Activity 30 – Introduction to Proof

properties of relations

(1) Name a relation that is irreflexive, anti-symmetric, and transitive.

(2) (Exercise # 4 in Section 6.2 of GIAM) For each of the five properties, name a relation that has it and a relation that doesn't.

(3) Consider the relation R defined by

$$x\mathsf{R}y \quad \iff \quad x \pmod 3 = y \pmod 3.$$

Show that R is reflexive.

(4) Show that R from the previous problem is symmetric.

(5) To show that R (defined in problem 2) is transitive we need to assume that there are 3 arbitrary integers, a, b, and c such that $a\mathsf{R}b$ and $b\mathsf{R}c$. From that setup we must deduce that $a\mathsf{R}c$.

Use the definition of R to write down precisely what needs to be shown to prove that this relation is transitive.

(6) Critique the following proof that R is transitive.

> *Proof:* Suppose that a, b and c are integers such that $a \pmod 3 = b \pmod 3$ and $b \pmod 3 = c \pmod 3$. It follows that $a \pmod 3 = c \pmod 3$ because of "reasons."
>
> Q.E.D.

(7) In the last worksheet we defined a relation on $\mathbb{R}$ by $x\mathsf{R}y \iff x^2 - y^2 = 0$. Verify that this relation is reflexive and symmetric by looking at its graph. Prove that it is also transitive.

(8) Define a relation L on the set of all words in the English language via

$$xLy \iff x \text{ comes before } y \text{ in the dictionary.}$$

Which of the 5 properties does L have?

(9) Let S be the power set of $\{1, 2, 3\}$ and consider the $\subseteq$ relation on S. Informally verify that this is an ordering relation. (Which three properties must we check?)