MSc Computational Cognitive Neuroscience

Cortical Modelling

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Coursework 2

Two rate neurons:

$$\tau \frac{dr_1}{dt} = -r_1 + \alpha r_2 + I_1 \qquad (1)$$

$$\tau \frac{dr_2}{dt} = -r_2 + \beta r_1 + I_2 \qquad (2)$$

If h is the time step

$r1(t)$ and $r2(t)$ are values at time t

$r1(t+h)$ and $r2(t+h)$ are values at time $t+h$

Using the first order derivative forward difference approximation:

$$\frac{dr_1}{dt} \approx \frac{(r1(t+h) - r1(t))}{h} \qquad (3)$$

$$\frac{dr_2}{dt} \approx \frac{r2(t+h) - r2(t)}{h} \qquad (4)$$

Subsituting (3) and (4) in (1) and (2)

$$\frac{r1(t+h) - r1(t)}{h} = \frac{-r1(t) + A r_2(t) + I_1}{\tau} \qquad (5)$$

$$\frac{r2(t+h) - r_2(t)}{h} = \frac{-r2(t) + B r_1(t) + I_2}{\tau} \qquad (6)$$

On multiplying both sides by h in (5) and (6) and rearranging:

$$r_1(t+h) = r_1(t) + \frac{h}{\tau}\left[-r_1(t) + A r_2(t) + I_1\right]$$

$$r_2(t+h) = r_2(t) + \frac{h}{\tau}\left[-r_2(t) + B r_1(t) + I_2\right]$$

2. Matlab Code

File name: CW2OsheenCorticalModelling

```
% Define parameters
tau = 0.02; % time constant
A = 0.5; % Alpha-coupling parameter for neuron 1
B = 0.5; % Beta-coupling parameter for neuron 2
U = 0.5; % mean of input current
SD = 0.1; % standard deviation of input current
h = 0.00001; % time step
tsim = 0.25; % simulation time

% Initialize vectors
t = 0:h:tsim;
n = length(t);
r1 = zeros(n, 1);
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r2 = zeros(n, 1);
I1 = normrnd(U, SD, n, 1);
I2 = normrnd(U, SD, n, 1);

% Euler's method
for i = 2:n
   r1(i) = r1(i-1) + h*(-r1(i-1) + A*r2(i-1) + I1(i-1))/tau;
   r2(i) = r2(i-1) + h*(-r2(i-1) + B*r1(i-1) + I2(i-1))/tau;
end

% Plot results
plot(t,r1,'k',t,r2,'g--', LineWidth=2);
xlabel('Time');
ylabel('Rate');
title(sprintf('Rate: r1 = %.1f, r2 = %.1f', r1(i), r2(i)), 'FontSize', 12);
legend('Neuron 1','Neuron 2','Location', 'best','FontSize', 14);
```

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3. Simulating the code with the given parameters. All units have been converted to standard units.

