Variance measurements – EFTS 2025

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Warning

For this lab session, you will use a Graphical User Interface version of SigmaTheta (see § Resources, page 6) which is still under test. Please be lenient if you encounter any bugs while running this version, and provide feedback to the authors for correction!

1 Getting started

At start, your configuration screen should look like figure 1. In this case, close the window “`untitled – GNU Radio Companion`” which will be used in another laboratory session.

If this is not the case, open two file manager windows, one containing the datasets, the other which will be your working folder (“`playground`” in the figure) and make sure to copy the content of “datasets” to “playground” to always keep a safe copy of the original files. In the parent folder (“`variance_lab`”), there is the PDF file corresponding to this booklet “`variance_lab_2025.pdf`” that you already have since you are reading these lines. You can read it by using the command line :

```
xpdf ../variance_lab_2025.pdf
```

On the other hand, all the data are in the folder “`dataset`”.

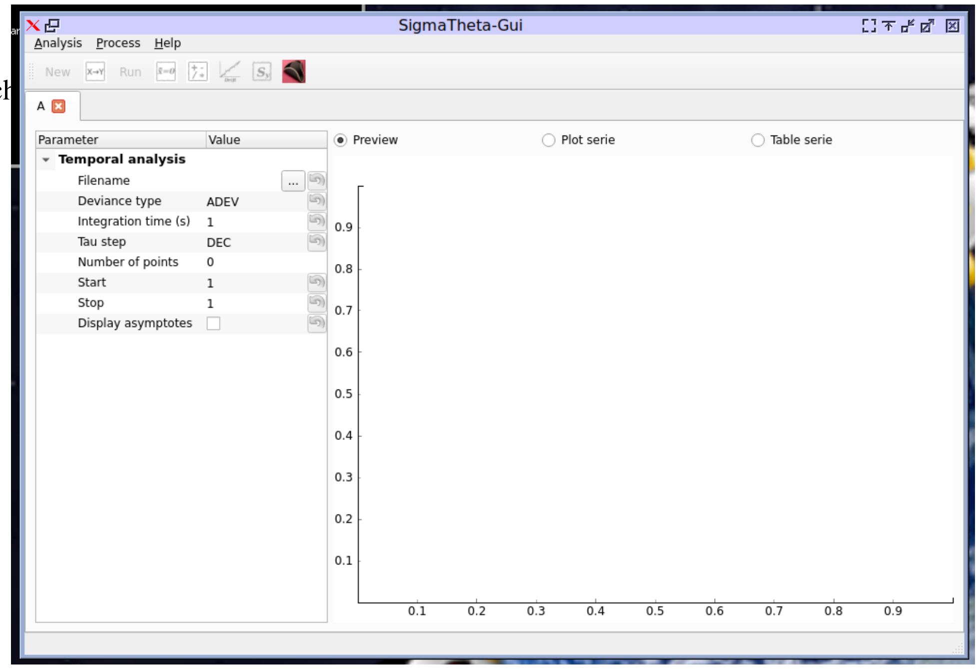


Figure 1: Startup screen graphical user interface.

You can use “Shift-Alt” to switch between English (“QWERTY”) and French (“AZERTY”) keyboard layouts.

2 Example of Allan variance computation

The Allan variance (AVAR) computation is quite simple:

$$\sigma_y^2(\tau) = \frac{1}{2}\left\langle (\bar{y}_2 - \bar{y}_1)^2 \right\rangle , \tag{1}$$

where $< \cdot >$ stands for mathematical expectation. In order to prove this assertion, let us compute the Allan variance with a spreadsheet software (`gnumeric`) on a very small 10 item-file: `tuto_Allan.ykt`.

