On the Limitations of Government Borrowing: A Framework for Empirical Testing

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How long can government budget deficits continue unchecked? This question raises two separate issues. First, are perpetual deficits desirable—are the effects on inflation, investment, and the balance of payments ones that we can live with? Second, are perpetual deficits feasible—even if the government wanted to run a budget deficit forever, is this something it really could do?

If we were talking about the budget plans of a private household, clearly the question of feasibility would be paramount, for we entertain little doubt that households would like to run a permanent deficit if they could get away with it, continually rolling over debt without having to pay anything back and enjoying the associated free lunch. We presume that households do not generally engage in such behavior primarily for the reason of feasibility—no one would be willing to continue lending money to such a household. For this reason, we usually specify that households are subject to the borrowing constraint that the expected present value of expenditures (exclusive of interest payments) not exceed the expected present value of receipts.

The question we pose in this paper is whether governments are subject to an analogous constraint—when a government runs a deficit, is it making an implicit promise to creditors that it will run offsetting surpluses in the future? If governments are subject to this constraint, which we will term the present-value borrowing constraint, the policy of running a permanent deficit (exclusive of interest payments) is infeasible; (though, as we shall see below, a permanent deficit when interest payments are counted as part of the deficit may still be feasible). The question of feasibility of a permanent deficit (exclusive of interest payments) holds profound implications for macroeconomic theory and practice. If governments intend to raise the needed revenues with future tax increases, then government deficits may have no stimulative effect on aggregate demand,[1] but can have significant distortionary effects on private incentives if the future tax increases are large (see Robert Barro, 1984b). On the other hand, if the revenues are to be raised implicitly through money creation, budget deficits can be a principal cause of inflation, as suggested by Thomas Sargent (1982) and Sargent and Neil Wallace (1981).

Whether governments can continually run a budget deficit remains an unsettled theoretical question. If the government borrows at an interest rate that equals or exceeds the economy's growth rate, then a continuing unpaid deficit implies that the debt must

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[1] If government spending (G_t), taxes (T_t) and debt (B_t) are related by

$$B_t + \sum_{j=1}^{\infty} (1+r)^{-j} G_{t+j} = \sum_{j=1}^{\infty} (1+r)^{-j} T_{t+j}$$

then any proposal that changes taxes but leaves spending the same must be such that the right-hand side of the above expression remains constant. Thus, permanent income defined by

$$\sum_{j=1}^{\infty} (1+r)^{-j} \left(W_{t+j} L_{t+j} - T_{t+j} \right)$$

would be completely unaffected by any such tax policy. See Robert Barro (1974, 1984a) and Olivier Blanchard (1985) for further discussion.

grow to become an infinite multiple of GNP.[2] Equilibrium models in which investors would continue to buy government debt under such circumstances have proven difficult to develop; see Bennett McCallum (1984) for a clear discussion of the issues. If the real interest rate is less than the growth rate, by contrast, deficits could continue forever without an increase in the ratio of debt to GNP. Theoretical models that seem to allow this possibility have been explored by Willem Buiter (1979), Jonathan Eaton (1981), and Jeffrey Carmichael (1982), though these results have not gone unchallenged (see, for example, John Burbidge, 1983).

In any case, it seems desirable to supplement these theoretical considerations with empirical evidence. David Aschauer (1985) and John Seater and Roberto Mariano (1985), among others, have tested the hypothesis that the government's receipts must equal its expenditures in present-value terms jointly with a permanent income hypothesis, and accepted. Paul Evans (1985) documented the absence of statistical correlation between U.S. budget deficits and interest rates, which he interpreted as evidence in support of this same joint hypothesis. Barro (1984b) tested the hypothesis that the government is subject to the present-value borrowing constraint jointly with the assumption that taxation and deficit policies have historically been optimal, and again accepted. However, to our knowledge there has been no direct empirical test of the present-value borrowing constraint itself.

At first glance, a test of the present-value borrowing constraint might seem straightforward enough. The U.S. government, for example, has run more or less a chronic deficit since 1930, suggesting that permanent deficits are quite feasible and practical. However, this simple argument ignores the potential role of debt retirement through monetization and likewise ignores capital gains on bonds or tangible assets through inflation. Moreover, the official government deficit includes interest payments, whereas we will argue below that the correct magnitude for purposes of testing the present-value borrowing constraint should exclude such payments. Finally, a formal test of whether historical deficits were rationally anticipated and allowance for what might rationally be expected to happen out of sample seems necessary to evaluate this hypothesis adequately.

