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Inverse Kinematics Problem for 6-DOF Space Manipulator Based On the Theory of Screws

Jian Xie and Wenyi Qiang
Department of Control Science and Engineering
Harbin Institute of Technology
Harbin, 150001, P.R. China
xjbob_hit@163.com qiangwy@hit.edu.cn

Bin Liang and Cheng Li
Shenzhen Space Technology Center
HIT Research Institute in Shenzhen
Shenzhen, 518057, P.R. China
{bliang & cli}@robotsat.com

***Abstract-* Space manipulators will play a significant role in the maintenance and repair of space stations and satellites, and other space missions. The inverse kinematics is an important problem in the automatic control of space manipulator. The paper presents a novel method for solving the inverse kinematics problem of 6-DOF space manipulator based on the theory of screws. To solve the inverse kinematics problem, we formulate the kinematics equations of 6-DOF space manipulator and study a novel method. The method can give the analytic solutions of the inverse kinematics problem for 6-DOF space manipulator. The virtual prototyping technology is used to model the 6-dof mechanical arm system of Free-Flying space Robot (FFR) and simulate the space manipulator system, it shows the validity of the method which is presented in this paper. Compared with other methods, the method based on the theory of screws just establish two coordinates, is applicable to real-time control, allows one to directly choose the desired configuration from the multi-solutions and its geometry meaning is obvious.**

Index Terms –Space Manipulator, Inverse Kinematics

I. INTRODUCTION

Space exploration is a new frontier in current science and engineering [1]. Benefits from the space exploration are enormous, however, the stake is also very high. Space missions most are hazardous to astronauts, so space manipulators are expected to play an increasingly important role in the future space activity. One broad area of application is in the servicing, construction, and maintenance of satellites and large space structures in orbit. In hostile environments, space manipulators are very useful for performing complex tasks in stead of astronauts and enhance the efficiency and quality of tasks completion.

The Shuttle Remote Manipulator System (SRMS) is the first operational space robotic system [2], which is developed by Canadian Space Agency. Since its first mission aboard U.S. Space Shuttle Columbia in 1981, SRMS has completed many important missions: position an astronaut for Extra Vehicular Activities tasks, place satellites into the orbit and retrieve satellites for repairing.

In 1993, ROTEX was launched by Germany. Many experiments in ROTEX were to demonstrate space technologies, such as grasping the target with a multi-sensor gripper, teleoperation from the ground, visual simulation, and so on [3]. All these space experiments are performed in an experiment rack in the space shuttle. Thus, ROTEX is the first remotely controlled and an Inter Vehicular Activity space robot.

European Space Agency (ESA) has investigated the potential use of Geostationary Service Vehicle (GSV) which provide in orbit inspection of geostationary satellites and intervention when nessary . The manipulator of GSV will be used to capture the target sallite, and dock it to GSV [4].

The Engineering Test Satellite VII (ETS-VII) was launched by National Space Development Agency of Japan (NASDA) in 1997. ETS-VII is the first free-flying space robot system. Its purpose is to develop and demonstrate the rendezvous/docking, teleoperation and space robotic technologies. It demonstrated autonomous capture of the target satellite and a small metal ball [5][6][7][8].

The Chinese Experimental Space System or On orbit Robotic Services (CESSORS) is now being developed by Shenzhen Space Technology Development Center of China (SSTC). The purpose of this project is to develop a small space robotic system, which is capable of changing orbit and implementing un-manned robot servicing, such as repairing or retrieving malfunctioning satellites.

Free float that the attitude and position of the spacecraft all are not controlled is the most extreme status, the motion of manipulator arm will disturb the attitude of the spacecraft, and the coupling between the manipulator and the spacecraft causes control technology of the ground manipulators to be unable completely to apply on the space manipulators. But for normal working, for example the spacecraft should keep radio signal of communication or continuous working of the solar energy board, it needs guarantee the spacecraft attitude stabilization, this status that the attitude of the spacecraft is under controlled and the position of the spacecraft is uncontrolled is more common than the free float status.

