Activity 33 – Introduction to Proof

functions

(1) (Exercise 1 in GIAM section 6.5)

(2) Verify that A is reflexive, symmetric, and transitive.

(3) Consider the generic "equivalence mod m" relation:

$$x\mathsf{R}y \iff x \mod m = y \mod m.$$

Show that R is reflexive, symmetric and transitive.

(4) In the book we defined the function $sf(n)$ which returns n divided by the largest perfect square that divides n. For example $sf(20) = 5$ since 4 is the largest perfect square that divides 20, and $20/4 = 5$

Use the sf function to define a relation:

$$x\mathsf{S}y \quad \Longleftrightarrow \quad sf(x) = sf(y)$$

Show that S is an equivalence relation on $\mathbb{N}$.

(5) Continuing with the relation S defined in problem 4, notice that for any number x that actually is a perfect square, $sf(x) = 1$. Given this, what is $\overline{1}$?

(6) (Still working with the relation from problem 4.) Many different natural numbers can be used as a "label" for the equivalence classes under S. For instance $\overline{2}$, $\overline{8}$ and $\overline{18}$ all denote the same equivalence class – let's follow the convention of always using the smallest such label. Characterize these reduced representatives for the equivalence classes.

(7) Define a relation Q on the set of all finite sets by

$$A \mathsf{Q} B \iff |A| = |B|$$

What are the equivalence classes under Q?

(8) Do problem 3 in GIAM section 6.3.

(9) Do problem 4 in GIAM section 6.3.