Present Status of Muon Number

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Abstract

This work surveys the present experimental and theoretical status of muon number. The development of our ideas regarding flavour numbers is discussed using muon number as an example. The role of muon number in different models that are currently being studied is investigated. The phenomenological and experimental status of different muon number violating processes is reviewed. The predictions of different models for muon number violating processes is discussed.

Contents

I. Introduction

The weak interactions have shown a notorious disrespect for our ideas of what the symmetries of nature ought to be. The violation of parity and the even more startling violation of CP provide examples of this notoriety. In the field of flavour numbers the perversity of the weak interactions manifests itself from the opposite direction, namely, nature presents physicists with a lot of symmetries which have no good reason for existing. For example, since the time the muon was found to behave like a heavy electron physicists have been debating over the question of why it exists. The symmetries of nature which account for its existence and the existence of other flavour numbers like electron number, strangeness, charm etc. are still not completely understood, and seem to be fortuitous symmetries. A number of papers dealing with possible small violations of muon number, baryon number, lepton number, etc. continue to appear in the literature. The motivation for possible violations of baryon number and lepton number is provided by Grand Unified theories, which are reviewed by Langacker (1980). In this paper I study the role of muon number in present ideas of particle physics. A pre-gauge theory review of muon number can be found in Frankel, 1974 and Pontecorvo, 1967. Cheng and Li (1977a) and Weinberg (1977a and 1978) give a large number of references to the early literature on muon number violation after the advent of spontaneously broken gauge theories. In the present work the development of our ideas regarding flavour numbers is briefly reviewed by looking at muon number as an example. I then look at the role of muon number in different extensions of the Glashow-Weinberg-Salam model that are currently of interest. After a review of the phenomenological and experimental status of different muon number violating processes, I conclude by discussing which of these processes are favoured by different gauge models.

The idea of flavour numbers is in essence a description of the ultimate building blocks of nature. There is a close similarity between the basic elements of the ancient Greeks and Indians or the immutable chemical elements of Dalton and flavour numbers. However, the latter term is used only in relation to the modern concepts regarding the ultimate structure of matter. The discoveries at the end of the nineteenth century (discoveries of the electron, of the transmutability of elements, etc.) indicated that the basic building blocks of nature were far fewer than the ninety or so elements discovered till then. In the early decades of this century two fundamental building blocks, the proton and electron, were believed to be the constituents of all matter. With the realisation that nuclear beta decay involved a new massless particle, the neutrino, a study of its nature was taken up. Racah (1937) and Pontecorvo (1950) suggested experiments to see if the neutrino and its antiparticle were identical. The radiochemical experiments of Davis (1955) established the separate existence of the neutrino and antineutrino. It now seemed as if all interactions in nature conserved two quantities, baryon number and lepton number. This simple situation did not last long and the discovery of the neutron, muon, pion and strange particles in the first half of this century showed that nature was more complex. Two new flavours, strangeness (Gell-Mann, 1953; Nishijima, 1955) and muon number, were introduced within a short time of each other. The introduction of these new flavours was phenomenological, i.e., it merely served to explain the absence of certain particle reactions and did not lead to any new insights into the details of weak interactions. Since that time our understanding of flavour numbers and their conservation schemes has developed quite a lot, but we still do not understand why so many flavours

exist in nature (in fact, many new flavours have been discovered since then and we have no idea how many more flavours may exist). The brief historical sketch of the muon that I now present serves to illustrate the puzzlement that physicists felt, and still feel, about the existence of so many flavours.

The muon was discovered in cosmic rays by Anderson and Neddermeyer (1938) and by Street and Stevenson (1937). At first it was confused with the particle predicted by Yukawa as mediating strong interactions. However, its stability in nuclear environments showed that it did not have a strong interaction with nucleons and hence could not be the particle predicted by Yukawa. In the forties it was established that the muon was very similar to the electron and seemed to differ from it only in mass. Hence physicists expected it to decay into the electron without accompanying neutrinos, in addition to its decay modes involving final state neutrinos. However, searches of increasing accuracy in the fifties and sixties for processes like $\mu \to e\gamma$, $\mu^+ \to e^+e^-e^-$ and μe conversion failed to find them. The theoretical predictions for the rates of such processes were beset with problems of divergent integrals and artificial cutoffs because these decays proceeded in second order in the old non-renormalisable models of weak interactions [current X current model or the old intermediate vector boson model with only one type of neutrino, reviewed in Frankel (1974)]. These problems made it difficult to decide if the experimental upper limits suggested modifications in the theory of weak interactions. There seemed to be growing evidence for the existence of two types of neutrinos and for a new quantum number, "muon number". This idea gained support with the introduction of intermediate vector boson theories of the weak interaction (Feinberg, 1958; Schwinger, 1957).

Experimental tests for the new quantum number were discussed by Pontecorvo (1959), Schwartz (1960) and Lee and Yang (1960). The need for introducing a new flavour was definitely established by the experiments of Danby *et al.* (1962), who showed that when neutrinos produced from the decay of pions into muons interacted with nuclei, they produced muons but not electrons. The rate for production of electrons and muons in these processes is unambiguously calculable even in the old models of weak interactions and a conflict between theory and experiment clearly emerged. Thus, it became necessary to introduce a new type of flavour.

The phenomenological nature of the old theories is well illustrated by the many different types of lepton number schemes that were proposed. The only way of choosing among them was to look for processes that were permitted in some schemes but not in others. The lepton number schemes did not predict rates for the "exotic" processes. The need to probe the validity of the different schemes was one reason for the experimental interest in lepton number violating processes in the seventies. Three lepton number conservation schemes have become commonly known (Frankel, 1974; Pontecorvo, 1967). The additive lepton number scheme (Nishijima, 1957; Bludman, 1958; Schwinger, 1957) is the most widely known and also the most restrictive. In this scheme there are two lepton numbers, L_e and L_μ, with e^- and ν_e having $L_e=1$ and $L_\mu=0$, and μ^- and ν_μ having $L_\mu=1$ and $L_e=0$. The antiparticles have the opposite sign for these quantum numbers. These lepton numbers are additively conserved, i.e., $\sum L_e$ and $\sum L_\mu$ do not change in any interaction. Another lepton number scheme, actually the first lepton number scheme, is due to Konopinski and Mahmoud (1953). In this scheme there is only one lepton number L^* which is additively conserved. μ^- and e^+ have $L^*=1$, there is

only one neutrino which can exist in both negative and positive helicity states and has $L^*=1$, and the antiparticles have $L^*=-1$. The V-A character of the weak interactions plays an important role in suppressing unwanted reactions like μ^-e^+ conversion and $K^+ \rightarrow \mu^+e^+\pi^-$. When the mass of the neutrino is zero all reactions forbidden by the additive scheme are also forbidden in this scheme. The relation between the suppression of unwanted processes and the V-A character of the weak currents is nicely illustrated by the work of Primakoff and Rosen (1972) who introduce V+A currents and calculate the rates for anamolous processes. A third scheme is the multiplicative scheme introduced by Feinberg and Weinberg (1961a). In this scheme there is an additively conserved lepton number L and a multiplicatively conserved lepton number L_p. All reactions satisfy $(\sum L)_{initial}=(\sum L)_{final}$ and $(\pi L_p)_{initial}=(\pi L_p)_{final}$. μ^-, e^-, ν_μ, and ν_e have $L=1$ and their antiparticles have $L=-1$. e^+, e^-, ν_e and $\overline{\nu}_e$ have $L_p=1$ and μ^+, μ^-, ν_μ and $\overline{\nu}_\mu$ have $L_p=-1$. This scheme is less restrictive than the previous ones and allows reactions like $\mu^++e^- \rightarrow \mu^-+e^+$, $\mu^+ \rightarrow e^+\overline{\nu}_e\nu_\mu$, etc. The limits on the phenomenological coupling constants are comparable to the weak interaction coupling constant G_F if one uses phenomenological currents for these processes. It is not straightforward to extend the Konopinski-Mahmoud scheme or the Feinberg-Weinberg scheme to take into account the τ lepton discovered in 1975 (Perl *et al.*, 1975).

The advent of spontaneously broken renormalisable gauge theories of weak and electromagnetic interactions (Abers and Lee, 1973; Taylor, 1976) led to constraints on the kinds of lepton number schemes allowed. I describe these theories now in some detail because of their importance and because they form the framework for the rest of the discussion. Readers familiar with these theories may want to skip this paragraph and the next. Gauge theories belong to the Lagrangian approach to particle physics. In this approach particles are described by local fields (wave functions) and particle interactions are derived according to certain prescriptions from a Lagrangian which is a function of the particle fields. The prescriptions include an arbitrary subtraction of certain infinite quantities to give finite values, a procedure called renormalisation. Field theories in which the renormalisation procedure is well defined are called renormalisable theories. Renormalisable theories are very important because they are the only type of theories which seem to be able to make non-trivial predictions for particle interactions. In Lagrangian field theories there is a close relation between conserved quantities and symmetries of the Lagrangian. For every conserved quantity there exists a continuous transformation which leaves the Lagrangian invariant. Conversely, for every continuous transformation which leaves the Lagrangian invariant one can find a conserved quantity. This is known as Noether's theorem and is reviewed in (Hill, 1957). In general the transformation is a global transformation, i.e., the wave functions in the Lagrangian mix among themselves in the same way at every point of space and time. In the gauge theories particles are assigned to representations of some gauge group. The Lagrangian remains unchanged when the wave functions of the particles transform among themselves under a gauge transformation according to the group representations to which the particles are assigned. To extend this symmetry to local gauge transformations (i.e., the gauge group parameters are functions of space-time coordinates) one has to introduce massless gauge bosons. These gauge bosons interact with the fermions in such a way as to make the Lagrangian invariant under local gauge transformations. Local gauge theories are

appealing because particle interactions are not arbitrary but are fixed by the choice of the gauge group and the particle representations. In addition, when calculating S-matrix elements one can use the gauge symmetries to prove that these theories are renormalisable. At present local gauge theories seem to provide a correct description of particle interactions. Since the only massless gauge boson known to us is the photon one has to find a mechanism for giving masses to the other gauge bosons occurring in the theories. A mechanism must also exist to break the mass degeneracy between fermions belonging to the same representation of the gauge group. Both can be achieved by introducing scalar bosons (Higgs bosons) which interact in such a way that the theory is spontaneously broken, i.e., the ground state of the theory does not have the symmetry of the Lagrangian. An example of spontaneous symmetry breaking in solid state physics is provided by a ferromagnetic crystal. In a ferromagnetic crystal the spins of the atoms have a tendency to align themselves in one direction. A priori, there are many equivalent directions in which they can align themselves. However, if a few spins get aligned in one of these directions (due to thermal fluctuations, a magnetic field which is momentarily applied, or for any other reason), the other spins will follow suit very quickly and the crystal does not fully represent the symmetry of the spin-spin interaction. It is this reduction in symmetry which sometimes inhibits phase transitions in ultra-pure systems. The system must choose one of several allowed configurations, and in condensed matter physics this choice is often made by the impurities (nucleation centres for condensation, the magnetic field in the above example, and so on).