Regarding the space manipulators which the attitude is under controlled, we can transform the space manipulators to the ground manipulators using virtual manipulator method [9], and apply the inverse kinematics solution method that is used on the ground manipulators with fixed base on the space manipulators. Generally, there are many solution methods to solve the robot inverse kinematics problem, for example, Paul's analytic solution [10][11], Lee and Ziegler's geometric method[12], Dinesh Manocha's efficient inverse kinematics method [13], Gregory Z.Grudic's iterative inverse kinematics method [14].

The elements of screw theory can be traced to the work of Chasles and Poinsot in the early 1800s. Using the theorems of Chasles and Poinsot as a starting point, Robert S. Ball developed a complete theory of screws which he published in 1900. There are two main advantages of using screw, twists and wrenches for describing rigid body kinematics. The first is that they allow a global description of rigid body motion that does not suffer from singularities due to the use of local coordinates. For example, such singularities are inevitable

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when one chooses to represent rotation via Euler angles. The second advantage is that the screw theory provides a geometric description of rigid motion which greatly simplifies the analysis of mechanisms. M.Murray studied this method and proposed to solve the inverse kinematic problem, but he just gived the inverse kinematic problem solution of a 3-DOF manipulator [15].

This paper deals with a 6-DOF space manipulator, provides the inverse kinematics solution procedure and method. We develop a method based on the theory of screws for solving the inverse kinematics problem. This method has an advantage that avoids a large amount of matrixes inverse multiply operation, establish just two coordinates and the expression is simple that it is convenient for the trajectory planning and simulation.

This paper also develops a virtual model of the 6-DOF space manipulator based on virtual prototyping technology. The simulation verifies solutions of the method.

II. Exponential coordinates for rigid motion and twists

We can describe rigid motion with twists. $q\in\Re^3$ is a point on a rigid body, we represent the axis l as a directed line and $\omega\in\Re^3$ is a unit vector specifying the direction. Consider a rigid body motion which consists of rotation about the axis l through an angle of θ radians, followed by translation along the same axis by an amount d as shown in Fig. 1. We define the pitch of the screw to be a the ratio of translation to rotation $h := d/\theta$.

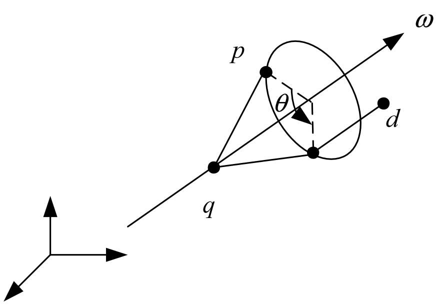


Fig. 1. Screw motion

Let the initial configure of the rigid body relative to the fixed coordinate is $g_{ab}(0)$, $g_{ab}(\theta)$ is the configure after motion, then:

$$g_{ab}(\theta)=e^{\hat{\xi}\theta}g_{ab}(0) \tag{1}$$

in the above equation

$$e^{\hat{\xi}\theta}=\begin{bmatrix} e^{\hat{\omega}\theta} & (1-e^{\hat{\omega}\theta})q+h\theta\omega \\ 0 & 1 \end{bmatrix} \tag{2}$$

with

$$\hat{\omega}=\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}\in so(3)=\left\{S\in\Re^{3\times 3}: S^T=-S\right\}$$

$e^{\hat{\omega}\theta}\in SO(3)=\left\{R\in\Re^{3\times 3}: R^TR=R^TR=I, \det R=1\right\}$, is matrix exponential, we define:

$$\begin{aligned} e^{\hat{\omega}t} &= I+\hat{\omega}t+\frac{(\hat{\omega}t)^2}{2!}+\frac{(\hat{\omega}t)^3}{3!}+\cdots \\ &= I+\hat{\omega}\sin\theta+\hat{\omega}^2(1-\cos\theta) \end{aligned} \tag{3}$$

$\omega\theta\in\Re^3$ is exponential coordinate of rotation matrix $R=e^{\hat{\omega}\theta}$, formula (3) is commonly referred to as Rodrigues' formula [15].