1. Open a terminal (fourth icon on the menu bar at the bottom of the window manager named "XTerm") and go to the `playground/` folder (`cd ~/variance_lab/playground`).
2. Copy the `tuto_Allan.ykt` file from the `dataset/` folder to the `playground/` folder:
   `cp ../dataset/tuto_Allan.ykt .`
3. First, have a look on this file:
   `less tuto_Allan.ykt` exhibits two columns after a header defining the measurement conditions
4. Load it with `gnumeric`: in the terminal, after quitting `less` with `q`, type
   `gnumeric tuto_Allan.ykt &`
5. Use the definition of AVAR (1) in `gnumeric` to compute the estimator for $\tau = 1, 2, 3$ and 5 s.
6. Compare these Allan variance results with the Allan deviation results obtained by the command-line software `ADev`:
   `ADev tuto_Allan.ykt`
   Are they compatible?

3 About the software `SigmaTheta-gui`

In the command line window, type "`SigmaTheta-gui`" to run the software. You will get a new window like the figure 2. The important features are

- the top banner which displays several commands (e.g. '`New`' for adding a new frame, '`Run`' for computing the deviation, etc.)
- the left window which allows modifying some parameters (e.g the deviation type, the display of asymptote, etc.)
- the main right window showing either a plot of the input data (`Preview`), a plot of the results (`Plot serie`) or a numerical table of the results (`Table serie`).

More details will be given in the following.

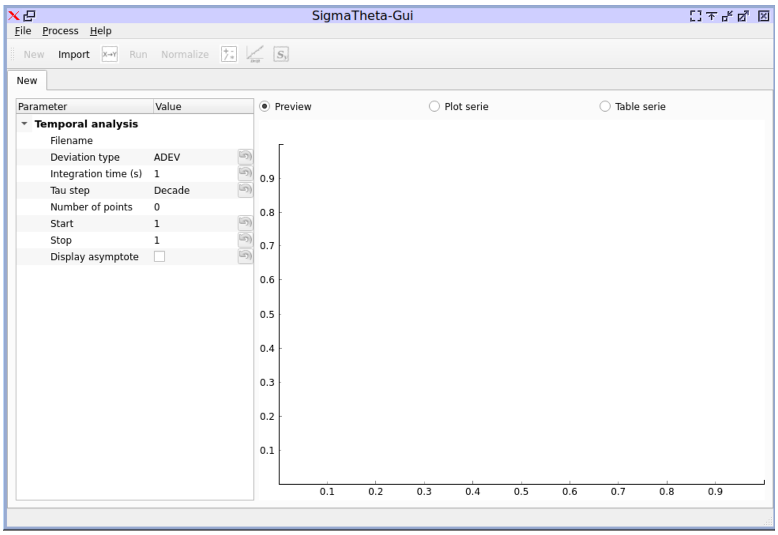


Figure 2: The `SigmaTheta-gui` graphical user interface.

4 Analysis of the file `Cs_hour.ykt`

- Clock: Cs clock
- Data: frequency deviation samples
- Sampling period: 1 hour
- Number of samples: 100 000

4.1 Preliminary observation

Import the file `variance_lab/datasets/YkGraph Cs_hour.ykt` and look at the preview.

4.2 Frequency stability assessment

1. Check that the `Deviation Type` is set to `ADEV` and compute the Allan deviation of this file by clicking on the `Run` button.
2. Observe the asymptote coefficients which are displayed on the graph if the box `Display asymptote` is enabled. Be aware that these asymptotes fit the ADev curve and not the AVar curve...
3. Translate these coefficients in terms of noise levels of the Power Spectral Density (PSD) thanks to Table 1.
4. Observe the PSD by clicking on the S_y button on the top banner. Is it compatible with the noise levels that you estimated ?

5 Analysis of the file `HM_minute.xtt`

- Clock: hydrogen Maser
- Data: time errors
- Sampling period: 1 minute
- Number of samples: 200 000

Click on the `New` button to get a new frame.

5.1 Preliminary observations

1. Import and observe `HM_minute.xtt`.
2. Convert the phase data into frequency data thanks to the `X→Y` button and observe them.
3. What do you observe?

