Single Asset Optimal Allocation

Allocation

1. **Allocation** is done by using MVO for a single asset.

$$E[R] \cdot \theta - \frac{\lambda}{2}\theta^2\sigma(R)^2 - tc|\Delta\theta|$$

where $\lambda =$ Risk Aversion, where $\Delta\theta = \theta - \theta_0$

2. **Portfolio Optimization -**
   Vol Scaling It is typical to consider scaling returns forecasts. This is covered in Grinold-Kahn and their various publications. Why? Because the volatility of out-of-sample forecasts is far smaller than the volatility of returns. Try it for yourself. Two methods:
   a. Find a model for forecasts $\hat{R}$ and scale by the historic volatility of returns to forecast, i.e., $\frac{\sigma(R)}{\sigma(\hat{R})} \cdot \hat{R}$
   b. Use a multiplier, like η here i.e.,
   $$\eta E[R] \cdot \theta - \frac{\lambda}{2}\theta^2\sigma(R)^2 - tc|\Delta\theta|$$

3. **Other hyperparameters** - There are several hyperparameters for portfolio optimization,(possible) multipliers for transaction costs, for risk aversion, for alpha, etc. Noting that even a perfect forecast could result in negative PnL with the wrong hyperparameters.
4. For purposes of this document we will assume $\eta=1$

5. **Solutions**
   Solutions to this optimization are explicit.
   Let θ_0 be the previous period's allocation. First let $\mu = E[R]$
   $\theta^{upper} = \frac{\mu+tc}{\lambda\sigma^2}$,$\theta^{lower} = \frac{\mu-tc}{\lambda\sigma^2}$, $\theta^* = \frac{\mu}{\lambda\sigma^2}$
   Where θ^* is the optimal allocation when there are no transaction costs.

$$\theta = \theta^{upper} \ if \ \theta_0 > \theta^{upper}$$

$$\theta = \theta^{lower} \ if \ \theta_0 < \theta^{lower}$$

$$\theta = \theta_0 \ if \ \theta_0 \in [\theta^{lower}, \theta^{upper}] \text{ also known as the } \textit{no-trade-zone}$$

The no-trade-zone is centred around θ^*, the optimal allocation if there were no transaction costs. Effectively, the change in allocation must *pay* for transaction costs

Scaled with Hyperparameters. We note that, in terms of hyperparameters, this is particularly simple - we have a shift multiplier and a width multiplier

$\nu_s = \frac{\eta}{\lambda} \wedge \nu_w = \frac{1}{\lambda}$

with

$\theta = \nu_s\left(E\frac{[R]}{\sigma^2}\right) \pm \nu_w\left(\frac{tc}{\sigma^2}\right)$

NOTE: What is tc?

If we can assume we are small and can always trade at the top-of-book, then

$$tc = \frac{|ask-bid|}{2}$$

We are assuming that we are measuring all prices at mid, so we incur only half the bid-ask spread for each transaction.

If it is a large order size then we will have to measure the possible impact and it is typical to add a term of the form

$$t|\Delta\theta|^a$$

For 1<a<= 2. Economists like a=2 since it works well with Kyle's model. Practitioners tend to take a=1.5 ('square root' impact). The problem with all impact based frameworks is they do not have no trade zones. In reality it should be feasible to consider costs to be linear for small enough size and then a power of 1.5 if the size is large enough to move the market.