Neil Chriss sets out to codify the game theory of trading

The co-author of the benchmark Almgren-Chriss model has updated his thinking on market impact

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The study of so-called market impact – how an investor's trades move market prices – is a field Neil Chriss knows well. He was one half of the duo that more-or-less invented it. In 2000, together with **Robert Almgren**, Chriss developed the foundational model for what became a Wall Street cottage industry of formulating **execution algorithms**.

The Almgren-Chriss model describes how trading a security moves its price in the short- and the long-term. In the years that followed, others built on the work. Chriss, meanwhile, went on to set up the quantitative investing unit at SAC Capital Advisors (now Point72), then to launch his own hedge fund and more recently to work for a short time as co-CEO of Paloma Partners.

Now Chriss has returned to publishing, and to his earlier research. In three working papers released in recent weeks, Chriss melds his original market impact ideas with game theory.

One market participant describes this as the biggest idea in the field in years. Most market impact models act as if traders work in a vacuum – pushing prices up or down themselves, but with no allowance for what others are doing. Chriss has set out to codify how a portfolio manager might trade optimally, bearing in mind that competitors are trying to do the same. The framework he has created gives portfolio managers “a dispassionate, repeatable, analysable way of trading”, he says.

Some of the things the maths brings into focus are intuitive, others surprising. Chriss shows that consolidating trading in a so-called centre book, for example, can increase rather than reduce trading implementation costs. This means that firms should sometimes disguise how many of their traders are pursuing an idea, making it seem to others that more – or fewer – players are making the same bet.

“It all depends on the details,” Chriss says. But it’s not necessarily the details that traders might expect, such as how fast alpha in a trade is decaying or the volume of trading getting done. In many scenarios, what truly matters is how many independent traders are competing for a stock.

The framework sounds like a blueprint a machine could follow, and that’s precisely what Chriss is contemplating. Chriss says he is considering the launch of a new venture in which the role of portfolio managers would be largely automated – something like a 'pod shop' without PMs.

Game plan

Chriss joins a body of practitioners who have applied game theory to markets. Trading firm **Susquehanna trains its employees** in the field. Quants at the Oxford-Man Institute of Quantitative Finance used game theory to analyse whether machine learning algorithms might **learn to collude**. Others have called for its use in **modelling geopolitical risk**. The theory itself, developed largely by Nobel winner John Nash in the 1950s, sets out the maths of optimising behaviour in games where participants compete over many iterations.

Chriss's **first paper**, then, works through the best responses to known ways that market participants trade. He shows that if a rival trades cautiously, for example, the best response is to trade eagerly – to get ahead of the competitor. A PM's best strategy versus a fast-trading, more aggressive rival, can be either to trade eagerly or cautiously depending on the pace of the adversary's own trading and the level of permanent impact on price. Chriss shows that in some cases the best choice is to sell a stock short to begin with, and buy heavily later.

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Neil Chriss

Several of the conclusions from the work tally with common trading strategies. Others, though, cast doubt on market heuristics. The industry's default strategy, VWAP – where PMs trade in line with a stock's volume-weighted average price – proves to be the wrong go-to choice, Chriss says.

"To trade VWAP, you have to assume nobody is trading more aggressively – that no-one's getting ahead of the trade. That's definitely not true." A better defensive approach would be to trade eagerly, the maths shows, though doing so means taking on market risk, which would have to be hedged.

In the **second paper**, Chriss formulates a version of the framework using quantitative programming, a mathematical technique that makes the necessary computations simpler and quicker. Then, in the **third**, he analyses the behaviour of the market as a whole and considers the implications of the findings for an individual firm. For this, Chriss assumes a game-theoretic equilibrium, which is to say, that all players reach a balanced state in which their strategies are optimised.

Here the results took Chriss by surprise. Simulations to calculate potential savings for a firm that centralised trading showed costs going up. But slicing orders into ever smaller pieces eventually caused costs to go up, too. "When you consolidate, you become the big trader that everyone else can pick off," Chriss says. "But if the rest of the market believes there are a million little traders out there, the market has a defence for that." Centralisation "requires a strategy", he says. "It turns out there's an optimal middle ground of how much you should split your order up, and it depends on the details."

The sweet spot, according to the maths is to appear to have roughly the same number of traders as the rest of the market. "If there are 30 traders competing for a stock and 10 in the firm, then the firm should consolidate its trades and split them three for one, so that what they represent to the market is 30 traders, matching what's outside the firm," Chriss says. "If the firm has 20 traders and the rest of the market is 10, then the firm should consolidate its trades one for two and represent 10 traders. These are the optimal centralising policies."

Masking the number of independent traders changes the number of players in the game, Chriss explains. On one hand, adding more 'players' increases implementation costs across all participants. On the other, how the burden is shared depends on the fraction of the trade taken up by individual traders. If the firm represents a large fraction of the trade, for example, its share of the total cost will generally decrease. The opposite would hold if the firm reduced the apparent number of traders.

The scale of the effect grows with higher rates of permanent impact, cutting implementation costs by as much as 10% in some of Chriss's simulations.

None of this is to suggest that markets exist in a state of optimal equilibrium, he acknowledges. But studying the maths allows a PM to understand their rivals' best moves. Chriss draws a parallel with poker players that calculate their opponent's so-called GTO — their 'game theoretic optimal' plays. "You need to know what the baseline is, to check when people aren't playing GTO and to know the proper response."

At the same time, investors still have to figure out through "detective work" what they can about who else is trading and in what size, Chriss says. Inferring such details is difficult, he adds. But doing so, even approximately, is powerful. "Our aim is to provide a framework for developing trading strategies to be applied in the 'real-world' taking the actions of other traders into account," Chriss states in the third paper. "Of course, trading in the real-world is what mathematicians call a 'messy real-world problem' and such problems require many tools applied in combination to reach a satisfactory conclusion… Nevertheless, the direction set forth here… provides a set of tools that provide a first-line approach."

Theory into practice

The treatment of markets as if participants see everything that others are doing is clearly very different from how markets truly work, says Yuriy Nevmyaka, who heads machine learning research at Morgan Stanley and provided feedback to Chriss on the research. "If everybody played poker with all cards up, the strategies would be very different."

That said, the work is a "big idea" and "bold", he says, and the findings were surprising enough that he was reluctant to believe them to begin with. The next step will be to apply the theory in situations where such information is unavailable, Nevmyaka says, suggesting that machine learning might be used to estimate the number of other players in a trade.

Alvaro Cartea, director of the Oxford-Man Institute and professor of mathematical finance at Oxford University, who has also published widely on market impact, echoes the point. "It's impossible to know what other people are doing," he says. "But these types of models can give you an anchor point to say how to respond in a market where others are trading. You expect to be pulled towards equilibrium. A stylised model, with a few assumptions, tells you roughly where the market should be going."

Chriss, for his part, reckons market impact often holds the key to a PM's success. While managing SAC's centre book, he says, he realised that some portfolio managers made more money than others even when trading the same stocks and ideas. "The people that were amazing got into their trades a little bit cheaper and got out a little bit richer," he says. "Once you started a trading position, it really mattered what your execution price was."

Editing by Kris Devasabai