In-place Haar processing

$$\mathbf{c}_k^{n-1} = \mathbf{s}_{2k}^n - \mathbf{s}_{2k+1}^n$$

$$\mathbf{s}_k^{n-1} = \frac{\mathbf{s}_{2k+1}^n + \mathbf{s}_{2k}^n}{2}$$

$$\mathbf{c}_k^{n-1} \rightarrow \frac{1}{\sqrt{2}}\mathbf{c}_k^{n-1}$$

$$\mathbf{s}_k^{n-1} \rightarrow \sqrt{2}\,\mathbf{s}_k^{n-1}$$

$$\mathbf{s}_{2k}^n \,,\ \mathbf{s}_{2k+1}^n \rightarrow \mathbf{s}_k^{n-1} \,,\ \mathbf{c}_k^{n-1}$$

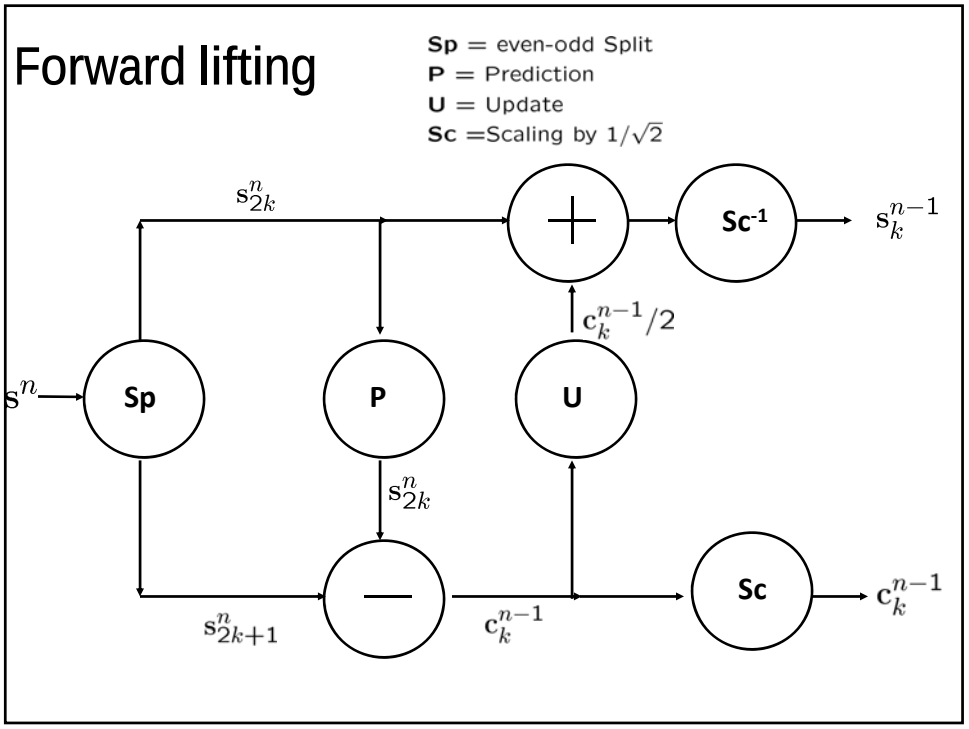
Basic operations

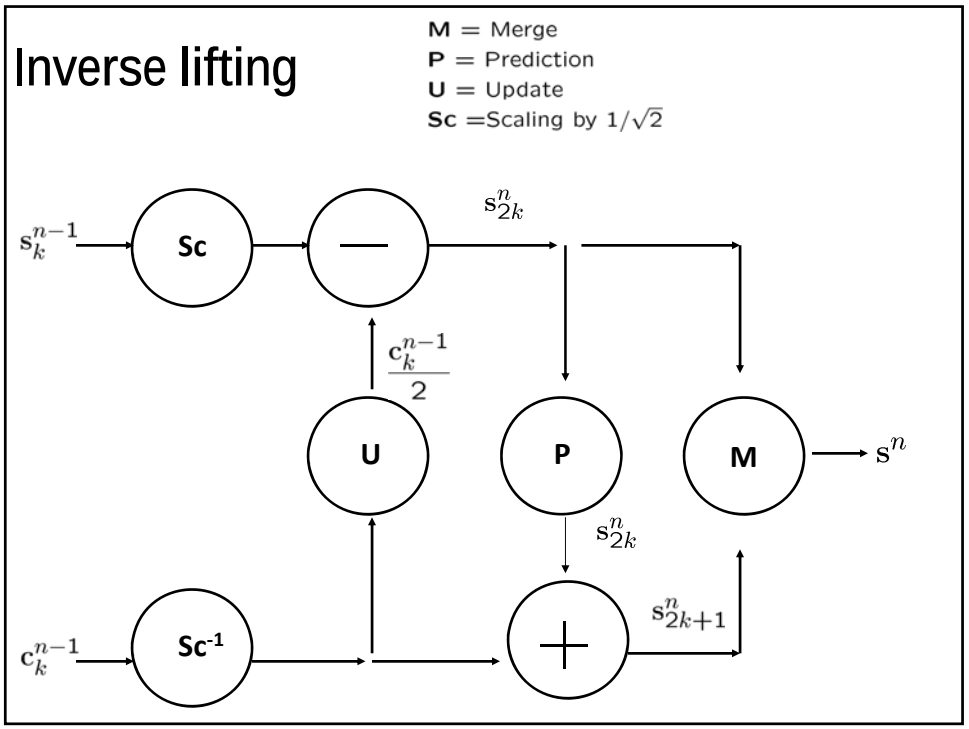
Split. Separates the even-indexed and odd-indexed elements of the input sequence.