5.2 Frequency stability assessment

1. Plot the Allan deviation of the frequency data.
2. Determine the power law asymptotes.
3. Estimate the coefficients of the power laws.
4. Calculate the corresponding noise levels of the PSD.
5. What is the meaning of the τ^{+1} asymptote?
6. Compute the drift level from the corresponding asymptote.

Table 1: Response of the Allan variance, Modified Allan variance and Hadamard variance for the different types of noises and a linear drift. f_h represents the high cut-off frequency.

$S_y(f)$ (s)	$\sigma_y^2(\tau)$	Mod$\sigma_y^2(\tau)$	$\sigma_H^2(\tau)$
$h_{-2}f^{-2}$	$\frac{2\pi^2 h_{-2}\tau}{3}$	$\frac{11\pi^2 h_{-2}\tau}{20}$	$\frac{\pi^2 h_{-2}\tau}{3}$
$h_{-1}f^{-1}$	$2\ln(2)h_{-1}$	$\frac{[27\ln(3) - 32\ln(2)]\,h_{-1}}{8}$	$\frac{[8\ln(2) - 3\ln(3)]\,h_{-1}}{2}$
$h_0 f^0$	$\frac{h_0}{2\tau}$	$\frac{h_0}{4\tau}$	$\frac{h_0}{2\tau}$
$h_{+1}f^{+1}$	$\frac{[1.038 + 3\ln(2\pi f_h \tau)]\,h_{+1}}{4\pi^2\tau^2}$	$\frac{[24\ln(2) - 9\ln(3)]\,h_{+1}}{8\pi^2\tau^2}$	$\frac{5\,[0,964 + \ln(\pi\tau f_h)]\,h_{+1}}{6\pi^2\tau^2}$
$h_{+2}f^{+2}$	$\frac{3h_{+2}f_h}{4\pi^2\tau^2}$	$\frac{3h_{+2}}{8\pi^2\tau^3}$	$\frac{5h_{+2}f_h}{6\pi^2\tau^2}$
Drift: $y(t) = D_1 t$	$\frac{1}{2}D_1^2\tau^2$	$\frac{1}{2}D_1^2\tau^2$	0

5.3 Drift removal and variance insensitive to drifts

1. Estimate the slope of the linear frequency drift by clicking on the  button and note the drift parameter ‘`degree 1`’ in the ‘`Drift coefficients`’ section of the left window.
2. Is it compatible with the drift estimation obtained from the ADev asymptote?
3. Plot the Allan deviation of `HM_without_drift.ykt`.
4. Estimate the $\tau^{1/2}$ coefficient and convert it into the corresponding noise level. What do you observe?
5. in a new frame, reload `HM_minute.xtt` and convert it into frequency data.
6. Plot the Hadamard deviation of these frequency data (change the `Deviation type` in the left window).
7. Estimate the $\tau^{1/2}$ coefficient and convert it into the corresponding noise level. What do you observe?
8. What do you think of the Allan variance as a tool for estimating drifts?
9. Is the drift removal method recommended?

6 Analysis of the file `VCO_1m3.xtt`

- Clock: VCO
- Data: time errors
- Sampling period: 1 ms
- Number of samples: 200 000

6.1 Preliminary observations

Observe `VCO_1m3.xtt` and transform it into frequency deviation data.

6.2 Allan variance vs Modified Allan variance

1. Plot the Allan deviation of the data.
2. Determine the power law asymptotes.
3. Estimate the coefficients of the power laws.
4. Calculate the corresponding noise levels of the PSD.
5. Perform the same calculation with the Modified Allan deviation.

What is the advantage of this variance? Estimate carefully the white PM level as well as the flicker PM level.

7 Analysis of a three-cornered system

Unfortunately, the Graphical User Interface is not available for this section and the study must be conducted by using command lines (sorry for mouse-click fans!).

