Assignment 2: Advection-Diffusion-Reaction equation

Consider the linear advection diffusion problem described in the lectures (see also Section 12.5 of the book "Numerical Mathematics" by Quarteroni, Sacco, Saleri), i.e.

$$-\alpha u'' + \beta u' = 0, \qquad u(0) = 0,\ u(L) = 1 \tag{1}$$

and the linear diffusion reaction equation

$$-\alpha u'' + \gamma u = 0, \qquad u(0) = 0,\ u(L) = 1 \tag{2}$$

in the domain $\Omega = [0, L]$.

For equation (1), define the so-called Péclet number Pe through

$$Pe = \frac{|\beta| L}{2\alpha}\,, \tag{3}$$

and for equation (2) the so-called Damköhler number

$$Da = \frac{\gamma}{\alpha}\,. \tag{4}$$

Boundary value problem and finite differences Discretise the equation by finite differences, with simple centred schemes for the second and first order derivatives. This consists of assembling a finite difference matrix (depending on the number of points $N = \frac{1}{h}$, and the equation parameters). Create a new class for these types of equations, that will contain a `SparseMatrix`, the number of points and parameters, as well as methods to assemble the matrix, to solve the system (using your solver from the previous assignment, or any other external solver), and to compute the error with respect to the analytical solution inserted as arguments of the executable code). Note the boundary condition $u(L) = 1$. This boundary condition will change the right hand side b of the linear problem $Ax = b$ resulting from the finite difference discretisation.

Testing Pick either the advection diffusion or the diffusion reaction. For the advection diffusion problem, consider at least three different values of Péclet number, first zero, then around unity, and than large. Similarly, for the diffusion reaction problem, consider again three similar values of the Damköhler number. Plot the solution for a few different mesh sizes and parameters. Analyze the error with respect to the analytical solutions using the maximum or L_2 norm. Try to find the order of convergence of the method and a theoretical result justifying it.

Submission requirements

You should submit your **code** together with a **report** that discusses the problem setup, underlying theory, all tests you performed, answers the questions outlined in the assignments, and describes your observations. Remember that your code should include some **comments** to make it readable.

Prepare a **folder** with all your files related to the assignment such that everything you have done can be reproduced, including the plotting and the report LaTeXcompilation. You have to provide a potential user with a simple way to reproduce your results. Provide a `README` with instructions about how to generate your report. Please also put a **separate pdf version** of your report into this folder, so that marking can be done even if the reproduction should fail.

Combine all files together into a single **archive**, the filename of which is `u123123123-assignment2.zip` (or other ending), where the first part should be replaced by your **user-id**. Then upload the archive file to

`https://files.warwick.ac.uk/ma913_2018/sendto`

Please let us know in the covering note for the upload the amount of time you have spent on this assignment.

Deadline: Wed 21 November 2018, 6:00pm