Activity 10 – Introduction to Proof

logical equivalence

(1) What are the contrapositive and the equivalent disjunction of

$$\neg P \implies Q?$$

(2) Suppose X and Y are compound statements that are equivalent. (They may involve many variables so the truth table could be enormous.) What will a truth table for $X \iff Y$ look like?

(3) The patterns of T's and ϕ's that we use in the initial columns of a truth table has a recursive structure.

When there is a single variable it look like $\left|\begin{matrix} T \\ \phi \end{matrix}\right|$. To build the next one (two variables) you put two copies of this above one another and prefix the top one with T's and the bottom one with ϕ's.

Complete the setup of the following truth table with 3 variables using this recursive idea.

A	B	C	?
	T	T	
	T	ϕ	
	ϕ	T	
	ϕ	ϕ	

Challenge: can you create a truth table with 4 variables (16 rows!)

(4) The following truth table has all the elements you'll need to verify the distributive property of $\wedge$ over $\vee$. Fill it in.

A	B	C	$B \vee C$	$A \wedge B$	$A \wedge C$	$A \wedge (B \vee C)$	$(A \wedge B) \vee (A \wedge C)$
T	T	T					
T	T	ϕ					
T	ϕ	T					
T	ϕ	ϕ					
ϕ	T	T					
ϕ	T	ϕ					
ϕ	ϕ	T					
ϕ	ϕ	ϕ					

(5) Verify both versions of DeMorgan's law using truth tables.

(6) Use DeMorgan's law(s) to find negations of the following.

(a) $A \vee \neg B$

(b) $\neg A \wedge B$

(c) $\neg A \vee \neg B$

(7) Each of the following can be evaluated using one of the Domination, Identity or Complimentarity laws. Evaluate and also state which law was used. (The last one needs two.)

(a) $A \vee \neg A$

(b) $c \wedge B$

(c) $c \vee \neg B$

(d) $t \wedge C$

(e) $B \wedge \neg B$

(f) $(A \vee \neg A) \vee C$

(8) Here's a statement that can be massively simplified using the absorption law (if you look at it the right way).

$$(A \wedge B) \vee ((\neg A \vee (A \wedge C)) \wedge (A \wedge B)).$$