Activity 20 – Introduction to Proof

intro to set theory

(1) What is the truth set of $P(x)$ = "x is divisible by 3 and x is a square" ?

(2) What logical open sentence corresponds to the set

$$A = \{1, 5, 9, 13, 17, 21, \ldots\}?$$

(3) What are the cardinalities (i.e. sizes, i.e. how many elements?) of the following sets?

$\{1, 2, 3, 4\}$ $\{1, 2, \{3, 4\}\}$ $\{\{1, 2\}, \{3, 4\}\}$ $\{\{1, 2, 3, 4\}\}$

(4) Which of the following are equal?

(i)	$\{1,2\}$	(ii)	$\{2\}$	(iii)	$\{1,2,3,4\}$	(iv)	$\{1,2,\{3,4\}\}$
(v)	$\{2,2,2\}$	(vi)	$\{1,3,2,4\}$	(vii)	$\{2,\{2,2\}\}$	(viii)	$\{1,2,1,2\}$
(ix)	$\{2,1\}$	(x)	$\{1,2,\{4,3\}\}$	(xi)	$\{4,3,2,1,2,3,4\}$	(xii)	$\{\{2\},2\}$
(xiii)	$\{\{4,3\},2,1\}$	(xiv)	$\{1,2,1\}$	(xv)	$\{2,2\}$	(xvi)	$\{2,1,\{3,4\}\}$

(5) In Logic we defined two special statements, c and t, contradiction and tautology. Supposing we are working in an unspecified universal set U, what are the Set Theory equivalents to c and t?

(6) Complete the suggested exercise in the text by writing out the power sets of $\{1,2,3\}$, $\{1,2\}$, $\{1\}$ and $\emptyset$. Conjecture a formula for the cardinality of $\mathcal{P}(\{1,2,3,\ldots n\})$.

(7) The power set of $\{1, 2, 3, 4, 5\}$ will consist of sets having cardinalities between 0 (the empty set) and 5 (the entire set). For each cardinality, give an example of a subset of $\{1, 2, 3, 4, 5\}$ having that cardinality and state how many subsets of $\{1, 2, 3, 4, 5\}$ will have that cardinality.

(8) Some of the sets in a power set are contained in others. Create a graph (the sort of diagram we used in the pebbling number problems) that has nodes labelled by the elements of the power set of $\{1, 2\}$ and edges (i.e. connections) between nodes where one set is contained in the other.

(9) Make a diagram similar to the one in the previous problem, but with nodes labelled by divisors of 15, and edges where one number divides another.

(10) Is there a sets/inclusion graph[1] that corresponds to the numbers/divisibility graph[2] for the divisors of 12? If so, what is a set? If not, explain why not.

[1] as in problem 8

[2] as in problem 9