$\hat{\zeta}\in se(3)=\left\{(v \quad \hat{\omega}): v=-\omega\times q+h\omega\in\Re^3, \hat{\omega}\in so(3)\right\}$, we write ξ as:

$$\hat{\xi}=\begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}\in\Re^{4\times 4} \tag{4}$$

An element of $se(3)$ is referred to as a twist, and we call $\xi := [v,\omega]$ the twist coordinates of $\hat{\xi}$.

$g=e^{\hat{\xi}\theta}\in SE(3)=\{(P,R), P\in\Re^3, R\in SO(3)\}$, it is rigid body transformation described by twist.

We consider an arbitrary open-chain manipulator with n degrees of freedom, the joint angles $\theta=\begin{bmatrix}\theta_1 & \theta_2 & \cdots & \theta_n\end{bmatrix}^T$, the initial configuration of the manipulator $g(0)$. For joint i, construct a twist ξ_i which corresponds to the screw motion for the i^{th} joint with all other joint angles held fixed at $\theta_j=0$, where $\omega_i\in R^3$ is a unit vector in the direction of the twist axis and $q\in R^3$ is any point on the axis. For a revolute joint,

$$\xi_i=\begin{bmatrix} -\omega_i\times q_i \\ \omega_i \end{bmatrix} \tag{5}$$

For a prismatic joint,

$$\xi=\begin{bmatrix} v_i \\ 0 \end{bmatrix} \tag{6}$$

Combining the individual joint motions of the open-chain manipulator with n degrees of freedom, the forward kinematics map, $g_{st}: Q\to SE(3)$, is given by

$$g_{st}(\theta)=e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}\cdots e^{\hat{\xi}_n\theta_n}g_{st}(0) \tag{7}$$

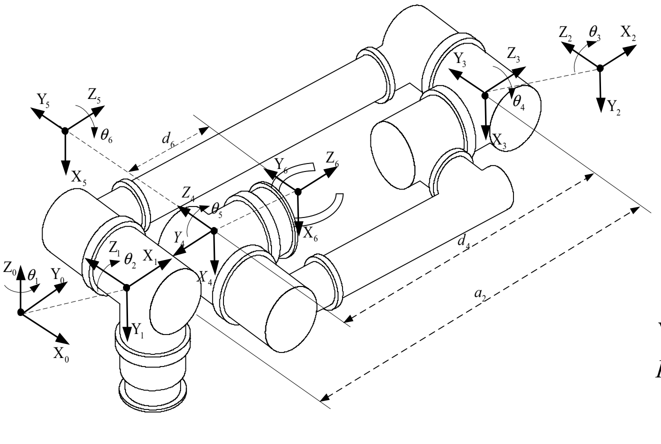
Equation (7) is called the product of exponentials formula for the manipulator forward kinematics.

Using the product of exponentials formula for the forward kinematics map, we develop a geometric algorithm to solve the inverse kinematics problem [16]. To solve the inverse kinematics problem, we reduce the full inverse kinematics problem into appropriate sub-problems whose solutions are known. There are some sub-problems [15]:

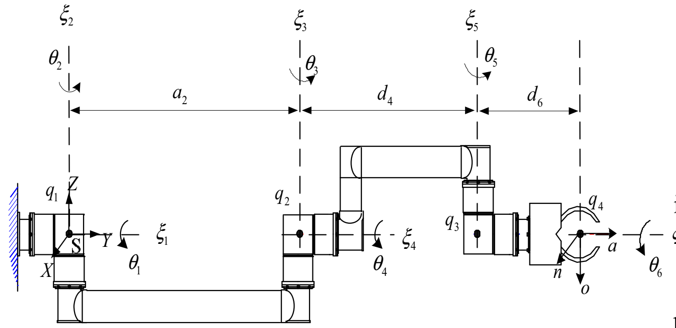
1. Rotation about a single axis.
2. Rotation about two subsequent axes.
3. Rotation to a given distance.