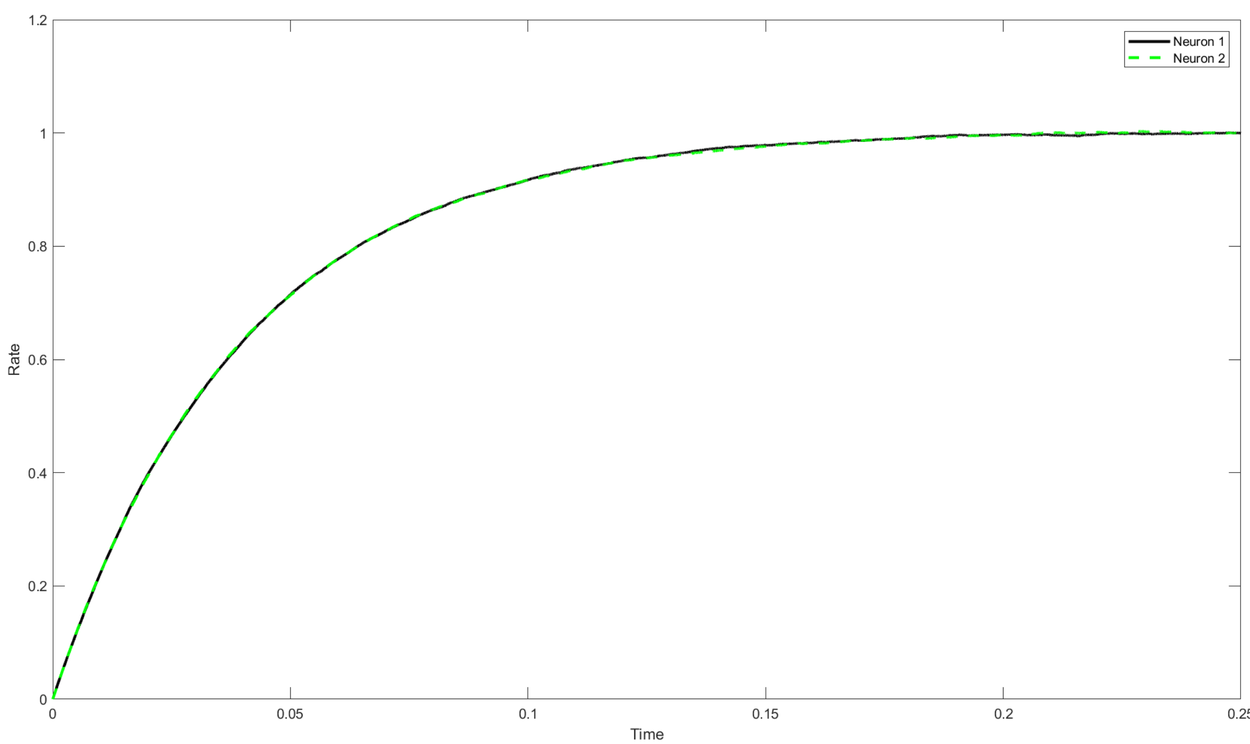

Figure 1

4. Changing the initial activity of neurons with the given values and adding the responses for each case:

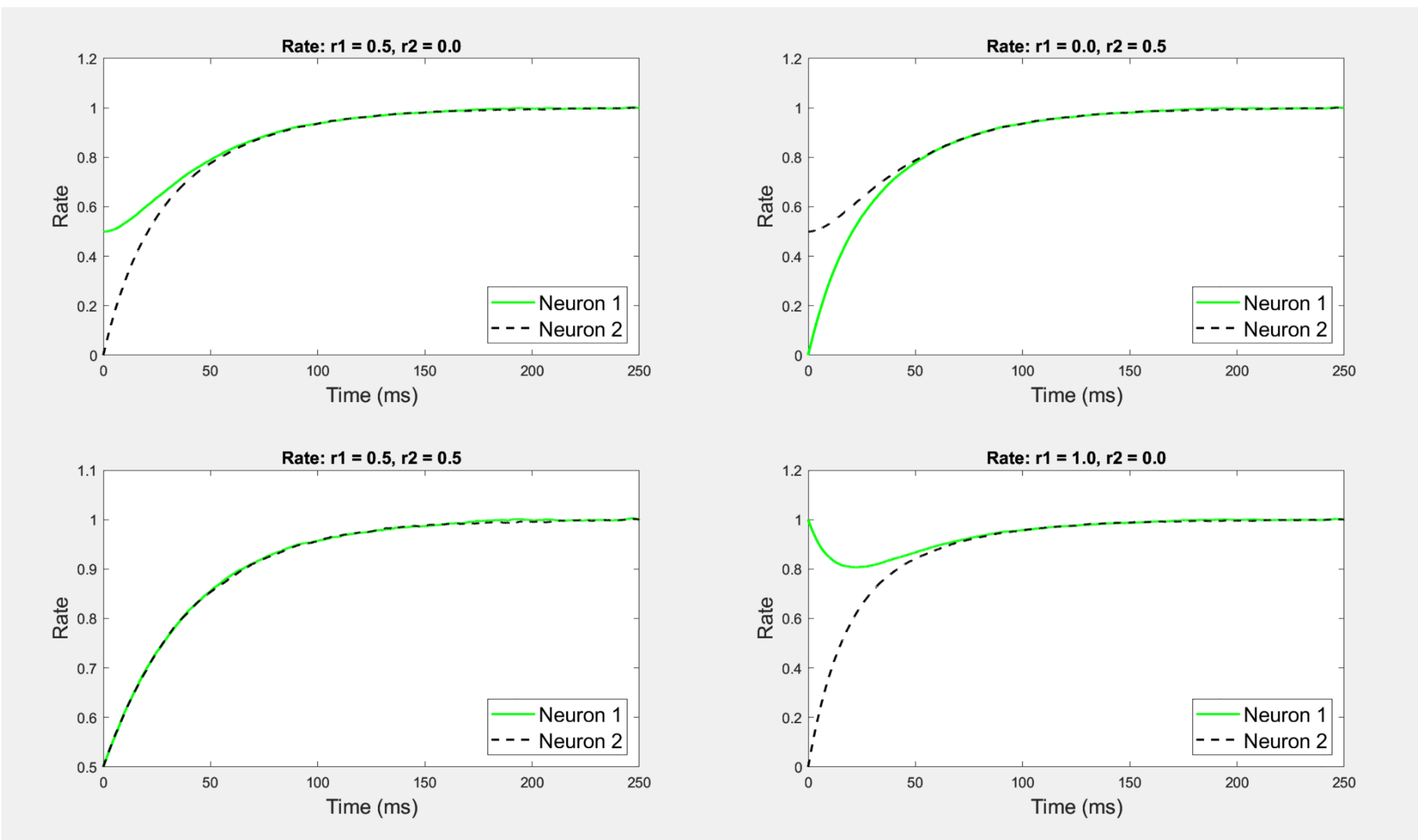

Figure 2

5. Observations regarding the steady-state response of the system of coupled neurons

Here are the observations based on the simulations from various initial conditions in Q4:

- When both neurons are initially at zero activity, (r1=0, r2=0), they exhibit similar transient behaviour, the activity of both neurons has a similar frequency and amplitude, with a gradual increase in their activity until they reach a steady state.
- For the initial conditions where only one neuron is active (r1=0.5 or r2=0.5), the active neuron drives the other neuron to become active as well, resulting in both neurons reaching a steady-state firing rate, which is again, nearly identical after the transient behaviour.
- For the initial condition where both neurons are active (r1=0.5, r2=0.5), the system quickly reaches a steady state where both neurons have nearly similar firing rates.
- For the initial condition where only one neuron has a high initial activity (r1=1, r2=0), Neuron 1 has a high firing rate, and Neuron 2 has a low firing rate. After the transient behaviour, the firing rates of the two neurons become synchronised and the system reaches a steady state. This is due to the coupling between them, A = B = 0.5.

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6. Steady State Derivation

Q6. Steady State Solution

To find steady state, set the derivates to 0

$$\tau \frac{dr_1}{dt} = -r_1 + Ar_2 + I_1 = 0 \qquad (1)$$

$$\tau \frac{dr_2}{dt} = -r_2 + Br_1 + I_2 = 0 \qquad (2)$$

Solving both equations (1) and (2)

$$r_1 = Ar_2 + I_1 \qquad (3)$$

