Activity 21 – Introduction to Proof

containment

(1) Name three elements of $\mathbb{N}$.

(2) Name three subsets of $\mathbb{N}$.

(3) The empty set, $\emptyset$ is contained in every set A (regardless of A's contents). Why?

(4) Let A be the set $\{\emptyset, 1, 2, \{3, 4, 5\}\}$. Mark the following statements True or False.

a) $\emptyset \subseteq A$

b) $\emptyset \in A$

c) $\{1\} \subseteq A$

d) $\{3, 4, 5\} \subseteq A$

e) $\{\emptyset\} \subseteq A$

f) $\{1, 2\} \in A$

(5) Insert either $\in$ or $\subseteq$ in the blanks in the following sentences (in order to produce true sentences).

i) 1 _____ $\{3, 2, 1, \{a, b\}\}$

ii) $\{a\}$ _____ $\{a, \{a, b\}\}$

iii) $\{a, b\}$ _____ $\{3, 2, 1, \{a, b\}\}$

iv) $\{\{a, b\}\}$ _____ $\{a, \{a, b\}\}$

(6) A number is called *doubly even* if it is divisible by 4. Let D denote the set of doubly-even natural numbers, and let E denote the set of even natural numbers. Which set is a subset of the other?

(7) The membership criteria for the sets D and E in the previous question are $M_D = 4 \,|\, x$ and $M_E = 2 \,|\, x$ respectively. Which logical statement implies the other?

(8) The following incomplete diagram illustrates the correspondence between Set Theory (in the top row) and Logic (in the bottom row). Fill in the missing symbols.

$$\begin{array}{ccc} D & & \subseteq \\ \updownarrow & \updownarrow & \updownarrow \\ & & M_E(x) \end{array}$$

(9) Name a superset of $\mathbb{N}$.

(10) Name a superset of $\mathbb{R}$.

(11) In the next lecture we'll talk about set equality. How would you describe a reasonable way to determine if two sets are "the same"?