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Source: *Journal of Political Economy*, Vol. 92, No. 1 (Feb., 1984), pp. 123-135

Published by: [University of Chicago Press](#)

Stable URL: <http://www.jstor.org/stable/1830549>

Accessed: 26-10-2015 05:33 UTC

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# Are Bond-financed Deficits Inflationary? A Ricardian Analysis

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This paper considers the possible theoretical validity of the following “monetarist hypothesis”: that a constant, positive government budget deficit can be maintained permanently and without inflation if it is financed by the issue of bonds rather than money. The question is studied in a discrete-time, perfect-foresight version of the competitive equilibrium model of Sidrauski, modified by the inclusion of government bonds as a third asset. It is shown that the monetarist hypothesis is invalid if the deficit is defined exclusive of interest payments, but that it is valid under the conventional definition. It is also shown that the stock of bonds can grow indefinitely at a rate in excess of the rate of output growth, provided that the difference is less than the rate of time preference. These formal conclusions do not take account of possible limitations on taxing capacity arising from default incentives that tend to grow with the stock of bonds.

## I. Introduction

Recent developments in U.S. monetary and fiscal affairs have led to a renewal of interest in issues reminiscent of the “monetarist vs. Keynesian” debates of previous decades. In particular, the Council of Economic Advisers’ 1982 and 1983 forecasts of a long string of unusually

This paper was prepared for the Second Peterkin Symposium, “Foundations of Monetary Policy and Government Finance,” held April 1–2, 1982, at Rice University. I am indebted to Robert Barro, John Bryant, Marvin Goodfriend, Peter Neary, Edmund Phelps, Paul Romer, José Scheinkman, Warren Weber, and two anonymous referees for helpful comments on earlier versions. Financial support was provided by the National Science Foundation (SES 82-08151).

[*Journal of Political Economy*, 1984, vol. 92, no. 1]  
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large federal budget deficits, together with the Federal Reserve's repeated avowals to keep monetary growth rates low, have conferred intense practical interest upon the question of whether bond-financed deficits have significant impact on aggregate demand and thereby on price level and/or output magnitudes. In addition, recent academic emphasis on improved choice-theoretic foundations for aggregate specifications has led to some new notions regarding monetarist models, most prominently in the work of Sargent and Wallace (1981, 1982).

The present paper seeks to contribute to the theoretical understanding of such issues by consideration of a precisely specified question that bears directly on the monetarist notion that bond-financed deficits are unimportant for aggregate demand. In particular, it asks whether it is possible that—in an equilibrium model of a nongrowing economy with maximizing agents and cleared markets—a positive government budget deficit can be maintained permanently and without inflation if financed entirely by bond sales, the money stock being held constant. As it transpires, the answer to this question depends on the way “deficit” is defined, the crucial distinction involving the inclusion or exclusion of interest payments on outstanding government debt.

## II. Preliminaries

In order to limit the issues at hand in a manageable fashion, it will be presumed throughout the analysis that the economy can be represented by a deterministic, aggregative, flexible-price, equilibrium model. For some issues, such a model might be inadequate or misleading. Our present concern, however, involves the influence on inflation rates of a policy stance maintained over an extended period of time. For that type of concern, a flexible-price equilibrium model—which presumes that aggregate demand effects are manifested primarily in price level or inflation responses—seems well suited.

As a matter of terminology—and to sharpen the issues—let us define a monetarist viewpoint as one that asserts that bond-financed deficits have no effect on aggregate demand. More precisely, our *monetarist hypothesis* is that, for given time paths of the money stock and government spending, it does not matter for aggregate demand whether the necessary revenue is raised by taxation or by bond sales. In other words, bond-financed changes in tax receipts have—according to the monetarist hypothesis—no effects on the price level or on output.

At this point, it perhaps needs to be asked whether there is *any* explicable reason to believe that the monetarist hypothesis (as defined) might be correct. Discussions in the most well-known refer-

ences notwithstanding,<sup>1</sup> the main intellectual support for such a position seems to be provided by the “Ricardian equivalence theorem,” which obtains in some models in which infinite-lived agents correctly take account of the effects on future budgets of current budgetary actions.<sup>2</sup> With fixed time paths of government spending and money creation, any bond-financed change in current taxes implies changes in future interest payments to be made by the government. If these are to be financed by lump-sum taxes and if the government and private agents face the same market interest rates, then the change will have no effect on a representative private agent’s intertemporal budget constraint. Under such conditions, then, a bond-financed change in taxes will have no effect on the agent’s supplies or demands and, consequently, no effect on the price level or output—just as predicted by the monetarist hypothesis.