$$r_2 = Br_1 + I_2 \qquad (4)$$

Subsituiting value of r_1 in (4)

$$r_2 = B(Ar_2 + I_1) + I_2$$

$$r_2 = ABr_2 + BI_1 + I_2$$

$$r_2 - ABr_2 = BI_1 + I_2$$

$$r_2(1 - AB) = BI_1 + I_2$$

$$r_2 = \frac{BI_1 + I_2}{1 - AB} \qquad (5)$$

Subsitutiing values of r_2 in (4) to (3)

$$r_1 = A\left(\frac{BI_1 + I_2}{1 - AB}\right) + I_1$$

$$r_1 = \frac{ABI_1 + AI_2}{1 - AB} + I_1$$

multiplying both sides by $(1 - AB)$

$$r_1(1 - AB) = ABI_1 + AI_2 + I_1 - ABI_1$$

$$r_1 = \frac{AI_2 + I_1}{(1 - AB)} \qquad (6)$$

Therefore, the steady state equations for r_1 and r_2 are:

$$r_1 = \frac{AI_2 + I_1}{(1 - AB)}$$

$$r_2 = \frac{BI_1 + I_2}{(1 - AB)}$$

On solving the equations (5) and (6) for the following values, we get:

For A = B = 0.5

r1 = (0.5*0.5 + 0.5) / (1 - 0.5*0.5)
r1 = 1

r2 = (0.5*0.5 + 0.5) / (1 - 0.5*0.5)
r2 = 1

In Figure 1, we can see that Neuron 1 and Neuron 2 show similar transient behaviour before reaching a steady state at a firing rate of r1 = r2 = 1.

This shows that for a mean value of I1 = I2 = 0.5 (as given in the question), the theoretical derivation of steady-state equations and the MATLAB code gives the same answer.

Part II (3.1)

1. Excitatory Coupling increases to A = B = 0.75

When the excitatory coupling of the two neurons is increased (i.e. A = B = 0.75) we observe that the activity of both neurons oscillates with a higher frequency compared to the original case where A and B were set to 0.5. The reason for the increase is that the excitatory coupling between the neurons strengthens, increasing their mutual excitation and causing them to fire more frequently, leading to a higher frequency of oscillations in the activity of both neurons.

If we increase the time to 0.4 seconds instead of 0.25, the system reaches the steady state at firing rate of 2.

