Activity 8 – Introduction to Proof

logic

(1) Which of the following sentences are statements? For those which are not, is there a way to modify them so that they are?

(a) The domain of the divisibility relation is $\mathbb{Z}$.

(b) x is a prime number.

(c) The range of the divisibility relation is $\mathbb{R}$.

(d) This sentence is false.

(2) Several compound sentences can be constructed using either the $\wedge$ or the $\vee$ operator together with $\neg$'s on the predicate variables. (For example, $\neg A \vee B$ or $\neg A \wedge \neg B$.) Assuming that A and B appear in that order, there are 8 such sentences. List them.

(3) A shortcut to figuring out the truth tables of sentences like the ones from the previous problem is this: "Or" statements are *false* in exactly one row, "And" statements are *true* in exactly one row.

We take the ordering of the rows of a 2-variable truth table to be:

	A	B
row 1:	T	T
row 2:	T	ϕ
row 3:	ϕ	T
row 4:	ϕ	ϕ

With the above convention what row number(s) are the following sentences true in?

(a) $A \wedge B$

(b) $A \wedge \neg B$

(c) $\neg A \wedge B$

What row noumbers are the following false in?

(a) $\neg A \vee B$

(b) $A \vee \neg B$

(c) $\neg A \vee \neg B$

(4) Now we should be ready to create all 8 truth tables for the statements from problem 2. I've pre-filled the skeletons of the tables to ease your burden.

A	B	$A \wedge B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$A \wedge \neg B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$\neg A \wedge B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$\neg A \wedge \neg B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$A \vee B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$A \vee \neg B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$\neg A \vee B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

A	B	$\neg A \vee \neg B$
T	T	
T	ϕ	
ϕ	T	
ϕ	ϕ	

(5) Create a truth table for $A \wedge (A \vee B)$

(6) Create a truth table for $A \vee (A \wedge B)$

(7) Comparing the last two problems, what do you notice?

(8) Draw the digital logic diagram for $A \wedge (A \vee B)$ and $A \vee (A \wedge B)$.
How could they be simplified?

(9) In the video we mentioned the *exclusive or* operator which is true when exactly one of its inputs is true. Usually, exclusive or is notated thusly: $A \oplus B$.

Using the notion of *disjunctive normal form*[1] determine an expression that only uses $\wedge$, $\vee$ and $\neg$ that is equivalent to $A \oplus B$.

(10) Construct a digital logic diagram – using the disjunctive normal form concept – for the following truth table. Challenge: A much simpler logic diagram is possible. Can you find it?

A	B	C	???
T	T	T	ϕ
T	T	ϕ	T
T	ϕ	T	ϕ
T	ϕ	ϕ	T
ϕ	T	T	ϕ
ϕ	T	ϕ	ϕ
ϕ	ϕ	T	ϕ
ϕ	ϕ	ϕ	ϕ

[1] Recall that disjunctive normal form consists of the "or" of a bunch of "and" statements that recognize the rows we want to be true.