Activity 19 – Introduction to Proof

proofs and disproofs of existentials

(1) Is there a Fibonacci number that is a perfect cube?

(2) Partition numbers count the number of ways we can write a number n as a sum (we don't count different orderings of the same sum). We denote the partition numbers by $p(n)$. By convention, $p(0)$ is defined to be 1 ($p(1)$ is also 1). As a more interesting example, $p(4) = 5$ since there are 5 distinct ways to write 4 as a sum: 4, $3+1$, $2+2$, $2+1+1$ and $1+1+1+1$. The partition numbers are A000041 in the OEIS.

After the first two 1's the values of $p(n)$ are all prime (at first). Prove that there is a non-prime partition number, $p(n)$ with $n \geq 2$.

(3) Show that there is a 4 digit vampire number.

Review problems for chapter 3.

(4) Use proof by exhaustion to show that there are no 2 digit vampire numbers.

(5) Prove

$$\forall a, b, c \in \mathbb{Z},\ (a \mid b \land b \mid c) \implies a \mid c.$$

(6) Prove

$$\forall a, b \in \mathbb{Z},\ (a \mid b \ \wedge\ b \mid a) \implies a = b.$$

(7) Prove

$$\forall x \in \mathbb{R},\ x^5 < x^4 \implies 8 > 5x + 3.$$

(8) Prove that for all integers a, b, and c, if $a \mid b$ and $a \mid (b + c)$, then $a|c$.

(9) Show that $\binom{n}{k} \cdot \binom{k}{r} = \binom{n}{r} \cdot \binom{n-r}{k-r}$ (for all integers r, k and n with $r \leq k \leq n$).

(10) Referring to the partition numbers defined in problem 2, we say a partition has "odd parts" if all the numbers in the sum are odd. We say a partition has "distinct parts" if all the numbers in the sum are different from one another. It is a general fact that the number of partitions of n into odd parts is equal to the number of partitions of n into distinct parts. (Frankly, that is really weird...)

Show exhaustively that the number of partitions of 7 into odd parts is equal to the number of partitions of 7 into distinct parts.