For the sake of completeness I should mention that not all local gauge theories are renormalisable. In the proof of renormalisability one uses certain identities, called the Ward identities, which are derived from gauge invariance. Sometimes the derivation of these identities (and hence the proof of renormalisability) cannot be carried through if the Feynman diagrams of the theory have triangle anomalies (Bardeen, 1974). (Feynman diagrams are diagrams which tell us how to calculate physical cross sections, distributions, etc. from the Lagrangian. I know of no simple way to describe triangle anomalies.) Stated another way, triangle anomalies imply that the gauge invariance of the Lagrangian is not maintained by the Feynman diagrams of the theory. This loss of symmetry comes about because the Feynman diagrams have to be renormalised, and no renormalisation procedure can be found which maintains gauge invariance. The triangle anomalies do not always spoil the renormalisability of the theory. If the reader will permit a non-serious digression I would like to mention another triangle anomaly discovered earlier by psychologists. The psychologists' triangle anomaly is illustrated in Fig. 1. A cursory examination shows that two shades of white have been used in the figure. However, a closer inspection reveals that the white triangle is actually the same shade as the background white! Thus, unlike the particle physics triangle anomaly where a symmetry was lost in going from the Lagrangian to the Feynman diagrams, this triangle anomaly reveals a hidden symmetry. A more serious (and technical) explanation of the particle physics triangle anomaly can be found in (Adler, 1970; Jackiw, 1970).

In the spontaneously broken gauge theories there is a close connection between the assignment of particles to group representations, and lepton number schemes. The gauge bosons induce transformations between particles belonging to the same group multiplet. Thus, one can introduce a "flavour number" to each multiplet and the gauge interactions will conserve this flavour number. For example, in the standard model (Glashow,

1961; Weinberg, 1967; Salam and Ward, 1964) the electron and the muon belong to different $SU(2)_{weak}$ doublets and the additive lepton number conservation scheme is realised by the model. In a slightly different model proposed by Derman (1979) some new interactions exist in addition to the usual weak and electromagnetic interactions. These new interactions result in small violations of the usual additive muon number and other lepton numbers. However, one can still define multiplicatively conserved lepton numbers and the model realises a multiplicative lepton number conservation scheme. Weinberg (1972) studied a gauge model where the Konopinski-Mahmoud lepton number scheme is realised. Thus, in spontaneously broken gauge theories flavour numbers are not arbitrary but have to be explained in terms of particle representations. Assigning fermions to different group representations is necessary but not sufficient to ensure flavour conservation, because eigenstates of the weak interactions are not eigenstates of the mass matrix. Due to spontaneous symmetry breaking, states with the same charge and helicity will mix and the weak interactions will violate flavour number, with the order of magnitude of the flavour violation depending on the mixing angles. An example is the Cabibbo angle and non-conservation of strangeness in weak interactions. Flavour violation is said to be suppressed naturally if the suppression is due to the representation content of the theory and not due to arbitrarily small parameters. For example, flavour non-conservation is suppressed naturally in the standard model because the GIM mechanism (Glashow *et al.*, 1976) explains the very small strength of the observed strangeness-changing neutral currents in terms of the gauge multiplet assignments of the quarks. Flavour number violation in gauge theories can occur due to lepton mixing and a non-diagonal mass matrix (Cheng and Li, 1977a; Kobayashi and Maskawa, 1973), due to flavour changing gauge bosons

(Deshpande, 1979; Mannheim, 1978; Maehara and Yanagida, 1979), and due to flavour number violating Higgs particle couplings (Bjorken and Weinberg, 1977; McWilliams and Li, 1980a). Flavour violation could also occur due to anomalies ('t Hooft, 1976). However, the effect is very small and has no practical consequences.

Thus, we see that spontaneously broken gauge theories provide the best framework to discuss lepton number conservation schemes and possible deviations from these schemes. It is very easy to incorporate a small amount of lepton number violation in gauge theories as shown by the avalanche of gauge theories which followed the rumour in 1975 that the $\mu \rightarrow e\gamma$ process had been observed (the rumour turned out to be wrong). If muon number is to be conserved exactly in a gauge theory, then some restrictive conditions must be satisfied, namely, the muon and electron must occur in separate multiplets, some mechanism must be found for ensuring that the lepton mixing does not result in violation of muon number, and the Higgs sector must also be chosen to conserve muon number. In the standard SU(2) × U(1) model with six quarks and six leptons all these conditions are satisfied. The muon and electron occur in different doublets, the absence of right handed neutrinos ensures their masslessness and hence all lepton mixing angles can be made zero by properly defining the muon and electron neutrino states (this can be done because of the degeneracy in their masses), and the presence of only one Higgs doublet ensures that the Higgs-lepton interactions conserve muon number even after the lepton mass matrix is diagonalised. While the Glashow-Weinberg-Salam model predicts exact conservation of muon number, any modification of this model usually results in a small amount of muon number violation.

The Glashow-Weinberg-Salam (GWS) model agrees very well with essentially all present experimental results. Hence one may ask if

there is any reason to expect muon number violation. The answer is that violation of muon number is not unreasonable, firstly, because the experimental data do not severely constrain the Higgs sector in the GWS model and hence the violation of muon number through the Higgs sector is not ruled out, and secondly, because there are many questions that the GWS theory doesn't answer, for example: why are right-handed neutrinos absent? Why does nature repeat herself, with the electron family, the muon family, the τ-lepton family and maybe more (Harari, 1979)? It would be nice to have a theory which explained such facts instead of taking them as assumptions, and one has to go beyond the GWS model for this. Any theory which attempts to answer these questions also generally predicts a small violation of muon number.

One popular reason to consider modifications of the elementary GWS model is the attempt to calculate quark mixing angles in terms of their masses. In these attempts additional discrete or continuous symmetries involving transformations among the different fermion generations are imposed on the particles in the theory. The additional symmetries can be chosen so that natural flavour conservation (NFC) is imposed on the theory. However, general theorems exist (Kang and Rothman, 1980; Segre and Weldon, 1979) which show that if one wishes to have calculable quark mixing angles one must also accept flavour violation at some level. No compelling model exists which predicts mixing angle relations.

Another reason to consider modification of the GWS model is to generate CP violation spontaneously (Lee, 1973 and 1974). This possibility is interesting because it explains naturally why we can presently measure CP violation only in the K^0-$\overline{K}^0$ system. G.C. Branco (1980) has shown that if NFC is imposed on the theory the Higgs exchanges provide the only source of CP violation. In this class of models the neutron electric dipole moment can be relatively large. The CP violating kaon decay parameter ϵ'/ϵ may also be very large (Deshpande, 1981; Sanda, 1981). If natural flavour conservation is not imposed one can get spontaneous CP violation in the Weinberg-Salam model with two Higgs doublets. Lahanas and Vayonakis (1979) have considered such a model and found that the Higgs masses are of the order of a TeV. Their model has an interesting feature, namely, the Higgs contribution to the K_L-K_S mass difference is proportional to Higgs particle mass differences.

$SU(2)_L \times SU(2)_R \times U(1)$ models have also been considered for generating spontaneous CP violation and to have a theory which is invariant under parity at high energies (Pati and Salam, 1974; Mohapatra and Pati, 1975; Senjanovic and Mohapatra, 1975). In the left-right symmetric models it is possible to identify the U(1) generator with (B-L), where B is baryon number and L is lepton number (Marshak and Mohapatra, 1980a; Mohapatra and Marshak, 1980a), and hence to study violation of (B-L). It is also possible to relate the small mass of the neutrinos to the large masses picked by the gauge bosons of $SU(2)_R$ (Mohapatra and Senjanovic, 1980b), and thus to the scale at which parity is spontaneously broken. These models have heavy right-handed Majorana neutrinos (with mass of order 100 GeV). Flavour violation is a very general feature of left-right symmetric models (Chanlowitz *et al.*, 1977). This has been studied by Marshak and Mohapatra, 1980a, Mohapatra and Marshak, 1980a, and Mohapatra and Senjanovic, 1980b. They study the neutrinoless double beta decay and the $\mu \to e\gamma$ process, but do not make a complete study of muon-number violating (lepton number conserving) processes.

It has also been suggested that elementary Higgs fields should be discarded and a dynamical mechanism (Weinberg, 1976; Susskind, 1980) for spontaneous symmetry breaking must be sought to avoid the gauge

hierarchy problem in grand unified theories (the gauge hierarchy problem is the problem of getting naturally two levels of spontaneous symmetry breaking at vastly different energies, one at the grand unification energy of about 10^{15} GeV and the other at present energies of a few hundred GeV.) In dynamically broken theories only fermions and gauge bosons are introduced. All interactions are determined by the local gauge principle. The role of the elementary Higgs particles of the usual theories is now played by bound fermion-anti-fermion composites. These theories are potentially attractive because in principle they have very few arbitrary parameters. No successful theory has been constructed as yet, but much work is being done on the problem. These theories predict flavour changing neutral currents leading to flavour violating rates at levels not far below present experimental upper limits (Eichten and Lane, 1980; Ellis *et al.*, 1980; Dimopoulos and Ellis, 1980).

