Activity 7 – Introduction to Proof

relations

(1) Which of the following are true?

$13 \mid 52$	$3 < 1$	$24 \geq 5$
$5 > 9$	$1 \in \{1, 2, 3\}$	$1 = 0.\overline{9}$
$243 \leq 243$	$3 \mid \frac{42}{7}$	$19 \equiv 13 \pmod 7$

(2) What are all of the ordered pairs in the $<$ relation restricted to the set $\{0, 1, 2, 3, 4\}$?

(3) What are the domain, range and codomain of the previous problem's relation?

(4) Let's create our own relation. Let R be defined by

$$x\mathsf{R}y \iff 7 \,|\, (x-y).$$

Let's suppose the domain and codomain of R are both $\mathbb{Z}$.

Name 5 pairs of integers that are in R and 5 pairs that are not.

(5) A fairly trivial relation on the real numbers might be stated in words as: "x and y are related when they have the same sign." For the sake of consistency, let's treat 0 seperately, so possible signs are $-$, $+$, and 0. What does the graph of this relation look like?

(6) Here's an example of a relation that works with words rather than numbers. Let $x\mathsf{L}y$ be the relation "x comes before y in the dictionary." Find three things (pairs of words) that are in L. (FYI, the official name for dictionary order is *lexicographic order* hence the choice of letter L.)

(7) Some relations are known as "reflexive" this means that a thing is always related to itself. Characterize the graphs of reflexive relations on the real numbers.

(8) There's a well-known property of $=$ that is often paraphrased as "two things that are equal to a third must be equal to one another." This is known as the transitive property. Which of the relations we've seen in this worksheet (including the ones we created ourselves) have the transitive property?

(9) The property that an integer is either even or odd is known as its *parity*. We can create a relation P that is true when its inputs have the same parity.

$$x\mathsf{P}y \quad \Longleftrightarrow \quad x \text{ and } y \text{ are both even or } x \text{ and } y \text{ are both odd.}$$

Try restating the lemma from the previous section

$$\big(\forall x \in \mathbb{Z}, \quad \text{if } x^2 \text{ is even, then } \ x \text{ is even.}\big)$$

using P.

(10) Of course the statement you created in the previous problem will be stronger since it deals with odd numbers as well as even ones. Is it still true?