Activity 11 – Introduction to Proof

logical equivalence

It may be useful to have a copy of the table of basic logical equivalences handy while working on these. In addition to the equivalences in the table, recall the following:

$A \iff B \cong (A \implies B) \wedge (B \implies A)$

and $A \implies B \cong \neg A \vee B$.

(1) Fill in the right-hand column with explanations for each step in the following two column proof.

We want to show that $(A \vee \neg B) \wedge (B \vee \neg A)$ is logically equivalent to $(A \wedge B) \vee (\neg B \wedge \neg A)$.

$(A \vee \neg B) \wedge (B \vee \neg A)$	
$((A \vee \neg B) \wedge B) \vee (A \vee \neg B) \wedge \neg A)$	
$((A \wedge B) \vee (\neg B \wedge B)) \vee (A \vee \neg B) \wedge \neg A)$	
$((A \wedge B) \vee (\neg B \wedge B)) \vee ((A \wedge \neg A) \vee (\neg B \wedge \neg A))$	
$((A \wedge B) \vee c) \vee (c \vee (\neg B \wedge \neg A))$	
$(A \wedge B) \vee (\neg B \wedge \neg A)$	

(2) Fill in the rest of the left-hand column with logical expressions in the following two column proof.

We want to show that $A \vee (\neg A \wedge B) \cong A \vee B$.

$A \vee (\neg A \wedge B)$	Given
	Distributive law
	Complementarity
	Identity law

(3) Create two column proofs for the following:

(a) $((A \wedge B) \vee (\neg A \wedge B)) \wedge (A \wedge B) \cong A \wedge B$

(b) $A \cong A \wedge ((A \vee \neg B) \vee (A \vee B))$

(c) $\neg(A \wedge B) \wedge \neg(A \wedge C) \cong \neg A \vee (\neg B \wedge \neg C)$

(d) $A \iff B \cong (A \wedge B) \vee (\neg A \wedge \neg B)$