Part I: Implementing a LIF Neuron

1. **Euler's Method**

Equation of LIF model:

$$\tau \frac{dV}{dt} = E - V + R_m I_e \qquad (1)$$

By using Euler's method,

$$\frac{dV}{dt} \approx \frac{V(t+\Delta t) - V(t)}{\Delta t} \qquad (2)$$

From (1)

$$\frac{dV}{dt} = \frac{E - V + R_m \times I_e}{\tau} \qquad (3)$$

From (2) and (3)

$$\frac{V(t+\Delta t) - V(t)}{\Delta t} \approx \frac{E - V + R_m \times I_e}{\tau}$$

Multiplying both sides by Δt, and on rearranging

$$V(t+\Delta t) \approx V(t) + \frac{\Delta t}{\tau}\left(E - V + R_m \times I_e\right)$$

On simplify, the equation (1) can be expressed in terms of

$$V(t+\Delta t) = V(t) + \frac{\Delta \tau}{\tau}\left[E - V(t) + R_m I_e\right]$$

2. This MATLAB code implements a simplified version of the **leaky integrate-and-fire (LIF)** neural network model. Following is the explanation of the complete code:

Section A:
This section initializes the values of all variables used in the code.

- **tau** represents the time constant of the LIF model.
- **dt** is the time step used in the simulation, which is set as **tau/50**.
- **Rm** is the membrane resistance.
- **V_thresh** is the threshold membrane potential.
- **V_reset** is the membrane potential reset value after a spike.
- **E** is the resting potential of the membrane.

Section B:
This section initializes other values used in the code.

- **Ie** is the external current applied to the neuron.
- **T** is the time vector for the simulation, ranging from 0 to 0.3 in increments of **dt**.
- **V_hat** is an array to store the neuron's membrane potential at each time step. It is initialized as a vector of zeros the same size as **T**.
- **S** is an array to store the spike train of the neuron, initialized as a vector of zeros with the same size as **T**.
- **V_0** is the initial membrane potential of the neuron.
- **V_hat(1)** is set equal to **V_0** to initialize the first value of the **V_hat** array.

Section C:
This section contains a for loop that updates the membrane potential and spike train of the neuron at each time step.

- The loop starts at index 2 because the initial values of **V_hat** and **S** have already been set.
- The if-else statement inside the loop determines if the membrane potential is below the threshold **(V_hat(t-1) < V_thresh**). If it is, the membrane potential evolves according to the LIF equation.
- If it is not, the neuron fires, the membrane potential is reset to the value of **V_reset**, and the spike train is updated to indicate that a spike has occurred.

3. **Complete the code. Here is the snapshot:**

```
% section A
tau = 0.015;
dt = tau/50;
Rm = 1.0e+07;
V_thresh = -0.05;
V_reset = -0.08;
E = -0.07;
```

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4. Membrane potential equation

```
%section C
for t=2:length(T)
   if V_hat(t-1)<V_thresh
       V_hat(t)= V_hat(t-1) + (dt/tau) * (E - V_hat(t-1) + Rm * Ie);
   else
       V_hat(t)= V_reset;
       S(t)=1;
   end
end
```

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5. Value of I_e

I_e = 3.5e-09A

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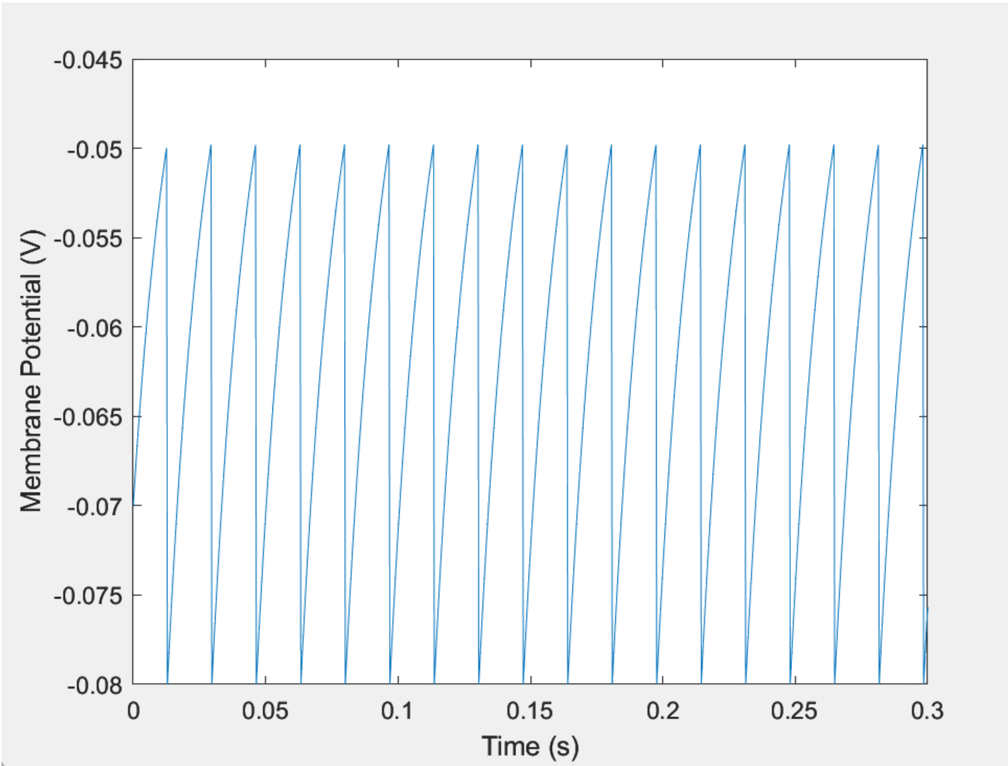
6. Plot the membrane potential



