Polytechnic University of the Philippines

Sta. Mesa, Manila

CALCULUS 1

Midterm Examination

Name: _____ Score: _____

Year and Section:_____ Date: _____

Professor: _____ Remarks: _____

Directions: Select the correct answer from the following options given and write your answer on the **ANSWER SHEET**. If there is no answer among the choices, write **N** (**there are only two**). You can use extra paper for your solution but never submit it. Only **ANSWER SHEET** will be accepted and submitted. Always double check your work. Keep cool!

1. Using Four-Step Rule or Increment Method in the equation $y = 6 - 2x + x^3$, which of the following suggests the Step 3?

   A. $\frac{\Delta y}{\Delta x} = 3x^2 - 3x\Delta x + \Delta x^2 + 4$ C. $\frac{\Delta y}{\Delta x} = 3x^2 + 3x\Delta x + \Delta x^2 - 4$

   B. $\frac{\Delta y}{\Delta x} = -2 + \Delta x^2 + 3x\Delta x + 3x^2$ D. $\frac{\Delta y}{\Delta x} = 2 + \Delta x^2 - 3x\Delta x + 3x^2$

2. Using Four-Step Rule or Increment Method in the equation $y = x^{\frac{-1}{2}}$, which of the following suggests the Step 2?

   A. $\Delta y = \frac{\Delta x}{(\sqrt{x}+\sqrt{x+\Delta x})+\sqrt{x^2+x\Delta x}}$ C. $\Delta y = \frac{\Delta x}{(\sqrt{x}+\sqrt{x+\Delta x})-\sqrt{x^2+x\Delta x}}$

   B. $\Delta y = \frac{-\Delta x}{(\sqrt{x}+\sqrt{x+\Delta x})+\sqrt{x^2+x\Delta x}}$ D. $\Delta y = \frac{-\Delta x}{(\sqrt{x}-\sqrt{x+\Delta x})+\sqrt{x^2-x\Delta x}}$

3. Using Four-Step Rule or Increment Method in the equation $x = 3t^2 - 2\sqrt{t}$, which of the following suggests the Step 3?

   A. $\frac{\Delta x}{\Delta t} = 6t + 3\Delta t - \frac{2}{\sqrt{t+\Delta t}+\sqrt{t}}$ C. $\frac{\Delta x}{\Delta t} = -6t + 3\Delta t + \frac{2}{\sqrt{t+\Delta t}+\sqrt{t}}$

   B. $\frac{\Delta x}{\Delta t} = 6t - 3\Delta t + \frac{2}{\sqrt{t+\Delta t}+\sqrt{t}}$ D. None of the above

4. Using Four-Step Rule or Increment Method in the equation $y = \sqrt{a^2 + x^2}$, which of the following suggests the Step 1?

   A. $y + \Delta y = \sqrt{(a + \Delta a)^2 + (x + \Delta x)^2}$ C. $y + \Delta y = \sqrt{(\Delta a)^2 + (\Delta x)^2}$

   B. $y + \Delta y = \sqrt{(\Delta a)^2 + (x + \Delta x)^2}$ D. $y + \Delta y = \sqrt{a^2 + (x + \Delta x)^2}$

5. Using Four-Step Rule or Increment Method in the equation $y = \frac{3x-1}{2x+5}$, which of the following suggests the Step 2?

   A. $\Delta y = \frac{3(x+\Delta x)-1}{2(x+\Delta x)+5} - \frac{3x+1}{2x-5}$ C. $\Delta y = \frac{3(x+\Delta x)-1}{2(x+\Delta x)-5} + \frac{3x+1}{2x+5}$

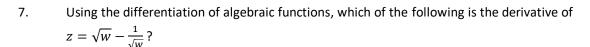   B. $\Delta y = \frac{17\Delta x}{(2x+2\Delta x+5)(2x+5)}$ D. None of the above

6. Using the differentiation of algebraic functions, which of the following is the derivative of $y = 7x^5 - 2x^3 + x^2 - 8$?

   A. $y' = 35x^4 - 6x^2 + 2x$ C. $y' = 35x^4 - 6x^2 + 2x + C$

   B. $y' = 35x^4 - 6x^2 + 2x - 8$ D. $y' = 35x^4 - 6x^2 + 2x \pm 0$

7. Using the differentiation of algebraic functions, which of the following is the derivative of
$z = \sqrt{w} - \frac{1}{\sqrt{w}}$?