Horizontal gauge models, in which neutral "horizontal" gauge bosons mediate transitions between members of different generations, have been considered by many authors (Maehara and Yanagida, 1979; Wilczek and Zee, 1979a; Ramond, 1979; Davidson and Wali, 1980; Chikashige *et al.*, 1980; Davidson *et al.*, 1979a, and 1979b; Barr and Zee, 1978; Kane and Thun, 1980; Cahn and Harari, 1980; Shanker, 1980 and 1981; Sato, 1980; Ong, 1980; Ecker *et al.*, 1981; Chakrabarti *et al.*, 1980, and references therein, and references in Langacker, 1980). In such models the repetition of generations is explained naturally, and the families are identified by the representations of the horizontal gauge group to which they are assigned. The motivations for introducing horizontal gauge symmetries have been many and varied. They include attempts to calculate the e-μ mass ratio (Barr and Zee, 1978), fermion mixing angles (Wilczek and Zee, 1979), to get CP violation with only four quarks (Maehara and Yanagida, 1979; Davidson *et al.*, 1979a), etc. Horizontal gauge bosons also occur in the dynamically broken theories, and this has motivated their study (Cahn and Harari, 1980). Horizontal symmetries have also been studied to avoid the strong CP problem (Davidson and Wali, 1980). The main motivation for considering horizontal symmetries seems to be the existence of the generation puzzle (Harari, 1979; Davidson *et al.*, 1979b). Attempts to explain the repeating generation structure have also been made by assuming that quarks and leptons are composite objects and that generations represent excitations. The problems in constructing a satisfactory theory seem to be far from solved.

Muon number is also violated in grand unified theories (in fact, both lepton number and baryon number are themselves violated in these theories). Grand unified theories (GUTs) seek to unify strong, weak and electromagnetic interactions by finding a simple group which contains the $SU(3)_C \times SU(2)_L \times U(1)$ group of strong, weak and electromagnetic interactions as a subgroup (Langacker, 1980; Goldhaber, 1977). A striking prediction of these theories is proton decay. Early experiments on proton decay have been done by Gurr *et al.*, 1967 and Reines and Crouch, 1974. Weinberg (1979) and Wilczek and Zee (1979b) have noted that if the only new masses (in addition to the masses in the usual $SU(3)_C \times SU(2)_L \times U(1)$ phenomenology) are grand unification masses (the grand desert scenario) then baryon and lepton number violation is mediated by local four-fermion operators that respect the $SU(3)_C \times SU(2)_L \times U(1)$ symmetry. This implies that (B-L) is conserved to leading order in the grand unification mass (where B is baryon number and L is lepton number). Lipkin (1980) has related this conservation of (B-L) to properties of the usual weak isospin assignments of the fermions in the Weinberg-Salam model and to properties of four-fermion operators. If there is indeed no new physics between 100 GeV and 10^{15} GeV then the only interesting processes are

proton decay and (possible) neutrino oscillations. It has also been noted (Marshak and Mohapatra, 1980b; Kuo and Love, 1980; Marshak *et al.*, 1980) that if there are intermediate mass scales between present energies and grand unification energies then even in leading order in the grand unification mass (B-L) violating processes like neutron oscillations (n-$\bar{n}$) and no-neutrino double beta decay could occur. Interesting new phenomenology, including muon number violation, could occur in the Pati-Salam GUTs, which have low mass unification at a scale of about 100 TeV. They are reviewed in Pati (1978 and 1979).

In addition to the above motivations for modifications to the GWS model leading to muon number violation it has been argued by some physicists that the presence of any absolutely conserved quantity not related to local gauge invariance is not satisfactory (DeRejula, Georgi and Glashow, 1975; Mack, 1979). This is because such conserved quantities (eg. baryon number, lepton number, etc.) predict global symmetries in the Lagrangian. These global symmetries require gauge transformations independent of the coordinates, i.e., they require the comparison of directions in internal charge spaces at widely separated space points simultaneously. DeRejula, Georgi and Glashow (1975) also argue that a global gauge invariance (and the corresponding conserved quantity) is not satisfactory for the following reason: If an electrically charged particle falls into a black hole, its memory is preserved by the electric field outside the black hole. However, the absorption of a neutrino leaves no trace outside, and it doesn't seem possible to talk about a conservation law for lepton number in the space outside the black hole. The difference between the electric charge and lepton number lies in the fact that the former is a local gauge symmetry, and the electric field which played a crucial role in the argument is the wave function of the gauge boson corresponding to this symmetry. The conservation of lepton number is related to a global gauge invariance and hence there is no gauge boson corresponding to this symmetry.

In concluding the general discussion we can say that the usual $SU(3)_C \times SU(2)_L \times U(1)$ phenomenology describes present experiments very well, and predicts muon number conservation. While there is no overwhelming reason to expect muon number violation, there are enough motivations to make muon number violation an intriguing possibility to consider. The repeating generation structure of the GWS model and the proliferation of flavours still remains an unanswered mystery and there are many open questions regarding flavours. The search for violations of established conserved quantum numbers is interesting for another reason: If a very weakly coupled force exists in nature, it may not be discovered for a long time because stronger interactions may mask it. However, if the weak force violates some conservation law valid for the stronger interactions, then it would be revealed in processes which violate the conservation law. This argument is due to Lincoln Wolfenstein. In a certain sense one may say that the weak interactions were discovered because they violated the law of immutability of elements.

I will now discuss the phenomenology of different muon number violating processes. Readers not interested in details may wish to skim through these sections and read the conclusions. In most theories with muon number violation the cause is (a) lepton mixing which arises if the lepton mass matrix is not diagonal with respect to the gauge eigenstates (Cheng and Li, 1977; Kobayashi and Maskawa, 1973; Marciano, 1978), (b) existence of exotic gauge bosons with muon number violating couplings (Deshpande *et al.*, 1979; Mannheim, 1978; Maehara and Yanagida, 1979) and

(c) scalar Higgs couplings which violate muon number (Bjorken and Weinberg, 1977; McWilliams and Li, 1980a). The Feynman diagrams contributing to muon number violating processes belong typically to one of three classes: (a) tree diagrams resulting from the exchange of a heavy gauge boson or Higgs boson (eg. Fig. 2a. This diagram could contribute to $\overline{K}^o \to \overline{\mu}e$ in a horizontal gauge model), (b) box diagrams as in Fig. 2b (this diagram contributes to the process $\overline{K}^o \to \overline{\mu}e$ in the standard model with a massive ν_τ), and (c) one loop diagrams like the one in Fig. 2c (this diagram contributes to the process $\mu \to e\gamma$ in the same model). These diagrams are usually suppressed by some heavy masses and reduce to local interactions to leading order in the heavy masses. Hence muon number violating processes can be described by local effective phenomenological currents in a way similar to the phenomenological current X current description of the usual weak interactions. This is the approach I use in the discussion. The phenomenological coupling constants described here will be calculable in different gauge theories. When a photon mediates muon number violating processes (eg. $\mu \to e\gamma_{virtual} \to ee\overline{e}$) the effective Hamiltonian describing such processes (eg. processes like $\mu \to ee\overline{e}$) will contain non-local terms involving the four-momentum transfer. Such terms can be discussed in terms of the effective coupling constants defined in the section on $\mu \to e\gamma$. Muon number violating processes which have been discussed in the literature can be classified as follows: 1. Leptonic processes: (a) $\mu \to e\gamma$, (b) $\mu \to ee\overline{e}$, (c) $\mu \to e\gamma\gamma$, (d) $\overline{\mu}e \to \overline{e}\mu$, and (e) neutrino processes. 2. Semi-leptonic processes: (a) μ^-e^- conversion, (b) μ^-e^+ conversion, (c) kaon decays, and (d) no-neutrino double beta decay. The phenomenology and current experimental limits on these processes are discussed below.

II. Leptonic processes

A. $\mu \to e\gamma$

The phenomenology of this process was first discussed by Weinberg and Feinberg (1959). The general form of the matrix element for the electromagnetic current operator between the states of an electron and a muon is determined by Lorentz invariance and electromagnetic current conservation to be of the form (Cheng and Li, 1977a; Weinberg and Feinberg, 1959; Lee and Shrock, 1977a):

$$T_\lambda = \langle e(p_e)|J_\lambda(0)|\mu(p_\mu)\rangle$$
$$= \frac{G_F}{\sqrt{2}}\bar{u}_e(p_e)\left[f_{ML}(1+\gamma_5)im_\mu\sigma_{\lambda\nu}q^\nu+f_{EL}(1+\gamma_5)(\gamma_\lambda q^2-q_\lambda\gamma\cdot q)\right]u_\mu(p_\mu)$$
$$+ \text{ right-handed terms}, \qquad (1)$$

where $q=p_\mu-p_e$ and m_μ is the muon mass. The form factors f_{ML}, f_{EL} etc. could be functions of q^2 but the q^2 dependence is generally suppressed by powers of some heavy masses occurring in the gauge theories. Also, in many gauge theories only electrons of a particular helicity are involved in this process (see, for example, Cheng and Li, 1977a; Bjorken and Weinberg, 1977). This means that only left- or right-handed coupling constants are non-zero in Eq. 1. When the photon is on its mass shell (i.e., $q^2=0$) only f_{ML} and f_{MR} contribute to the amplitude for $\mu \to e\gamma$. The amplitude is of the form:

$$m(\mu \to e\gamma) = \frac{G_F}{\sqrt{2}}\bar{u}_e\Big(f_{ML}(1+\gamma_5)+f_{MR}(1-\gamma_5)\Big)im_\mu\sigma_{\lambda\nu}q^\nu u_\mu\varepsilon^\lambda, \qquad (2)$$

where ε^λ is the photon polarisation vector, $\varepsilon^\lambda q_\lambda=0$. The rate for $\mu \to e\gamma$ is

$$\Gamma(\mu^+ \to e^+\gamma) = \frac{G_F^2 m_\mu^5}{8\pi}\Big(|f_{ML}|^2+|f_{MR}|^2\Big) . \qquad (3)$$

A complete phenomenological analysis of this decay (including a study of CP violating correlations) was made by Tung (1977). Recent calculations are done by Cheng and Li (1980a), Inami and Lin (1980) and Ma and Pramudita (1980). If this process occurs then a measurement of the electron asymmetry using polarised muons would give information on the helicity structure of the lepton number violating current. A search for this process is quite useful. When electrons of a particular helicity are produced in the decay the amplitude contains only one term whose phase is unmeasurable and hence CP violating effects will be absent (Treiman *et al.*, 1977). Also, in this case the angular distribution of the outgoing electron with respect to the initial muon polarisation is of the form $[1 \pm \cos(\theta)]$ where θ is the angle between the electron three-momentum and the muon spin (+ for left-handed electrons and - for right-handed ones). The experimental upper limit on the rate for this process is normally quoted relative to the ordinary muon decay rate $\Gamma(\mu^+ \to e^+\nu_e\bar{\nu}_\mu) = G_F^2 m_\mu^5/(192\pi^3)$. The present upper limit on the branching ratio $R_{e\gamma}$ is 2×10^{-10} and this limit has been set at Los Alamos (Bowman *et al.*, 1979). The experiment consists of looking for an e^+ and a γ-ray in time coincidence with the three-momenta being collinear and of magnitude 52.8 MeV. The main background for this experiment came from random coincidences between the e^+ from normal muon decay and a γ-ray from the bremsstrahlung process $\mu^+ \to e^+\nu_e\bar{\nu}_\mu\gamma$. The latter process also contributed to the background because in some of the events the electron and the photon were almost collinear and the magnitudes of the three-momenta were close to 52.8 MeV. There were about a hundred events which satisfied the conditions on the momenta within experimental resolutions. All of them could be attributed to background. A new experiment is under way at Los Alamos to improve the limits on the branching ratio for $\mu \to e\gamma$, $\mu \to ee\bar{e}$ and $\mu \to e\gamma\gamma$, using the crystal box detector.

