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Kinematics Solution Analysis of 6R Robot Based on Spinor Exponential Product

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Abstract: In the kinematics research of multi-degree-of-freedom robots, D-H(Denavit-Hartenber) parametric modeling is the most commonly used. However, the D-H parameter method needs to establish a complex coordinate system relationship, and it is difficult to avoid singular solutions. To solve these problems, a new forward and inverse motion solution method based on the exponential product form of the spinor theory is proposed. In the model, the rigid body motion equation based on the spinor theory is described based on the Plücker coordinate system. And a mathematical model for the robot's forward kinematics that is based the product of the spinor and the exponential is proposed. Based on the mathematical model of positive kinematics, the derivation operation is used to construct the inverse kinematics equation of the robot, and solve the complex multivariate equation system expanded by inverse kinematics. The mathematical symbolic operation method combined with MAPLE and MATLAB is used to solve the complex multivariate equations in inverse kinematics, and avoid the complexity of the Paden-Kahan subproblem process used in inverse kinematics based on spinor theory. The kinematics analysis and solution of the KUKA-KR5 arc 6R robot are carried out by the proposed method of rotational volume exponential product modeling. Through example calculation, the correctness and effectiveness of the proposed forward and inverse kinematics solution algorithm are verified.

Key Words: Spinor theory, Product of exponentials (POE), 6R serial robot, Inverse kinematics analysis

1 Introduction

The research on the solution of forward and inverse kinematics has always been an eternal subject in the robot kinematics research. As the basis of trajectory planning and motion control of robotics, forward kinematics is used to study the variation of each joint angle for the robot. Based on the variation of the joint angle and the relevant parameters of the robot arm, the pose corresponding to the robot end effector is derived. And the positive kinematics is the only solution. However, for the expected robot end pose, the real problem is to solve the corresponding joint rotation angle change by inverse derivation, which is the inverse kinematics solution. Certainly, the difficult problem is that the solution of the joint angles is not unique. And the inverse kinematics equation is complex and difficult to solve. The fast inverse kinematics solution is more practical in engineering applications.

At present, the D-H (Denavit-Hartenber) parameter method is extremely common for solving inverse kinematics problems in robot kinematics modeling applications，e.g. [1, 2, 3]. The D-H parameter method needs to continuously establish a new coordinate system according to structural changes, which increase the complexity of the coordinate system establishment. And it has no universal practicability for the current robot D-H parameter method. The spinor theory can be traced back to the research of Plücker and Klein in the 18th and 19th

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centuries. After Ball systematically studied this theory, the spinor theory was established, e.g. [4]. In 1997, Huang Zhen, Kong Lingfu and Prof. Fang Yuefa systematically applied spinor theory to analyze and perform study the parallel mechanism for the first time, e.g. [5]. Thus, the application of the spinor theory to describe the kinematics of robots was developed. The form of the spinor quantity exponential product based on the spinor theory provides a new idea for robot kinematics modeling, e.g. [6, 7, 8]. The spinor matrix exponential form (POE) is essentially different from the traditional D-H parametric method. The former only needs to establish two relative coordinate systems, one is the base coordinate system (*S*), and the other is the end tool coordinate system (*T*). The Plücker coordinate system is established through each joint, analyzing the transformation relationship between coordinate systems to simplify the structure of the robot. And the geometric meaning of the robot configuration can be more intuitively understood, thereby the problems of complicated D-H parameter operation and singularity are avoided, e.g. [9].

In this paper, based on the theory of spinor, the POE kinematics model of 6 degree of freedom robot is established. It uses Maple software and MATLAB software to derive calculations, and solves the inverse kinematics equations of the robot. Thus, the complexity of the inverse modeling of the robot and improves the efficiency of the solution is reduced.

2 Spinor Theory of Rigid Bodies and Model Analysis

To facilitate the understanding of inverse kinematic algorithm, the basic mathematical knowledge of spinor theory is introduced in this section.

2.1 Plücker Coordinate System

Any straight line in three-dimensional space can be regarded as a connection of two points. If there are two points $r_1 = (x_1, y_1, z_1, w_1)^T$ and $r_2 = (x_2, y_2, z_2, w_2)^T$ in the space, a straight line will be determined. And the Plücker coordinates of the linear ray form are thus shown as in Fig. 1.