The foregoing statement of the Ricardian result presumes that agents have infinite life spans, which is obviously untrue. But, as is well known, Barro (1974) has demonstrated that an economy of finite-lived agents who care about the utility of their offspring or parents may, under reasonably general conditions, be treated for analytical purposes as one with infinite-lived agents.<sup>3</sup> Consequently, this feature of the analysis seems acceptable, given the aims of the investigation.

A second crucial assumption of the Ricardian analysis is that agents are cognizant of effects on their own intertemporal budget constraints of governmental debt issues. But this assumption is merely a particular application of the hypothesis of rational expectations, the merits of which have been detailed extensively elsewhere.

Other complicating aspects of reality—uncertainty, distribution effects, multiple interest rates—are also ignored in the Ricardian equivalence argument. But the same is true of most policy-oriented theoretical analyses of macroeconomic phenomena. As there is no apparent reason why the issue at hand requires a different type of treatment, it would seem satisfactory to neglect them here, as elsewhere.<sup>4</sup>

<sup>1</sup> Many of these are included in Gordon (1974) and Stein (1976). The models typically assume that consumption or private expenditure depends directly upon private wealth, with government debt included as a component of the latter (see, e.g., Brunner and Meltzer 1976, p. 72). In these models, then, a bond-financed tax reduction directly increases aggregate demand.

<sup>2</sup> In this analysis, and in what follows here, taxes are assumed to be of the lump-sum variety.

<sup>3</sup> This statement presumes that intergenerational transfers are operative. A discussion of circumstances under which Barro’s result is inapplicable is provided by Drazen (1978).

<sup>4</sup> Barro (1974, 1983) argues that neglect of these complicating features does not serve to distort the results in a predictable direction.

The discussion thus far seems to suggest that bond-financed deficits could indeed be noninflationary. Each bond issue/tax reduction package has no impact on aggregate demand or the price level, so a sequence of bond-financed tax reductions should apparently have no impact on the inflation rate. But it is possible that the situation is different in the case of a *permanent* deficit, financed by indefinitely continuing issuance of bonds. In that case, as the deficit continues the outstanding bond stock continues to grow. So, accordingly, does the interest that must be paid each period on the outstanding stock. If, for example, the magnitude of the real deficit (net of interest payments) is kept at  $d$ , the real bond stock  $b_t$  will be required to grow according to

$$b_t = (1 + r)b_{t-1} + d(1 + r) \quad (1)$$

if the real rate of interest is constant at the value  $r$ . Thus the bond stock will in this case grow without bound; if  $d > 0$ ,  $b_t \rightarrow \infty$  as  $t \rightarrow \infty$ .

Barro (1976) has argued that under these circumstances the Ricardian equivalence argument breaks down. In particular, he suggests that the rate of growth of the bond stock cannot exceed the economy's rate of output growth—here temporarily taken to be zero—because “the value of the outstanding stock of debt at any point in time is bounded by the government's . . . present value of future taxing capacity” (1976, p. 343, n. 2).<sup>5</sup> In a similar vein, Sargent and Wallace (1981) have argued that “if the interest rate on bonds is greater than the economy's growth rate, the real stock of bonds will grow faster than the size of the economy. This cannot go on forever, since the demand for bonds places an upper limit on the stock of bonds relative to the size of the economy” (1981, p. 2).<sup>6</sup>

These arguments do not, however, seem entirely convincing. First, if the bond-issuance policy is permanently maintained, then taxes will never have to be collected to retire the debt so the difficulty concerning taxing capacity may not be decisive. And under the Ricardian view, government bonds are not regarded as net wealth to the private sector, so the size of the bond stock seems to be potentially irrelevant.<sup>7</sup> In any event, it would appear that a more formal analysis might be worthwhile. As it happens, that conjecture is verified in the following section: while the basic idea of the Barro and Sargent-Wallace conten-

<sup>5</sup> Taxing capacity in each period is assumed by Barro to be some exogenously given fraction of current output.

