AI5031: Machine Learning, exercise sheet 1

1 Notation and math primitives

Compute the resulting value of the following mathematical expressions:

1. $\sum_{i=0}^{10} i$

2. $\prod_{i=1}^{5} i$

3. $\sum_{i=0}^{4} i^2$

4. $\sum_{i=1}^{3} x_i y_i$ if $\vec{x} = (1, 2, 3)^T$ and $\vec{y} = (2, 1, 0)^T$

5. $\sum_{i=1}^{3} (x_i + y_i)$ if $\vec{x} = (1, 2, 3)^T$ and $\vec{y} = (2, 1, 0)^T$

6. $\frac{1}{3} \sum_{i=1}^{3} x_i^2$ if $\vec{x} = (1, 2, 3)^T$

2 Matrix multiplication

We have column vectors $\vec{x} = (1, 2, 3, 0)^T$, $\vec{y} = (2, 3, 4, 5)^T$, $\vec{z} = (2, 3, 0)^T$ and a

matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$. Give the result of the following matrix multipli-

cations if they are legally possible:

1. $A\vec{y}$

2. $A\vec{x}$

3. $\vec{x}^T \vec{y}$ (note: this is another way of writing the scalar product)

4. $\vec{x} A$

5. $A\vec{y}^T$

6. $\vec{x}^T A \vec{y}$ (note: this is called the *Mahalanobis distance* of two vectors given a matrix A)

7. $\vec{x}\vec{y}^T$ (note: this is called the *outer product* of two vectors)

8. $A\vec{z}$

9. $\vec{z}\vec{x}^T$

10. AA

11. AA^T

3 Functions

The Rectified Linear Unit (ReLU) function $f : x \in \mathbb{R} \mapsto y \in \mathbb{R}$ is defined as
$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{else} \end{cases}$ Assuming the quantities from the previous exercise and

the matrix $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, compute:

1. $f(\vec{x})$

2. $f(B\vec{x})$

3. $f(B)$

4. $f(\vec{x}^T \vec{y})$

4 Vector-scalar and vector-vector functions

Let $f(x) = <\vec{x}, (1, 1, 0, 0)^T>$ be a vector-scalar function and $g_i(\vec{x}) = x_i^2$ a vector-vector function. Please compute:

1. $f\left((1, 1, 1, 1)^T\right)$

2. $f(B)$ using the matrix B from the previous exercise

3. $f(B^T)$ using the matrix B from the previous exercise

4. $g\left((1, 2, 3)^T\right)$

5. $g(B)$ using B from the previous exercise

5 Bonus: A simple machine learning model

A classical problem in machine learning is the OR-problem. We want a ML model which returns 0 (false) or 1 (true), $f : \vec{x} \in \mathbb{R}^2 \mapsto y \in \mathbb{R}$, to learn the logical OR mapping, that is:

$$0, 1 \rightarrow 1 \tag{1}$$
$$1, 0 \rightarrow 1 \tag{2}$$
$$0, 0 \rightarrow 0 \tag{3}$$
$$1, 1 \rightarrow 1 \tag{4}$$

a) Formulate data and target matrices for this problem!
b) Can you find parameters $\vec{w} = (w_1, w_2, w_3)^T$ for the following model function

that solve the problem? It may be helpful to draw the data points into a 2D plot.

$$f(\vec{x}) = \begin{cases} 1 & w_1 x_1 + w_2 x_2 + w_3 > 0 \\ 0 & w_1 x_1 + w_2 x_2 + w_3 \leq 0 \end{cases} \tag{5}$$