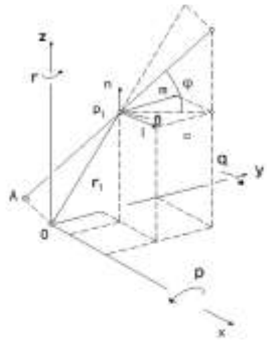

Fig. 1: Plücker coordinate system

The concept of spatial straight lines was first proposed by Cayley, and it can also be analyzed by Cayley transformation expression in the rotation matrix, e.g. [10]. Based on this concept, we can describe the joint axes of the robot by using the Cayley space line and the Plücker coordinate system to represent the line. The Plücker coordinate system is modeled to describe the various joint axes in robotics. Combining the Plücker coordinate system with the Cartesian coordinate system can theoretically avoid the action constraints caused by the specific configuration between the joint links, which provide a universal theory for the kinematics modeling and analysis of n-degree-of-freedom robots.

2.2 Spinor Theory

The spinor that is a line vector with a spinor pitch is a geometric quantity that describes the geometry. As the projection of the spinor sub part on the main part, the spinor pitch is the coefficient of the spinor. The spin algebra is a vector algebra describing the geometry, and is also a sub-algebra of the Lie algebra *se* (3), that is the ray passing through the origin in the Lie algebra, and an element of the *se* (3) of the projective Lie algebra. The foundation of the spinor theory is based on two basic theories: Poinsot force central axis theory and Chasles motion theory, e.g. [11, 12]. The resulting spinor theory accelerates the development of robotics. It has the advantages of clear and concise description of robot geometry, efficient and convenient calculation, and clearly describes the motion characteristics of rigid bodies. Which is more convenient to theoretically study robotics.

- Lie group in three-dimensional space

For the description of the motion pose in space, let *{S}* be the inertial coordinate system, and *{T}* be the coordinate system fixed on the rigid body, thus combining the bits described by the *{S} {T}* two relative coordinate systems. Through the relationship between the poses, it can get the transformation matrix:

$$SE(3) = \left\{ \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} | R \epsilon \Re^{3\times3}, t \in \Re^3, R^t R = I, det(R) = 1 \right\} \quad (1)$$

SE(3) is a special European group of rigid body transformations in three-dimensional space, which is a subgroup of Lie group, e.g. [13]. In the set, *R* is the attitude of the object coordinate system relative to the inertial coordinate system, and *t* is the position of the object coordinate system relative to the inertial coordinate system. The Lie algebra of *SE*(3) is denoted as *se*(3) and defined as:

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \in se(3) \mapsto (\omega^T; v^T)^T \in R^6 \quad (2)$$

The Chasles theorem mentioned above applies to rigid body motion. Any object moving from one configuration to another can be simplified as a rotation around an axis and a movement along a certain line. Because it involves two concepts of rotation around an axis and parallel movement, it can be summarized as a spiral motion. The study shows that there is an inseparable relationship between the spiral motion and the Lie algebra, which leads to the concept of the motion spinor, e.g. [14].

- The motion spin exponential product form describes the rigid body transformation

In the kinematics modeling research of tandem robots, the error modeling method based on exponential product formula and the exponential product based on spinor theory have been widely used, e.g. [15]. The former has a clear differential idea, and its differential form expression contains definite integral terms, which cannot be well applied in practical engineering applications. The latter's exponential product modeling has more simplified kinematic problems for ease of modeling.

The motion rotation coordinate is $\xi = (\omega^T, v^T) \in R^6$ so it can be expressed as Plücker coordinates:

$$\xi = \begin{pmatrix} \omega \\ v \end{pmatrix}^T \quad (3)$$

Where, $\hat{\xi}$ represents the motion spin and ω is the vector axis.

Reintroduction of the form of the motion spin matrix:

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \in se(3) \quad (4)$$

Where $v = -\omega \times q$; q is the positional variable describing the rotation of the rigid body; *v* is the combined motion of *ω* and *q*, and *se*(3) is the Lie algebra belonging to *SE*(3).