<sup>6</sup> This assumption, it should be noted, plays an important role in the Sargent-Wallace analysis.

<sup>7</sup> Irrelevance of continued bond growth is assumed, without any utility-maximizing justification, in McCallum (1978).

tions receives support from the theory, the precise statements concerning bond demand limitations do not.

### III. Analysis

Our first task is to specify a maximizing model that incorporates the crucial component of the Ricardian view, namely, infinite-lived agents who correctly take account of the government budget constraint.<sup>8</sup> The model must also be one that accommodates money, bonds, and a physical asset. To that end, let us adopt a discrete-time, perfect-foresight version of the well-known model of Sidrauski (1967), modified to include government bonds.<sup>9</sup> In order to keep matters as simple as possible, let us first consider a version with no depreciation or population growth.<sup>10</sup>

Formally, we imagine an economy composed of a large number of similar households, each of which seeks at period  $t$  to maximize

$$u(c_t, m_t) + \beta u(c_{t+1}, m_{t+1}) + \beta^2 u(c_{t+2}, m_{t+2}) + \dots \quad (2)$$

Here  $c_t$  is consumption in period  $t$  and  $m_t = M_t/P_t$ , with  $M_t$  the household's nominal money stock at the start of  $t$  and  $P_t$  the price of the (single) consumption good in  $t$ . The within-period utility function  $u$  is assumed to be "well-behaved"—see Sidrauski (1967, p. 535)—so unique, positive values are chosen for  $c_t$  and  $m_{t+1}$ ,  $t = 1, 2, \dots$ . The discount factor  $\beta$  equals  $1/(1 + \delta)$ , with the time-preference parameter  $\delta$  positive.

Each household has access to a production function that is homogeneous of degree one in its inputs, labor and capital. But since labor is supplied inelastically, this function can be written as  $f(k_t)$ , where  $k_t$  is the stock of capital held at the start of  $t$ . The function  $f$  is assumed to satisfy the conditions  $f' > 0$ ,  $f'' < 0$ ,  $f'(0) = \infty$ , and  $f'(\infty) = 0$ . Thus a unique, positive value will be chosen for  $k_t$  in each period. Capital is

<sup>8</sup> It should be pointed out that I am using the term "Ricardian" to refer primarily to these characteristics of the private economy. An entirely different use of the term occurs in Sargent (1982, p. 385), where it refers to a type of government policy regime—one that does not encompass policy variable paths such as those in our examples.

<sup>9</sup> The Sidrauski model is, of course, one in which bonds and money can easily coexist because real money balances appear as an argument of the household's within-period utility function. The rationale for this appearance—which has been severely criticized by Bryant and Wallace (1980) and others—is that transaction costs are reduced by money balances, so that more preferred bundles of consumption and leisure can be obtained. For a rather lengthy discussion of related issues, see McCallum (1983).

<sup>10</sup> A notable feature of the Sidrauski model is the invariance of the steady-state capital-labor ratio to expected inflation rates. This invariance (or superneutrality) does not survive minor modifications, such as making utility dependent upon leisure. It is my impression that the superneutrality property is not crucial for the issues under discussion here; the superneutral version of the model has been adopted for simplicity.

simply unconsumed output, so its price is the same as that of output and its real rate of return between  $t$  and  $t + 1$  is  $f'(k_{t+1})$ .

Each household has the opportunity in  $t$  of purchasing government bonds at a money price of  $Q_t$ . Each bond is redeemed in  $t + 1$  for one unit of money, so the nominal rate of return on bonds between  $t$  and  $t + 1$  is  $R_t = (1 - Q_t)/Q_t$ . The real rate  $r_t$  is then defined by  $1 + r_t = (1 + R_t)/(1 + \pi_t)$ , where  $\pi_t = (P_{t+1} - P_t)/P_t$  is the inflation rate. Finally, lump-sum transfers net of taxes in the amount  $v_t$  are distributed to (or, if negative, collected from) the household in period  $t$ . Consequently, the household's budget constraint for period  $t$  can be written as

$$f(k_t) + v_t = c_t + (1 + \pi_t)m_{t+1} - m_t + (1 + r_t)^{-1}b_{t+1} - b_t + k_{t+1} - k_t, \quad (3)$$

where  $b_t = B_t/P_t$ , with  $B_t \geq 0$  the number of bonds held at the start of  $t$ .