In this paper we propose an empirical framework for testing the practical limits to public borrowing which addresses these criticisms. We show that the proposition that the government can accumulate ever-growing debt through perpetual deficit financing has a mathematical parallel in the proposition that prices can rise continually in a self-fulfilling speculative bubble. Thus, empirical tests that have been developed for the latter hypothesis may also be fruitfully applied to study the limits of government borrowing.

The conclusions we draw from these tests complement those of Barro (1984a) and Robert Eisner and Paul Pieper (1984), who have noted that while the official budget has registered a chronic deficit, the real value of government debt fell substantially in the postwar period, suggesting that the official accounts have grossly misstated the true fiscal posture of the government. Once an economically reasonable definition of the government budget deficit is adopted, the data seem fully compatible with the assertion that the government budget historically has been balanced in expected present-value terms. Our tests show this conclusion to be reasonably robust with respect to specification of the information on which creditors were basing their forecasts of future surpluses.

[2]Again we are referring to the government deficit exclusive of interest payments. To take a simple example, let output $Q(t)$ grow at the rate $q\,(Q(t)=Q_0e^{qt})$, and let spending $gQ(t)$ and taxes $xQ(t)$ be fixed multiples of GNP. If r is the cost of borrowing and $B(t)$ is the debt, then debt accumulates according to

$$dB/dt=(g-x)Q(t)+rB(t)$$

implying $\quad B(t)=\{B_0+tQ_0(g-x)\}e^{qt} \quad \text{if } q=r$

$$=\{(g-x)/(q-r)\}Q_0(e^{qt}-e^{rt})+B_0e^{rt} \quad \text{otherwise,}$$

which explodes relative to Q_0e^{qt} whenever $g>x$ and $q\leq r$.

Past deficits have been followed by increases in revenues which covered the government's interest obligations, though the implicit "inflation tax" has historically made an important contribution.

Section I provides a formal statement of the present-value borrowing constraint to be tested. Section II briefly discusses some issues of data and measurement. In Section III we present the results of alternative empirical tests of whether the postwar record of budget deficits in the United States could be consistent with the present-value borrowing constraint under rational expectations, with brief conclusions offered in Section IV.

I. The Present-Value Government Borrowing Constraint

Suppose we collected all government debt of a given coupon and maturity into group j. Let $d_{j,t}$ denote the nominal market value of such debt at the end of period t and $\theta_{j,t}$ the total nominal coupon payments between dates $t-1$ and t. We further let P_t denote an aggregate price index of goods in the economy and r the *ex post* real interest rate that is earned on one-period government bonds during an average year.

Suppose no new bonds of type j are issued or redeemed during period t. Then changes in the market value of group j debt can be evaluated using a simple term-structure argument. Define $v_{j,t}$ to be the real excess one-period holding yield of j bonds relative to the average earned on a comparable investment in one-period bonds:

$$(1) \qquad v_{j,t} \equiv \frac{d_{j,t}+\theta_{j,t}}{P_t} - \frac{(1+r)d_{j,t-1}}{P_{t-1}}.$$

If real interest rates were white noise (or constant) and if the expectations theory of the term structure held, then $E_{t-1}v_{j,t}$ would equal zero. In general, positive values of $v_{j,t}$ mean bondholders have made a capital gain on long-term government debt, or that short-term rates are higher than average.

The market value of outstanding government debt will also change due to operations of the Treasury and Federal Reserve. Let T_t denote real tax revenues, G_t real government purchases of goods (excluding interest payments on the debt), and R_t the dollar flow of nominal interest payments made to the public divided by the price level. Thus, the official deficit in constant dollars is measured by

$$(2) \qquad G_t + R_t - T_t,$$

and new debt must be issued in this amount during the year. Changes in the stock of high-powered money (M_t) likewise retire an amount of debt whose market value is[3]

$$(3) \qquad (M_t - M_{t-1})/P_t.$$

Finally, let B_t denote the real market value of debt held by the public ($B_t = \sum_j d_{j,t}/P_t$). From (1)–(3), its value is given by

$$(4) \qquad B_t = (1+r)B_{t-1} - \sum_j \theta_{j,t}/P_t + \sum_j v_{j,t} + G_t + R_t - T_t - (M_t - M_{t-1})/P_t + U_{1,t}.$$

In principle, equation (4) might be thought of as an accounting identity. In practice, we must append the error term $U_{1,t}$ since bond purchases or sales towards the beginning of the period would have taken place at market prices closer to those characterizing B_{t-1} rather than B_t, and since measurements of the real market value of net government indebtedness are necessarily imperfect (see Section II below). Again abstracting from this issue of intraperiod timing, note further that $\sum_j \theta_{j,t}/P_t + U_{2,t} = R_t$. Thus

$$(5) \qquad B_t = (1+r)B_{t-1} - S_t + V_t,$$

where

$$(6) \qquad S_t \equiv T_t + (M_t - M_{t-1})/P_t - G_t$$

$$V_t \equiv \sum_j v_{j,t} + U_{1,t} + U_{2,t}.$$

[3]We are formally modelling assets held by the government as negative values of $d_{j,t}$, so that foreign exchange transactions by the Federal Reserve are also included in (3). Details of the variables used in the empirical analysis are provided in Section II.