Prediction. Predicts the n'th element of the input sequence based on its predecessors.

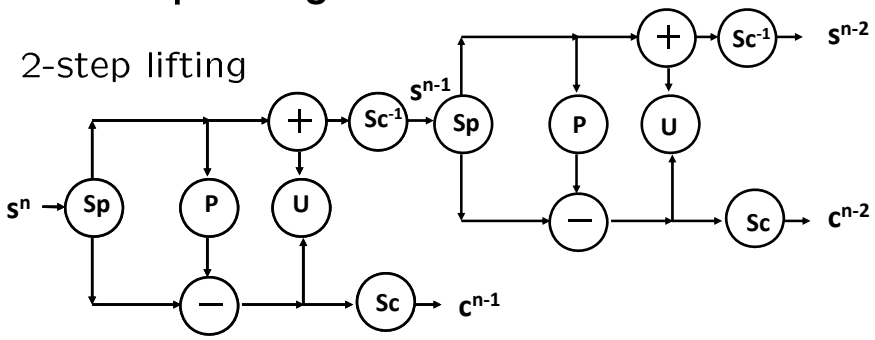
Update. The objective is that the output stream have the same mean as the input stream.

Scaling. Is applied so the combined output streams have the same energy as the input stream.


Forward lifting
Sp = even-odd Split
P = Prediction
U = Update
Sc =Scaling by $1/\sqrt{2}$
s_{2k}^n
Sc-1
s_k^{n-1}
$c_k^{n-1}/2$
s^n
Sp
P
U
s_{2k}^n
Sc
c_k^{n-1}
s_{2k+1}^n
c_k^{n-1}


Inverse lifting
M = Merge
P = Prediction
U = Update
Sc =Scaling by $1/\sqrt{2}$
s_k^{n-1}
Sc
s_{2k}^n
$\frac{c_k^{n-1}}{2}$
U
P
M
s^n
s_{2k}^n
s_{2k+1}^n
c_k^{n-1}
Sc-1

Multistep lifting



Sp = even-odd Split
P = Prediction
U = Update
Sc =Scaling by $1/\sqrt{2}$

$$\begin{array}{ccccccc} s^n & \rightarrow & s^{n-1} & \rightarrow & s^{n-2} & \rightarrow \\ & \searrow & c^{n-1} & \searrow & c^{n-2} & \searrow \end{array}$$

JPEG 2000

- Introduction
- The JPEG 2000 Standard
  - Preprocessing
    - Color Component Transformation
    - Image Tiling
    - DC level shifting
  - Core Processing
    - DWT (Wavelet Transform)
    - Quantization (Lossy Compression)
    - Entropy Coding (Lossless compression)
- Conclusion

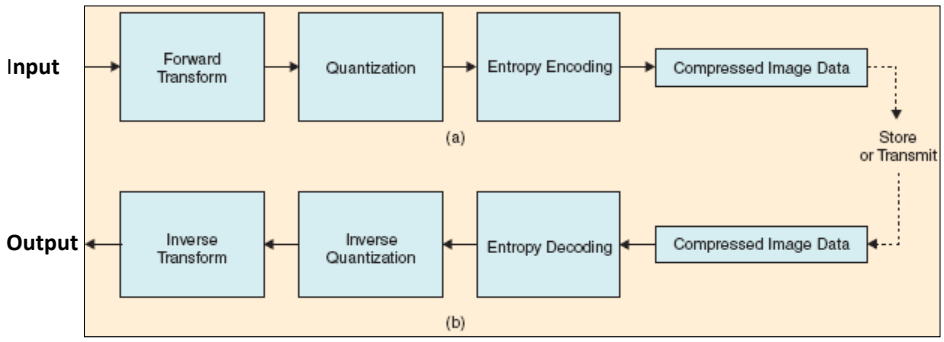
Introduction

- Joint Photographic Expert Group (JPEG) is an International Organization of Standards (ISO) and International Telecommunications Union (ITU) standard for compression of grayscale and color still images.
- JPEG initially was based on a simple, efficient 8-by-8 DCT compression algorithm that uses Huffman coding, operated in sequential mode, and was restricted to 8-bit/pixel input.

