Geometría Básica

Capítulo II: Axiomas para la Geometría Euclidiana Plana

Jackie Harjani y Belén López.

UNED, C.A. Las Palmas

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Ejercicio 2.6.

Sea $r \subset P$ una recta y $A, B \in r$, $A \neq B$. Demostrar que para todo $t \in \mathbb{R}$, existe un punto $P_t \in r$, único, verificando:

$$d(P_t, A) = |t|$$
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Solución:

Capítulo II: Ejercicio 2.5

Existe $\rho : r \longrightarrow \mathbb{R}$ cumpliendo el Axioma $P3$ y tal que:

$i)$ $\rho(A) = 0$

$ii)$ $\rho(B) = d(A, B)$

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Única solución del sistema $\rho(P') = t \quad \overset{\rho \text{ biy.}}{\Rightarrow} \quad P' = P_t$.