III. The space manipulator kinematics model

The space manipulator is with six degrees, and six joints all are revolute. The structure and coordinate of the space manipulator are shown as Fig.2.



(a) The space manipulator structure



(b) Body coordinate frames in initial configuration

Fig.2 The space manipulator structure and body coordinate frames

We let the initial configure of the space manipulator corresponding to $\theta = 0$, and attach base and tool frames shown as Fig. 2 (b). The transformation between tool and base frames at $\theta = 0$ is given by

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & a_2 + d_4 + d_6 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

To construct the twists for each joints

$$\omega_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{9}$$

and we choose axis points

$$q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; q_2 = \begin{bmatrix} 0 \\ a_2 \\ 0 \end{bmatrix} \; q_3 = \begin{bmatrix} 0 \\ a_2 + d_4 \\ 0 \end{bmatrix} \; q_4 = \begin{bmatrix} 0 \\ a_2 + d_4 + d_6 \\ 0 \end{bmatrix} \tag{10}$$

This yields twists

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_3 = \begin{bmatrix} a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_5 = \begin{bmatrix} a_2 + d_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{11}$$

The forward kinematics problem of the space manipulator is

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} g_{st}(0) = \begin{bmatrix} R(\theta) & p(\theta) \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

IV. THE INVERSE KINEMATICS SOLUTION PROCEDURE AND METHOD

The inverse kinematics problem of the manipulator is that we calculate the joint angles $\theta_1, \theta_2, \ldots, \theta_6$, when *n*, *o*, *a* and *p* are given.

We rearrange equation (12) as

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} = g_d g_{st}(0)^{-1} =: g_1 \tag{13}$$

Apply both sides of equation (13) to a point $q_3 \in R^3$ which is the common point of intersection for axis ξ_4, ξ_5 and ξ_6, this yield:

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} q_3 = g_1 q_3 \tag{14}$$

Subtract for both sides of equation (14) a point q_1 which is at the intersection of axis ξ_1 and ξ_2:

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} q_3 - q_1 = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} q_3 - q_1) = g_1 q_3 - q_1 \tag{15}$$

Using the property that the distance between points is preserved by rigid motions, take the norm of both sides of equation (15):

$$\left\| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} q_3 - q_1) \right\| = \left\| e^{\hat{\xi}_3\theta_3} q_3 - q_1 \right\| = \left\| g_1 q_3 - q_1 \right\| \tag{16}$$

Applying sub-problem 3, we solve θ_3:

$$\theta_3 = \pm \arccos\left(\frac{(p_x - a_x d_6)^2 + (p_y - a_y d_6)^2 + (p_z - a_z d_6)^2 - d_4^2 - a_2^2}{2 d_4 a_2}\right) \tag{17}$$

Since θ_3 is known, equation (14) becomes:

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} q_3) = g_1 q_3 \tag{18}$$

Applying sub-problem 2 with $p = e^{\hat{\xi}_3\theta_3} q_3, q = g_1 q_3$, we find

$$\alpha = \omega_1^T (q - q_1) = p_y - a_y d_6 \tag{19}$$

$$\beta = 0 \tag{20}$$

$$\gamma = \pm\sqrt{d_4^2 + a_2^2 + 2a_2 d_4 \cos\theta_3 - (p_y - a_y d_6)^2} \tag{21}$$

We let

$$c = [\gamma \quad \alpha \quad 0]^T \tag{22}$$

We use sub-problem 1 to solve $e^{\hat{\xi}_2\theta_2} p = c$ and $e^{-\hat{\xi}_1\theta_1} q = c$, find θ_1 and θ_2:

$$\theta_1 = \mathrm{atan}\,2(-(p_z - a_z d_6), p_x - a_x d_6) \tag{23}$$

$$\begin{aligned} \theta_2 = a\tan 2(&-d_4 \sin\theta_3 (p_y - a_y d_6) \\ &-(a_2 + d_4\cos\theta_3)\sqrt{d_4^2 + a_2^2 + 2a_2 d_4 \cos\theta_3 - (p_y - a_y d_6)^2}, \\ &(p_y - a_y d_6)(a_2 + d_4\cos\theta_3) \\ &- d_4 \sin\theta_3 \sqrt{d_4^2 + a_2^2 + 2a_2 d_4 \cos\theta_3 - (p_y - a_y d_6)^2}) \end{aligned} \tag{24}$$