Given this setup, we can find the optimality conditions for the household's problem by considering the Lagrangian expression

$$L = \sum_{t=1}^{\infty} \beta^{t-1} \{ u(c_t, m_t) + \lambda_t [f(k_t) + v_t - c_t - (1 + \pi_t)m_{t+1} + m_t - (1 + r)^{-1}b_{t+1} + b_t - k_{t+1} + k_t] \}.$$

Because of our assumptions on  $u$  and  $f$ , which assure that  $c_t$ ,  $m_{t+1}$ , and  $k_{t+1}$  will be strictly positive, the first-order Euler conditions associated with those variables can be written as equalities holding for all  $t = 1, 2, \dots$ . They are

$$u_1(c_t, m_t) - \lambda_t = 0 \quad (4)$$

$$\beta u_2(c_{t+1}, m_{t+1}) - \lambda_t(1 + \pi_t) + \beta \lambda_{t+1} = 0 \quad (5)$$

$$-\lambda_t + \beta \lambda_{t+1} [f'(k_{t+1}) + 1] = 0. \quad (6)$$

The condition associated with  $b_{t+1}$  must, however, be written in two parts, as follows:

$$\frac{-\lambda_t}{(1 + r_t)} + \beta \lambda_{t+1} \leq 0 \quad (7a)$$

$$b_{t+1} \left[ \frac{-\lambda_t}{(1 + r_t)} + \beta \lambda_{t+1} \right] = 0. \quad (7b)$$

In addition to these first-order conditions, we also have the infinite-horizon transversality conditions

$$\lim_{t \rightarrow \infty} m_{t+1} \beta^{t-1} \lambda_t (1 + \pi_t) = 0 \quad (8)$$

$$\lim_{t \rightarrow \infty} k_{t+1} \beta^{t-1} \lambda_t = 0 \quad (9)$$

$$\lim_{t \rightarrow \infty} b_{t+1} \beta^{t-1} \frac{\lambda_t}{(1 + r_t)} = 0. \quad (10)$$

Conditions (3)–(7) are, of course, necessary for optimality while (3)–(10) are jointly sufficient.<sup>11</sup> Thus if (8)–(10) are satisfied, the household's choice of  $c_t$ ,  $k_{t+1}$ ,  $m_{t+1}$ ,  $b_{t+1}$ , and  $\lambda_t$  will be—given initial asset stocks and time paths of prices and transfers—described by (3)–(7).

Before continuing, let us pause to note that, because of our assumptions on  $u$ ,  $\lambda_t$  will be positive for all  $t$ . Also, (6) and (7) together imply that  $r_t = f'(k_{t+1})$  whenever  $b_{t+1} > 0$ ; the rates of return on bonds and capital are the same if any bonds are demanded. Furthermore, if it happens that  $\lambda_{t+1} = \lambda_t$ , as will be the case in a steady state, then each of these rates of return will equal  $\delta$  (provided that  $b_{t+1}$  is positive).

Next we consider the government's budget. Expressing all quantities in per capita terms and letting  $g_t$  denote government purchases of output, we have the identity

$$M_{t+1} - M_t + Q_t B_{t+1} - B_t = P_t(g_t + v_t) \quad (11)$$

or, in real terms,

$$(1 + \pi_t)m_{t+1} - m_t + (1 + r_t)^{-1}b_{t+1} - b_t = g_t + v_t. \quad (12)$$

The government's choices of time paths for  $M_t$ ,  $B_t$ ,  $g_t$ , and  $v_t$  must conform to (11) and (12). Note that together (3) and (12) imply the national income identity:

$$f(k_t) = c_t + k_{t+1} - k_t + g_t. \quad (13)$$

Given time paths for three of the policy variables, equilibrium values are determined by conditions (3)–(11) and the relevant identities. In particular, if the time paths for  $M_t$ ,  $g_t$ , and  $v_t$  are selected by the government, conditions (3), (4), (5), (6), (7), and (12) plus  $m_t = M_t/P_t$  and  $\pi_t = (P_{t+1} - P_t)/P_t$  will determine paths for  $c_t$ ,  $k_t$ ,  $b_t$ ,  $\pi_t$ ,  $r_t$ ,  $\lambda_t$ ,  $m_t$ , and  $P_t$ .