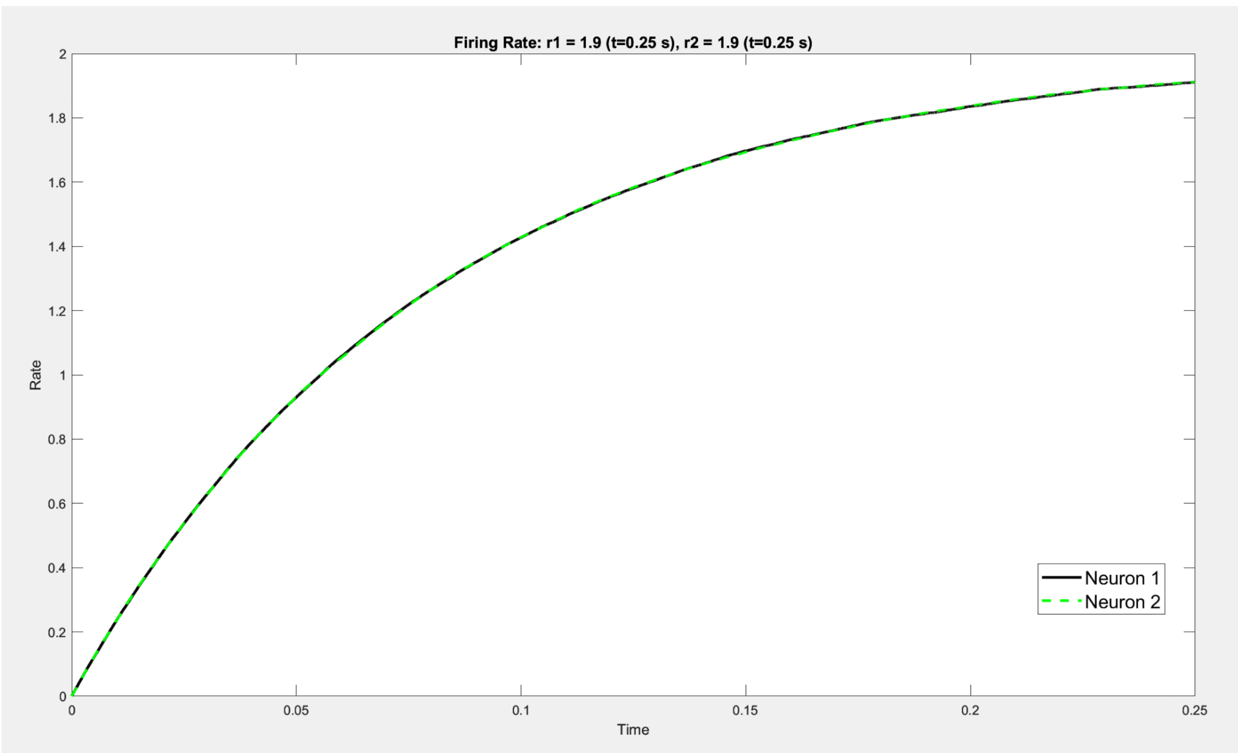

Figure 4

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2. Steady-state theoretical solutions for A = B = 0.75

On solving the equations (5) and (6) from Q6 for the following values, we get:

For A = B = 0.5

r1 = (0.75*0.5 + 0.5) / (1 - 0.75*0.75)
r1 = 2

r2 = (0.75*0.5 + 0.5) / (1 - 0.75*0.75)
r2 = 2

Yes, the theoretical derivation for the steady state values of r1 and r2 still match with simulated results. As can be seen in Figure 4, the system reaches the steady state at a value of 1.9 for both r1 and r2, which is close to the value of r1 and r2 we get on solving the steady state equations which is 2.

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3. Exitatory Coupling increases to A = B = 1

