# Stat421 3/7

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## 1) Give the one-step transition probability matrix P

```
P <- matrix(c(0,1,0,0,0,0,
              0.5,0,(1/3),0,(1/6),0,
              0,(2/3),0,(1/3),0,0,
              0.5,0,(1/3),0,(1/6),0,
              0,0,0,0,0.5,0.5,
              0,0,0,0,0.5,0.5), nrow = 6, byrow = TRUE)
P
```

```
##      [,1]      [,2]      [,3]      [,4]      [,5] [,6]
## [1,]  0.0 1.0000000 0.0000000 0.0000000 0.0000000  0.0
## [2,]  0.5 0.0000000 0.3333333 0.0000000 0.1666667  0.0
## [3,]  0.0 0.6666667 0.0000000 0.3333333 0.0000000  0.0
## [4,]  0.5 0.0000000 0.3333333 0.0000000 0.1666667  0.0
## [5,]  0.0 0.0000000 0.0000000 0.0000000 0.5000000  0.5
## [6,]  0.0 0.0000000 0.0000000 0.0000000 0.5000000  0.5
```

## 2) Find the communication classes and state whether each is closed or not-closed.

The communitcation classes are: {5, 6} (Closed) and {1,2,3,4} (Not Closed).

## 3) Give the period of each state

| State | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Period | 2 | 2 | 2 | 2 | 1 | 1 |

## 4) Find $lim_{n \to \infty} P(n)$

```
Pn <- P%*%P
for(i in seq(1:30000))
{
  Pn <- Pn%*%P
}
round(Pn, 2)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    0    0    0  0.5  0.5
## [2,]    0    0    0    0  0.5  0.5
## [3,]    0    0    0    0  0.5  0.5
## [4,]    0    0    0    0  0.5  0.5
## [5,]    0    0    0    0  0.5  0.5
## [6,]    0    0    0    0  0.5  0.5
```

**Assume that X_0 = 1.**

## 5) Calculate $P(X_4 = 4)$

$P(X_4 = 4) = 0$

You can never reach state 4 in 4 time steps given you started at 1.

## 6) Write a program which will simulate the DTMC for a fixed number of time steps. Run the program repeatedly to verify your answer in #5.

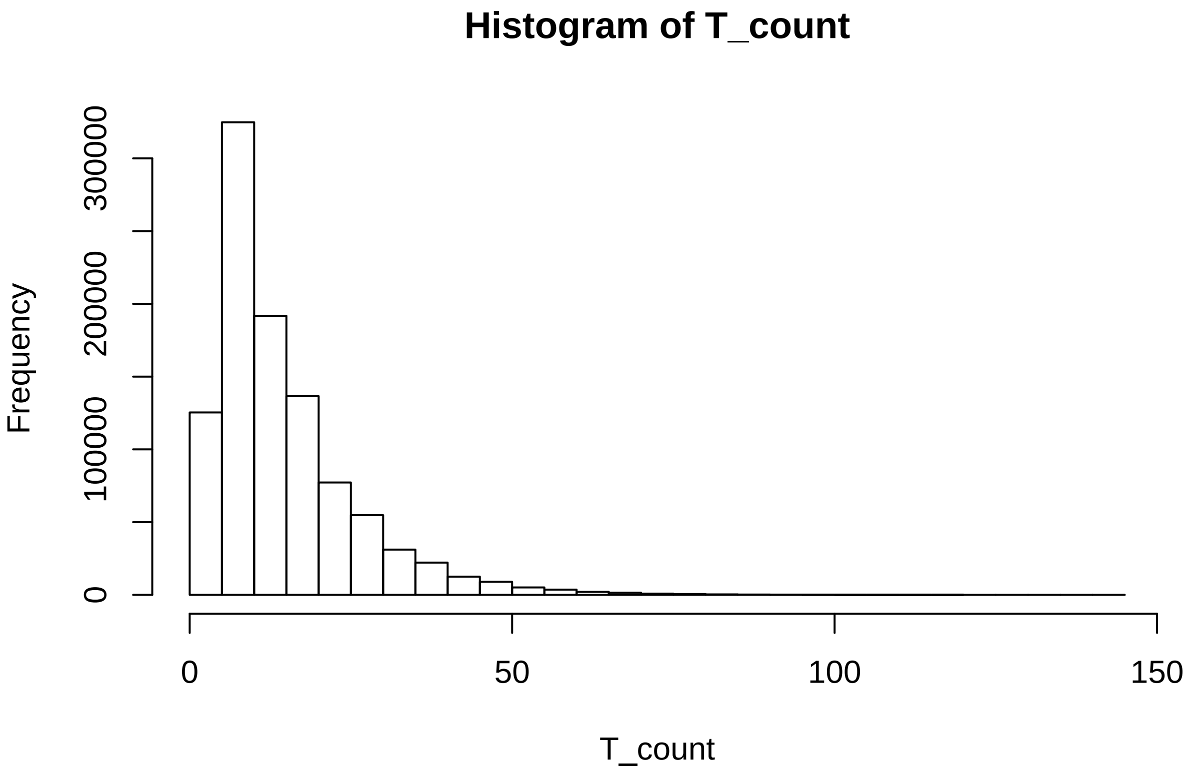
```
end_state <- numeric()

for(j in seq(1,1000)){
  xCur <- 1
  for(i in seq(1:4))
  {
    if(xCur == 1){
      xCur <- 2
    }
    else if(xCur == 2){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0.5,0,(1/3),0,(1/6),0))
    }
    else if (xCur == 3){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,(2/3),0,(1/3),0,0))
    }
    else if (xCur == 4){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE,prob = c(0.5,0,(1/3),0,(1/6),0))
    }
    else if (xCur == 5){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,0,0,0,0.5,0.5))
    }
    else if (xCur == 6){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,0,0,0,0.5,0.5))
    }

  }
  end_state[j] <- xCur
}

sum(end_state == 4)/length(end_state)
```

```
## [1] 0
```

## 7) Let T denote the first passage time to state 6. That is, $T = min\{n > 0 : X_n = 6|X_0 = 1\}$.

```
T_count <- numeric()

for(j in seq(1:999999)){
```

```r
  xCur <- 1
  i <- 1

  while(TRUE){
    if(xCur == 1){
      xCur <- 2
      i<-i+1
    }
    else if(xCur == 2){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0.5,0,(1/3),0,(1/6),0))
      i<-i+1
    }
    else if (xCur == 3){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,(2/3),0,(1/3),0,0))
      i<-i+1
    }
    else if (xCur == 4){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE,prob = c(0.5,0,(1/3),0,(1/6),0))
      i<-i+1
    }
    else if (xCur == 5){
      xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,0,0,0,0.5,0.5))
      i<-i+1
    }
    else if (xCur == 6){
      #xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,0,0,0,0.5,0.5))
      T_count[j] <- i
      break
    }

  }
}

hist(T_count)
```




```r
mean(T_count)
```

```
## [1] 14.99528
```

```r
sum(T_count > 10)/length(T_count)
```

```
## [1] 0.5497955
```