We now have enough apparatus to demonstrate that the Barro-Sargent-Wallace claim is correct, provided that the deficit is defined

<sup>11</sup> That eqq. (3)–(10) are jointly sufficient under our assumptions on  $u$  and  $f$  is well known from various papers including Weitzman (1973). As the applicability of published results to our problem is not immediately apparent, an outline of a proof is available from the author.



exclusive of interest payments on current debt. More specifically, we can show that the model at hand will not support a zero-inflation equilibrium in which a permanently maintained positive deficit of  $g_t + v_t = d$  is financed entirely by bond sales. To do so, we first observe that this monetarist hypothesis implies that  $(1 + \pi_t)m_{t+1} - m_t = 0$  and that steady-state conditions prevail for all variables except  $b_t$  so that  $r_t$  and  $\lambda_t$  are constant, with  $r = \delta > 0$  and  $\lambda > 0$ .<sup>12</sup> Then we insert the hypothesized constant values in (12), obtaining

$$b_{t+1} = (1 + r)b_t + (1 + r)d, \quad t = 1, 2, \dots \quad (14)$$

Next we note that the latter implies

$$b_{t+1} = (1 + r)b_1 + (1 + r)d[1 + (1 + r) + \dots + (1 + r)^{t-1}], \quad (15)$$

which in turn implies

$$b_{t+1}\beta^{t-1} \frac{\lambda_t}{(1 + r_t)} = \lambda b_1 + \frac{\lambda d[(1 + r) - (1 + r)^{1-t}]}{r}. \quad (16)$$

But it can then readily be seen that the expression in (16) approaches  $\lambda b_1 + [\lambda d(1 + r)/r]$  as  $t \rightarrow \infty$ , which violates condition (10). Equivalently, but in different words,  $b_{t+1}$  grows at the rate  $r$  while  $\beta^{t-1}$  decays at the rate  $\delta = r$ , so their product grows at the rate zero—that is, does not vanish as  $t$  increases. Thus one of the jointly sufficient conditions for household optimality is not satisfied by the proposed path. Now, since transversality conditions are not in all cases necessary for optimality,<sup>13</sup> we must also show that the proposed path is in fact inconsistent with optimality. To do so, we note from (12) that  $(1 + r)^{-1}b_{t-1} - b_t = d$  is positive in each period,  $t = 1, 2, \dots$ . Then inspection of (3) indicates that by setting  $b_{t+1} = 0$  the household could increase its consumption in each period,  $t = 1, 2, \dots$ , without changing its holdings of capital or real money balances. Thus the proposed monetarist path cannot be an equilibrium.

Our next object is to show that a constant, positive deficit can, however, be financed entirely by bond sales with no resulting inflation if “deficit” is defined—as it typically is—to include current interest payments. To make this argument, let us define the issue value of bonds outstanding at  $t$  by  $\bar{B}_t = B_t/(1 + R_{t-1})$ . Then with  $M_{t+1} - M_t = 0$ , the government budget identity becomes

$$\bar{B}_{t+1} - \bar{B}_t = P_t(g_t + v_t) + \bar{B}_t R_{t-1}. \quad (17)$$

<sup>12</sup> In this experiment (and those that follow) it is assumed that steady-state values for  $k$  and  $m$  prevail in period 1. Since the crucial aspects of the analysis involve limiting conditions, this simplification seems unobjectionable for the issues at hand.

<sup>13</sup> An example in which the transversality condition is not necessary for optimality was mentioned by Arrow and Kurz (1970, p. 46).

Now assume that policy keeps the real value of the right-hand side of (17) constant at  $\bar{d}$ , so that

$$\bar{B}_{t+1} - \bar{B}_t = \bar{d}P_t. \quad (18)$$

Next we conjecture that, with  $\bar{d}$ ,  $g$ , and  $M$  held constant, the price level will also be constant so that (18) becomes

$$b_{t+1} - b_t = \bar{d}(1 + r). \quad (19)$$

But in this case we have

$$b_{t+1} = b_1 + \bar{d}(1 + 1 + \dots + 1^{t-1})(1 + r) = b_1 + \bar{d}t(1 + r), \quad (20)$$

so that

$$b_{t+1}\beta^{t-1} \frac{\lambda_t}{(1 + r_t)} = \frac{\lambda b_1 + \lambda \bar{d}t(1 + r)}{(1 + r)^t}, \quad (21)$$

which does approach zero as  $t \rightarrow \infty$ .