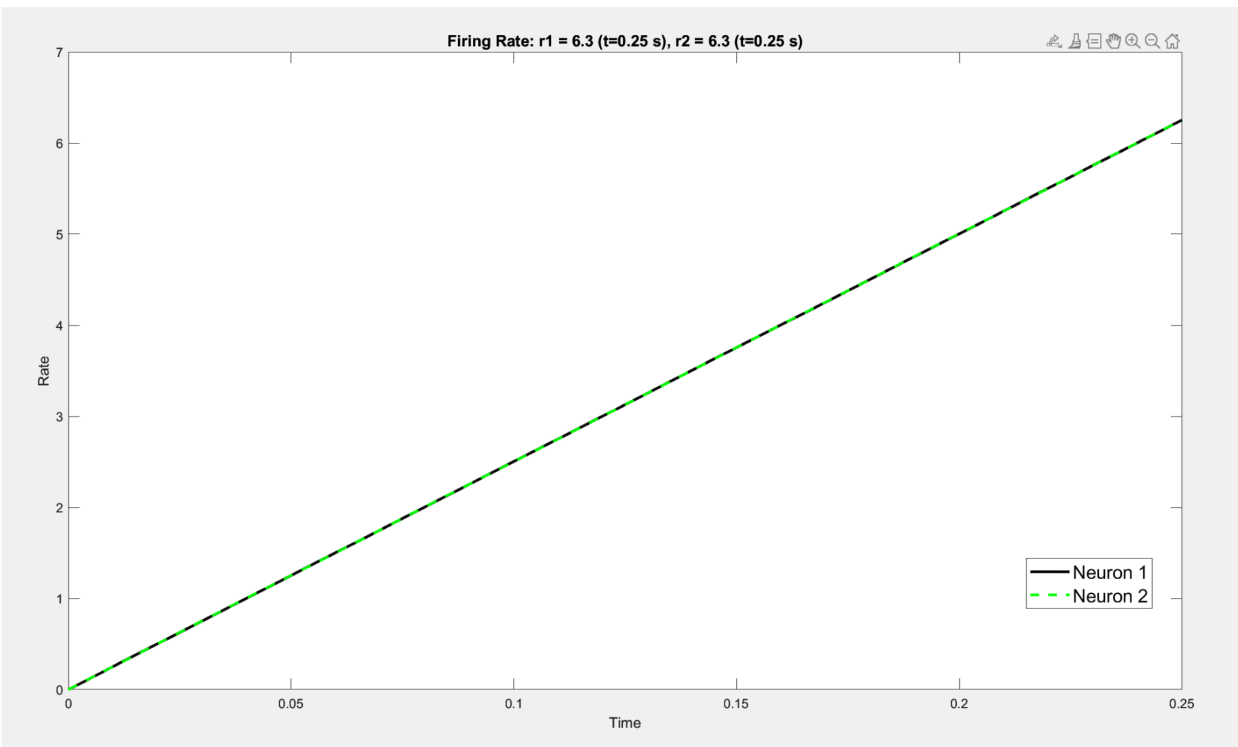

Figure 5

The graph of A = 1 and B = 1 is linear. The system does not appear to reach a steady state because the feedback loop between the two neurons become self-reinforcing and also because there is no mechanism to inhibit this positive feedback loop. As a result, the firing rates of the neurons keep increasing without bounds and the graph of the firing rates in Figure 5 confirms this behaviour.

When A = 1 and B = 1, the graph shows a straight line that does not converge, as the firing rates of both neurons keep increasing without bounds. The value of r1 and r2 in the graph at 0.25 seconds being 6.3 indicates that the firing rates of both neurons are increasing rapidly, and will continue to do so indefinitely as long as the input current to the neurons remains constant.

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4. Mathematical analysis for A = B = 1

When solving for the steady-state equations with A = B = 1, the denominator of each equation becomes zero, which means that the solutions are undefined. This occurs because when A and B are both 1, the feedback between the neurons becomes too strong and the system becomes unstable, which prevents it from reaching a stable state.

Mathematical analysis shows that the system will never reach a steady state, regardless of time. In Figure 5, the firing rate is graphed up to time t = 0.25 seconds, but the mathematical analysis demonstrates that the graph will never converge and the system will never reach a steady state.

On solving for A = B = 1, we also infer that for the system to reach a steady state, AB must be less than 1.

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5. Dynamics of the system for values more than 1

