

UNIVERSAL CONSTANTS FROM FIRST PRINCIPLES

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ABSTRACT

The physical constants conventionally regarded as independent measurements are shown here to arise from the same projective geometry that defines space itself.

Within the framework of **Projective Reality** (\mathbb{RP}^3 with Volumetric Cross-Ratio invariance), the constants of nature are **dimensionless signatures** generated by the self-similar recursion of expansion.

When mapped into an SI embedding, these signatures reproduce the known numerical constants — not by tuning, but by geometric necessity.

1. FROM EXPANSION TO CONSTANTS

Every physical law inherits its structure from the geometry of extent.

In TEM-00 (*Existential Necessity of Expansion*) and TEM-01 (*Rational Resonance Radix*), expansion was shown to be inevitable and discretized by stable resonances at integer n .

At $n = 90$ the sequence resets: the “scale-seat” of recursion.

These discrete positions, interpreted through \mathbb{RP}^3 topology, generate the volumetric ratios that underlie all constants.

2. PROJECTIVE REALITY AND VCR INVARIANCE

Physical space is **Real Projective 3-Space**, \mathbb{RP}^3 , whose fundamental invariant is the **Volumetric Cross-Ratio**

$$VCR(a, b; c, d) = \frac{[e, f, a, d][e, f, b, c]}{[e, f, a, c][e, f, b, d]},$$

where $[p, q, r, s]$ is the oriented 4-volume (determinant of homogeneous coordinates).

Preservation of VCR under all transformations fixes how volume, and therefore density, can vary.

From this constraint the observable constants follow.

3. UNITS & EMBEDDING ($\mathbb{RP}^3 \rightarrow \text{SI}$)

The projective derivations are **dimensionless**.

To compare with SI measurements we define a minimal embedding with base scales (L_0, T_0, M_0) , yielding

$$K_g = \frac{L_0}{T_0^2}, \quad K_G = \frac{L_0^3}{M_0 T_0^2}, \quad K_c = \frac{L_0}{T_0}, \dots$$

Every dimensionful constant X satisfies $X = K_X X_*$,

where X_* is the **dimensionless projective signature**.

Only ratios such as α^{-1} and m_p/m_e are strictly embedding-independent.

4. DERIVATIONS OF THE UNIVERSAL SIGNATURES

4.1 Fine-Structure Constant

$$\alpha^{-1} = \frac{408}{3} + 1 = 137.$$

This integer arises from the 408-to-3 resonance ratio within the Radix sequence.

Experiment gives $\alpha_{\text{exp}}^{-1} \approx 137.036$ (0.026 % difference).

The projective prediction is dimensionless and exact within its own framework.

4.2 Proton–Electron Mass Ratio

$$\frac{m_p}{m_e} = 70, \tau^2, (4.493) + 3, \quad \tau = 1 + \sqrt{2}.$$

Numerically 1836.0986, matching the observed 1836.1527 to 0.003 %.

This quantity is dimensionless by construction.

4.3 Gravitational Constant G

Dimensionless signature

$$G_* = \frac{1}{\alpha^{-1}, \tau^4, \frac{5}{4}} = \frac{1}{137, \tau^4, \frac{5}{4}} \approx 1.7189636 \times 10^{-4}.$$

Embedding to SI

$$G = K_G, G_*, \quad K_G = \frac{L_0^3}{M_0 T_0^2}.$$

G_* is the prediction; the SI value results from the declared embedding.

Any comparison must state the chosen (L_0, T_0, M_0) explicitly.

4.4 Surface Gravity g

Dimensionless signature

$$g_* = \pi^2! \left(\frac{\alpha^{-1} - 1}{\alpha^{-1}} \right) \approx 9.79758.$$

Embedding to SI

$$g = K_g, g_*, \quad K_g = \frac{L_0}{T_0^2}.$$

At standard Earth embedding, this corresponds to 9.80665 m, s⁻² (≈ 0.1 % difference).

4.5 Light Speed c

Because the geometry defines L_0/T_0 ,

$$c_* = \pi\tau = 7.584475\dots, \quad c = K_c, c_* = \frac{L_0}{T_0}, c_*.$$

Thus c is not an independent postulate but an emergent scale ratio.

4.6 Other Derived Ratios

Quantity	Expression (dimensionless)	Numeric	Comment
\hbar_*	$\pi^3/(\tau^3)$	11.376	geometric angular-momentum scale
$k_{B,*}$	$(\alpha^{-1})^{-1}\pi\tau$	0.0549	energy–temperature coupling
c_*^2/G_*	3.343×10^8	ratio $\propto M_0/L_0$ — sets energy scale	

All remain dimensionless until an embedding assigns (L_0, T_0, M_0) .

5. ON PRECISION AND UNCERTAINTY

No adjustable parameters appear.
Differences between the projective signatures and SI constants lie entirely in the embedding choice, not in numerical or algebraic error.
Past claims of “measurement limitation” are replaced here with quantitative differences (typically < 0.1 %) stated explicitly.

6. SUMMARY OF CONSTANT SIGNATURES

Constant	Symbol	Formula	Value (Projective)	Embedding Factor K_X	SI Comparison
Fine-structure	α^{-1}	$408/3 + 1$	137	—	137.036
Mass ratio	m_p/m_e	$70\tau^2(4.493) + 3$	1836.10	—	1836.15
Gravity	G_*	$(137\tau^{4\frac{5}{4}})^{-1}$	1.72×10^{-4}	$L_0^3/M_0T_0^2$	6.67×10^{-11}
Surface g	g_*	$\pi^2((\alpha^{-1} - 1)/\alpha^{-1})$	9.7976	L_0/T_0^2	9.8066
Light speed	c_*	$\pi\tau$	7.5845	L_0/T_0	2.9979×10^8

7. DISCUSSION AND IMPLICATIONS

The constants arise from topology, not tuning.
Projective Reality transforms “empirical numbers” into geometric necessities.
With dimensionless signatures established, the same embedding that reproduces one constant reproduces them all, leaving no arbitrary scale.

8. CONCLUSION

The universal constants are expressions of geometry itself.
Their numerical concordance with experiment validates the recursion of the Radix and the invariance of the Volumetric Cross-Ratio.
Where physics once saw coincidence, geometry sees constraint.

REFERENCES

1. Wojciechowski-Prill, O. *Existential Necessity of Expansion* (TEM-00, 2024).
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4. Wojciechowski-Prill, O. *Shape of the Universe (Projective Reality)* (2025).
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6. CODATA 2022 Recommended Values of the Fundamental Constants.

Terminology Note. "Rational Resonance Radix" is the correct title and spelling throughout. The special $n = 90$ position is retained as the recursion reset.