To verify that a noninflationary steady state also satisfies the other relevant conditions, we argue (constructively) as follows. First, with constant  $m$  and  $\pi = 0$ ,  $v_t = (1 + r)^{-1}b_{t+1} - b_t - g$  by (12), so (3) reduces to (13). Thus, given values for  $\bar{d}$  and  $g$ , steady-state values of  $k$ ,  $m$ ,  $c$ , and  $\lambda$  are determined by (4), (5), (6), and (13). With  $b_t > 0$  for all  $t$ , then, equation (7a) holds as an equality and determines  $r > 0$ . Finally, conditions (8) and (9) are satisfied with constant values for  $m$ ,  $\lambda$ , and  $k$  since  $\beta^{t-1} \rightarrow 0$  as  $t \rightarrow \infty$ .

The policy rule in the last example is one that makes  $b_t$  grow at a rate that decreases over time, approaching zero in the limit. Let us next consider an example in which  $b_t$  grows at a constant, positive rate that is numerically smaller than  $\delta$ . For simplicity, let us suppose that the rate is  $\delta/2$ . From our previous discussions it is clear that in this case condition (10) is not violated. Furthermore, it can be readily verified that conditions (3)–(9) are all satisfied by constant values of  $c$ ,  $k$ ,  $m$ ,  $r$ ,  $\lambda$ , and  $\pi = 0$ . The behavior of  $v_t$  in this case satisfies

$$\begin{aligned} g + v_t &= (1 + r)^{-1} \left( 1 + \frac{\delta}{2} \right) b_t - b_t \\ &= \left[ \left( 1 + \frac{\delta}{2} \right) (1 + \delta)^{-1} - 1 \right] b_t = - \left( \frac{\delta}{2} \right) (1 + \delta)^{-1} b_t, \end{aligned} \quad (22)$$

so  $d_t = g + v_t$  is negative and decays at the rate  $\delta/2$ . The alternative (conventional) deficit measure  $\bar{d}_t = d_t + \delta b_t$  is positive, however, and grows at the rate  $\delta/2$ . Thus we see that the conventionally defined deficit can—in the Ricardian/monetarist Sidrauski model—grow forever without causing inflation. Furthermore, we see that the real

stock of bonds can grow forever at a rate exceeding zero—which is, in this instance, the growth rate of the economy.

From these three examples, then, we obtain the following conclusions regarding bond finance of deficits in the basic, zero-growth version of our Ricardian/monetarist economy: (i) a permanent deficit cannot be financed solely with bonds if the deficit is defined exclusive of interest payments; (ii) a permanent deficit can be financed solely with bonds, and without inflation, if the deficit is defined inclusive of interest payments; and (iii) a positive growth rate of bonds outstanding can be maintained permanently, but only if the growth rate is smaller than the rate of time preference.

#### IV. Extensions and Qualifications

The purpose of this section is to describe both extensions and qualifications to the three basic conclusions above. We begin by developing extensions involving population growth and depreciation and then turn to some considerations not reflected in the formal model—default incentives, for example—that have the effect of limiting the applicability of the basic conclusions.

First, let us suppose that—as in Sidrauski (1967)—the size of each household grows at the rate  $n$  and that the utility function remains as in (2) with  $c_t$  and  $m_t$  now measuring per capita values. Then, with all other quantities also expressed in per capita terms, the household budget constraint becomes

$$f(k_t) + v_t = c_t + (1 + n)(1 + \pi_t)m_{t+1} - m_t + (1 + n)(1 + r_t)^{-1}b_{t+1} - b_t + (1 + n)k_{t+1} - k_t. \quad (3')$$

One relevant effect is that the counterpart of (7) now implies that  $(1 + n)(1 + r_t)^{-1} = (1 + \delta)^{-1}$  if  $b_t > 0$  and  $\lambda_t = \lambda_{t+1}$ . Another is that condition (10) is replaced with

$$\lim_{t \rightarrow \infty} b_{t+1} \beta^{t-1} \frac{\lambda_t(1 + n)}{(1 + r_t)} = 0. \quad (10')$$