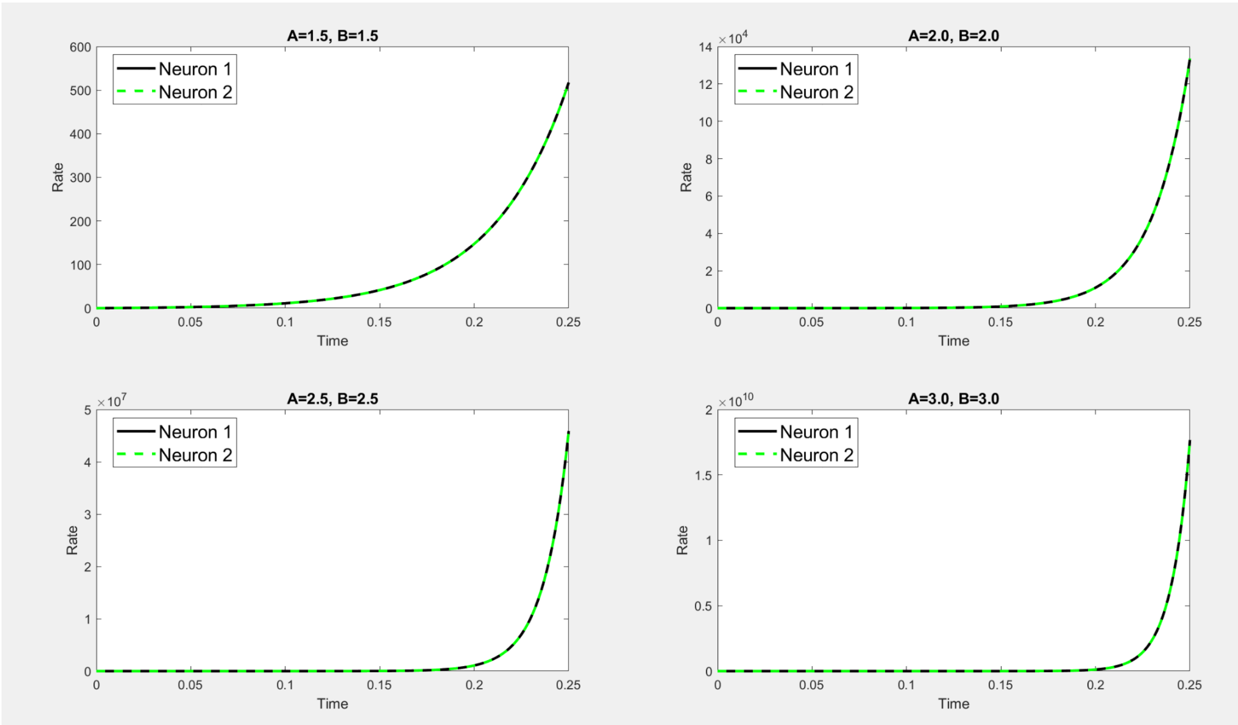

Figure 6

I tried more coupling values such as 1.2, 1.5, 2, 2.5, and 3 to observe the dynamic of the system, as can be seen in Figure 6. For values greater than 1, we can see that the graphs take an exponential curve and not reaching a steady state. For steady-state solutions as well, the value of r1 and r2 becomes negative when AB > 1.

Part II (3.2)