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7. Compute the firing rate in Hz of the neuron using the array containing spikes (S).

```
% Compute the firing rate
spike_times = find(S==1);
firing_rate = length(spike_times)/(T(end)-T(1));
```

This approach uses the 'find' function to locate the indices of all spikes in the array 'S'. The number of spikes is then divided by the total time of the simulation (T(end) - T(1)).

The firing rate is 60Hz

Part II: Influence of Hyper Parameters

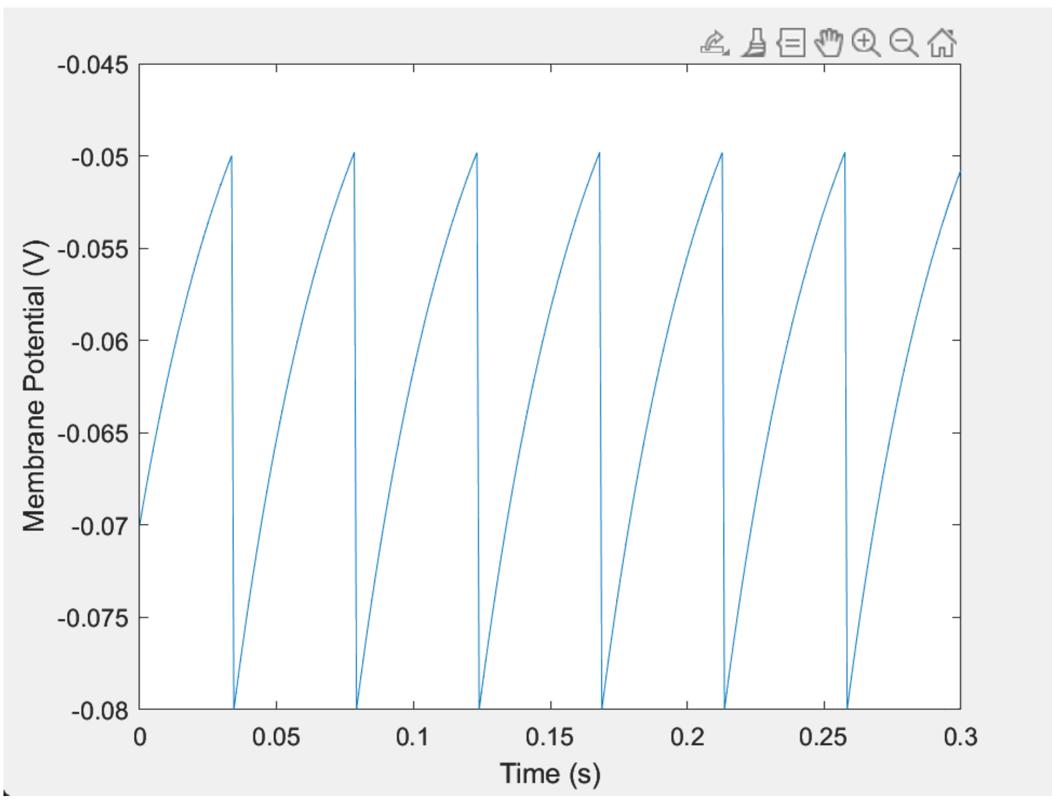
The Time Constant

1. Time constant

Time constant
`tau = 0.04`

Firing rate
The firing rate is 20Hz

Membrane potential plot



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2. The effect of changing tau on the firing rate

The firing rate decreased from 60 Hz with tau = 0.015 to 20 Hz with tau = 0.04 due to the larger time constant causing a slower response of the membrane potential to changes in input current.

The smaller tau in the previous section allowed for a faster response and higher firing rate, while the larger tau caused a slower response and lower firing rate.

The plot with tau = 0.04 has a slower rise time and shallower slope compared to the plot with tau = 0.015 due to the slower response of the membrane potential caused by the larger time constant. The plot with tau = 0.015 also shows more spiking behavior, with higher peak voltage and more frequent spikes.

