Stat421 3/7

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1) Give the one-step transition probability matrix P

```
P <- matrix(c(0,1,0,0,0,0,
              0.5,0,(1/3),0,(1/6),0,
              0,(2/3),0,(1/3),0,0,
              0.5,0,(1/3),0,(1/6),0,
              0,0,0,0,0.5,0.5,
              0,0,0,0,0.5,0.5), nrow = 6, byrow = TRUE)
P
```

```
##      [,1]      [,2]      [,3]      [,4]      [,5] [,6]
## [1,]  0.0 1.0000000 0.0000000 0.0000000 0.0000000  0.0
## [2,]  0.5 0.0000000 0.3333333 0.0000000 0.1666667  0.0
## [3,]  0.0 0.6666667 0.0000000 0.3333333 0.0000000  0.0
## [4,]  0.5 0.0000000 0.3333333 0.0000000 0.1666667  0.0
## [5,]  0.0 0.0000000 0.0000000 0.0000000 0.5000000  0.5
## [6,]  0.0 0.0000000 0.0000000 0.0000000 0.5000000  0.5
```

2) Find the communication classes and state whether each is closed or not-closed.

The communitcation classes are: {5, 6} (Closed) and {1,2,3,4} (Not Closed).

3) Give the period of each state

State	1	2	3	4	5	6
Period	2	2	2	2	1	1

4) Find $lim_{n \to \infty} P(n)$

```
Pn <- P%*%P
for(i in seq(1:30000))
{
  Pn <- Pn%*%P
}
round(Pn, 2)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    0    0    0  0.5  0.5
## [2,]    0    0    0    0  0.5  0.5
## [3,]    0    0    0    0  0.5  0.5
## [4,]    0    0    0    0  0.5  0.5
## [5,]    0    0    0    0  0.5  0.5
## [6,]    0    0    0    0  0.5  0.5
```

Assume that X_0 = 1.

5) Calculate $P(X_4 = 4)$

6) Write a program which will simulate the DTMC for a fixed number of time steps. Run the program repeatedly to verify your answer in #5.

7) Let T denote the first passage time to state 6. That is, $T = min\{n > 0 : X_n = 6|X_0 = 1\}$.

```r
xCur <- 1
for(i in seq(1:4))
{
  if(xCur == 1){
    xCur <- 2
  }
  else if(xCur == 2){
    xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0.5,0,(1/3),0,(1/6),0))
  }
  else if (xCur == 3){
    xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,(2/3),0,(1/3),0,0))
  }
  else if (xCur == 4){
    xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE,prob = c(0.5,0,(1/3),0,(1/6),0))
  }
   else if (xCur == 5){
    xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,0,0,0,0.5,0.5))
   }
  else if (xCur == 6){
    xCur <- sample(c(1,2,3,4,5,6),size = 1, replace = TRUE, prob = c(0,0,0,0,0.5,0.5))
   }

}
xCur
```

```
## [1] 1
```

7)