal demand variations. By introducing constraints that reflect fluctuations in demand throughout the year, such as increased demand for jerseys and footballs during the football season or major tournaments, the optimization model can better align production schedules with market dynamics. This could involve adjusting production quantities for different items based on anticipated demand levels during peak periods, ensuring that resources are allocated efficiently to meet customer requirements.
Another extension could involve integrating constraints related to environmental regulations and sustainability standards. This aspect is becoming increasingly significant as environmental concerns become more prominent. Ensuring compliance with waste disposal regulations, emission limits, and other environmental requirements mandated by regulatory bodies is of utmost importance. With consideration for the energy consumption and pollution generated during production processes, we may encounter constraints and obligations imposed by government regulations. Therefore, we have to explore alternative methods of production to determine if we can still meet the demand while minimizing pollution.

References

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