The value of tau has a significant impact on the behavior of the neuron, affecting the firing rate and spiking behavior. A smaller tau results in a higher firing rate and more pronounced spiking behavior due to the neuron's faster response to changes in input current, while a larger tau leads to a lower firing rate and less pronounced spiking behavior due to the neuron's slower response.

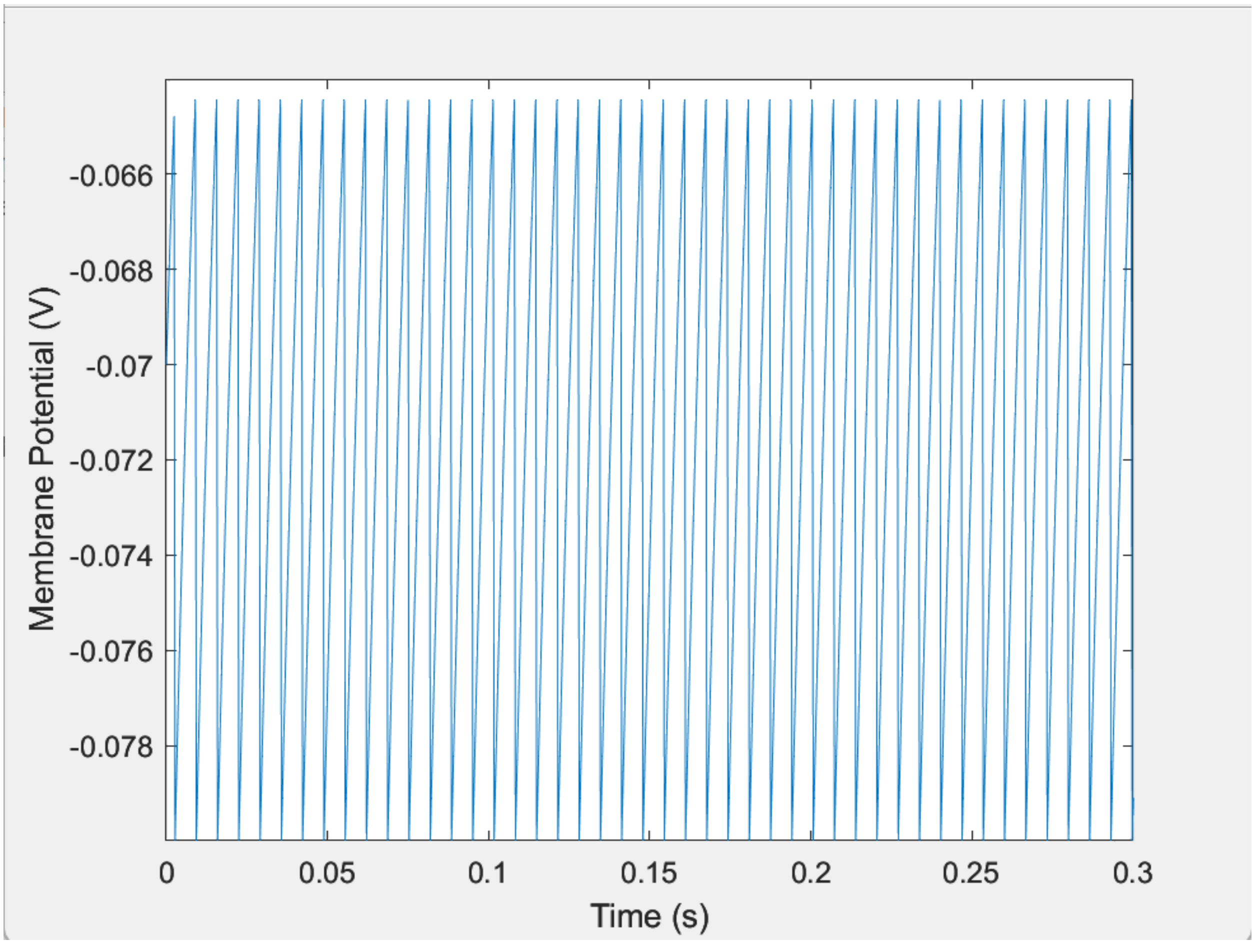
The threshold voltage V_{thresh}

1. Plot the membrane potential and compute the firing rate for this threshold.

V_{thresh}
`-0.065`

Firing rate
The firing rate when tau is kept at intial value is 153.3333

Membrane potential plot



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2. The effect of changing tau on the firing rate

Lowering the threshold from -0.05 to -0.065 results in more frequent firing because the membrane potential is more likely to cross the threshold, and it takes longer to reach the reset voltage. The threshold is an important parameter that affects the firing rate, with a lower threshold resulting in more spikes being generated.

However, in the simulation, we observed a decrease in firing rate when the threshold was lowered, likely due to the fact that the new threshold was closer to the resting potential of the neuron, resulting in fewer spikes being generated overall.
