

# STAT 421 Homework 14

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1)

As the state  $i$  increases the birth rate increases linearly. Since the rate of death is a square function, the larger state  $i$ , the larger the death rate. When the state is above 25 the death rate is larger than the birth rate, which means the population is dying at a faster rate than it is growing. When  $i$  is under 25, the birth rate is larger than the death rate and the population is growing more than it is dying. This process will grow between 0 and 25, and grow the fastest at around state 10. The process will decay anywhere above state 25. It will decay the fastest when  $i$  tends to infinity.

2)

```
t <- 0
i <- 5
lambda_i <- (1/5)*i + 1
mu_i <- (1/100)*i^2

pop_list <- list()
time_list <- list()

for( j in seq(1:1000)){

  t <- 0
  i <- 5

  q <- 1

  event_vec <- numeric()
  time_vec <- numeric()

  while(t<30){

    lambda_i <- (1/5)*i + 1
    mu_i <- (1/100)*i^2
    gamma_i <- lambda_i + mu_i

    arrival <- rexp(1, gamma_i)
    t <- t + arrival
    time_vec[q] <- t

    birth <- rbinom(1, 1, lambda_i/gamma_i)

    if (birth == 1){
      i <- i + 1
      event_vec[q] <- i
      q <- q + 1
    }
  }
}
```

```

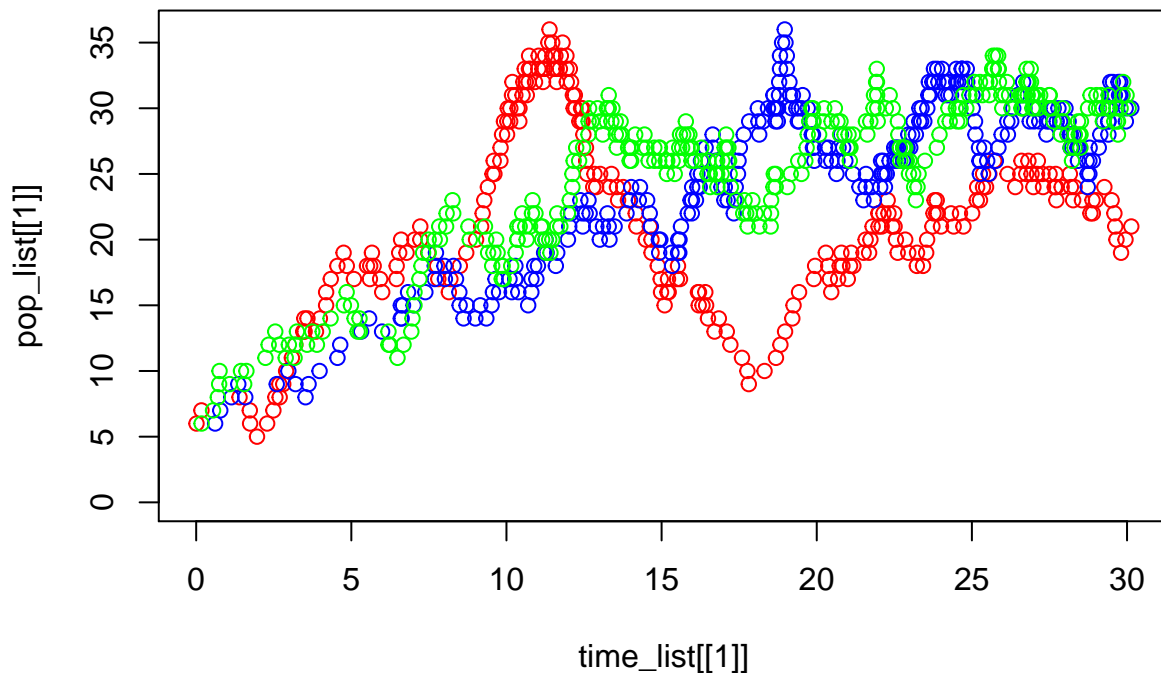
    } else {
      i <- i - 1
      event_vec[q] <- i
      q <- q + 1
    }
  }

  pop_list[[j]] <- event_vec
  time_list[[j]] <- time_vec
}

y_lim = c(0, max( pop_list[[1]], pop_list[[2]],pop_list[[3]] ) )
x_lim = c(0,max( time_list[[1]] ) )

plot(time_list[[1]], pop_list[[1]], col = "red", type = "b", xlim = x_lim, ylim = y_lim)
points(time_list[[2]], pop_list[[2]], col = "blue", type = "b")
points(time_list[[3]], pop_list[[3]], col = "green", type = "b")

```



3)

The realizations in 2 do agree with the explanation in 1. Since the two rates cross at about 25 we should see the population “even” out at around 25 as  $t$  goes to infinity.

4)

```
max_vec <- NULL

for (i in seq(1:length(pop_list))){
  max_vec[i] <- (max(pop_list[[i]]) > 40)
}

mean(max_vec)

## [1] 0.012
```