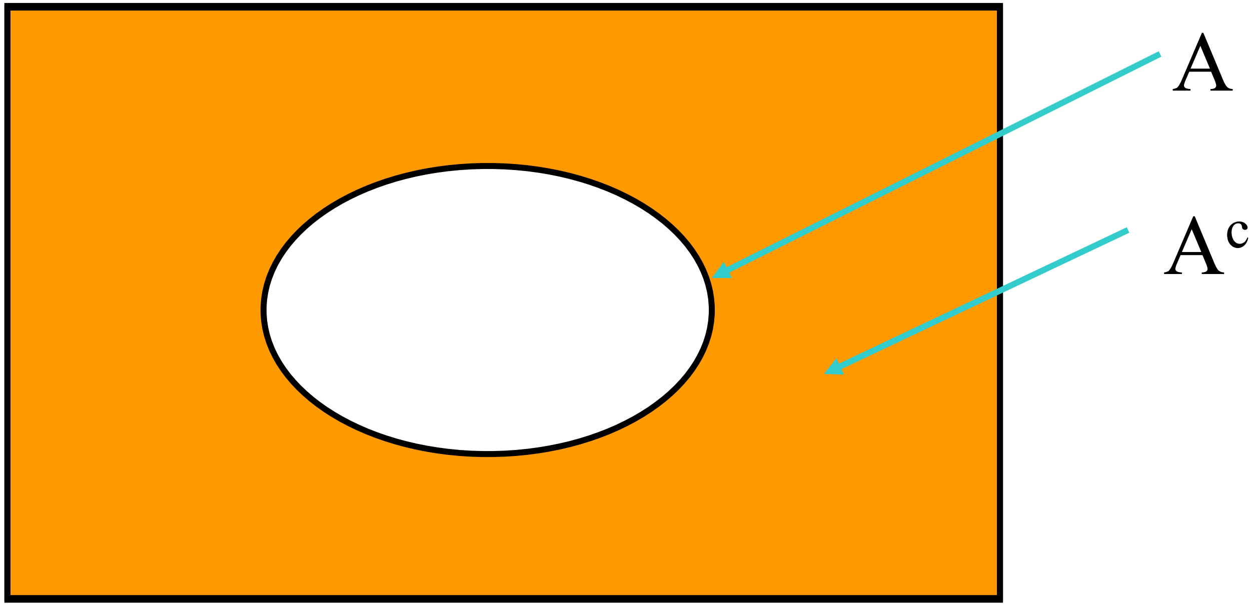
Mathematics for Computing - I

Sets (07 hrs.)

Set Operations - Complement

- The (absolute) complement of a set ‘A’ is the set of elements which belong to the universal set but which do not belong to A. This is denoted by A^c or $\bar{A}$ or A' .
- In other words we can say:
- $A^c = \{x : x \in U \wedge x \notin A\}$