Why JPEG2000 ?

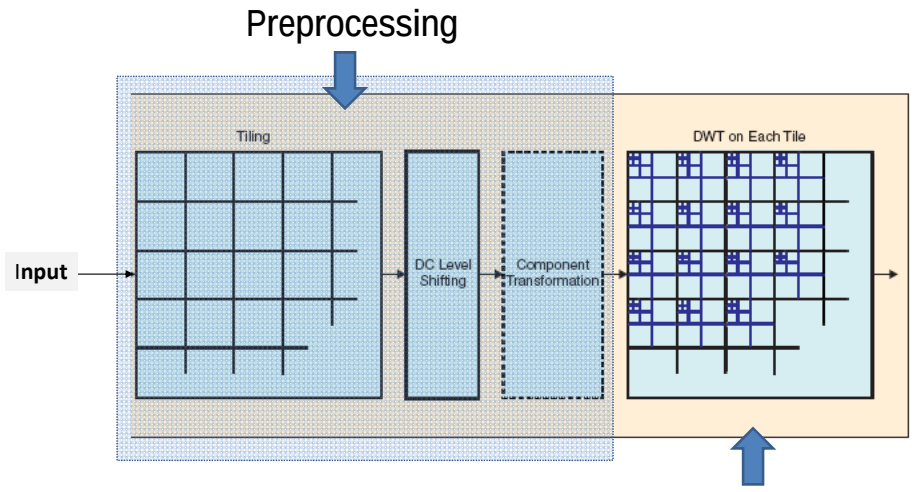
- To satisfy the growing need for high-resolution images, digital libraries, high fidelity color, multimedia, the internet, wireless communication, medical imaging, etc.
- To improve over the original JPEG (based on the Discrete Cosine Transform), by:
  - Reducing distortions and artifacts.
  - Improving compression.
  - Providing effective color-space support.
  - Allowing a multiple resolution representation of images.

Basic flowchart



Options for Transform Encoding/Decoding :

1. Lossy mode (CDF 9.7 biorthogonal wavelets)
2. Lossy or Lossless modes (Le Gall 5.3 integer wavelets)



Preprocessing

1. Tiling
2. DC Level Shifting
3. Color Component Transformation

Preprocessing

Tiling

The partitioning of the original (source) image into rectangular non-overlapping blocks (tiles) which are compressed independently. Has the advantages of reduces memory requirements, and of allowing selective decoding of specific parts of the image instead of the whole image.

DC Level Shifting

JPEG 2000 expects its input to have a dynamic range that is approximately centered about zero. To this effect, the nominal dynamic range of the samples is adjusted by subtracting $2^{(p-1)}$ from each sample value, where "p" is the component's precision.

For example range "0 to 255" is changed to "-127 to +128".

Color Component Transformation

The objective is to reduce the correlations between (R, G, B)color components. This results in less redundancy and higher compression ratios. In "irreversible" mode, the uncompressed RGB color image is transformed to YCbCr color space (Y: luminance, Cb: blue difference chrominance Cr: red difference chrominance).

Color component transformations

In JPEG2000 the input color images are transformed from the RGB colorspace to another color space, leading to three *components* that are handled separately. There are two possible choices:

1. **Irreversible Color Transform** uses the Y CB CR color space. It is called "irreversible" because it has to be implemented in floating or fix-point and causes round-off errors.

$$\begin{pmatrix} Y \\ C_b \\ C_r \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.16875 & -0.33126 & 0.5 \\ 0.5 & -0.41869 & -0.08131 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix} \qquad \begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 1.0 & 0 & 1.402 \\ 1.0 & -0.34413 & -0.71414 \\ 1.0 & 1.772 & 0 \end{pmatrix} \cdot \begin{pmatrix} Y \\ C_b \\ C_r \end{pmatrix}$$

2. **Reversible Color Transform** uses a modified Y U V color space that does not introduce quantization errors, so it is fully reversible. Implementation of the RCT requires that numbers be rounded. The transformations are:

$$\begin{pmatrix} Y_r \\ V_r \\ U_r \end{pmatrix} = \begin{pmatrix} \left\lfloor \frac{R+2G+B}{4} \right\rfloor \\ R-G \\ B-G \end{pmatrix} \qquad \begin{pmatrix} G \\ R \\ B \end{pmatrix} = \begin{pmatrix} Y_r - \left\lfloor \frac{U_r+V_r}{4} \right\rfloor \\ V_r+G \\ U_r+G \end{pmatrix}$$

Wavelet transform encoding/decoding

- Decomposes the color components of each tile into horizontal, vertical , and diagonal “details” at different levels of resolution.
- This is accomplished via recursive filtering, which is performed in both vertical and horizontal directions.
- While access in one direction is contiguous in memory and efficient, access in the other direction is strided, requiring that the each image color component tile be transposed (and buffered) for quicker access .
- An “untiled” approach is difficult to implement on hand-held devices because of limited memory.