Another solution is:

$$\theta_1 = \arctan 2(p_z - a_z d_6, -(p_x - a_x d_6)) \tag{25}$$

$$\theta_2 = a\tan 2(-d_4 \sin\theta_3 (p_y - a_y d_6) + (a_2 + d_4 \cos\theta_3)\sqrt{d_4^2 + a_2^2 + 2a_2 d_4 \cos\theta_3 - (p_y - a_y d_6)^2}, \quad (p_y - a_y d_6)(a_2 + d_4 \cos\theta_3) + d_4 \sin\theta_3 \sqrt{d_4^2 + a_2^2 + 2a_2 d_4 \cos\theta_3 - (p_y - a_y d_6)^2}) \tag{26}$$

(atan2(y, x) returns the arc tangent of the two variable x and y in degree, which is between -180° and 180°)

The remaining equation we write as:

$$e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} = e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1} g_d g_{st}(0)^{-1} =: g_2 \tag{27}$$

Apply both sides of equation (27) to a point q_4 which is on axis ξ_6 but not on axis ξ_4. This gives:

$$e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} q_4 = g_2 q_4 \tag{28}$$

Applying sub-problem 2, we let:

$$\alpha' = \omega_4^T (q' - q_3) = -d_4 - a_2 \cos\theta_3 - p_x \sin(\theta_2 + \theta_3)\cos\theta_1 + p_z \sin(\theta_2 + \theta_3)\sin\theta_1 + p_y \cos(\theta_2 + \theta_3) \tag{29}$$

$$\beta' = 0 \tag{30}$$

$$\gamma' = \pm\sqrt{d_6^2 - \alpha'^2 - \beta'^2} = \pm\sqrt{d_6^2 - (-d_4 - a_2 \cos\theta_3 - p_x \sin(\theta_2 + \theta_3)\cos\theta_1 + p_z \sin(\theta_2 + \theta_3)\sin\theta_1 + p_y \cos(\theta_2 + \theta_3))^2} \tag{31}$$

and

$$c' = [\gamma' \quad \alpha' + a_2 + d_4 \quad 0]' \tag{32}$$

We apply sub-problem 1 to equations $e^{\hat{\xi}_5\theta_5} p' = c'$ and $e^{-\hat{\xi}_4\theta_4} q' = c'$, solve θ_4 and θ_5:

$$\theta_4 = \operatorname{atan2}(-(p_x \sin\theta_1 + p_z \cos\theta_1), \; -p_z \cos(\theta_2 + \theta_3)\sin\theta_1 + p_x \cos(\theta_2 + \theta_3)\cos\theta_1 + p_y \sin(\theta_2 + \theta_3) - a_2 \sin\theta_3) \tag{33}$$

$$\theta_5 = \operatorname{atan2}(-\sqrt{d_6^2 - (-d_4 - a_2 \cos\theta_3 - p_x \sin(\theta_2 + \theta_3)\cos\theta_1 + p_z \sin(\theta_2 + \theta_3)\sin\theta_1 + p_y \cos(\theta_2 + \theta_3))^2}, \; -d_4 - a_2 \cos\theta_3 - p_x \sin(\theta_2 + \theta_3)\cos\theta_1 + p_z \sin(\theta_2 + \theta_3)\sin\theta_1 + p_y \cos(\theta_2 + \theta_3)) \tag{34}$$

