Activity 4 – Introduction to Proof

Definitions of Elementary Number Theory

(1) What integer must exist in order to show that 42 is even?

(2) What integer must exist to show that 17 is odd?

(3) Try writing a formal argument that the sum of an even and an odd integer must be odd.

(4) If we want to write numbers in base-12, we'll need have single digits for all of the numbers from 0 to 11. We can certainly reuse 0 through 9, let's use δ and ϵ for 10 and 11. In the little 12 toes schoolhouse rock episode these are pronounced "dec" and "ell" respectively.

   (a) What base-12 number comes immediately after $\epsilon\epsilon\epsilon_{12}$?

   (b) What base-12 number comes immediately before $\epsilon\epsilon\epsilon_{12}$?

   (c) Suppose I tell you that the decimal number 99 translates to 83_{12} in base-12. What is 100 in base-12? In contrast, what decimal corresponds to 100_{12}?

(5) In using the definition of place value to reinterpret a base-12 number in decimal, we have to use the base-10 equivalents for the digits.

$$\delta_{12} = 10_{10} \quad \text{and} \quad \epsilon_{12} = 11_{10}$$

So what is the decimal value of $89\delta\epsilon_{12}$?

(6) Recall that in hexadecimal, the digits are in

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

which correspond to the decimal numbers 1 through 15. In hexadecimal, F "acts like 9." Use this to say what the decimal value of $FFFF_{16}$ is.

(7) Circle the divisibility statements that are true:

$5 \mid 100$	$7 \mid 43$	$11 \mid 77$
$1 \mid 2$	$21 \mid 0$	$3 \mid 322$
$8 \mid 64$	$22 \mid 11$	$3 \mid 321$

(8) Is it true that 0 divides all integers? Is $0|0$ true?

(9) Suppose you are taking down Christmas decorations and you have 79 ornaments that need to be stored in boxes that hold 12 ornaments each. Use either a floor or a ceiling computation to determine the number of boxes that will be needed.

(10) What is the value of $\lfloor \lceil \pi \rceil \rfloor$? Can you simplify $\lfloor \lceil x \rceil \rfloor$ in general?

(11) Calculate the following:

(a) 17 mod 5

(b) 77 mod 5

(c) 25 mod 12

(d) 99 mod 7

(e) 10003 mod 1000

(12) Recall that the notation $n \mod d$ is used for the remainder obtained when we divide n by d. What is $(n \mod d) \mod d$?

(13) Complete the table:

n	$n!$
0	
1	
2	
3	
4	
5	
6	
7	

(14) Which binomial coefficient would be obtained by simplifying the following fraction?

$$\frac{10!}{4! \cdot 6!}$$

Is there another correct answer?