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Inlämning G.2

a, Sats 1.14: $\sum_{j=1}^{n} ar^{j-1} = a\frac{1-r^n}{1-r}$, $r \in \mathbb{C}$, $n, k \in \mathbb{Z}$

r i polär form: $e^{i\theta}r$

$$\sum_{j=1}^{n} a(e^{i\theta}r)^{j-1} = a\frac{1-(e^{i\theta}r)^n}{1-e^{i\theta}r} \quad *$$

1. Bassteg: $n=1$

$$\left.\begin{aligned} VL &= a(e^{i\theta}r)^{1-1} = a \\ HL &= a\frac{1-(e^{i\theta}r)^1}{1-(e^{i\theta}r)} = a \end{aligned}\right\} \text{OK!}$$

2. Induktionsantag: anta att $*$ gäller då $n=p$, $p \in \mathbb{Z}_+$, d.v.s.

$$\sum_{j=1}^{p} a(e^{i\theta}r)^{j-1} = a\frac{1-(e^{i\theta}r)^p}{1-e^{i\theta}r}$$

3. Induktionssteg: visa att $*$ gäller då $n=p+1$ d.v.s.

$$\sum_{j=1}^{p+1} a(e^{i\theta}r)^{j-1} = a\frac{1-(e^{i\theta}r)^{p+1}}{1-e^{i\theta}r}$$

enl 2.

$$VL = \sum_{j=1}^{p+1} a(e^{i\theta}r)^{j-1} = a\frac{1-(e^{i\theta}r)^p}{1-e^{i\theta}r} + a(e^{i\theta}r)^p = a\left(\frac{1-(e^{i\theta}r)^p + a(e^{i\theta}r)^p - (e^{i\theta}r)^{p+1}}{1-e^{i\theta}r}\right)$$

$$= a\frac{1-(e^{i\theta}r)^{p+1}}{1-e^{i\theta}r} = HL$$

4. Induktionsprincipen ger att $*$ gäller för alla $n \in \mathbb{Z}$ & $r \in \mathbb{C}$

b, $\sum_{k=0}^{6} e^{2\pi i k/7}$ kan skrivas med formeln $\sum_{j=1}^{n} ar^{j-1}$:

$$\sum_{j=1}^{7} e^{\frac{2\pi i}{7}(j-1)} = \frac{1-\left(e^{\frac{2\pi i}{7}}\right)^7}{1-e^{\frac{2\pi i}{7}}} = \frac{1-(e^{2\pi i})}{1-e^{\frac{2\pi i}{7}}} = \frac{1-1}{1-e^{\frac{2\pi i}{7}}} = 0$$

enl Euklides identitet