The key points to note about (5) are: (a) it describes the market rather than the par value of government debt, since open market operations by the Federal Reserve retire debt at market value; (b) the measure of the surplus S_t *excludes* interest payments from government spending; and (c) money seigniorage $(M_t - M_{t-1})/P_t$ is added to taxes T_t as a source of revenue for retiring outstanding government debt.

By recursive substitution forwards, equation (5) is seen to imply

$$(7)\quad B_t = \sum_{i=t+1}^{N} \frac{(S_i - V_i)}{(1+r)^{i-t}} + \frac{(1+r)^t B_N}{(1+r)^N}.$$

Equation (5) and its implication (7) cannot be a point of serious controversy, for they do little more than summarize the definitions of monetary and fiscal policy. What is of economic interest (and subject in principle to empirical refutation) is what creditors expect to happen to the second term in (7) as N gets large. Indeed, letting E_t denote the expectations of creditors based on information available at date t, it is clear from (7) that the hypothesis that the government is subject to the present-value borrowing constraint,

$$(8a)\qquad H_0\colon\ B_t = E_t \sum_{i=t+1}^{\infty} \frac{(S_i - V_i)}{(1+r)^{i-t}},$$

is mathematically equivalent to the restriction that the real supply of bonds held by the public is expected to grow no faster on average than the rate of interest:

$$(8b)\qquad H_0\colon\ E_t \lim_{N\to\infty} \frac{B_N}{(1+r)^N} = 0.$$

Using the equivalence between (8a) and (8b), we are now in a position to comment in more detail on the precise nature of the present-value borrowing constraint. Note first that condition (8) can be consistent with a permanent government deficit as conventionally measured, that is, inclusive of interest rates. If a constant (interest inclusive) deficit of $-S_t + V_t + rB_{t-1} = k$ were maintained forever, for example, we see from (5) that $B_N = Nk + B_0$ and $\lim_{N\to\infty} B_N/(1+r)^N = 0$. Thus a policy of keeping the interest component of the deficit from rising will ultimately force the government to pay off its debts in present-value terms. On the other hand, a permanent deficit exclusive of interest payments $-S_t + V_t = k$ is not consistent with (8), for then $B_N = k[(1+r)^N - 1]/r + (1+r)^N B_0$ and $\lim_{N\to\infty} B_N/(1+r)^N = k/r + B_0$. Recall McCallum's demonstration that a permanent deficit inclusive of interest payments could be consistent with optimizing behavior by bondholders (and would satisfy our condition (8), whereas a permanent deficit exclusive of interest payments is not consistent with optimizing behavior in his model (and would violate (8)). Finally, we note that condition (8) does not imply that the national debt must eventually be paid off. In fact, (8) is consistent with a constantly increasing stock of debt, as long as the rate of increase is less than the government's borrowing rate. Rather, the question we pose is whether interest on this debt is to be paid with future tax increases or instead with continual issue of new debt.

If (8) represents the null hypothesis that the government budget must be intertemporally balanced, how may we usefully frame the alternative possibility that government deficits (i.e., negative values of S_t) need not be balanced with future surpluses? One interesting class of alternative hypotheses is obtained by assuming that

$$E_t \lim_{N\to\infty} \left[B_N/(1+r)^N \right] = A_0 > 0.$$

Thus, we allow the possibility that a certain annual amount of real government expenditures $r(A_0 - B_0)$ need never be paid for with taxes. From equation (7) we then obtain

$$(9)\quad B_t = E_t \sum_{i=t+1}^{\infty} \frac{(S_i - V_i)}{(1+r)^{i-t}} + A_0(1+r)^t$$

as a general class of solutions to (5). The hypothesis H_0 that the government budget must be balanced in present-value

TABLE 1—ILLUSTRATIVE SUMMARY OF ADJUSTMENTS TO OFFICIALLY REPORTED SURPLUS FOR FISCAL YEAR 1974 AND FOR DEBT AT END OF FISCAL YEAR 1974
(Millions of current dollars)