In addition, the government budget identity becomes, in per capita terms,

$$(1 + n)(1 + \pi_t)m_{t+1} - m_t + (1 + n)(1 + r_t)^{-1}b_{t+1} - b_t = g_t + v_t. \quad (12')$$

Consequently, if the per capita magnitudes  $g_t$ ,  $v_t$ , and  $M_t$  are held constant over time—so that the aggregate deficit and money stock

each grow at the rate  $n$  in every period—the following equation will govern the behavior of  $b_t$  under the monetarist hypothesis:<sup>14</sup>

$$b_{t+1} = (1 + r)(1 + n)^{-1}b_t + (d - nm)(1 + r)(1 + n)^{-1}. \quad (14')$$

Thus the per capita bond stock grows at the rate  $(1 + r)(1 + n)^{-1} - 1$ . But since  $1 + r$  equals  $(1 + n)(1 + \delta)$  in the hypothesized steady state,  $b_{t+1}$  then grows at the rate  $\delta$ , which equals the rate at which  $\beta^{t-1}$  contracts. So the product  $b_{t+1}\beta^{t-1}$  grows at the rate zero. This violates (10') as in our first example with  $n = 0$ .<sup>15</sup> But it *just* violates (10'), so it is clear that the second and third examples, in which the per capita bond stock grows at a diminishing rate that approaches zero and at a constant rate less than  $\delta$ , respectively, will not violate (10'). Thus the conclusions above hold in per capita terms when the economy experiences population growth of the specified type.

Furthermore, allowing proportional depreciation of capital would leave the crucial relationship between  $\delta$  and the growth rate of bonds unchanged. The marginal product of capital would exceed  $r_t$  by the amount of the depreciation rate, but the steady-state condition  $1 + r = (1 + \delta)(1 + n)$  would continue to hold and it is the relationship between  $r$  and  $\delta$  that governs the relative growth rates of  $b_{t+1}$  and  $\beta^{t-1}$ .<sup>16</sup>

Now we turn to the qualifications. Given the rather surprising nature of conclusions ii and iii, it is important to recognize that some potentially significant aspects of actual economies are not reflected in the formal model. One of these involves the notion of "taxing capacity," mentioned above. The suggestion put forth by Barro (1976), namely, that government debt cannot permanently grow at a rate in excess of output growth, is based on the presumption that tax collections cannot forever grow faster than output. The feature of our analysis that enables the third example to contradict this apparently reasonable presumption is the recognition that households' disposable income includes interest payments from the government as well as income from production. Thus taxes can exceed output for each household and yet be smaller in magnitude than the household's disposable income. A reformulated version of Barro's suggestion, one stating that government debt cannot forever grow at a rate in excess of disposable income, would not be contradicted by any of our examples.

<sup>14</sup> Under that hypothesis, the inflation rate will be zero.

<sup>15</sup> Again we avoid reliance on this transversality condition, for which necessity has not been established. Instead we can show that the proposed path is nonoptimal by inferring from (3') and (14') that a zero value for  $b_{t+1}$  in each  $t = 1, 2, \dots$  would permit increases in  $c_t$  with the same paths for  $m_{t+1}$  and  $k_{t+1}$ .

<sup>16</sup> Incomplete investigations with technical progress lead to the conjecture that the results remain valid in cases in which steady-state growth is possible.

Next, the fact that both taxes and debt grow without bound in the second and third examples suggests that consideration should be given to the existence of incentives to default. In particular, there would seem to be at work an incentive on the part of the government to default on its outstanding debt. The strength of this incentive, moreover, would seem to be governed by the size of the debt in absolute (per capita) terms or in relation to output, rather than in relation to disposable income, for the latter is of little economic significance in the circumstances of the second and third examples. Consequently, our conclusions must be understood as ones that apply *conditionally* upon the permanent maintenance of the specified government policies. While there is nothing unusual about a strategy of studying the effects of specified policies, actual governments might be hard pressed to maintain the particular policies involved in our examples.