1. Inhibitory Coupling

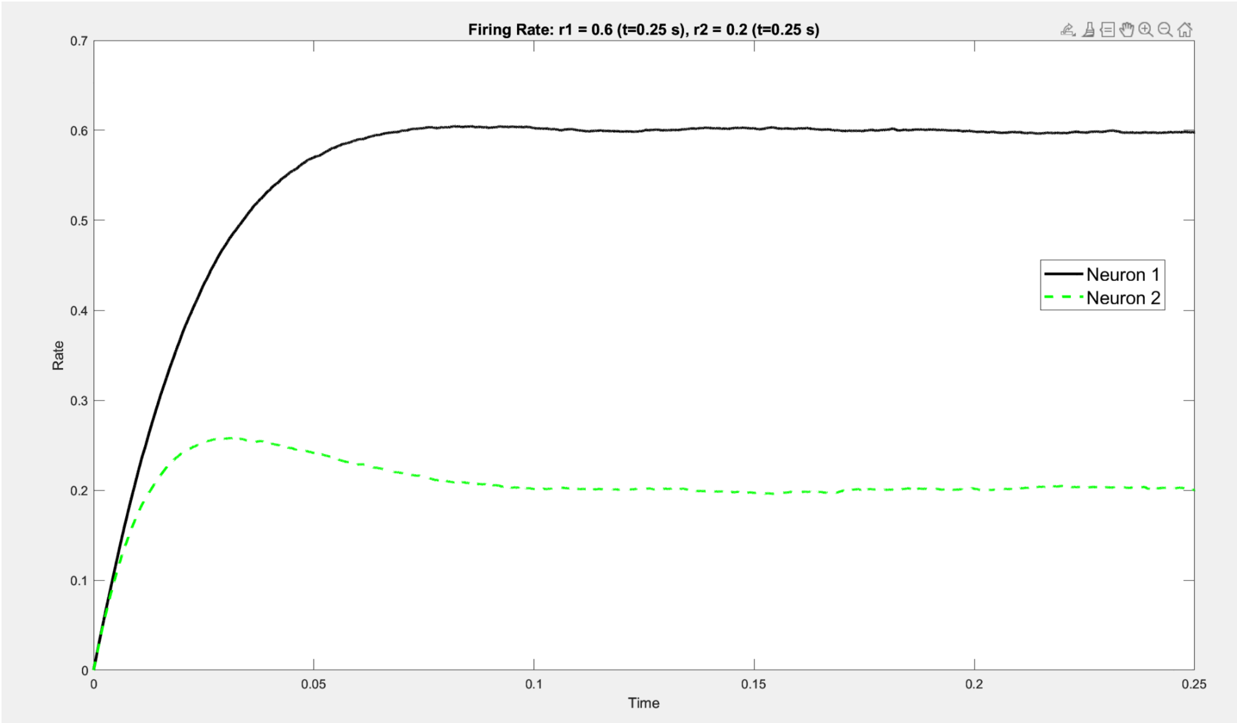

Figure 7

When neuron 2 has an inhibitory coupling, its activity suppresses the activity of neuron 1, which can be seen from the above figure. As a result, when neuron 2 becomes more active, it leads to a decrease in the activity of neuron 1 because the value of r1 drops to 0.6 at 0.25s, which is lower than the value of r1=1 when they had excitatory coupling. This is in contrast to an excitatory coupling where neuron 2 would enhance the activity of neuron 1.

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2. When AB > 1, the steady-state equation results become negative.

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3. Stronger values of coupling in (E-I) coupled system?

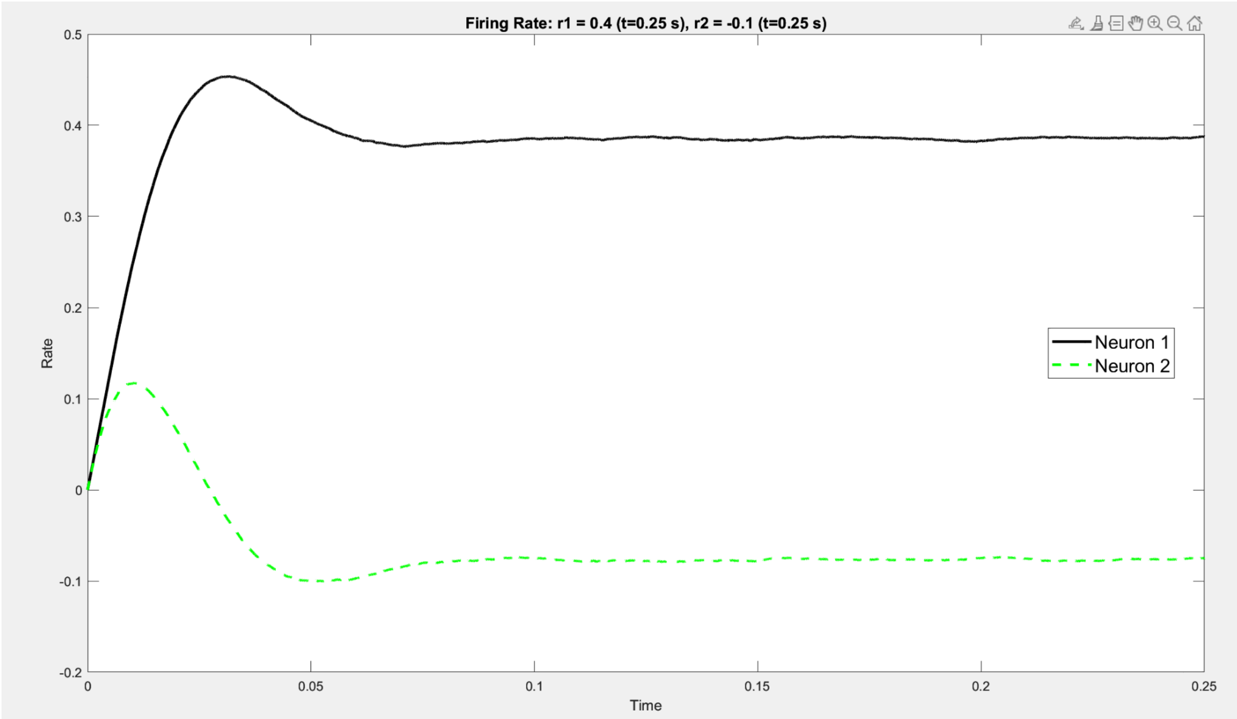

Figure 8

When both A and B are above 1 and there is negative coupling, the activity of one neuron will suppress the activity of the other neuron. For example, when A = 1.3 and B = -1.3, the activity of both neurons decreases over time and converge to a steady-state value that is close to zero. Similarly, when A = 1.7 and B = -1.7, the activity of both neurons decreases even faster and converges to a steady-state value that is even closer to zero, which can be seen from the Figure

This is because the negative coupling from neuron 2 to neuron 1 (B), cancels out the positive coupling from neuron 1 to neuron 2 (A). As a result, the net effect of the coupling is inhibitory, leading to a decrease in the activity of both neurons.

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4. No. the theoretical derivation doesn't capture the steady state values of the system during the strong E-I coupled regime.

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5. Effect of increasing excitatory-inhibitory coupling on the transient response of the system?