B. $\underline{\mu^- \to e^-e^-e^+}$

$\mu^- \to e^-e^-e^+$ occurs in all models in which $\mu \to e\gamma$ occurs, because of pair production by the photon. In most models there are additional Feynman diagrams which contribute to this process, for example, box diagrams similar to the one in Fig. 2. The pre-gauge theory discussions of this process were beset by difficulties because the additional diagrams often involved loop integrations and were calculable only in a renormalizable theory. Bander and Feinberg (1960) calculated the rate for $\mu^- \to e^+e^-e^-$ including only the contribution from $\mu \to e\gamma$ (they performed their calculations in terms of effective $\mu e\gamma$ couplings. Their effective couplings are essentially the same as those of Eq. 1). This process was also studied by Marciano and Sanda (1977a; 1977b) among others. The advent of renormalizable gauge theories made it possible to calculate the contributions other than the photon exchange contribution to this process. Because of the non-photon exchange contributions the relation between the branching ratios for $\mu \to ee\bar{e}$ and $\mu \to e\gamma$ becomes model dependent (Marciano, 1978). In general the amplitude for $\mu \to ee\bar{e}$ (excluding the photon exchange contribution) takes the form (Cheng and Li, 1977a; Altarelli *et al.*, 1977):

$$m(k_1,k_2) = \frac{G_F}{\sqrt{2}}\left[\bar{u}_e(k_1)(1+\gamma_5)\gamma_\lambda\, u_\mu(p)\right] \times$$
$$\times\left[\bar{u}_e(k_2)\left\{g_L\frac{(1+\gamma_5)}{2} + g_R\,\frac{(1-\gamma_5)}{2}\right\}\gamma^\lambda\, v_e(k_1)\right] \qquad (4)$$

where k_1 and k_2 are the four-momenta of the electrons, k_3 is the four-momentum of the final state positron and p is the four-momentum of the initial muon. For the sake of simplicity I haven't written down the

terms involving a V+A μe current. In many theories the μe current is either V-A or V+A and tensor terms are also suppressed by heavy masses (Cheng and Li, 1977a). The rate for $\mu \to ee\bar{e}$ has an interference term between the photonic and non-photonic amplitudes. In terms of the effective couplings of Eq. 1 and Eq. 4 the partial width for this process is

$$\Gamma(\mu\to 3e) = \frac{G_F^2 m_\mu^5}{768\pi^3}\left\{16\pi\alpha\left(4\ln\frac{m_\mu}{2m_e} - \frac{13}{6}\right)|f_M|^2 - 48\pi\alpha\,\mathrm{Re}(f_M^* f_E) + \right.$$
$$\left. 12\pi\alpha|f_E|^2 + |g_L|^2 + 2|g_R|^2 - 2\sqrt{4\pi\alpha}\,\mathrm{Re}\left[(f_E - 2f_M)^*(g_L + 2g_R)\right]\right\}. \qquad (5)$$

The rate is normally compared to the ordinary muon decay rate. In many theories one expects g_L and g_R to be somewhat larger than ef_E and ef_M as the former couplings get contributions from box diagrams which are enhanced by logarithmic factors (Marciano, 1978). The μ^-e^- conversion process which is described later is likely to be a more sensitive probe of lepton number violation than $\mu \to ee\bar{e}$, unless mixing angles conspire to suppress it. The best upper limit for the branching ratio for $\mu \to ee\bar{e}$ has been set by Korenchenko *et al.* (1976) at Dubna. They quote an upper limit of 1.9×10^{-9}. The experiment involved stopping positive pions in a target and looking for two positrons and an electron in time conicidence using a magnetic spark chamber detector. The process was searched for using kinematic reconstruction of the event. All the events which remained after imposing various cuts could be attributed to background. According to Korenchenko (1976) the possible sources of background are events from the photonic muon decay followed by internal or external conversion of the photon, events from accidental coincidences of positrons emitted by two decaying muons where one of the positrons is later scattered by an electron of the target material and transfers an exceptionally large energy to it, and finally events from the accidental coincidence between emission of a positron by a muon and passage through the chamber and target of a charged particle from the region outside the chamber (the passage of the charged particle simulates the production of a positron and an electron).

C. $\mu \to e\gamma\gamma$

$\mu \to e\gamma\gamma$ has been studied by Bowman *et al.* (1978). They point out that the rate for $\mu \to e\gamma\gamma$ may be larger than for $\mu \to e\gamma$ in some models, in particular for models in which the latter process is suppressed by a natural mechanism and in which charged exotic heavy leptons exist. They also point out that experiments setting limits on the branching ratio for $\mu \to e\gamma$ can be used to set limits on $\mu \to e\gamma\gamma$. In most of the extensions to the standard model that are presently studied the rate for this process would be of order α times the rate for $\mu \to e\gamma$.

D. $e\bar{\mu} \to \mu\bar{e}$

This process has been studied to test for the multiplicative lepton number scheme. It also occurs in various extensions of the GWS model. It has been studied by Feinberg and Weinberg (1961) and by Pontecorvo (1957). Present experiments can test for the effective coupling constant g_{eff} in the phenomenological Lagrangian:

$$\mathcal{L}_{eff} = g_{eff} \frac{G_F}{\sqrt{2}} \left(\bar{\mu}\gamma_\lambda(1+\gamma_5)e\right)\left(\bar{\mu}\gamma^\lambda(1+\gamma_5)e\right) \tag{6}$$

at the level $g_{eff} = 1$ (Frankel, 1974). The rate for the conversion in solids and in gases is quenched below the conversion rate in vacuum. The muonium-anti-muonium transition does not seem to be a sensitive test of lepton number violation.

E. Neutrino processes

These tests are difficult because the neutrinos are weakly interacting neutral particles and are difficult to detect and study. In

addition to the experiment of Danby *et al.* (1962) and Davis (1955) which established the existence of two kinds of neutrinos, neutrino experiments have been carried out to test the nature of the neutrinos from muon decay (Willis *et al.*, 1980) and to look for neutrino oscillations (Reines, Sobel and Pasierb, 1980; Blietschau *et al.*, 1978). The neutrinos from muon decay were studied by Willis *et al.* (1980) at Los Alamos. They stopped positive muons and looked for electron-type antineutrinos from the muon decay. They also looked for electron-type neutrinos. The former process is allowed by the multiplicative lepton number conservation scheme while the latter process is allowed by both schemes. They looked for $\bar{\nu}_e$ using the reaction $\bar{\nu}_e p \to ne^+$ and for ν_e using $\nu_e d \to ppe^-$ (both reactions are allowed by both lepton number schemes). They find that the branching ratio $\Gamma(\mu^+ \to e^+\bar{\nu}_e\nu_\mu)/\Gamma(\mu^+ \to \text{all})$ is -0.001 ± 0.04, thus providing strong evidence against the multiplicative scheme. They find good agreement with theory for the additive lepton number scheme.

The possibility of neutrino oscillations (i.e., the nature of the neutrino changing with time) has been studied for a long time (Maki, Nakagawa and Sakata, 1962; Nakagawa, Okonogi, Sakata and Toyoda, 1963; Pontecorvo, 1967; Gribov and Pontecorvo, 1969; Wolfenstein, 1980a; Wolfenstein, 1978; Cheng and Li, 1978; Marciano, 1980; Schechter and Valle, 1980; Bilenky and Pontecorvo, 1978, and other references in this section). Bilenky and Pontecorvo (1978) give a good review of neutrino oscillations. The recent literature on this process is vast and growing, and requires a review of its own. We will content ourselves here with an explanation of the basic aspects, which are very simple. Three experimental results motivate the present surge of interest in neutrino processes. They are the solar neutrino experiment (Davis Jr., Evans and Cleveland, 1978), the neutrino mass experiment (Lubimov *et al.*, 1980), and one of the neutrino oscillation experiments (Reines, Sobel and Pasierb, 1980). The solar neutrino experiment has been of interest for many years now. This is an experiment to measure the flux of neutrinos reaching the earth after they are produced in energy-releasing processes in the sun. The experiment detects a flux which is a factor of 3 or 4 below the theoretical prediction. The discrepancy seems to require modifying our model of the sun, or of modifying the standard model of weak interactions. For example, the discrepancy could be due to neutrino oscillations. The neutrino mass experiment measures the shape of the electron spectrum in tritium beta decay $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$. The shape is sensitive to the mass of the $\bar{\nu}_e$ and the experiment indicates that the mass lies between 14 eV and 45 eV. The neutrino oscillation experiment detects the nature of the neutrinos coming from a nuclear reactor. About half the neutrinos seem to change their type in traversing the 11.2 m from the reactor core (where they are produced) to the detector. These experiments seem to indicate the need for modifying the standard weak interaction model, but they are difficult experiments and are not conclusive.