Surplus:	
Officially Reported Surplus	−3,460
Plus: interest	+28,072
Minus: interest paid to government trust funds	−6,583
Minus: deposit of Fed earnings	−4,845
Plus: change in agency securities	+903
Equals: Measure of surplus in which interest payments are excluded	+14,087
Plus: net capital gains on gold stock	+3,250
Plus: money seigniorage	+11,331
Equals: true surplus (current dollars)	+28,668
Divided by: consumer price index	÷1.469
Equals: true surplus (1967 dollars)	+19,515
Debt:	
Officially Reported Debt	474,235
Minus: investments in government accounts	−140,194
Equals: par value of Treasury debt	334,041
Multiplied by: market-par ratio	0.951
Equals: market value of Treasury debt	317,740
Minus: currency in circulation	−73,833
Minus: member bank reserves	−30,086
Equals: market value of net interest-bearing debt held by public	213,821
Minus: Treasury operating balance	−9,159
Minus: market value of gold holdings	−39,951
Equals: adjusted government debt (current dollars)	164,711
Divided by: consumer price index	÷1.469
Equals: adjusted government debt (1967 dollars)	112,124

terms holds true if and only if $A_0 = 0$ in equation (9).

Equation (9) is mathematically equivalent to the models of self-fulfilling fads or speculative bubbles first explored by Robert Flood and Peter Garber (1980). We accordingly propose that such tests might also be fruitfully applied to understanding the limits of government borrowing. The next section discusses the data on which such tests might be based, while Section III summarizes our results.

II. Issues of Data and Measurement

This section, inspired in part by Barro (1984a) and Eisner and Pieper, briefly discusses how the theoretical magnitudes appearing in our equation (9) are related to the budget figures actually reported in the United States. Complete details are provided in a data appendix available from the authors on request.

A. *Interest Payments*

The theoretical measure of government spending (G_t) in the above derivation excludes outlays for interest payments. Correcting the officially reported surplus ($\hat{T}_t - \hat{G}_t$) to exclude interest payments from G_t requires three steps. 1) Add to ($\hat{T}_t - \hat{G}_t$) total interest paid by the government. 2) For data prior to the accounting change in 1982, avoid double counting by subtracting back out that part of this Treasury interest that was paid directly into government trust funds such as Social Security. 3) Likewise subtract the deposit of Federal Reserve earnings which are already included in $\hat{T}_t$ under miscellaneous receipts. Sample calculations for corrections to the deficit and debt are provided in Table 1; the actual series used in our empirical work are reported in Table 2.

Some debt is issued by various federal agencies in addition to that issued by the Treasury. Outlays for the service of such

TABLE 2—ADJUSTED VALUES FOR SURPLUS AND DEBT
(Millions of 1967 dollars)

Fiscal Year	Adjusted Surplus for Fiscal Year	Adjusted Debt for End of Fiscal Year
1960	+8,533	167,954
1961	+921	170,826
1962	+2,117	176,187
1963	+3,697	177,944
1964	+3,968	176,036
1965	+9,604	173,319
1966	+11,514	160,103
1967	+7,997	157,441
1968	−3,262	162,227
1969	+14,142	144,302
1970	+6,529	136,877
1971	−3,631	145,977
1972	+1,812	151,836
1973	+13,451	136,272
1974	+19,515	112,124
1975	−13,152	134,673
1976[a]	−35,480	185,230
1977	−4,162	191,707
1978	+3,022	183,956
1979	+28,211	148,641
1980	+20,506	121,432
1981	−27,747	145,895
1982	−22,147	209,445
1983	−38,336	265,164
1984	−28,675	303,205

[a]Data for fiscal year 1976 include the transition quarter (July 1, 1976 through September 30, 1976).

debt are presently included as expenditures of that agency, and should be subtracted from $\hat{G}_t$ to arrive at our measure of G_t. Unfortunately, such data are not readily available. Since agency debt is small (for example, \$12 B in 1974 compared with \$475 B in Treasury debt), little harm can come from *assuming* that the present-value relation holds for agency debt, in which case we could approximate the present value of later agency interest payments by the current market value of new agency debt issue. Thus, we subtract new issue of agency securities from $\hat{G}_t$, and do not count the market value of agency securities in our measure of public holdings of government debt, B_t.[4]

B. *Trust Funds*

Some might argue that trust fund holdings are to be used against the government's liabilities implied by future Social Security benefit payments. It seems to us that this is an inaccurate interpretation. Such programs are not a current liability in the sense that they can be associated with any concrete number. Rather, they represent the outcome of an uncertain political process, and the correct way to represent this "liability" is by the discounted cash flow of an entry on current account rather than any dubious imputation to capital account. For this reason, we follow the official accounts in registering net Social Security inflows or outflows on the deficit account, but differ from the official accounts on the debt account. The correct measure subtracts that money which is owed from one branch of government to another.

C. *Off-Budget Items*

Starting in 1971, the activities of certain agencies such as the Postal Service Fund and Rural Electrification and Telephone Revolving Fund are characterized as "off-budget." Any deficits of such operations require the issue of Treasury bonds, though they do not count in the officially measured deficits of the U.S. government.