Wavelet filters

- Floating point (Cohen-Daubechies-Feauveau) CDF 9.7 wavelet filter for lossy compression
  - Best performance at low bit rate
  - High implementation complexity
- Integer (LeGall) 5.3 wavelet filter for lossless coding
  - Integer arithmetic
  - Low implementation complexity
- The JPEG2000 standard supports both convolution-based and lifting-based implementations of filtering.

Table 1 Daubechies 9/7 Analysis Filters

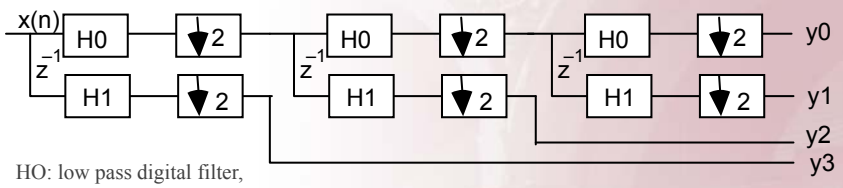
k	Lowpass Filter (h_k)	Highpass Filter (g_k)
0	0.6029490182363579	1.115087052456994
1	0.2668641184428723	-0.5912717631142470
2	-0.07822326652898785	-0.05754352622849957
3	-0.01686411844287495	0.09127176311424948
4	0.02674875741080976	

Table 2 Le Gall 5/3 Analysis Filters

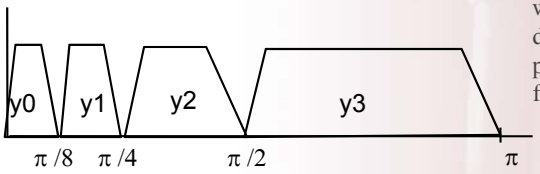
k	Lowpass Filter (h_k)	Highpass Filter (g_k)
0	6/8	1
1	2/8	½
2	-1/8	

Core Processing

1D Discrete Wavelet Transform



HO: low pass digital filter,
H1: high pass digital filter.
Z^{-1}: delay, ↓2: down-sample by 2



Recursive application of wavelet transform in spatial domain corresponds to dyadic partition of data in the frequency domain.

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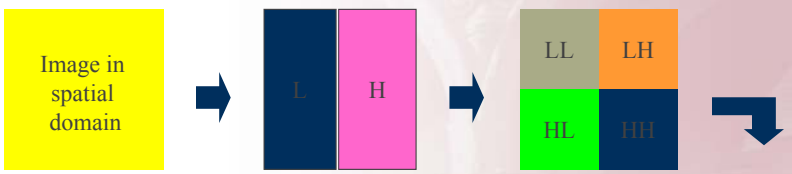
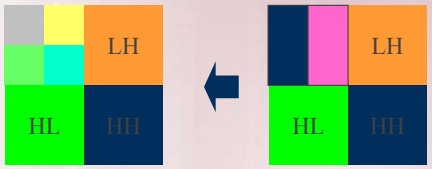
Core Processing

2D separate DWT



- 1D DWT applied alternatively to vertical and horizontal direction line by line.
- The LL band is recursively decomposed, first vertically, and then horizontally.
- This is Mallat method. Other methods have also been proposed.



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Quantization and lossless compression

Quantization

1. The purpose of quantization is to approximate each transformed coefficient with few prototypical values so that fewer bits will be needed to encode the transformed coefficients.
2. Quantization is a “binning” or clustering process.
3. Progressive process:
   - Start with few prototypes
   - Incrementally improve accuracy by adding new quantization levels.
   - When all bits are encoded, it becomes lossless quantization.

Lossless Compression (entropy encoding)

Uses an arithmetic coding system that compresses binary symbols relative to adaptive probability models (frequently occurring symbols are represented and stored with few bits; infrequently occurring symbols with more bits).

Conclusions

- JPEG2000 with extensions is an emerging image coding standard for the next generation of digital imaging.
- No IPR (intellectual property right) on part I of the standard exists (free licensing).
- JPEG2000 is based on the discrete wavelet transform.
- It is more complex than JPEG.
- Flexible design with hardware implementation in mind (FPGAs, GPUs, etc).