The presence of neutrino oscillations implies that the neutrinos have mass and mix among themselves. Neutrino oscillations are very similar to the K^0-$\bar{K}^0$ oscillations, the study of which led to the discovery of CP violation. They occur when the neutrino mass eigenstates are not eigenstates of the weak interaction. In this case the weak interaction properties of a neutrino are periodic functions of time. The interesting information about neutrinos is the nature of their mass matrix (i.e., masses, mixing angles and Dirac or Majorana nature). A discussion of this

question is given by Cheng and Li (1980b). The mass of a particle can be either a Dirac mass or a Majorana mass. If the particle has a Dirac mass the particle and its antiparticle have a separate identity. If the particle has a Majorana mass then it is its own charge conjugate. In general the Lagrangian will contain both Dirac and Majorana masses. The physical particles are mass eigenstates and one has to diagonalise the mass matrix in the Lagrangian to see the nature of these particles. For one particle the general mass term in the Lagrangian takes the form (Cheng and Li, 1980b):

$$\mathcal{L}_{mass} = D\bar{\psi}_L\psi_R + A\bar{\psi}^c_L\psi_L + B\bar{\psi}^c_R\psi_R + \text{Hermitean conjugate} \qquad (7)$$

where L and R denote left and right hand components of the fields and the superscript c refers to the charge conjugate field. When the fields are mixed among themselves to diagonalise the mass matrix and get physical states, one finds that in general the physical particles are Majorana particles. One gets the familiar Dirac particles if A=B=0. Thus, the Dirac mass is a special case of a Majorana mass. When A or B is not zero the Lagrangian describes two Majorana particles with different masses $1/2\{(A+B) \pm [(A-B)^2 + D^2]^{1/2}\}$. If the Lagrangian conserves some charge (say electric charge or lepton number) and if the particle has a non-zero eigenvalue for this charge then A and B are forced to be zero. This explains why Dirac particles are so ubiquitous though they form a particular case of a more general situation. This also explains why most extensions of the standard theory of weak interactions give Majorana neutrinos if lepton number conservation is not imposed a priori in the Lagrangian (Cheng and Li, 1980b). Phenomenological implications of non-zero neutrino masses and mixings have been studied by, among others, (Ng, 1980 and 1981; Barger *et al.*, 1980a, 1980b; Shrock, 1980;

Kalyniak and Ng, 1981; see also the references in Cheng and Li, 1980b). Rosen and Kayser (1981) and Frere (1981) mention that neutrino oscillations affect the interpretation of the $\sin^2\theta_W$ measurement. Kalyniak and Ng (1981) discuss the electron and neutrino correlation in muon decay as a possible probe of the Majorana or Dirac nature of the neutrinos. Grand unification models provide a motivation for non-zero neutrino masses and this has been studied in (Gell-Mann, Ramond and Slansky, 1979; Georgi and Nanopoulos, 1979; Barbieri, Ellis and Gaillard, 1980; Witten, 1980; Wolfenstein, 1980b). Zee (1980) considers the possibility of the neutrinos picking up Majorana masses from a charged $SU(2)_W$ singlet in the Weinberg-Salam model. He mentions that such a Higgs particle could arise in some grand unified theories. Many of the grand unified models predict neutrino masses in the range 10^{-5} to 10^2 eV (Langacker, 1980).

The experimental bounds on neutrino masses are usually given neglecting the possibility of neutrino mixing. The limits are then 14 eV < $m(\nu_e)$ < 46 eV (Lubimov *et al.*, 1980), $m(\nu_\mu)$ < 0.52 MeV (Lu *et al.*, 1980; Daum *et al.*, 1979) and $m(\nu_\tau)$ < 250 MeV (Bacino *et al.*, 1979). Simpson (1981) quotes a value for $m(\nu_e)$ of 20 eV, but with large errors. Information on neutrino masses can also come from astrophysical arguments. For example, standard cosmological models predict that the universe is permeated by neutrinos. If these neutrinos have non-zero masses, this would affect the expansion rate of the universe. From the measured expansion rate one gets the limit that the sum of all stable light neutrino masses should be less than about 60 eV (Cowsik and McClelland, 1972; Lee and Weinberg, 1977b; Tremaine and Gunn, 1979). Also, the helium abundance of the universe constrains the number of distinct light stable

neutrinos to be less than 4 (Shvartsman, 1969; Steigman *et al.*, 1977). More model-dependent cosmological arguments give more constraints (Dolgov and Zeldovitch, 1981; Steigman, 1979). I quote as an example an upper limit on the ν_τ mass of 35 eV calculated by Cowsik (1980). The experimental situation regarding neutrino oscillations is still ambiguous and many new experiments are being planned. A survey of some experiments is given by Mann (1981). Improving our experimental knowledge about neutrinos is very important for indicating the direction in which particle physics theory should develop. Neutrino experiments also have important implications for astrophysics and cosmology.

III. Semi-leptonic processes

A. μ^-e^- conversion

When negative muons are stopped in some material they are captured by atoms and form muonic atoms. In a time short compared to their lifetime they cascade down to the ground state atomic orbit. From this state they are either captured by the nucleus with the emission of a neutrino or they decay into an electron and two neutrinos. If muon number is not exactly conserved the μe conversion process $\mu^-+(A,Z) \to e^-+(A,Z)$ can occur some of the time. The phenomenological study of this process has a long history beginning with the work of Weinberg and Feinberg (1959). This process has some similarities to $\mu \to ee\bar{e}$. For example, the discussion of Marciano and Sanda (1978, 1977a, 1977b) on the relation between $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ is also relevant for this process. With a suitable choice of stopping material the rate for μe conversion becomes larger than the rate for $\mu \to e\gamma$ or $\mu \to ee\bar{e}$ in most theories. This is because the process can take place coherently (i.e., the nucleus is not excited and remains in the ground state). The overlap integral of the muon wave function with the nuclear density occurring in the matrix element further enhances the rate. Finally, just as in the case of $\mu \to ee\bar{e}$ the nonphotonic contributions to the rate are enhanced over the photonic contribution because of logarithmic factors coming from box Feynman diagrams (Marciano, 1978). Thus, μe conversion seems theoretically to be a good place to look for muon number violation. This has also been emphasized by Altarelli *et al.* (1977). An interesting aspect of the μe conversion process is that a careful measurement of the branching ratio for different elements reveals the nature of the current mediating the process, and hence throws light on the flavour violating mechanism. For example, flavour violation due to gauge bosons, Higgs bosons or lepto-quark pseudoscalars (occurring in dynamically broken theories) leads to vector currents, scalar currents or a combination respectively. In all three cases the calculations of a recent phenomenological study (Shanker, 1979) can be used to get the coupling constants of the theory from the μe conversion branching ratios for a few elements.

Experimentally also the μe conversion process has many advantages as a probe for muon number violation. The experimental signal for the coherent conversion process is very clear because it involves the detection of a single monoenergetic electron of energy E_μ (where E_μ is the muon energy, i.e., muon mass minus the muon binding energy). The coherent process dominates the rate in general, and since it involves the detection of a single particle in contrast to $\mu \to e\gamma$ or $\mu \to ee\bar{e}$, the experimental search is facilitated. The background to the coherent process (from electrons coming from bound muon decay and from conversion of photons coming from radiative pion and muon capture) is negligible.

All these features make the process an attractive probe for muon number violation.

The μe conversion rate is normally compared to the ordinary muon capture rate. The present limits on the branching ratios are 7×10^{-11} for sulphur (Badertscher *et al.* quoted in Egger, 1979; see also Badertscher, 1977) and 1.6×10^{-8} for copper (Bryman *et al.*, 1972). An experiment to measure the branching ratios for different elements with greater accuracy is under way at TRIUMF in Vancouver. The TRIUMF group use a Time Projection Chamber to increase the solid angle acceptance and to improve the energy resolution over previous experiments.

B. μ^-e^+ conversion

In μ^-e^+ conversion the muon in a muonic atom gets converted into a positron while two of the protons in the nucleus become neutrons. It thus involves a double charge exchange between leptons and hadrons. It is allowed in the Konopinski-Mahmoud scheme of lepton numbers. In this scheme it is suppressed due to the helicity assignments of the neutrino and the V-A nature of the weak interaction (Frankel, 1974; Pontecorvo, 1967). The process could also occur due to Majorana neutrinos (Kamal and Ng, 1979) or Higgs particles (Vergados, 1981). It was first studied in detail by Kamal and Ng (1979). The calculations of the nuclear matrix elements needed for the process are similar to those done for the no-neutrino double beta decay, which is reviewed in a later section. The similarity of the calculations stems from the fact that both processes involve double charge exchange. The experimental limit on the rate for μ^-e^+ conversion is normally quoted as a fraction of the ordinary muon capture rate. Experiments which set limits on μ^-e^- conversion also set limits on the branching ratio for this process. Bryman *et al.*

(1972) report a limit $R(\mu^-e^+) < 2.6 \times 10^{-8}$ for copper. A limit $R(\mu^-e^+) < 9 \times 10^{-10}$ has been set by the Bern group (Badertscher *et al.*, 1978) for sulphur. For experiments which detect the final state positron the background comes from radiative muon capture followed by photon pair production. An interesting new way to observe this reaction is to look for the recoiling nucleus. Such an experiment was done by the Basel-Karlsruhe group at SIN (Abela *et al.*, 1980). They searched for ^{127}Sb from μ^-e^+ conversion in ^{127}I using radiochemical techniques. They get the limit $R(\mu^-e^+) < 3 \times 10^{-10}$ for iodine. The background for this experiment is production of ^{127}Sb through pionic double-charge exchange. For the pion contamination of $<5 \times 10^{-4}$ in the μ^- beam this was found to be negligible.

C. Muon number violating kaon decays

The decays normally investigated are the decay of K_L or K_S into $\mu\bar{e}$ or $\bar{\mu}e$ with or without a pion and the decays of charged kaons into pions and an electron and a muon. Experimental limits are available only for $K_L \to \mu\bar{e}$ or $\bar{\mu}e$, $K^+ \to \bar{e}\mu\pi^+$ and $K^+ \to \pi^+\bar{\mu}e$. Muon number violating decays can be discussed in terms of the effective Lagrangian:

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}}\left[\bar{e}\Gamma^i\left(f_{i(s)}+f'_{i(s)}\gamma_5\right)\mu J^i_{(s)}+\bar{e}\Gamma^i\left(f_{i(a)}+f'_{i(a)}\gamma_5\right)\mu J^i_{(a)}\right] + \text{Hermitian conjugate} \tag{8}$$

where

$$J^i_{(s)} = \bar{s}\Gamma^i d + \bar{d}\Gamma^i s,$$
$$J^i_{(a)} = \bar{s}\Gamma^i d - \bar{d}\Gamma^i s, \tag{9}$$

and Γ^i are 1, γ_5, γ^λ, $\gamma^\lambda\gamma_5$ and $\sigma^{\mu\nu}$ for i = S,P,V,A,T respectively. Herczeg (1979) pointed out in a phenomenological study that muon number violating kaon decays are likely to be small unless the flavour violating

mechanism is suppressed for the K^o-$\bar{K}^o$ transition. Models are known in which the latter process is suppressed and in which muon number violating kaon decay rates can be interesting (Maehara and Yanagida, 1979; Cahn and Harari, 1980; Shanker, 1980 and 1981).

