

# A Gentle Introduction to Linear Algebra

Version 0.1

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The latest version of this book is available (without charge) in portable document format at

<http://osj1961.github.io/gila/>.

The source code repository can also be found on GitHub.

This is version 0.1 of *A Gentle Introduction to Linear Algebra*.



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