In summary, the matrix exponential form of rigid body transformation can be obtained:

$$g = exp(\hat{\xi} \cdot \theta) \quad (5)$$

The above formula can be discussed when *ω≠0*:

$$g = exp(\hat{\xi} \cdot \theta) = \begin{bmatrix} exp(\hat{\omega}) & (E\text{-}exp(\hat{\omega} \cdot \theta))(\omega \cdot v) + \omega \cdot \omega^T \cdot \theta \cdot v \\ 0 & 1 \end{bmatrix} \quad (6)$$

If *ω=0*, then there is:

$$g = exp(\hat{\xi} \cdot \theta) = \begin{bmatrix} I & v \cdot \theta \\ 0 & 1 \end{bmatrix} \quad (7)$$

In the calculation and derivation of the above exponential matrix, the Rodrigues formula is applied, which is used to calculate a new vector obtained in a three-dimensional space after the vector is rotated by a certain angle around the rotation axis. This formula uses a two-vector cross product to represent the new vector after the rotation. Because it can be rewritten as a matrix form, it is widely used in the field of spatial analytic geometry and computer graphics, and becomes the basic calculation formula of rigid body motion. Below the concept of the Rodrigues transformation formula is briefly introduced.

Let *v* be a three-dimensional space vector, *k* is the unit vector of the rotation axis. Then *v* rotates around the rotation axis *k* in the sense of right-handed spiral rule, and the rotation angle is *θ*. The resulting vector can be composed of three non-coplanar vectors *v*, *k* and the frame formed by $k \times v$ indicates:

$$v_{rot} = Rv \quad (8)$$

$$v_{rot} = cos\theta v + (1 - cos\theta)(v \cdot k)k + sin\theta k \times v \quad (9)$$

Among them:

$$R = Ecos\theta + (1 - cos\theta) \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} (k_x, \ k_y, \ k_z) + sin\theta \begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix} \quad (10)$$

It should be noted that E is a 3rd order unit matrix, and the second term in the formula is not a dot product, but a tensor product, and finally a 3rd order matrix is obtained.

3 Analysis of The Establishment of The Positive Kinematics Model Of 6R Robot

In this paper, the KUKA-KR5 arc model 6-DOF robot is used as the test object of this paper for kinematics modeling analysis and verification. The front 3R joint axes of this model 6R robot do not intersect, and the rear 3R joint axes intersect at one point, which is in accordance with the Pieper criterion. So the closed solution of the forward and inverse kinematics can be calculated. This is a robot sample of the KUKA-KR5 arc model, as in Fig. 2. Basic information has been marked in the figure. A coordinate system schematic diagram based on the Plücker coordinate system is established for each joint angle, and the inertial coordinate system S and the tool coordinate system T are respectively established according to the structure, as in Fig. 3.

Fig. 2: KUKA-KR5 arc model robot

1. First revolute joint 2. Second revolute joint 3. Third revolute joint 4. Fourth revolute joint 5. Fifth revolute joint 6. Sixth revolute joint first

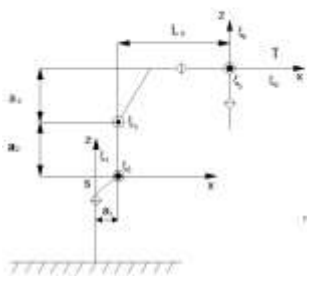

Fig. 3 Diagram of each joint coordinate system and its Plücker coordinate system

According to the parameters marked in the figure, the initial pose matrix of the robot under natural static and the initial value of the joint angle is 0:

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & b \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The values of b and c are $\mathrm{b} = \mathrm{a}_1 + \mathrm{l}_4$; $\mathrm{c} = \mathrm{a}_2 + \mathrm{a}_3$. b and c are the x and z coordinates of the tool coordinate system T relative to the inertial coordinate system S.

Below the solution in the forward kinematics of the robot for the motion transformation of the rigid body are analyzed. The concept of a unit vector axis $\omega = (\omega_1 \omega_2 \omega_3)^T$ is introduced when describe the rotation of a rigid body around an axis. That is, the unit vector axis of each joint angle can be obtained according to the schematic diagram:

$$\begin{cases} \omega_1 = (0 \quad 0 \quad 1)^T \omega_2 = (0 \quad 1 \quad 0)^T \omega_3 = (0 \quad 1 \quad 0)^T \\ \omega_4 = (1 \quad 0 \quad 0)^T \omega_5 = (0 \quad 1 \quad 0)^T \omega_6 = (0 \quad 0 \quad 1)^T \end{cases} \tag{12}$$