- Clock: 3 Cryogenic Sapphire Oscillators (A, B and C)
- Data: frequency deviation samples
- Sampling period: 1 s
- Number of samples: 469 999
- Measuring device: K&K (Lange-Electronic GmbH)
- Files:

  `KnK_AB.ykt:` comparison of B versus A ($\bar{y}_B - \bar{y}_A$)
  `KnK_BC.ykt:` comparison of C versus B ($\bar{y}_C - \bar{y}_B$)
  `KnK_CA.ykt:` comparison of A versus C ($\bar{y}_A - \bar{y}_C$)

7.1 Preliminary observations

Observe the Allan deviation of these 3 files by using `SigmaTheta`:
`SigmaTheta KnK_AB.ykt KnK_AB.adev`
What do you think of the stability of the CSOs A, B and C?

7.2 Using the Groslambert Covariance

1. Compute the Groslambert Deviation (square root of the Groslambert Covariance) of CA and AB and store the result in a file (e.g. `KnK_A.gcod`):
   `GCoDev KnK_CA.ykt KnK_AB.ykt > KnK_A.gcod`

2. Observe the results:
   `cat KnK_A.gcod`
   What is the sign of the GCOD? Why?

3. Compute the Groslambert Deviation of AB and BC and store the result in a file (e.g. `KnK_B.gcod`).

4. Compute the Groslambert Deviation of BC and CA and store the result in a file (e.g. `KnK_C.gcod`).

5. Observe the stability of each clock on the same graph and store the result in a file (e.g. `KnK_ABC.gcod`):
   `DevGraph KnK_ABC.gcod -KnK_A.gcod -KnK_B.gcod -KnK_C.gcod`
   (the dash "-" before a file name causes `DevGraph` to take the opposite of the data from this file).

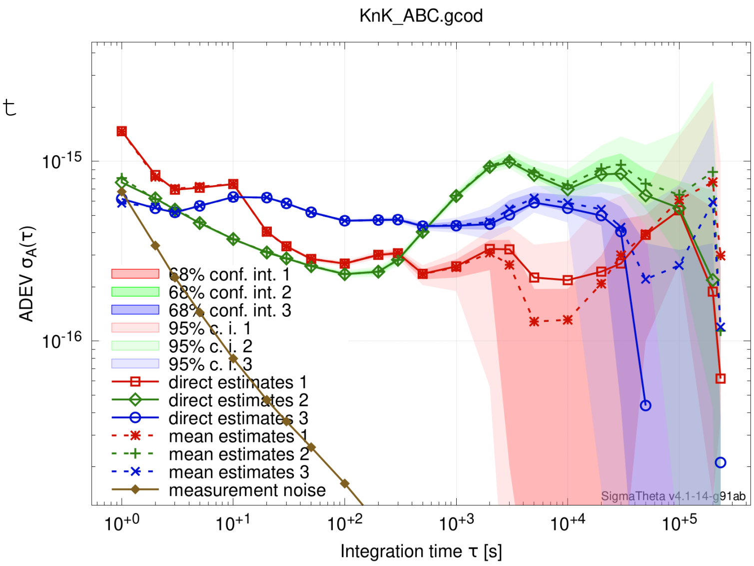
6. What is the meaning of the negative CoDeviations (e.g. C at $\tau = 10^5$ and $2 \cdot 10^5$ s).

The Groslambert Deviations can also be computed all at once.

7. You can also type directly:
   `3CorneredHat KnK_AB.ykt KnK_BC.ykt KnK_CA.ykt KnK_ABC.gcod`
   (it could take a while...).

8. The different colored areas correspond to the uncertainty domains for each clock stability. Which clock (red, green or blue) is the most stable at $\tau = 10^2$ s?

9. Same question at $\tau = 10^4$ s? What is the 95% confidence interval of the ADEV estimate of the red clock? What is your conclusion regarding this clock?



Resources

SigmaTheta is a collection of numerical programs for time and frequency metrology. It is free software, you can redistribute it and/or modify it under the terms of the *CeCILL* License. It may be downloaded from the following `git` repository:

`git clone https://gitlab.com/fm-ltfb/SigmaTheta`

Contributors are welcome!
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