Venn Diagram for the Complement



Set Operations - $\cup$nion

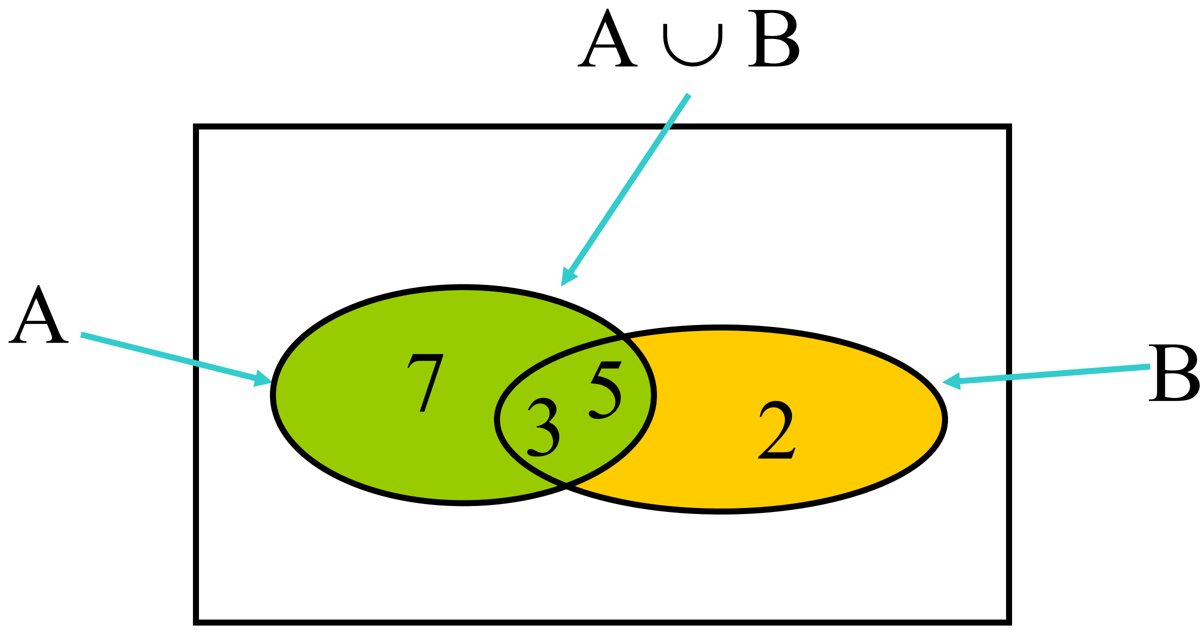
- Union of two sets 'A' and 'B' is the set of all elements which belong to either 'A' or 'B' or both. This is denoted by $A \cup B$.
- In other words we can say:

$$A \cup B = \{x : x \in A \vee x \in B\}$$

- E.g. $A = \{3, 5, 7\}$, $B = \{2, 3, 5\}$

$$A \cup B = \{3, 5, 7, 2, 3, 5\} = \{2, 3, 5, 7\}$$

Venn Diagram Representation for Union



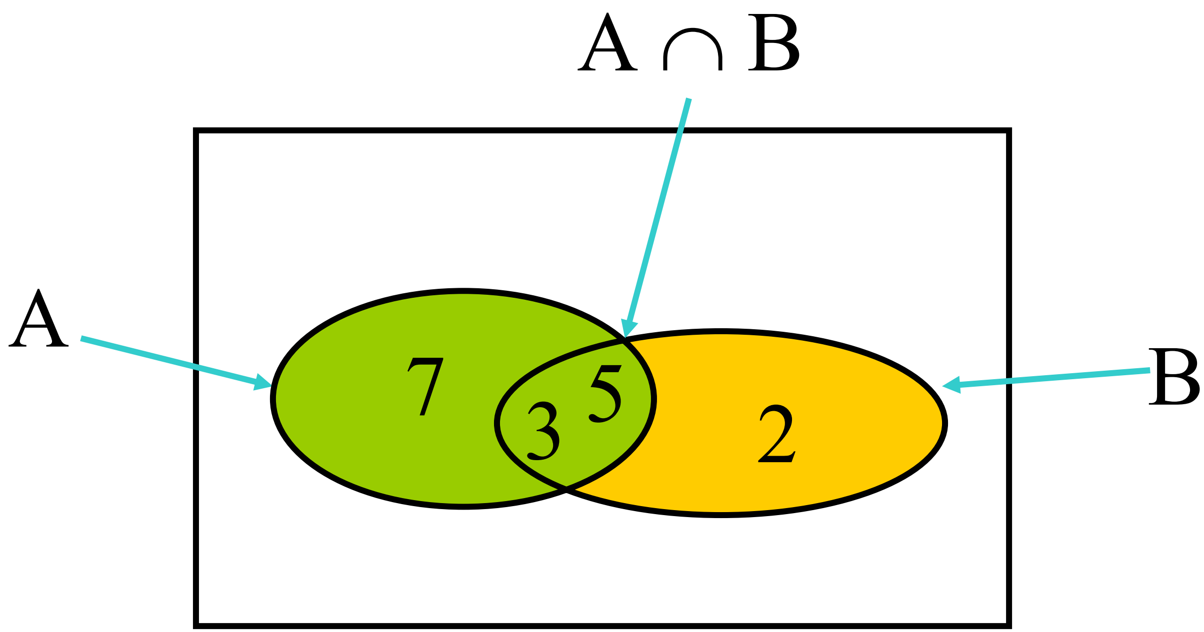
Set Operations - I$\cap$tersection

- Intersection of two sets 'A' and 'B' is the set of all elements which belong to both 'A' and 'B'. This is denoted by $A \cap B$.
- In other words we can say:

  $A \cap B = \{x : x \in A \wedge x \in B\}$
- E.g. $A = \{3, 5, 7\}$, $B = \{2, 3, 5\}$

  $A \cap B = \{3, 5\}$

Venn Diagram Representation for Intersection



Set Operations - Difference

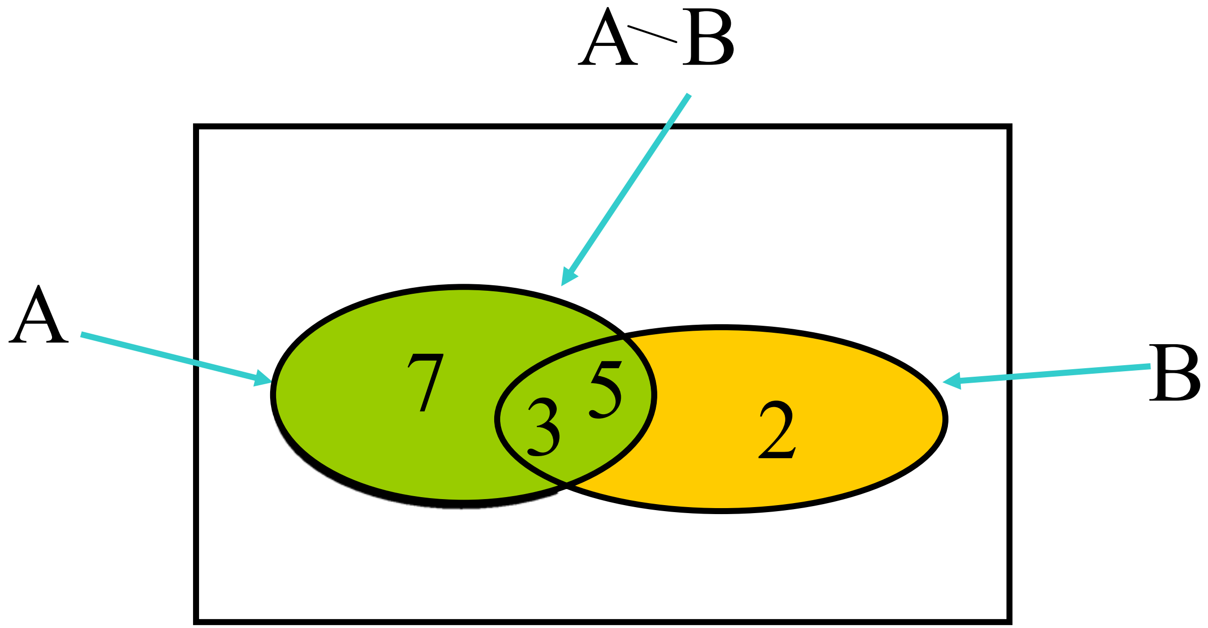
- The difference or the relative complement of a set 'B' with respect to a set 'A' is the set of elements which belong to 'A' but which do not belong to 'B'. This is denoted by $A \setminus B$.
- In other words we can say:

$$A \setminus B = \{x : x \in A \land x \notin B\}$$

- E.g. $A = \{3, 5, 7\}$, $B = \{2, 3, 5\}$

$$A \setminus B = \{\cancel{3}, \cancel{5}, 7\} \setminus \{2, 3, 5\} = \{7\}$$

Venn Diagram Representation for Difference



Some Properties

- $A \subseteq A \cup B$ and $B \subseteq A \cup B$
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$
- $|A \cup B| = |A| + |B| - |A \cap B|$
- $A \subseteq B \Rightarrow B^c \subseteq A^c$
- $A \setminus B = A \cap B^c$
- If $A \cap B = \Phi$ then we say 'A' and 'B' are disjoint.