Another solution is:

$$\theta_4 = \operatorname{atan2}(p_x \sin\theta_1 + p_z \cos\theta_1, \; -(-p_z \cos(\theta_2 + \theta_3)\sin\theta_1 + p_x \cos(\theta_2 + \theta_3)\cos\theta_1 + p_y \sin(\theta_2 + \theta_3) - a_2 \sin\theta_3)) \tag{35}$$

$$\theta_5 = \operatorname{atan2}(\sqrt{d_6^2 - (-d_4 - a_2 \cos\theta_3 - p_x \sin(\theta_2 + \theta_3)\cos\theta_1 + p_z \sin(\theta_2 + \theta_3)\sin\theta_1 + p_y \cos(\theta_2 + \theta_3))^2}, \; -d_4 - a_2 \cos\theta_3 - p_x \sin(\theta_2 + \theta_3)\cos\theta_1 + p_z \sin(\theta_2 + \theta_3)\sin\theta_1 + p_y \cos(\theta_2 + \theta_3)) \tag{36}$$

After finding θ_4 and θ_5, we rearrange the kinematics equation as:

$$e^{\hat{\xi}_6\theta_6} = e^{-\hat{\xi}_5\theta_5} e^{-\hat{\xi}_4\theta_4} e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1} g_d g_{st}(0)^{-1} = \begin{bmatrix} n'_x & o'_x & a'_x & p'_x \\ n'_y & o'_y & a'_y & p'_y \\ n'_z & o'_z & a'_z & p'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} =: g_3 \tag{37}$$

Apply both sides of equation (37) to a point $p = [0 \quad 0 \quad 1]^T$ which is on axis ξ_6 but not on axis ξ_5, we find the solution of θ_6:

$$\theta_6 = \operatorname{atan2}(p'_x, p'_z) \tag{38}$$

At the end of this procedure, θ_1 through θ_6 are solved. There are a maximum of eight possible solutions, due to multiple solutions for equations (16), (18) and (28).

V. Simulation results

Experiments based on the mentioned method were performed on the 6-DOF space manipulator test table. The photo of the manipulator and the test table is shown in Fig. 3.The experiments were to verify validity of the method with the 2-step method [17]. It can't show the real attitude of manipulator, because the 2-step method is designed for the ground experiment of the space manipulator.

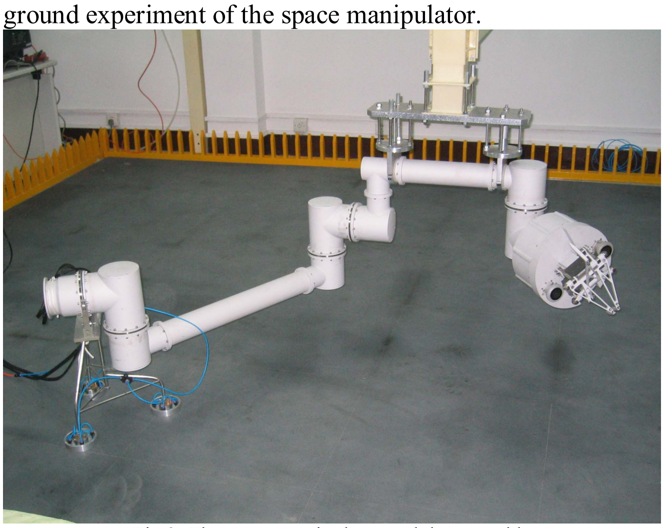


Fig.3 The space manipulator and the test table

The virtual prototyping technology is used to model the 6-dof mechanical arm system of Free-Flying space Robot (FFR) and simulate the space manipulator system. To make the simulation to be more suitable for the actual case, we apply ADAMS as the main simulating software. ADAMS is one of the world's most widely used mechanical system simulation software. It enables users to produce virtual prototypes, realistically simulating the full-motion behavior of complex mechanical systems on their computers. Model of the Satellite and space manipulator is created in the ADAMS. The configuration, relative position, and initial state are shown in Fig.4.

