Activity 33 – Introduction to Proof

functions

(1) When a function is presented without specifying the domain, the default rule is to assume that the domain is the largest possible set of real numbers for which the function is defined. By the default rule, what is the domain of $f(x) = 1/x$?

(2) (Exercise 1 in section 6.5 of GIAM) For each of the following functions, give its domain, range and a possible codomain.

(a) $f(x) = \sin(x)$

(b) $g(x) = e^x$

(c) $h(x) = x^2$

(d) $m(x) = \frac{x^2+1}{x^2-1}$

(e) $n(x) = \lfloor x \rfloor$

(f) $p(x) = \langle \cos(x), \sin(x) \rangle$

(3) Deduce a formula for a bijective function that gives a correspondence between the odd numbers and the positive squares:

$$\{1, 3, 5, 7, 9, \ldots\}$$

and

$$\{1, 4, 9, 16, 25, \ldots\}$$

(4) Consider the function

$$m : \mathbb{Z} \longrightarrow \{0, 1, 2, 3, 4, 5, 6\}$$

defined by

$$m(z) = z \mod 7.$$

What is the image of the set of perfect squares ($S = \{0, 1, 4, 9, 16, \ldots\}$) under m?

What is the preimage 2?

(5) Show that the natural exponential function is a surjection onto $(0, \infty)$.

(6) Recall that the hyperbolic sine function, $\sinh(t)$, is defined by

$$\sinh(t) = \frac{e^t - e^{-t}}{2}.$$

Show that $\sinh$ is an injection.

(7) Show that $\sinh$ is also a surjection.