An antisymmetric matrix of ω is required for the calculation of the rotation matrix, namely:

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{13}$$

Combine the unit vector axis $\omega_1 = (0 \quad 0 \quad 1)^T$ to obtain the antisymmetric matrix of ω_1:

$$\hat{\omega} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

According to the actual link parameters of the robot on the schematic diagram, the points on the axis can be obtained:

$$\begin{cases} q_1 = (0 \quad , \quad 0 \quad , \quad 0)^T \\ q_2 = (a_1 \quad , \quad 0 \quad , \quad 0)^T \\ q_3 = (a_1 \quad , \quad 0 \quad , \quad a_2)^T \\ q_4 = q_5 = q_6 = (a_1 + l_4 \quad , \quad 0 \quad , \quad a_3 + a_2)^T \end{cases} \tag{15}$$

The known parameters can be brought in by the formula $v = -\omega \times q$ and formula (3) to obtain the spin coordinate of each joint angle. Combining with formula (7), the positive kinematics model of the 6R robot can be established as:

$$g_{st}(\theta_i) = exp\,(\hat{\xi}_i, \theta_i) g_{st}(0) \tag{16}$$

Where i=*1, 2, 3, 4, 5, 6*. $g_{st}(\theta_i)$ is the end pose matrix of the robot end effector after the movement, and its matrix form can be expressed as:

$$g_{st}(\theta)=\begin{bmatrix} n_x & o_x & \alpha_x & p_x \\ n_y & o_z & \alpha_x & p_y \\ n_z & o_y & \alpha_x & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

4 Paper Submission Inverse Kinematics Analysis of 6R Industrial Machine

Based on the establishment of the forward kinematics model, the inverse kinematics solution equation can be derived and calculated. The pose matrix $g_{st}(\theta)$ of the robot end effector is preset in advance and used as a known quantity to solve the set of θ_i solution. The idea of this inverse solution is based on the forward kinematics model formula of the 6R robot. The solution process and the derivation of the symbol formula are solved by using MATLAB and the function package built in the Maple software. The process is as follows:

4.1 Solve $\theta_1, \theta_2, \theta_3$

The first is to sort out the forward kinematics model, and simultaneously multiply both sides of the robot kinematics model by the initial pose $g_{st}{}^{-1}(0)$, and get:

$$g = exp(\hat{\xi}_1 \cdot \theta_1)\, exp(\hat{\xi}_2 \cdot \theta_2) \cdots\cdots exp(\hat{\xi}_6 \cdot \theta_6) \tag{18}$$

Where: $g = g_{st}(\theta) \cdot g_{st}^{-1}(0)$

Using the spiral theory, the forward kinematics model $exp(\hat{\xi} \cdot \theta)q = q$ can be simplified, and the three joint intersection points q_6 can be multiplied on both sides to obtain:

$$\begin{cases} exp(\hat{\xi}_1 \cdot \theta_1)\, exp(\hat{\xi}_2 \cdot \theta_2)\ exp(\hat{\xi}_3 \cdot \theta_3) q_6 = g \cdot q_6 \\ g \cdot q_6 = (m, n, o, 1)^T \end{cases} \tag{19}$$

Express the above exponential product in matrix form as:

$$exp(\hat{\xi}_1 \cdot \theta_1)\, exp(\hat{\xi}_2 \cdot \theta_2)\ exp(\hat{\xi}_3 \cdot \theta_3) = \begin{bmatrix} h_{11} & h_{121} & h_{13} & r_1 \\ h_{21} & h_{22} & h_{23} & r_2 \\ h_{31} & h_{32} & h_{33} & r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

Combining (15) to expand into a matrix operation form:

$$\begin{bmatrix} h_{11} & h_{121} & h_{13} & r_1 \\ h_{21} & h_{22} & h_{23} & r_2 \\ h_{31} & h_{32} & h_{33} & r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 + l_4 \\ 0 \\ a_2 + a_3 \\ 1 \end{bmatrix} = \begin{bmatrix} m \\ n \\ o \\ 1 \end{bmatrix} \tag{21}$$