Suppose that these funds were indeed used primarily to issue market-interest loans to the private sector. Imagine the agency issuing a \$1 loan financed through a \$1 sale of Treasury bonds, and so running an off-budget \$1 deficit for that year. In the following year, the agency receives \$$r$ as interest payments from the private sector, but the Treasury pays \$$r$ back to the public as interest on the T bond. For this year, the official budget and off-budget items would accordingly sum to a zero net deficit. Thus, the

[4]To take a simple example, suppose that in year 1 spending exceeds taxes by \$1, with the shortfall made up by \$1 issue of new agency debt. In all subsequent years, interest spending is \$$r$ and taxes are correspondingly higher by \$$r$. In the accounts as actually reported, this policy would be associated with a \$1 deficit in year 1 and no surplus in subsequent years. In our proposed measure, by contrast, the deficit is zero in all years; i.e., agency debt has no effect on the present-value calculation for Treasury debt, as it should not in this case.

present-value budget across the two years would register a deficit if one combined the official budget and off-budget items into a single account, whereas the operation itself is clearly fiscally neutral—the government issued a \$1 Treasury bond but acquired a \$1 private bond, and has simply swapped like assets with the public. A correct measure would be obtained in this case if we adopt our convention of excluding Treasury interest payments from G_t and simply ignore the off-budget surplus or deficit altogether.

Of course, these programs are not pure market loans but in fact have a substantial subsidy aspect. Our only justification for ignoring this is that it is difficult to quantify and presumably small relative to the complete budget.

D. *Net Government Indebtedness*

Eisner and Pieper have begun the difficult task of quantifying the market value of various tangible assets owned by the government. The question for purposes of the present study is, do government bondholders believe that future interest payments will really be met through sale of such assets, rather than by more conventional means such as tax revenues or monetization? Our own view is that, for the vast majority of these assets, the promise of substantial liquidation of government tangible assets is not a politically credible backing for U.S. Treasury debt.

One important exception is the government's gold holdings. There is abundant historical evidence that governments willingly draw down or deplete these stocks in the wake of fiscal crises; this indeed is presumably the primary purpose of holding such stocks in the first place. Let Au_t denote the government's gold holdings in ounces, P_t^{Au} the price per ounce of gold, and D_t the nominal value of debt, all measured at the beginning of period t. Let P_tS_t denote the correctly measured surplus during period t (excluding interest payments from spending but making no correction for gold flows), and let i_t be the one-period interest rate, which for simplicity we assume is the same for all bonds. If at the end of period t the government sells off some amount of gold $(Au_t - Au_{t+1} > 0)$ at price P_{t+1}^{Au} and uses the proceeds to retire debt, then next period's debt will be given by

$$D_{t+1} = (1+i_t)D_t - P_tS_t - (Au_t - Au_{t+1})P_{t+1}^{Au}$$

which can be written as

$$\left(D_{t+1} - P_{t+1}^{Au}Au_{t+1}\right) = (1+i_t)\left(D_t - P_t^{Au}Au_t\right) + \left(i_t - \pi_t^{Au}\right)P_t^{Au}Au_t - P_tS_t$$

where $\pi_t^{Au} \equiv (P_{t+1}^{Au} - P_t^{Au})/P_t^{Au}$. Thus, if we define true government indebtedness as the stock of Treasury debt held by the public less the current market value of the government's gold holdings (i.e., as $D_t - P_t^{Au}Au_t$), the equation governing the evolution of true government indebtedness is obtained by subtracting $(i_t - \pi_t^{Au})P_t^{Au}Au_t$ from the officially measured surplus $(\hat{T}_t - \hat{G}_t)$.

For fiscal year 1974, gold prices increased by 17.16 percent, whereas the nominal interest rate on one-year government bonds was 7.56 percent. Based on a market value of the government's gold holdings of \$33.8 B, a sum of $(.1716 - .0756)(33.8) = \$3.2$ B should be added to the surplus for fiscal year 1974 to represent capital gains from gold.

We also need to subtract liquid assets held by the Federal Reserve from our measure of the government debt (see fn. 3). Since these are all carried on the books at par value, the simplest way to construct the correct measure of the market value of these assets is from the liabilities side, namely, by subtracting high-powered money from outstanding government debt to get a measure of net interest-bearing debt held by the public. The Treasury operating cash balance must also be subtracted to arrive at the figure B_t appearing in equation (5).

Painstaking calculations of the true market value of government debt based on actual market quotations of outstanding securities have been updated by W. Michael Cox (1985). At the end of fiscal year 1974, outstanding government debt was trading at a market value of only 95 percent of its par value.