A. $z' = \frac{1}{2\sqrt{w}} - \frac{\sqrt{w}}{2w}$

C. $z' = \frac{1}{2\sqrt{w}} + \frac{1}{2w\sqrt{w}}$

B. $z' = \frac{\sqrt{w}}{2w} + \frac{1}{2\sqrt{w}}$

D. $z' = \frac{1}{2\sqrt{w}} - \frac{1}{2w\sqrt{w}}$

8. Using the differentiation of algebraic functions, which of the following is the derivative of
$y = [(4x + 3)(7x - 1)]$?

A. $y' = 65x + 71$

C. $y' = 17x - 56$

B. $y' = 56x - 17$

D. $y' = 56x + 17$

9. Using the differentiation of algebraic functions, which of the following is the derivative of
$y = \left(\frac{5x+3}{4x^2-7}\right)$?

A. $y' = \frac{-20x^2 - 24x - 35}{(4x^2 - 7)^2}$

C. $y' = \frac{-(20x^2 + 24x - 35)}{(4x^2 - 7)^2}$

B. $y' = \frac{-(20x^2 - 24x - 35)}{(4x^2 - 7)^2}$

D. $y' = \frac{(4x-7)(5x+5)}{(4x^2 - 7)^2}$

10. Using the differentiation of algebraic functions, which of the following is the derivative of
$y = (32x^2 - 9)(5x^5 - 7x^2)^{-1}$?

A. $y' = \frac{x(480x^5 - 225x^3 - 126)}{(5x^5 - 7x^2)^2}$

C. $y' = \frac{x(-480x^5 - 225x^3 + 126)}{(5x^5 - 7x^2)^2}$

B. $y' = \frac{x(-480x^5 + 225x^3 - 126)}{(5x^5 - 7x^2)^2}$

D. None of the above

11. Which of the following is the equation of tangent line to the curve $y = 3x^2 + 4x + 9$ at $(0,9)$?

A. $y = 9x + 4$

C. $y = 4x + 9$

B. $y = 4x - 9$

D. $y = 9x - 4$

12. Which of the following is the equation of normal line to the curve $y = \frac{(x+2)^2}{\sqrt{3x^2+1}}$ at the point $(0,4)$?

A. $y = \frac{x-16}{4}$

C. $y = \frac{-(16+x)}{4}$

B. $y = \frac{x+16}{4}$

D. $y = \frac{16-x}{4}$

13. Which of the following is the equation of the tangent line to the curve $y = (4x + 7)(x^2 - 1)$ at the point $(0,0)$?