Let us consider the experimental constraints on the phenomenological coupling constants in Eq. 8 from the process $K_L \to \mu^+e^-$. Only the axial vector and pseudoscalar "symmetric" currents with coupling constants $f_{A(s)}$, $f'_{A(s)}$, $f_{P(s)}$ and $f'_{P(s)}$ will contribute to the process. If we define the K_L to vacuum matrix elements

$$<0|J^A_{\lambda(s)}|K_L(P)> = P_\lambda m_K a_A/\sqrt{2m_K}$$
$$<0|J^P_{(s)}|K_L(P)> = -m_K^2 a_P/\sqrt{2m_K}, \tag{10}$$

the rate for $K_L \to \mu^+e^-$ can be written (Herczeg, 1979)

$$\Gamma(K_L \to \mu^+e^-) = m_K(1-m_\mu^2/m_K^2)^2(|A|^2+|B|^2)/(8\pi) \tag{11}$$

where

$$A \approx (4.2 \times 10^{-7})f_{A(s)}a_A - (2 \times 10^{-6})f_{P(s)}a_P,$$
$$B \approx (4.2 \times 10^{-7})f'_{A(s)}a_A + (2 \times 10^{-6})f'_{P(s)}a_P. \tag{12}$$

The branching ratio for $K_L \to \mu^+e^-$ is

$$B(K_L \to \mu^+e^-) = \Gamma(K_L \to \mu^+e^-)/\Gamma(K_L \to \text{all})$$
$$\approx (1.4 \times 10^{15})(|A|^2+|B|^2). \tag{13}$$

The experimental upper limit for the branching ratio (Clark *et al.*, 1971) is 2×10^{-9} and implies that the absolute value of A (or B) should be less than 1.2×10^{-12}. The form factor a_A can be related to the leptonic kaon decay form factor f_K using isospin symmetry:

$$<0|J^A_{\lambda(s)}|K_L(P)> = \sqrt{2}<0|\bar{s}\gamma_\lambda\gamma_5 u|K^+> = \sqrt{2}\, f_K P_\lambda/\sqrt{2m_K}\ , \tag{14}$$

which gives $a_A = \sqrt{2}\, f_K/m_K$. a_P can be estimated from a_A using the current algebra relation (Branco *et al.*, 1976)

$$\partial^\lambda J^A_{\lambda(s)} = i(m_s+m_d)J^P_{(s)} \tag{15}$$

which gives

$$a_P = a_A\, m_K/(m_s+m_d) \tag{16}$$

The above relations give (Herczeg, 1979)

$$a_A \approx 0.48$$

$$a_P \approx 1.5 \tag{17}$$

if we take $f_K = 1.23\, m_\pi$ and $m_s = 150$ MeV, $m_d = 7.5$ MeV (Weinberg, 1977b). The decay could occur due to neutrino mixing, for example it could occur in the standard model with six leptons and six quarks if the τ neutrino were not massless and mixed with the other neutrinos. In the standard model all muon number violating effects depend on the parameter $\beta\gamma$ where β and γ measure the amount of ν_τ mass eigenstate in the gauge group eigenstates ν_e' and ν_μ' respectively (Lee and Shrock, 1977a; Cheng and Li, 1977b). The $K_L \to \mu e$ rate in this model has been calculated by Lee and Shrock (1977a) and by Cheng and Li (1977b). Lee and Shrock find that the main contribution to the rate comes from the free quark diagram Fig. 2b and the contribution from the intermediate two photon decay $K_L \to 2\gamma \to \mu e$ is small. This is in contrast to the situation for the decay $K_L \to \mu\bar{\mu}$ where the dominant contribution comes from the 2γ intermediate state. The ratio of the free quark contributions to $K_L \to \mu e$ and to $K_L \to \mu\bar{\mu}$ is of the order of $(\beta\gamma)^2$. Hence the ratio of the rate for $K_L \to \mu e$ to the total rate for $K_L \to \mu\bar{\mu}$ is of the order of $10^{-3}(\beta\gamma)^2$. Since the branching ratio for $K_L \to \mu\bar{\mu}$ is very small it is clear that the rate for $K_L \to \mu e$ is negligible in the standard model. With the experimental limits $(\beta\gamma)^2 < 2 \times 10^{-3}$ (Lee and Shrock, 1977a; Altarelli *et al.*,

1977; Cheng and Li, 1977b; Bleitschau *et al.*, 1978) and $m_{\nu_\tau} < 250$ MeV one obtains

$$\frac{\Gamma(K_L \to \mu^+e^-)}{\Gamma(K_L \to \text{all})} < 5 \times 10^{-16}, \tag{18}$$

which is far below the present experimental limit. More interesting limits are obtained in horizontal gauge models and technicolour models.

The only other muon number violating kaon decays for which significant limits exist are $K^+ \to \pi^+\bar{\mu}e$ and $K^+ \to \pi^+\mu\bar{e}$. These processes could occur due to "symmetric" and "antisymmetric" scalar, vector and tensor currents in the effective Lagrangian Eq. 8. Using isospin symmetry the K^+ to π^+ vector matrix element can be related to the K^+ to π^0 matrix element which is known for vector currents from $K^+ \to \pi^0\mu^+\nu_\mu$ and $K^+ \to \pi^0 e^+\nu_e$ decays. For scalar currents the matrix element can be related to the vector matrix element using current algebra (Branco *et al.*, 1976). The present limits on the branching ratios are (Diamant-Berger *et al.*, 1976):

$$\frac{\Gamma(K^+ \to e^+\mu^\pm\pi^\mp)}{\Gamma(K^+ \to \text{all})} < 7 \times 10^{-9},$$

$$\frac{\Gamma(K^+ \to \pi^+\mu^+e^-)}{\Gamma(K^+ \to \text{all})} < 5 \times 10^{-9}. \tag{19}$$

For illustrative purposes I will describe the phenomenological calculation of Herczeg (1979) on the rate for $K^+ \to \pi^+\mu\bar{e}$ mediated by a scalar (Higgs) particle. He assumes that a scalar particle ϕ couples to fermion currents with coupling constants $(g'\bar{\mu}e\phi + g''J^S_{(s)}\phi)$, where $J^S_{(s)}$ is defined by Eq. 9. With these couplings the effective muon number violating Lagrangian due to the exchange of the scalar particle is given by Eq. 8 with $G_F f_{S(s)}/\sqrt{2} = g'g''/m_\phi^2$. The branching ratio for $K^+ \to \pi^+\mu\bar{e}$ is then given by $8|f_{S(s)}|^2$. One can get some constraints on $f_{S(s)}$ because the scalar particle exchange also contributes to the K_L-K_S mass difference.

The effective $\Delta S = 2$ Lagrangian due to the scalar particle exchange is of the form $(g''/m_\phi)^2(\bar{s}d)(\bar{s}d)$ and this contributes to the K_L-K_S mass difference an amount

$$\Delta m_K = 2(g''/m_\phi)^2 \langle \bar{K}^o|\bar{s}d\bar{s}d|K^o\rangle \ . \tag{20}$$

Herczeg had to estimate the K^o-$\bar{K}^o$ transition matrix element in Eq. 20. A study of the K^o-$\bar{K}^o$ transition matrix elements for a general combination of the $\Delta S = 2$ currents now exists (McWilliams and Shanker, 1980b), and I will use the results for the analysis.

I will describe the calculation of the K^o-$\bar{K}^o$ transition matrix elements briefly in view of their importance. Three methods have been used to calculate them. One method is to use the bag model (Shrock and Trieman, 1979). The remaining two methods are both called the vacuum insertion method. To avoid confusion I will use different names. The first evaluation of the K^o-$\bar{K}^o$ matrix element was done for the (V-A) $\Delta S = 2$ current which occurs in the standard weak interaction model (Gaillard and Lee, 1974). They used what is called the vacuum insertion method, and I will call this method the Fierz transform method. In this method the kaon is treated as a two quark object (i.e., the effect of sea quarks and gluons are ignored). The four quark $\Delta S = 2$ operator is broken up into creation and annhilation components, and these are Fierz transformed such that the operators destroy the quark-antiquark pair in the K^o meson before creating those in the $\bar{K}^o$ meson. If a complete set of states is now introduced between the operators, only the vacuum state will contribute provided the K^o meson is really made up of only a quark and an anti-quark. That is why this method is called the vacuum insertion method. This method relates the K^o-$\bar{K}^o$ matrix

element to the K^o to vacuum matrix element which is experimentally determined. The third approach is also called the vacuum insertion method because it is often confused with the method which I described above, the Fierz transform method. In this method a complete set of states is introduced without making the Fierz transformations. Since only the kaon to vacuum matrix element is known to reasonable accuracy, the sum over complete states is truncated with the vacuum contribution. Hence the third approach can be called a truncated complete states method. In the study of McWilliams and Shanker the Fierz transform method of calculation and the bag model calculation are used for a general combination of $\Delta S = 2$ currents, unlike the previous studies which were limited to a (V-A) combination of currents. This study finds that the bag model and Fierz transform methods are in reasonable agreement.[1] However, the truncated complete states method, which is commonly used, can give very misleading results due to the neglect of states other than the vacuum in the sum.

I will use the Fierz transform value for the matrix element in Eq. 20. The Fierz transform calculation is sensitive to current algebra quark masses, and I use the values $m_s = 150$ MeV, $m_d =$ 7.5 MeV (Weinberg, 1977b). The matrix element takes the value $-0.01\ \mathrm{GeV}^3$. Assuming that the scalar particle contribution to the

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[1]The bag model matrix elements are approximately 0.7 times the Fierz transform matrix elements, and this is within the expected accuracy of the methods. Note that for the (V-A) case the bag model calculation is 0.4 times the Fierz transform calculation. The agreement is poorer for the (V-A) case because a cancellation occurs between the vector and axial vector matrix elements, and hence the results are more sensitive to the bag model parameters.

K_L-K_S mass difference is comparable to the experimental value of 3.5×10^{-15} GeV, we find that $m_\phi > g''(2 \times 10^6)$ GeV. Often the coupling of scalar (Higgs) particles to fermions is proportional to the fermion masses. Hence, if we assume that $g'/g'' = m_\mu/m_s$ we find that the above result constrains the $K^+ \to \pi^+\mu\bar{e}$ branching ratio to be less than 1.5×10^{-15}. For $g'/g'' = m_\mu/m_d$ the branching ratio is less than 5×10^{-13}. These are well below the experimental value quoted in Eq. 19, illustrating the strong constraints provided by the K_L-K_S mass difference on lepton number violation in some models. The branching ratios calculated above are two orders of magnitude larger than the results of Herczeg. This is because I used the result of a K^0-$\bar{K}^0$ matrix element calculation instead of an estimate. The large change reflects the fact that the K_L-K_S mass difference is proportional to the amplitude of the flavour-violating current and hence is inversely proportional to m_ϕ^2 while the rate for $K^+ \to \pi^+\mu\bar{e}$ is proportional to the square of the amplitude and hence is inversely proportional to m_ϕ^4. Thus, an order of magnitude change in the value of the K^0-$\bar{K}^0$ transition matrix element that was used changed the estimate for the $K^+ \to \pi^+\mu\bar{e}$ rate by two orders of magnitude. The use of better K^0-$\bar{K}^0$ transition matrix elements does not change the results of the other cases considered by Herczeg so drastically.