E. *Money Seigniorage*

When the Federal Reserve acquires a Treasury bond, future interest payments on that bond accrue to the Fed and are counted as part of tax revenues under "Miscellaneous Receipts" in the official budget. Since the present value of this tax benefit is equal to the Fed's initial cost of the bond, seigniorage is in this sense theoretically already included in the budget. In practice, however, any finite-sample estimate of the discounted value of these miscellaneous receipts must be less than the discounted value of $M_{t+i} - M_{t+i-1}$, because open market purchases towards the end of the sample period have not yet been amortized. For this reason, we exclude deposits of Fed interest from our measure of T_t and include the seigniorage measure $(M_t - M_{t-1})$, where M_t denotes high-powered money (the sum of currency in circulation plus reserves of member banks).

III. Empirical Tests

We are interested in the question of whether the U.S. government's creditors could rationally expect that the government budget would be balanced in present-value terms. In the light of the discussion of Section I, we state this hypothesis as the restriction $A_0 = 0$ in the formulation

$$B_t = A_0(1+r)^t + E_t \sum_{j=1}^{\infty} (1+r)^{-j} S_{t+j} + n_t \tag{10}$$

where B_t and S_t are the adjusted debt and surplus series reported in Table 2 and n_t is a regression disturbance term reflecting expected changes in real short-term interest rates, the term structure of long rates, and measurement error. The operator E_t denotes the expectations of creditors, which we assume are formed rationally.

Equation (10) is mathematically equivalent to the model proposed by Flood and Garber for studying self-fulfilling hyperinflations. However, Hamilton and Charles Whiteman (1985) expanded on the caveat stated by Flood and Garber that their technique implicitly imposes strong restrictions on the variables used by agents in forming expectations E_t and on the dynamics allowed for n_t. Behzad Diba and Herschel Grossman (1984) and Hamilton and Whiteman suggested that a more general test should first be considered which is more robust with respect to such restrictions. In particular, for *any* stationary process for $(n_t, E_t\sum_{j=1}^{\infty}(1+r)^{-j}S_{t+j})$, when $A_0 = 0$, B_t will be stationary, whereas for $A_0 > 0$, B_t will not be stationary. We accordingly initially examine two simple tests.

A. *Dickey-Fuller Test for Unit Roots*

David Dickey and Wayne Fuller (1979, p. 431) suggested the following test of the null hypothesis that a series z_t is nonstationary with unit roots. Estimate

$$z_t - z_{t-1} = \psi_0 + \psi_1 z_{t-1} + \psi_2(z_{t-1} - z_{t-2}) + e_{z,t}$$

by ordinary least squares, and calculate $\hat{\psi}_1/\hat{\sigma}_1$ where $\hat{\sigma}_1$ is the *OLS* standard error for $\hat{\psi}_1$. The null hypothesis (nonstationarity) says this statistic should be zero; the alternative (stationarity) says less than zero.

Using the data in Table 2 we estimated the following equation by *OLS* for $t = 1962$ to 1984 (standard errors in parentheses):

$$S_t - S_{t-1} = \underset{(3.27)}{-0.53} - \underset{(0.24)}{0.70}S_{t-1} + \underset{(0.24)}{0.38}(S_{t-1} - S_{t-2}) + \hat{e}_{S,t}$$

$$B_t - B_{t-1} = \underset{(28.13)}{79.63} - \underset{(0.17)}{0.48}B_{t-1} + \underset{(0.22)}{1.02}(B_{t-1} - B_{t-2}) + \hat{e}_{B,t}$$

The Dickey-Fuller test statistics are -2.92 in the case of the surplus and -2.82 in the case of debt, to be compared with a 5 percent critical value of -3.00 and 10 percent value of -2.63 reported in Fuller (1976, Table 8.5.2, p. 373). The data thus favor rejection of the null hypothesis of nonstationarity in both cases; that is, the data seem fully compatible with the assertion that investors rationally expected the budget to be balanced in present-value terms.

B. *Generalized Flood-Garber Test*

If expectations of future surpluses are conditioned in part on past surpluses and if we include lagged debt to eliminate the serial correlation of the resulting error term, then equation (10) takes the form

$$(11)\quad B_t = c_0 + A_0(1+r)^t + c_1 B_{t-1} + \dots + c_p B_{t-p} + b_0 S_t + b_1 S_{t-1} + \dots + b_{p-1} S_{t-p+1} + \varepsilon_t.$$

