SVM

osimpson

2021-04-16

1 Basics

Theorem 1. *The distance between two parallel hyperplanes,* $a^T x = b_1, a^T x = b_2$ *is* $\frac{|b_1 - b_2|}{\|a\|}$.

Proof. Let p_1 be a point on the first plane, and let p_2 be the point on the second plane closest to p_1. The distance between p_1 and p_2 is the distance between the hyperplanes. This distance is along the normal vector which passes through p_1 to p_2. The direction of the normal vector is a. For a quick proof of this, consider two points x_1, x_2 on the hyperplane. Both points satisfy the equation $a^T x = b_1$, so we have that $a^T(x_2 - x_1) = 0$, and since $x_2 - x_1$ defines an arbitrary line on the hyperplane, a is orthogonal to the hyperplane. Therefore, a vector orthogonal to the hyperplane that goes through p_1 to p_2 is given by $p_1 + a\lambda = p_2$.

Then $\|p_2 - p_1\| = \|p_1 + a\lambda - p_1\| = |\lambda| \cdot \|a\|$. We can solve for λ using the hyperplane equations:

$$\begin{aligned} a^T(p_2 - p_1) &= b_2 - b_1 \\ a^T(a\lambda) &= b_2 - b_1 \\ \lambda \|a\|^2 &= b_2 - b_1 \\ \lambda &= \frac{b_2 - b_1}{\|a\|^2} \end{aligned} \tag{1}$$

Thus, $\|p_2 - p_1\| = \frac{|b_2 - b_1|}{\|a\|}$

$\square$