The search for muon number violating kaon decays is useful as a probe for horizontal gauge bosons or the leptoquark pseudoscalars of dynamically broken theories. Doubly charged muon number violating kaon currents have also been discussed in the context of the Konopinski-Mahmoud lepton number scheme. However, μ^-e^+ conversion and no-neutrino double beta decays seem to provide more stringent tests of doubly charged muon number violating currents (Frankel, 1974; Diamant-Berger *et al.*, 1976).

D. Neutrinoless double beta decay

The neutrinoless double beta decay process is the emission of two electrons by a nucleus, with no accompanying neutrinos. Two neutrons in the nucleus get converted into protons. This process violates lepton number. It could occur in gauge theories if the neutrinos pick up Majorana masses or it could occur due to Higgs particle interactions. A recent review of the process is given in Rosen (1980). If the neutrino is a Majorana particle and has zero mass the process is not allowed because of the (V-A) nature of the weak interaction. This is because the process occurs in two steps, i.e., $n \to p+e^-+\nu_M$ followed by $\nu_M+n \to p+e^-$, and the neutrino emitted in the first step has the wrong helicity to participate in the second step. The process could occur if the weak interaction had right-handed components, or if the neutrino had non-zero mass, since mass terms lead to violation of helicity. It is sometimes stated that a Dirac particle and Majorana particle are equivalent in the zero mass limit. This is only true for the equation of motion, i.e., the Dirac and Majorana equations become equivalent for zero mass. As the above discussion shows, even in the zero mass limit the Dirac or Majorana nature of the neutrinos could affect their interactions.

The weak interaction Hamiltonian for double beta decay with Majorana neutrinos has been discussed by Pauli (1957), Pursey (1957), Lüders (1958), Enz (1957). The effect of neutrino mass in overcoming the suppression due to the helicity of the weak current has been discussed by Enz (1957), Greuling and Whitten (1960) and Halprin *et al.* (1976). Calculations of the lifetimes for the process in terms of weak interaction parameters were performed by Furry (1939), Touschek (1949), Primakoff (1952), Konopinski (1955) and Primakoff and Rosen (1959, 1965 and 1969). There are very large uncertainties (about two orders of magnitude) in the calculations due to uncertainties in the knowledge of the nuclear matrix elements. Recent calculations of the matrix elements for the tellurium nucleus were performed by Haxton *et al.* (1979). All the above work assumed that no-neutrino double beta decay occurred due to a Majorana neutrino. The process must involve two nucleons because one nucleon cannot change its charge by two units (a nucleon has isospin 1/2 and hence only two isospin components). Experimental limits constrain the neutrino mass to be either very small or very large. In the former case the helicity of the weak interaction operates in suppressing the rate for the process and in the latter case the suppression is due to the Yukawa-type force generated by the exchange of the neutrino between two nucleons. If the nucleus contains nucleon resonances in addition to unexcited nucleons, the process could involve only one resonance of isospin greater than 1/2. This is because such a resonance would have more than two components and could change its charge by two units. The process is assumed to involve two quarks within the resonance (Primakoff and Rosen, 1969; Halprin *et al.*, 1976). The lower limit on the neutrino Majorana mass for the heavy neutrino case becomes much more stringent if such a nucleon resonance is contained in the nucleus. This is because the quarks participating in the process come closer together than in the case involving two nucleons. Also, unlike nucleons the quarks are expected to have no hard-core potential between them. Pontecorvo (1968) discussed a $\Delta L = 2$ superweak interaction mediating the process, in analogy with the superweak CP violating interaction proposed by Wolfenstein (1964).

The experimental search for the process involves both geo-chemical and direct detection methods. The lepton number conserving two neutrino double beta decay process does occur and has to be separated experimentally or subtracted theoretically. A review of the experiments is given in Bryman and Picciotto (1978). They find that phase space calculations do not account for the ratio of Te^{130} and Te^{128} lifetimes, assuming that the nuclear matrix elements for $Te^{130} \to Xe^{130}$ and $Te^{130} \to Xe^{128}$ are equal and that the decays are due to lepton number conserving two neutrino double beta decay modes. They find that the existence of a small number of no-neutrino decay mode events could remove the discrepancy. The uncertainties in the nuclear matrix elements are too large to draw any definite conclusions regarding lepton number violation (Haxton *et al.*, 1979).

IV. Model predictions

Having seen the types of models which predict muon number violation and the different muon number violating processes that have been studied, it is natural to ask which of the processes are most sensitive to violation of muon number. Unfortunately, the answer to this question is model-dependent. At present we cannot convincingly say whether muon number violation should occur at all, the scale at which possible violations may be expected, or the process most sensitive to muon number violation. However, one can look at the different types of models and draw some conclusions.

A. Horizontal gauge models

Let us first look at horizontal gauge models. In these models the muon number violating rates will be uninterestingly small unless the contribution of the horizontal gauge bosons to the K_L-K_S mass difference is suppressed. This is because the K_L-K_S mass difference is sensitive to the matrix element of the flavour violating current, unlike the rates which are sensitive to the square of the matrix element. A class of models is known (Maehara and Yanagida, 1979; Cahn and Harari, 1980; Shanker, 1980 and 1981) in which the K_L-K_S mass difference is suppressed and in which the muon number violating rates could be close to the experimental upper limits. The suppression mechanism depends on the fact that the horizontal gauge bosons are almost degenerate in mass. Table 1 shows a representative calculation of the constraints imposed on the parameters of this class of theories by muon number violating processes. The cases referred to in the table are described in Shanker (1981) and correspond to different choices of a discrete symmetry in the model. The muon number violating processes constrain two parameters in the theory, which are essentially the horizontal gauge boson mass M and the gauge boson degeneracy breaking, $\delta = \Delta M/M$. δ is expected to be of order M_W^2/M^2 if all gauge coupling constants are comparable (M_W is the mass of the ordinary W boson mediating weak interaction processes). In reading the table it should be kept in mind that all mixing angles are assumed to be of order one and possible suppressions due to mixing angles can change some of the conclusions. For comparison with the table we note that the imaginary part of the K_L-K_S mass difference constrains δ to be less than $4 \times 10^{-13}(M/M_W)^2$. Since δ is expected to be of order M_W^2/M^2, we see that μ^-e^- conversion provides the most stringent constraint on the theory, i.e., $M > 3000\ M_W$, unless mixing angles suppress the rate for this process. The pattern in the table can be understood quite simply: the mechanism which suppresses the K_L-K_S mass difference also suppresses muon number violating rates involving only two flavours, i.e. $\mu \to e\gamma$ and $\mu \to ee\bar{e}$. However, the mechanism works only when certain

restrictions are satisfied by the fermion mixing angles, and hence $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ are not suppressed in some cases. In these cases they give strong constraints on the horizontal gauge boson mass M. In all cases μe conversion provides one of the best limits on M. This is a general property of μe conversion and is not limited to the class of horizontal gauge models we are considering. The enhancement of μe conversion rates in many models reflects the fact that the quarks in the nucleus contribute coherently to the μe conversion amplitude. It is possible that the coupling constants conspire to reduce the rate for μe conversion.[2] This means that all the processes in the table must be investigated to look for muon number violation.

B. Hypercolour models

I now consider hypercolour models, in which the symmetry is dynamically broken. In these models muon number violation can occur due to light scalar particles (called pseudo-Goldstone bosons. They are actually bound states of hyperfermions. However, they behave like spin zero particles at low energies.) Muon number violation could also occur due to horizontal guage bosons which have to be introduced to give masses to the ordinary fermions. In hypercolour models the masses of the particles mediating flavour changing processes cannot be made arbitrarily large. They are constrained by the (ordinary) fermion masses and mixing angles. This causes some difficulty for hypercolour models, and they tend to predict too large a value for the K_L-K_S mass difference. A

[2] In general this is unlikely, but one can adjust parameters to build a model where this happens. Since the process involves both up and down quarks, it is difficult to adjust parameters such that the process is completely suppressed.

suppression mechanism for this process seems to be required. In the case of the flavour changing scalar particles Ellis *et al.* (1980) achieve this by what they call "monophagy", namely, each charge sector of ordinary fermions gets its mass from a unique effective scalar (hyperfermion condensate). This is similar to the standard model of weak interactions with two or more ordinary Higgs doublets and natural flavour conservation. Ellis *et al.* note that even in monophagous models coloured leptoquark scalars exist which mediate the $K_L \to \mu e$ process. Because of fermion mixing one would expect other processes like μe conversion to also occur, and this is now being studied (Ng and Shanker, work in progress). The leptoquark nature of the intermediate boson implies that the flavour violating current has both scalar and vector components, and this is a distinctive signal. The nature of the current would reveal itself in the variation of the μe conversion rate for different elements, for example.

For the horizontal gauge bosons in hypercolour models no suppression mechanism is known for the K_L-K_S mass difference, but such a mechanism seems to be needed. One would then expect the phenomenology of flavour violation due to the gauge bosons to resemble the phenomenology of the horizontal gauge models considered above. Dimopoulos and Ellis (1980) have made a useful study of flavour violation in hypercolour models. They arrive at different conclusions. They argue that in the absence of fermion mixing the mass of the horizontal gauge boson which gives mass to a particular quark is inversely proportional to the square root of the quark mass. Because of the large hierarchies in quark masses this leads to gauge bosons with very different masses. They then assume that flavour violating processes are mediated by gauge bosons with masses of

the same order as those which give masses to the quarks participating in the process. Because fermions do mix, it may be too restrictive to use flavour changing gauge bosons with very different masses for different flavour changing processes, as is done by Dimopoulos and Ellis. The lighter horizontal gauge bosons, which give masses to the heavier quarks (say to the bottom quark), may contribute to flavour changing processes involving light quarks only (say to μe conversion with a current coupling to the down quark), because of fermion mixing. In fact, a mechanism which generates both quark masses and quark mixing angles may not even have a large hierarchy of horizontal gauge boson masses. The work of Dimopoulos and Ellis seems to indicate that $K_L \to \mu e$ is the most sensitive muon number violating process in hypercolour theories. However, as I have just argued, a more detailed understanding of the mechanism generating fermion masses and mixing angles seems to be required before reliable conclusions can be drawn. Normally quark mixing angles do not change the phenomenology by many orders of magnitude. In the work of Dimopoulos and Ellis, however, the hierarchy of gauge boson masses makes the phenomenology sensitive to the mixing angles. The mixing angles which occur in the phenomenology are not necessarily the ones which occur in the ordinary weak interaction (the Kobayashi-Maskawa or generalised Cabibbo angles).