Expand the above formula:

$$\begin{cases} (a_1 + l_4)c1c2c3 - (a_1 + l_4)c1s2s3(a_2 + a_3)c1s2c3 \\ +(a_2 + a_3)c1c2s3 + c1c2(a_1 - a_1c3 - a_2s3) + c1s2 \\ \cdot (a_1s3 - a_2 - a_2c3) + c1(a_1 - a_1c2) = m \\ (a_1 + l_4)s1c2c3 - (a_1 + l_4)s1s2s3(a_2 + a_3)s1c2s3 \\ +(a_2 + a_3)s1s2c3 + s1c2(a_1 - a_1c3 - a_2s3) + s1s2 \\ \cdot (a_1s3 - a_2 - a_2c3) + s1(a_1 - a_1c2) = n \\ -(a_1 + l_4)s2c3 - (a_1 + l_4)c2s3 - (a_2 + a_3)s2s3 \\ +(a_2 + a_3)c2c3 - s2(a_1 - a_1c3 - a_2s3) + c2(a_1s1 \\ -a_2 - a_2c3) + a_1c2 = o \end{cases} \tag{22}$$

The trigonometric functions contained in the equation can take advantage of the trigonometric function relationship:

$$\begin{cases} sin^2\theta_1 + cos^2\theta_2 = 1 \\ sin^2\theta_3 + cos^2\theta_4 = 1 \\ sin^2\theta_5 + cos^2\theta_6 = 1 \end{cases} \tag{23}$$

To facilitate the calculation of the solution using Maple software, seting $\sin\theta_1 = x_1$, $cos\theta_1 = x_2$, $sin\theta_2 = x_3$, $cos\theta_2 = x_4$, $sin\theta_3 = x_5$, $cos\theta_3 = x_6$, the simultaneous (5) (6) formula has *s*:

$$\begin{cases} a_1x_2 + a_2x_2x_3 + l_4x_2x_4x_6 + a_3x_2x_3x_6 \\ +a_3x_2x_4x_5 - l_4x_2x_3x_5 = m \\ a_1x_1 + a_2x_1x_3 + l_4x_1x_4x_6 + a_3x_1x_3x_6 \\ +a_3x_1x_4x_5 - l_4x_1x_3x_5 = n \\ a_2x_4 - a_3x_3x_5 + a_3x_4x_6 - l_4x_3x_6 - l_4 \\ x_4x_5 = o \\ x_1^2 + x_2^2 = 1 \\ x_3^2 + x_4^2 = 1 \\ x_5^2 + x_6^2 = 1 \end{cases} \tag{24}$$

For the multivariate equations of the above formula, using the functions built into the Maple software for the elimination calculation. The values of $x_1 \sim x_6$ can be easily solved. because it is a multivariate equations system, the solution obtained is not unique. Therefore, θ_1, θ_2, θ_3 are also not unique.

4.2 Solve $\theta_4, \theta_5, \theta_6$

The obtained parameters θ_1, θ_2, θ_3 are combined with ω_4 and ω_5 in the formula (12) into (18) and converted into:

$$exp(-\hat{\xi}_1 \cdot \theta_1)\, exp(-\hat{\xi}_2 \cdot \theta_2)\ exp(-\hat{\xi}_3 \cdot \theta_3) g = exp(\hat{\xi}_4 \cdot \theta_4)\, exp(\hat{\xi}_5 \cdot \theta_5)\ exp(\hat{\xi}_6 \cdot \theta_6) \tag{25}$$

According to the relationship between ω_4, ω_5, ω_6, a point *k* can be randomly selected, which should fall on the ω_6 axis, but cannot intersect ω_4, ω_5. Both ends of the above formula are simultaneously multiplied by *k* and arranged:

$$exp(-\hat{\xi}_1 \cdot \theta_1)\, exp(-\hat{\xi}_2 \cdot \theta_2)\, exp(-\hat{\xi}_3 \cdot \theta_3) g * k = exp(\hat{\xi}_4 \cdot \theta_4)\, exp(\hat{\xi}_5 \cdot \theta_5)\, exp(\hat{\xi}_6 \cdot \theta_6) \mathrm{k} \quad (26)$$

The expressions of θ_4, θ_5 that have been obtained are rounded up according to the method of calculating θ_1, θ_2, θ_3:

$$exp(-\hat{\xi}_1 \cdot \theta_1)\, exp(-\hat{\xi}_2 \cdot \theta_2)\, exp(-\hat{\xi}_3 \cdot \theta_3)\, exp(-\hat{\xi}_4 \cdot \theta_4)\, exp(-\hat{\xi}_5 \cdot \theta_5) = exp(\hat{\xi}_6 \cdot \theta_6)\gamma \quad (27)$$

And then can find θ_6.

