

# A Gentle Introduction to Linear Algebra

Version 0.1

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The latest version of this book is available (without charge) in portable document format at

<http://osj1961.github.io/gila/>.

The source code repository can also be found on GitHub.

This is version 0.1 of *A Gentle Introduction to Linear Algebra*.



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Systems of equations . . . . .	1
1.2	Vector equations . . . . .	1
1.3	Transformational equations . . . . .	1
1.4	Matrix notation . . . . .	1
<b>2</b>	<b>Row reduction</b>	<b>3</b>
2.1	Triangular systems . . . . .	3
2.2	Echelon form . . . . .	3
2.3	RREF . . . . .	3
2.4	Row operations and Gaussian elimination . . . . .	3
2.5	Solving linear systems . . . . .	3
<b>3</b>	<b>Vectors</b>	<b>5</b>
3.1	Vectors and scalars . . . . .	6
3.1.1	vectors . . . . .	6
3.1.2	vector operations . . . . .	6
3.1.3	linear combinations . . . . .	6
3.1.4	the Euclidean basis . . . . .	6
3.1.5	parametric equations for lines and planes in $\mathbb{R}^3$ . . . . .	6
3.2	The matrix-vector product . . . . .	6
3.3	Homogeneous and non-homogeneous systems . . . . .	6

3.4	Matrix-matrix products . . . . .	6
3.4.1	basic transformations . . . . .	6
3.4.2	compositions . . . . .	6
3.4.3	the matrix of a composition . . . . .	6
3.4.4	matrix multiplication . . . . .	6
3.5	Vector spaces - an introduction . . . . .	6
3.6	Dependence and independence . . . . .	6
3.7	Bases and dimension . . . . .	6
<b>4</b>	<b>Transformations</b>	<b>7</b>
4.1	Linear and affine transformations . . . . .	7
4.2	Change of basis . . . . .	7
4.3	Rank and nullity . . . . .	7
4.4	Row operations and elementary matrices . . . . .	7
4.5	The LU decomposition . . . . .	7
<b>5</b>	<b>The spectral decomposition</b>	<b>9</b>
5.1	Diagonal and diagonalizable systems . . . . .	9
5.2	Eigenvalues and eigenvectors . . . . .	9
5.3	Jordan form and the singular value decomposition . . . . .	9
<b>6</b>	<b>Algebraic structures</b>	<b>11</b>
6.1	Groups, Rings and Fields . . . . .	12
6.1.1	fields . . . . .	12
6.1.2	the complex numbers . . . . .	12
6.1.3	rings . . . . .	12
6.1.4	groups . . . . .	12
6.2	Modules . . . . .	12
6.3	Algebras . . . . .	12
6.3.1	complex numbers . . . . .	12
6.3.2	the cross product in $\mathbb{R}^3$ . . . . .	12

*CONTENTS*

vii

6.3.3	quaternions . . . . .	12
6.4	Inner product spaces . . . . .	12





# List of Figures



# List of Tables



# Preface

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# Chapter 1

## Introduction

1.1 Systems of equations

1.2 Vector equations

1.3 Transformational equations

1.4 Matrix notation





# Chapter 2

## Row reduction

2.1 Triangular systems

2.2 Echelon form

2.3 RREF

2.4 Row operations and Gaussian elimination

2.5 Solving linear systems





# Chapter 3

## Vectors

### 3.1 Vectors and scalars

#### 3.1.1 vectors

#### 3.1.2 vector operations

#### 3.1.3 linear combinations

#### 3.1.4 the Euclidean basis

#### 3.1.5 parametric equations for lines and planes in $\mathbb{R}^3$

### 3.2 The matrix-vector product

### 3.3 Homogeneous and non-homogeneous systems

### 3.4 Matrix-matrix products

#### 3.4.1 basic transformations

#### 3.4.2 compositions

#### 3.4.3 the matrix of a composition

#### 3.4.4 matrix multiplication

# Chapter 4

## Transformations

4.1 Linear and affine transformations

4.2 Change of basis

4.3 Rank and nullity

4.4 Row operations and elementary matrices

4.5 The LU decomposition



# Chapter 5

## The spectral decomposition

5.1 Diagonal and diagonalizable systems

5.2 Eigenvalues and eigenvectors

5.3 Jordan form and the singular value decomposition







## Chapter 6

# Algebraic structures

### 6.1 Groups, Rings and Fields

#### 6.1.1 fields

#### 6.1.2 the complex numbers

#### 6.1.3 rings

#### 6.1.4 groups

### 6.2 Modules

### 6.3 Algebras

#### 6.3.1 complex numbers

#### 6.3.2 the cross product in $\mathbb{R}^3$

#### 6.3.3 quaternions

### 6.4 Inner product spaces