(Here ε_t is the residual from a projection of $[E_t \Sigma_{j=1}^{\infty}(1+r)^{-j} S_{t+j} + n_t]$ on $[S_t, S_{t-1}, \dots, S_{t-p+1}, B_{t-1}, B_{t-2}, \dots, B_{t-p}$, and a constant].) If one were willing to impose stronger restrictions on the dynamics of n_t and on the information set used by creditors, then one would want to estimate (11) jointly with cross-equation restrictions on the process followed by S_t, as in Flood and Garber's study of money demand. In the absence of such restrictions, Hamilton and Whiteman showed that the coefficients c_j, b_j are unrestricted, and a more general test of H_0 is obtained by simple *OLS* estimation[5] of (11):

$$B_t = \underset{(26.40)}{48.41} - \underset{(21.29)}{22.68}\,(1+r)^t + \underset{(0.21)}{0.69}\,B_{t-1} + \underset{(0.24)}{0.20}\,B_{t-2} - \underset{(0.13)}{1.30}\,S_t - \underset{(0.31)}{0.63}\,S_{t-1} + \hat{\varepsilon}_t.$$

We took $r = 0.0112$, the average *ex post* real rate over 1960–84. This equation clearly yields no indication that government debt tends to be growing at rate r; the coefficient $\hat{A}_0$ is statistically insignificant, and, if anything, negative in sign.

While the two tests employed above are more general than most of those appearing in the literature on speculative price bubbles, the motivating assumption that $\{E_t \Sigma_{j=1}^{\infty}(1+r)^{-j} S_{t+j}\}$ follows a stationary process is still not completely general. In particular, if one admits the possibility of out-of-sample changes in regime, the parameter A_0 in equation (10) would be unidentified (see Hamilton, forthcoming, and the references therein). One might further want to admit the possibility of a change in regime in which the government budget had been expected to be balanced in present-value terms up until some date t and only after that date was a permanent deficit introduced, in which case the corresponding "bubble" term would be zero up until date t and $A_t(1+r)^{\tau-t}$ for $\tau > t$.

One obvious candidate for a possible change in regime in both the $\{S_t\}$ series and the bubble term $A_0(1+r)^t$ is the inauguration of Ronald Reagan as president in 1981. As a general test of this possibility, one can imagine allowing all of the parameters in (11) to take on different values before and after 1981, and seeing whether the post-1981 $\hat{A}_0$ is still zero. Unfortunately, there are insufficient degrees of freedom to carry out such a test; if one believes that a change in regime occurred in 1981, it is impossible to determine statistically whether the change represents a new time series process for $\{S_t\}$ that is still consistent with (8), or instead represents a change to a nonzero value for the bubble term A_0. Distinguishing between these possibilities must await additional data. What one can say, however, is that the casual impression afforded by the official deficit series—that the government has been running a permanent deficit for 25 years in complete disregard of the present-value borrowing constraint—is not supported by a closer inspection of the data. For the sample taken as a whole, the data appear quite consistent with the assertion that the govern-

[5]Flood and Garber note in their fn. 18 (p. 754) that caution must be exercised in interpreting the usual t-test in this application. An added complication for our application is that under some specifications of the alternative hypothesis A_0 could be regarded as a random variable. A related issue arises in interpreting Flood and Garber's results if it were thought that some random economic event set off the speculative bubble. Our Dickey-Fuller test is likewise not without problems; if (8) fails, the nonstationary root is not unity as the Dickey-Fuller tests assume but rather $(1+r)$.

ment has historically operated subject to the constraint that expenditures not exceed receipts in expected present-value terms.

C. *Restricted Flood-Garber Test*

The test actually used by Flood and Garber (1980) would be valid for our application only under the further restrictions that n_t follows a white-noise process

$$(12) \qquad n_t = k + \varepsilon_{1t},$$

and that creditors' expectations of future surpluses are based solely on realizations of past surpluses.[6] A straightforward manipulation of the formula derived by Lars Hansen and Sargent (1981, p. 99) yields that for

$$(13) \quad S_t = k_2 + a_1 S_{t-1} + a_2 S_{t-2} + a_3 S_{t-3} + \varepsilon_{2t},$$

then

$$(14) \quad \hat{E}_t \sum_{j=1}^{\infty} b^j S_{t+j} = \frac{bk_2}{(1-b)(1-a_1 b - a_2 b^2 - a_3 b^3)} + \frac{(a_1 b + a_2 b^2 + a_3 b^3) S_t}{(1-a_1 b - a_2 b^2 - a_3 b^3)} + \frac{(a_2 b + a_3 b^2) S_{t-1}}{(1-a_1 b - a_2 b^2 - a_3 b^3)} + \frac{(a_3 b) S_{t-2}}{(1-a_1 b - a_2 b^2 - a_3 b^3)},$$

[6]The second assumption in particular is admittedly unrealistic. Indeed, it can be shown that expectations *must* be based on additional information besides S_t (namely, on the exogenous shocks to which the endogenous policy variable S_t responds) if equation (8) is to hold, because the forecast errors $(E_t - E_{t-1})S_{t+j}$ cannot be fundamental for S_t. It nevertheless seems of interest to see how good an approximation one gets to the data by ignoring this difference between the true forecasts of creditors $(E_t S_{t+j})$ and our econometric forecasts $(\hat{E}_t S_{t+j})$ based on a univariate autoregression for S_t.