A. $y = -x$

C. $y = 3x$

B. $y = 2x$

D. $y = -4x$

14. Which of the following is the equation of the tangent line to the curve $y = (3x + 2)^5$ at the point $(2,-2)$?

A. $y = 61,440x - 122,882$

C. $y = \frac{-(x+122,878)}{61,440}$

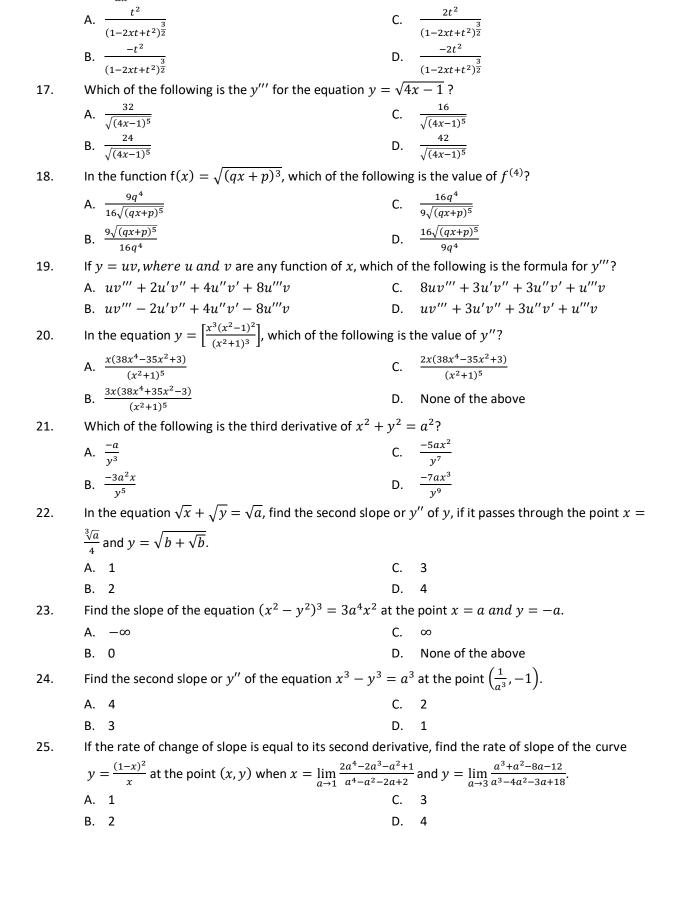B. $y = \frac{(x-122,878)}{61,440}$

D. $y = 61,440x + 122,882$

15. In the equation $y = -4x^2 + 3x - 2$, which of the following are the equations of the tangent and normal lines?

A. $y = 5x + 2 \text{ and } y = \frac{16x-1}{5}$

C. $y = 2 - 5x \text{ and } y = \frac{x-16}{5}$

B. $y = \frac{16x+1}{5} \text{ and } y = 5x - 2$

D. $y = \frac{1-16x}{5} \text{ and } y = 2x - 5$

16. Find $\dfrac{d^2y}{dx^2}$ from the equation $y = \sqrt{1 - 2xt + t^2}$ where t be held as constant.

A. $\dfrac{t^2}{(1-2xt+t^2)^{\frac{3}{2}}}$

C. $\dfrac{2t^2}{(1-2xt+t^2)^{\frac{3}{2}}}$

B. $\dfrac{-t^2}{(1-2xt+t^2)^{\frac{3}{2}}}$

D. $\dfrac{-2t^2}{(1-2xt+t^2)^{\frac{3}{2}}}$

17. Which of the following is the y''' for the equation $y = \sqrt{4x - 1}$?

A. $\dfrac{32}{\sqrt{(4x-1)^5}}$

C. $\dfrac{16}{\sqrt{(4x-1)^5}}$

B. $\dfrac{24}{\sqrt{(4x-1)^5}}$

D. $\dfrac{42}{\sqrt{(4x-1)^5}}$

18. In the function $f(x) = \sqrt{(qx + p)^3}$, which of the following is the value of $f^{(4)}$?

A. $\dfrac{9q^4}{16\sqrt{(qx+p)^5}}$

C. $\dfrac{16q^4}{9\sqrt{(qx+p)^5}}$

B. $\dfrac{9\sqrt{(qx+p)^5}}{16q^4}$

D. $\dfrac{16\sqrt{(qx+p)^5}}{9q^4}$

19. If $y = uv$, where u and v are any function of x, which of the following is the formula for y'''?

A. $uv''' + 2u'v'' + 4u''v' + 8u'''v$

C. $8uv''' + 3u'v'' + 3u''v' + u'''v$

B. $uv''' - 2u'v'' + 4u''v' - 8u'''v$

D. $uv''' + 3u'v'' + 3u''v' + u'''v$

20. In the equation $y = \left[\dfrac{x^3(x^2-1)^2}{(x^2+1)^3}\right]$, which of the following is the value of y''?

A. $\dfrac{x(38x^4-35x^2+3)}{(x^2+1)^5}$

C. $\dfrac{2x(38x^4-35x^2+3)}{(x^2+1)^5}$

B. $\dfrac{3x(38x^4+35x^2-3)}{(x^2+1)^5}$

D. None of the above

21. Which of the following is the third derivative of $x^2 + y^2 = a^2$?

A. $\dfrac{-a}{y^3}$

C. $\dfrac{-5ax^2}{y^7}$

B. $\dfrac{-3a^2x}{y^5}$

D. $\dfrac{-7ax^3}{y^9}$

22. In the equation $\sqrt{x} + \sqrt{y} = \sqrt{a}$, find the second slope or y'' of y, if it passes through the point $x = \dfrac{\sqrt[3]{a}}{4}$ and $y = \sqrt{b} + \sqrt{b}$.

A. 1

C. 3

B. 2

D. 4

23. Find the slope of the equation $(x^2 - y^2)^3 = 3a^4x^2$ at the point $x = a$ and $y = -a$.

A. $-\infty$

C. ∞

B. 0

D. None of the above

24. Find the second slope or y'' of the equation $x^3 - y^3 = a^3$ at the point $\left(\dfrac{1}{a^3}, -1\right)$.

A. 4

C. 2

B. 3

D. 1

25. If the rate of change of slope is equal to its second derivative, find the rate of slope of the curve $y = \dfrac{(1-x)^2}{x}$ at the point (x, y) when $x = \lim\limits_{a \to 1} \dfrac{2a^4-2a^3-a^2+1}{a^4-a^2-2a+2}$ and $y = \lim\limits_{a \to 3} \dfrac{a^3+a^2-8a-12}{a^3-4a^2-3a+18}$.