C. Grand unified theories

An interesting question is the implication of grand unified theories (GUTs) for muon number violation. The superfluous proliferation of families is as much of a mystery in these theories as it is in the low energy standard model. In GUTs also horizontal gauge symmetries have been considered to deal with the generation problem, and these attempts are reviewed in Langacker (1980). These attempts include the use of large simple groups or products of simple groups with discrete symmetries imposed to get one coupling constant. The implications of these models for the processes considered here depend on whether the horizontal gauge bosons get masses of the order of GUT masses, or whether they get masses low enough to give measurable rates for muon number violating processes. No satisfactory model which accounts for the repetition of generations exists. Many GUTs do predict non-zero mass for the neutrinos, and hence have implications for neutrino experiments. The discovery of muon number violation in other processes would have implications for GUTs, since the models would have to account for the new phenomena. A class of GUTs, the Pati-Salam models, have light (10^4-10^6 GeV) leptoquarks which could mediate muon number violating semileptonic processes like $K_L \to \mu e$ and μe conversion. While no detailed studies of this aspect have been made, the quark coherence effect in μe conversion should make it a favoured process. Because of possible mixing angle suppression in μe conversion muon number violating kaon decays could also be interesting processes.

D. Neutrino mass

Non-zero neutrino masses also imply the existence of flavour violating processes other than neutrino oscillations. However, the rates will be far below present experimental limits (Petkov, 1977). Cheng and Li (1980b) have studied the relation between $\mu \to e\gamma$ and neutrino mass terms in models with three generations of fermions, which reduce to the standard model at low energies. They find that in some cases these models can have detectable $\mu \to e\gamma$ rates if the model parameters are fine-tuned. The presence of new mass scales for the neutrinos (in the left-right symmetric models, or in a model with more than three generations, for

example) could lead to detectable flavour violating rates. Even in the left-right symmetric models, which have been thoroughly studied, no complete calculation of muon number violating (lepton-number conserving) rates have been made.

E. Higgs models

Let us finally look at extensions of the standard low energy model which introduce new scalar (Higgs) particles. The new particles have been most often introduced to get spontaneous CP violation, or are motivated because GUTs may reduce at low energies to theories which have these new particles. In theories which introduce new Higgs $SU(2)_W$ doublets one can impose a discrete symmetry which enforces natural flavour conservation. Even when natural flavour conservation is not imposed the stringent limit on the K_L-K_S mass difference is likely to constrain the Higgs particle masses so strongly that muon number violation is very small. However, if the contribution of the Higgs particles to the K_L-K_S mass difference is suppressed for some reason the muon number violating rates could be close to present experimental limits. For example, in the model of Lahanas and Vayonakis (1979) the Higgs particle contribution to the K_L-K_S mass difference is proportional to the Higgs particle mass differences. Hence muon number violating processes could have detectable rates if the Higgs scalars have almost degenerate masses. Bjorken and Weinberg (1977) have pointed out that in a model with Higgs-particle-mediated flavour violation two-loop diagrams may make a larger contribution to $\mu \to e\gamma$ than one-loop diagrams. In the model of Lahanas and Vayonakis $\mu \to e\gamma$ could be suppressed by Higgs particle mass differences since it involves only two kinds of flavours, like the K^0-$\overline{K^0}$ transition. Processes like μe conversion, on the other hand, may have detectable rates. The model of Lahanas and Vayonakis makes some assumptions regarding mixing angles (eg. that mixing angles in the left- and right-hand sectors are equal, up to a diagonal phase matrix). The role of these assumptions in the suppression of the K_L-K_S mass difference has to be studied.

After the detailed discussion of flavour violation in different models that has been presented we are in a position to make a general summary.

V. Conclusions

Many extensions of the standard model that are presently being studied have non-zero rates for muon number violating processes. A complete study of muon number violation has been made for only some of the models. On general grounds one finds that neutrino oscillations and μe conversion are sensitive probes of muon number violation. Neutrino oscillations are sensitive because the experiments measure the flavour violating amplitudes directly, while the experiments for the other processes measure the square of the amplitude. μe conversion is interesting because muon number violation could occur while the neutrinos remain massless. It is a sensitive process because the quarks in the nucleus contribute coherently to the amplitude. Since mixing angles may suppress μe conversion one should also investigate the other processes listed in Table I. For models where both muon number and total lepton number are violated (models with Majorana neutrinos, which have been investigated extensively, belong to this class) neutrino-anti-neutrino oscillations seem to be very sensitive tests. The $\mu^- e^+$ conversion and neutrinoless double beta decay processes should also be investigated because they may be sensitive to regions of the model parameters which have not been probed by neutrino oscillation experiments.

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Table I. Typical constraints on horizontal gauge models from muon number violating processes. M and M_W are the horizontal gauge boson mass and weak gauge boson mass respectively. δ is the horizontal gauge boson mass degeneracy breaking parameter, $\Delta M/M$. δ is expected to be of order M_W^2/M^2. See text for further details.

Process	Cases i, ii, v, vi Parameter	Limit	Limits on M for cases iii and iv	Experimental upper limit
$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to all)}$	δ	$<4 \times 10^{-7}(M/M_W)^2$	$1500\ M_W$	2×10^{-10}
$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to all)}$	δ	$<10^{-6}(M/M_W)^2$	$1000\ M_W$	2×10^{-9}
$\frac{\Gamma(\mu^- + A \to e^- + A)}{\Gamma(\mu^- + A \to \nu + A')}$	M	$>3000\ M_W$	$3000\ M_W$	7×10^{-11} (sulphur)
$\frac{\Gamma(K_L \to e\bar{\mu} \text{ or } \bar{\mu}e)}{\Gamma(K_L \to all)}$	M^a	$>100\ M_W$	$100\ M_W$	2×10^{-9}
$\frac{\Gamma(K^+ \to \pi^+ \bar{\mu} e)}{\Gamma(K^+ \to all)}$	M	$>300\ M_W$	$300\ M_W$	5×10^{-9}
$\frac{\Gamma(K^+ \to \pi^+ \mu \bar{e})}{\Gamma(K^+ \to all)}$	M^b	$>300\ M_W$	$300\ M_W$	7×10^{-9}

[a]These processes require axial vector quark currents. For the cases listed in the table such axial vector currents arise only at the one-loop level and hence the processes are suppressed by factors of coupling constants. When axial vector currents arise due to tree diagrams the limit on M is much more stringent, $M > 1000\ M_W$. For example, this would be the limit for the case in Shanker (1981) where the left and right mixing angles for the d type quarks differ by complex phase factors.

[b]For cases (i) and (v) some mixing angle relations result in a suppression of this process. The relevant bound in these cases is on δ, $\delta < 10^{-5}(M/M_W)^2$.

Addenda

In the section on μe conversion I should mention that Lincoln Wolfenstein [1977, in *Proc. of 7th Int. Conf. on High Energy Physics and Nuclear Structure*, edited by M.P. Locher (Birkhäuser Verlag, Basel), p. 363] emphasized the reliability with which information on gauge theory parameters can be deduced from experimental limits on the μe conversion rate. He played an important role in the nuclear physics calculations which are required for μe conversion. In the section on neutrino processes I should mention that Douglas R.O. Morrison (1980, CERN Report CERN/EP 80-190) gives a critical discussion of the experimental evidence for neutrino oscillations and mentions that explanations not involving neutrino oscillations seem to exist for all the experimental results. He also discusses anomalous results in the CERN beam dump experiments and mentions that explanations involving neutrino oscillations are unlikely for these results.

Epilogue

We began the discussion with the five elements of the ancients, progressed to ninety-two and beyond, went back to two, we are progressing again, and the end is not in sight. The riddle we have discussed is old, older still than the sphinx's riddle: what creature, with only one voice, has four legs in the morning, two at midday, and three in the evening, and yet is weakest when it has the most? Many weary travellers have attempted to answer the former riddle, and yet we wait for the Oedipus who will make the sphinx cast herself from her rock and cease to torment us!

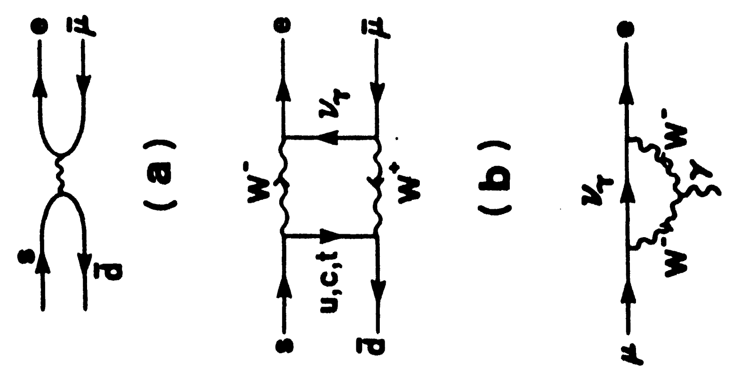

Fig. 2. Representative Feynman diagrams contributing to muon number violation in different models.

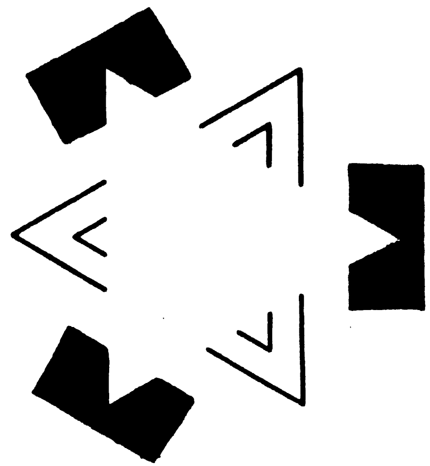

Fig. 1. A triangle anomaly.