TABLE 3—ESTIMATES OF PARAMETERS IN EQUATIONS (13) AND (15)

Parameter	Estimate	Standard Error
A_0	−61.52	(58.20)
k_1	241.51	(68.87)
k_2	0.90	(3.83)
a_1	0.15	(0.19)
a_2	−0.47	(0.22)
a_3	−0.51	(0.20)

where $\hat{E}_t(Y) \equiv E(Y|S_t, S_{t-1}, S_{t-2},\dots)$ and $b \equiv 1/(1+r)$. Substituting (12) and (14) into (10) yields

$$(15) \quad B_t = A_0(1+r)^t + k_1 + \frac{(a_1 b + a_2 b^2 + a_3 b^3) S_t}{(1-a_1 b - a_2 b^2 - a_3 b^3)} + \frac{(a_2 b + a_3 b^2) S_{t-1}}{(1-a_1 b - a_2 b^2 - a_3 b^3)} + \frac{(a_3 b) S_{t-2}}{(1-a_1 b - a_2 b^2 - a_3 b^3)} + \varepsilon_{1t}.$$

We estimated equations (13) and (15) jointly by nonlinear least squares for $t = 1963$ to 1984. Parameter estimates and standard errors are summarized in Table 3.

As in the less restrictive tests above, we note that there seems to be no role whatever for the bubble term; $\hat{A}_0$ is statistically insignificant, and, if anything, negative. The assumption that bondholders rationally expected the debt to be paid back in present-value terms fits the data better than the assumption that debt has simply accumulated with an ever-growing interest load. Moreover, for these parameter estimates, equation (15) has an R^2 of 0.53; that is, more than half of the observed variance in the market value of real government debt could be explained from a rational expectations forecast of the discounted value of future surpluses. Note such high explanatory power is achieved despite the fact that all of the other parameters that characterize the dynamics of outstanding debt in equation (15) are tied down by the univariate process for surpluses (13), and such parameters appear in (15) only to the extent that they could characterize rational expectations forecasts of future surpluses. The high R^2 is also achieved despite the omission of all of the additional variables besides past surpluses that would be used by agents to forecast future surpluses.

Of course, to achieve such a fit to the data, the regression is forced to fit strongly negative coefficients in the autoregressive process for surpluses at two- and three-year lags, as the values in Table 3 indicate. Bondholders must assume that big deficits typically only last a year or two, and will later be balanced out with surpluses. There is indeed moderate support for this position in a completely unconstrained *OLS* estimate of the latter regression:

$$S_t = \underset{(3.51)}{-0.30} + \underset{(0.23)}{0.62} S_{t-1} \underset{(0.28)}{-0.30} S_{t-2} - \underset{(0.27)}{0.20} S_{t-3} + \hat{\varepsilon}_{2t}$$

The restricted rational expectations estimates of Table 3 simply exaggerate this feature in the data. Overall, then, the present-value hypothesis seems to hold up quite well.

IV. Conclusions

In this paper we have examined the proposition that in order to be able to issue interest-bearing debt, a government must promise to balance its budget in expected present-value terms. We suggested a battery of empirical tests of this proposition, some of which are quite robust with respect to assumptions about the dynamics of variables that are seen by agents but not the econometrician, and others which are highly restrictive. The conclusion from all our tests, however, is the same—the proposition that the government must promise creditors that it will balance the budget in expected present-value terms seems largely consistent with postwar U.S. data.

This result might seem surprising since the official budget accounts register essentially uninterrupted deficits for the United States from 1960 to 1981. However, the real value of government debt held by the public actually fell during this period, indicating that the continuing reported deficits grossly misstated the true fiscal posture of the government. We suggested an alternative measure of the government deficit that takes into account revenues from monetization and capital gains on gold but excludes interest payments. From the time-series properties of the adjusted deficit series, one can construct a rational expectations forecast of the present value of future government budget surpluses. Such a forecast series can account for 53 percent of the observed variance of real government debt under the assumption that the government budget must be balanced in present-value terms.

If our conclusion on the limitations of government borrowing is correct, then the prevailing sentiment in Washington that current deficits can continue forever is wrong; the adjusted deficit series must soon turn to surplus. One policy change that could turn the adjusted series to surplus would be a resurgence of money growth.

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