A. 1

C. 3

B. 2

D. 4

26. Given the curve $x^4 + y^4 = a^4$ where a is a constant, find $\frac{d^2y}{dx^2}$ or the rate of change to its second derivative in simplest form at the point; $x = \frac{1}{a^2}$ and $y = 1$

A. -1 C. -3

B. -2 D. -4

For Problems 27 to 32:

The equation of the folium of Descartes is given as $x^3 + y^3 - 3axy = 0$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

27. Which of the following is the equation of the tangent line?

A. $x + y - 3a = 0$ C. $x - y + 3a = 0$

B. $2x + y - 3a = 0$ D. $x - 2y - 3a = 0$

28. Which of the following is the equation of the normal line?

A. $\frac{x}{y} = 0$ C. $\frac{y}{x} = 0$

B. $x + y = 0$ D. $x - y = 0$

29. Which of the following is the length of the tangent line?

A. $\frac{2\sqrt{3}}{3}a$ C. $\frac{3\sqrt{2}}{2}a$

B. $4\sqrt{2}a$ D. $\frac{5\sqrt{3}}{2}a$

30. Which of the following is the length of the normal line?

A. $\frac{5\sqrt{3}}{2}a$ C. $\frac{2\sqrt{3}}{3}a$

B. $\frac{3\sqrt{2}}{2}a$ D. $4\sqrt{2}a$

31. Which of the following is the length of the subtangent?

A. $\frac{-3a}{2}$ C. $\frac{-2a}{3}$

B. $\frac{-\sqrt{3}a}{2}$ D. $\frac{-\sqrt{2}a}{3}$

32. Which of the following is the length of the subnormal?

A. $\frac{-2a}{3}$ C. $\frac{-\sqrt{3}a}{2}$

B. $\frac{-\sqrt{2}a}{3}$ D. $\frac{-3a}{2}$

For Problems 33 to 35:

The curve of the trisectrix of Maclaurin is given by the equation $y^2 = \frac{x^2(3a-x)}{a+x}$ at the point (a, a).

33. Which of the following is the equation of the tangent line?

A. $2x - y + a = 0$ C. $x - y + a = 0$

B. $x - 2y + a = 0$ D. $x + y + a = 0$

34. Which of the following is the equation of the normal line?

A. $2x + y - 3a = 0$ C. $2x - y + 3a = 0$

B. $x + 2y - 3a = 0$ D. $2x + 2y + 3a = 0$

35. Which of the following is the length of the tangent?

A. $\sqrt{2}\,a$ C. $2a$

B. $\sqrt{3}a$ D. $\sqrt{5}a$

36. Which of the following is an example of Chain Rule functions?

A. $y = 5u^3 - u;$ $u = 2x + 10$

B. $x = 4t - 7t^3;$ $y = 3t^2 + 4$

C. $x = 2x^6 - 3x^2 - 4x + 3$

D. $x = \sqrt{y} + \sqrt[4]{y} : y = 2x + 3$

37. Which of the following is an example of the Parametric Function Rule?

A. $y = 5u^3 - u;$ $u = 2x + 10$

B. $x = 4t - 7t^3;$ $y = 3t^2 + 4$

C. $x = 2x^6 - 3x^2 - 4x + 3$

D. $x = \sqrt{y} + \sqrt[4]{y} : y = 2x + 3$

38. Which of the following is an example of the Inverse Function Rule?

A. $y = 5u^3 - u;$ $u = 2x + 10$

B. $x = 4t - 7t^3;$ $y = 3t^2 + 4$

C. $x = 2x^6 - 3x^2 - 4x + 3$

D. $x = \sqrt{y} + \sqrt[4]{y} : y = 2x + 3$

For Problem N0. 39 to 40. In the equation $4x^2 + 9y^2 = 25$ at $(2, -1)$

39. What is the slope of the equation at $(2, -1)$?

A. $\frac{9}{8}$ B. $\frac{-8}{9}$ C. $\frac{-9}{8}$ D. $\frac{8}{9}$

40. What is the equation of the tangent line that passes through the curve?

A. $8x - 9y = 25$

B. $8x + 9y = 25$

C. $-9x + 8y = 25$

D. $-8x - 9y = 25$

FORMAT OF ANSWERING

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Sta. Mesa, Manila

CALCULUS 1

Midterm Examination

Name: _____ Score: _____

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ANSWER SHEET

STRICTLY NO ERASURES

1.	11.	21.	31.
2.	12.	22.	32.
3.	13.	23.	33.
4.	14.	24.	34.
5.	15.	25.	35.
6.	16.	26.	36.
7.	17.	27.	37.
8.	18.	28.	38.
9.	19.	29.	39.
10.	20.	30.	40.