The code loops through the specified values of A and B and plots the dynamics of the two-neuron system for each combination of parameters. It can be seen from the plots that as the inhibitory coupling strength increases relative to the excitatory coupling strength, the activity of neuron 2 increasingly suppresses the activity of neuron 1. This leads to a more pronounced asymmetry in the activity of the two neurons and a slower response of neuron 1 to changes in input. Additionally, for very strong inhibitory coupling, the activity of neuron 1 can be completely suppressed by neuron 2.

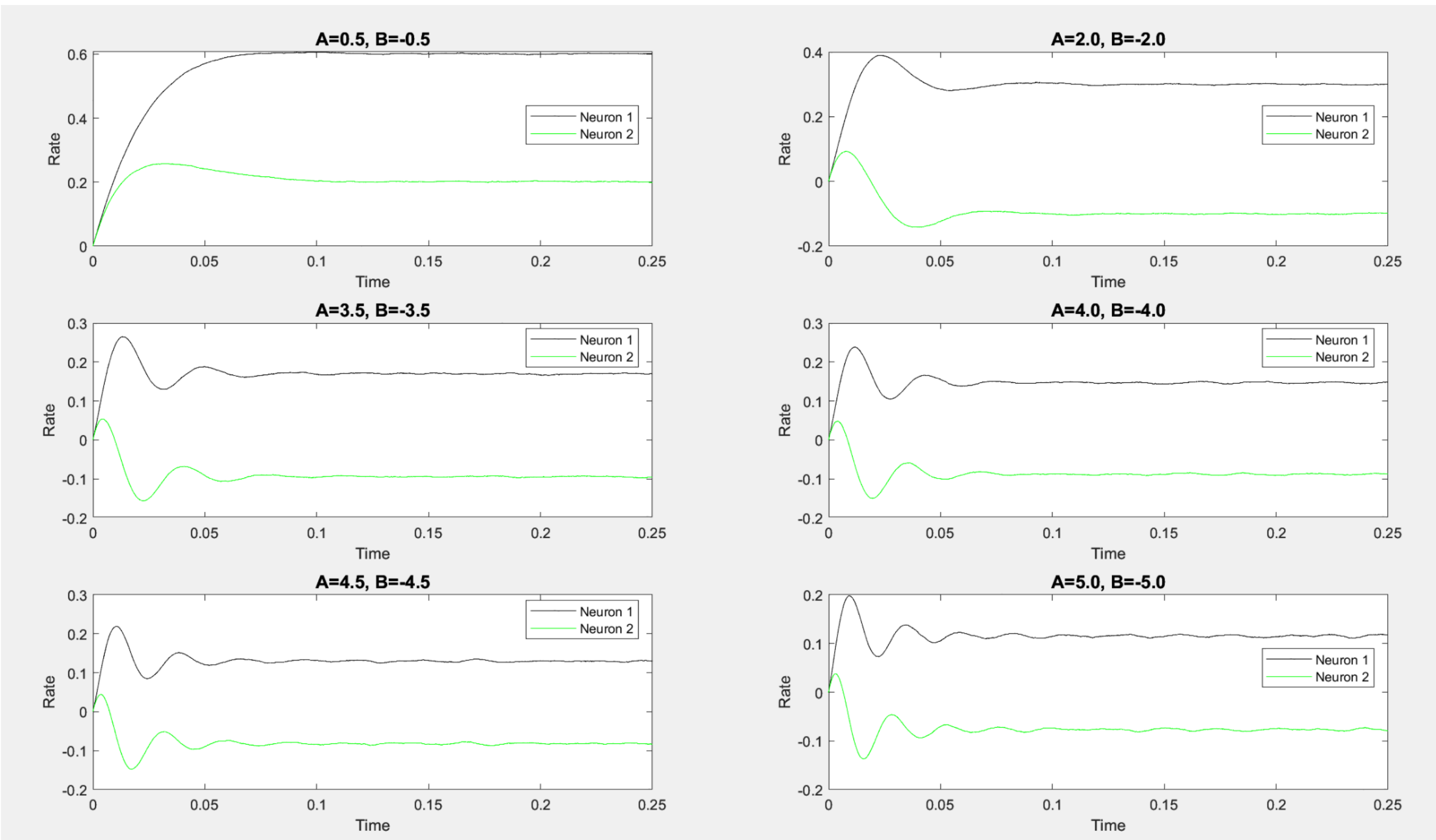

Figure 9

What we see is that the system settles quickly for values lower than 1, so for 0.5 and -0.5, the system settles quickly. For values greater than 1, it can be observed that the system sees multiple oscillations before settling down.