In addition, there is at work a sort of default incentive from the standpoint of individual households—namely, an incentive to evade taxes. Of course such an incentive always exists and is frequently ignored in the analysis of the inflationary effects of fiscal and monetary policies. It could be argued, however, that evasion incentives are more pertinent in the context of conclusions ii and iii than in most analyses, again because these incentives would presumably be governed by absolute (per capita) tax magnitudes.<sup>17</sup> In any event, it is clear that the existence of an upper bound on the absolute magnitude of per capita taxes—whether the bound is generated by default incentive or other reasons—would eliminate the possibility of equilibria like those in the examples yielding conclusions ii and iii.

## V. Conclusion

Our results can be summarized very briefly. The formal analysis is designed to investigate the theoretical validity of a “monetarist hypothesis” asserting that a constant, positive, per capita budget deficit can be maintained permanently and without inflation if it is financed by the issue of bonds rather than money. It is shown that, in a perfect-foresight version of the competitive equilibrium model of Sidrauski (1967), the hypothesis is invalid if the deficit is defined exclusive of interest payment but is valid under the conventional definition. It is also shown that the stock of willingly held government bonds can

<sup>17</sup> Robert Barro has mentioned to me that the model pertains to a closed economy. If debt and taxes grew drastically in an open economy, there would be an incentive for individuals to sell their bonds and emigrate. Analysis of such a possibility would require a model with several open economies and a specification of the policies—indeed, game-theoretic strategies—of the various governments.

grow indefinitely at a rate in excess of the rate of output growth, provided that the difference is smaller than the rate of time preference.

At the informal level, it is recognized that the results involving unbounded growth of bonds and taxes suggest that default incentives would also grow indefinitely. Consequently, from a practical point of view the results should not be interpreted as a claim that unbounded debt growth is possible in actual economies, but rather as a means of scrutinizing and identifying reasons why such growth is probably not feasible in actuality.

## References

- Arrow, Kenneth J., and Kurz, Mordecai. *Public Investment, the Rate of Return, and Optimal Fiscal Policy*. Baltimore: Johns Hopkins Press (for Resources for the Future), 1970.
- Barro, Robert J. "Are Government Bonds Net Wealth?" *J.P.E.* 82 (November/December 1974): 1095–1117.
- . "Reply to Feldstein and Buchanan." *J.P.E.* 84 (April 1976): 343–49.
- . "The Public Debt." In *Macroeconomic Analysis*. New York: Wiley, 1983.
- Brunner, Karl, and Meltzer, Allan H. "An Aggregative Theory for a Closed Economy." In *Monetarism*, edited by Jerome L. Stein. Amsterdam: North-Holland, 1976.
- Bryant, John, and Wallace, Neil. "A Suggestion for Further Simplifying the Theory of Money." Research Department Staff Report 62, Federal Reserve Bank of Minneapolis, December 1980.
- Council of Economic Advisers. *Economic Report of the President*. Washington: Government Printing Office, 1982.
- Drazen, Allan. "Government Debt, Human Capital, and Bequests in a Life-Cycle Model." *J.P.E.* 86 (June 1978): 505–16.
- Gordon, Robert J., ed. *Milton Friedman's Monetary Framework: A Debate with His Critics*. Chicago: Univ. Chicago Press, 1974.
- McCallum, Bennett T. "On Macroeconomic Instability from a Monetarist Policy Rule." *Econ. Letters* 1, no. 2 (1978): 121–24.
- . "The Role of Overlapping-Generations Models in Monetary Economics." *Carnegie-Rochester Conference Series on Public Policy* 18 (Spring 1983): 9–44.
- Sargent, Thomas J. "Beyond Demand and Supply Curves in Macroeconomics." *A.E.R. Papers and Proc.* 72 (May 1982): 382–89.
- Sargent, Thomas J., and Wallace, Neil. "Some Unpleasant Monetarist Arithmetic." *Federal Reserve Bank of Minneapolis Q. Rev.* 5 (Fall 1981): 1–17.
- . "The Real-Bills Doctrine versus the Quantity Theory: A Reconsideration." *J.P.E.* 90 (December 1982): 1212–36.
- Sidrauski, Miguel. "Rational Choice and Patterns of Growth in a Monetary Economy." *A.E.R. Papers and Proc.* 57 (May 1967): 534–44.
- Stein, Jerome L., ed. *Monetarism*. Amsterdam: North-Holland, 1976.
- Weitzman, Martin L. "Duality Theory for Infinite Horizon Convex Models." *Management Sci.* 19 (March 1973): 783–89.