Algebra of Sets

- Idempotent laws
  - $A \cup A = A$
  - $A \cap A = A$
- Associative laws
  - $(A \cup B) \cup C = A \cup (B \cup C)$
  - $(A \cap B) \cap C = A \cap (B \cap C)$

Algebra of Sets ctd…

- Commutative laws
  - $A \cup B = B \cup A$
  - $A \cap B = B \cap A$
- Distributive laws
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Algebra of Sets ctd…

- Identity laws
  - $A \cup \Phi = A$
  - $A \cap U = A$
  - $A \cup U = U$
  - $A \cap \Phi = \Phi$
- Involution laws
  - $(A^c)^c = A$

Algebra of Sets ctd…

- Complement laws
  - $A \cup A^c = U$
  - $A \cap A^c = \Phi$
  - $U^c = \Phi$
  - $\Phi^c = U$

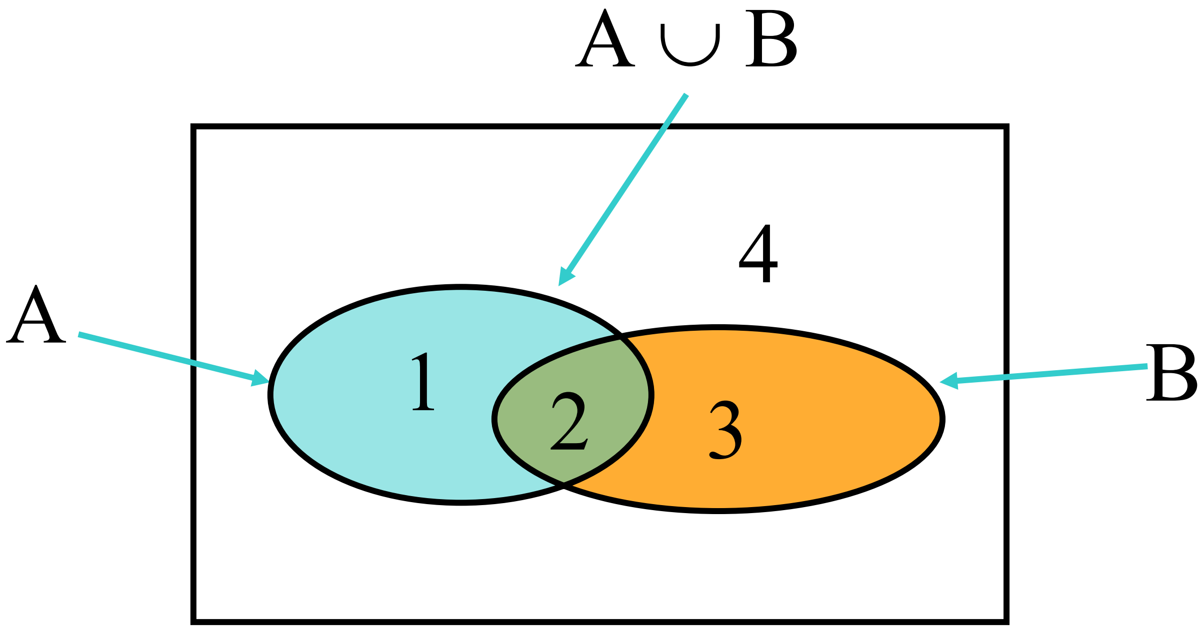
Algebra of Sets ctd…

- De Morgan's laws
  - $(A \cup B)^c = A^c \cap B^c$
  - $(A \cap B)^c = A^c \cup B^c$
- Note: Compare these De Morgan's laws with the De Morgan's laws that you find in logic and see the similarity.

Proofs

- Basically there are two approaches in proving above mentioned laws and any other set relationship
  - Algebraic method
  - Using Venn diagrams
- For example lets discuss how to prove
  - $(A \cup B)^c = A^c \cap B^c$

Proofs Using Venn Diagrams



- Note that these indicated numbers are not the actual members of each set. They are region numbers.

Proofs Using Venn Diagrams ctd…

U : 1, 2, 3, 4

A : 1, 2 (i.e. The region for 'A' is 1 and 2)

B : 2, 3

$\therefore$ $A \cup B$: 1, 2, 3

$\therefore$ $(A \cup B)^c$: 4 ———————————— (α)

Proofs Using Venn Diagrams ctd…

A^c : 3, 4

B^c : 1, 4

$\therefore A^c \cap B^c : 4$ ——— (β)

$$\underline{\underline{(\alpha) \wedge (\beta) \quad \Rightarrow (A \cup B)^c = A^c \cap B^c}}$$