In summary, the paper solved the variation of the six joint angles of the 6R robot by solving the mathematical model of forward kinematics and inverse kinematics. The method of modeling based on the spinor theory greatly reduces the complexity of the D-H parametric modeling, which is easier to solve and calculate.

5 Instance Calculation Verification

This article uses the detailed parameters given by the official KUKA-KR5 arc robot. The relevant parameters established according to the Plücker coordinate system are: $a_1 = 180, a_2 = 600, a_3 = 120, l_4 = 620$;

Use MATLAB's robot toolbox to import the relevant model parameters of KUKA-KR5 arc, given a set of random angles $(\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6)^T = (25\ 29\ 35\ 150\ 55\ 130)^T$, and the angle of the group conforms to the working range of the joint angle of the robot.

The end pose of the robot end actuator obtained from this set of random angles is:

$$g_{st}(\theta) = \begin{bmatrix} 0.2007 & 0.1684 & 0.2375 & 699.7469 \\ 0.4842 & 0.4064 & 0.1108 & 299.3618 \\ 0.6617 & 0.5553 & -0.7199 & -489.6031 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

According to the end pose, combined with the forward kinematics model and the calculation process of inverse kinematics, it is easy to prove the correctness of the algorithm.

$g_{st}(\theta)$ is as the known end pose to solve the combined solution of each joint angle, as shown in the following table.

Table 1: Eight Groups of Inverse Kinematics

$\theta_1 = 25°$	$\theta_2 = 29°$	$\theta_3 = 35°$
24.0836°	28.1776°	35.0646°
24.0836°	28.1776°	35.0646°
24.0836°	6.5906°	27.2476°
24.0836°	6.5906°	27.2476°
–155.9164°	–173.4092°	–144.9354°
–155.9164°	–173.4092°	–144.9354°
–155.9164°	–151.8224°	–152.7524°
–155.9164°	–151.8224°	–152.7524°
$\theta_4 = 150°$	$\theta_5 = 55°$	$\theta_6 = 130°$
149.9970°	55.0048°	130.1824°
30.0030°	124.9953°	49.8176°
–33.8830°	–59.6892°	81.9721°
146.1170°	–120.3108°	98.0279°
18.3246°	–51.9618°	–47.4256°
161.6754°	–128.0382°	–132.5744°
152.2390°	23.9852°	–55.3157°
27.7361°	156.0048°	–124.6843°

Note: θ_1, θ_2, θ_3, θ_4, $\theta_5$andθ_6 are corresponding to each joint angle of robot respectively.

From the data in table 1, it is not difficult to see that the inverse kinematics solution is not unique. Through the combination of Maple and MATLAB software to calculate the joint angles generated by each calculation, the data of each group are basically equal. The end pose matrix of the robot end actuator calculated after bringing in all the solutions is basically the same, the error can be controlled around 0.015. It can be proved that the robot kinematics modeling method and the kinematics calculation method are correct. Since the inverse kinematics solution is a multi-set solution, it is a solution to multi-conditions. To find a set of optimal solutions to match and generate sample angles from these real solutions, it is also necessary to satisfy the shortest motion path, smooth trajectory without bumps, and minimum energy consumption. And the solution after satisfying the above conditions can become the final solution of inverse kinematics.

6 Conclusions

The traditional D-H parameter method has the problem that it is difficult to avoid singular points. Solving robot kinematics based on spinor theory does not require the establishment of a complex coordinate system with the D-H parameter method, which the forward kinematics modeling of the exponential product form of the spinor theory has certain generality. Using the combination of Maple and MATLAB to deduce the mathematical symbolic operation, and solve the complex multi-variable equation system expanded by inverse kinematics. Which avoid over-reliance on the Paden-Kahan subproblem method used for inverse kinematics solutions based on spinor theory. Comparing with the D-H solution method, it can effectively solve the singularity avoidance problem, and finally introduces the example calculation to verify the accuracy of the method.

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