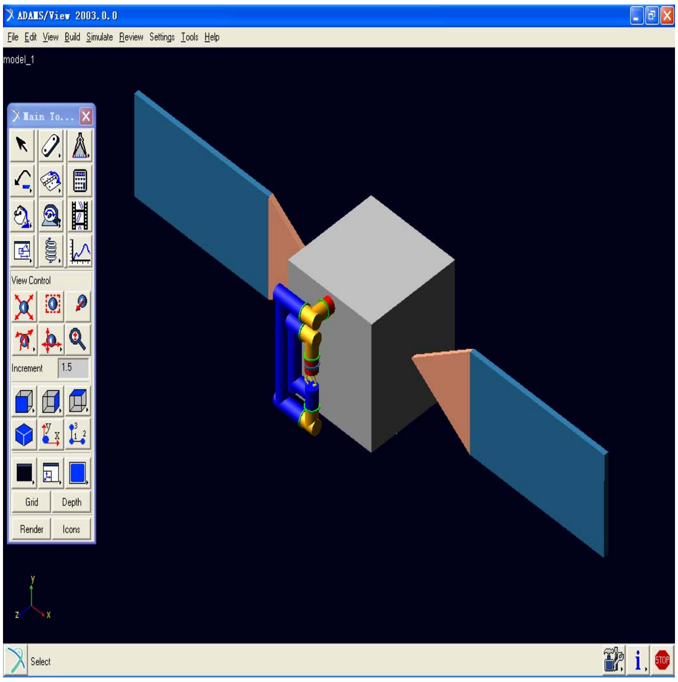


Fig.4 The virtual model of the space manipulator

The link parameters of the space manipulator are $a_2 = 0.83m$, $d_4 = 0.7m$ and $d_6 = 0.3345m$, when $p_x = 1.278m$, $p_y = 0.22m$ and $p_z = 0m$, the configure of the space manipulator is:

$$T = \begin{bmatrix} 0 & 1 & 0 & 1.278 \\ 0 & 0 & -1 & 0.22 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve the inverse kinematics problem of the above configure, we get the solutions:

$$\theta_1 = 0^\circ \quad \theta_2 = -54.92^\circ \quad \theta_3 = -66.28^\circ$$

$$\theta_4 = 0^\circ \quad \theta_5 = -58.81^\circ \quad \theta_6 = 90^\circ$$

We use the virtual prototyping to establish the space manipulator model and validate the correct of solutions. The attitude of the space manipulator is shown in Fig.5.

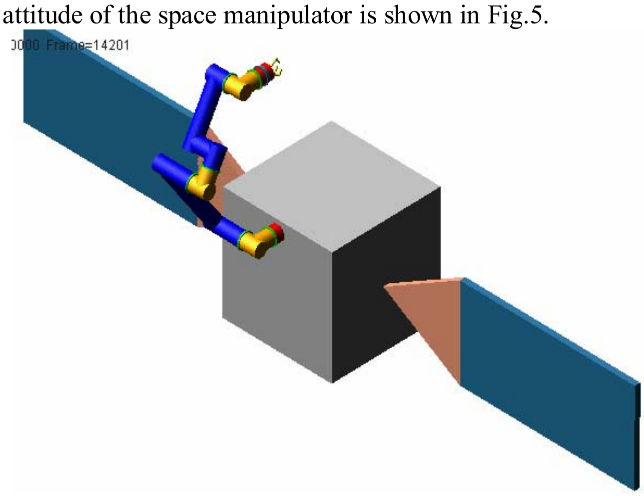


Fig.5 The attitude of the space manipulator

VI. Conclusion

The theory of screws is used to solve the inverse kinematics of the space manipulator, which is effective way to establish a global description of rigid body and avoid singularities due to the use of the local coordinates. We provide a geometric description of rigid motion which greatly simplifies the analysis of mechanisms. The theory has an advantage that does not need a large amount of matrixes inverse multiply operation, establish just two coordinates and the expression is simple that it is convenient for the trajectory planning and simulation. Moreover, the virtual prototyping technology we used is very effective to simulate the space manipulator motion, and is helpful to validate our algorithm in computer.

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