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Article *in* Journal of Economic Dynamics and Control · June 1988

DOI: 10.1016/0165-1889(88)90048-6

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Journal of Economic Dynamics and Control 12 (1988) 425–444. North-Holland

COMMON TRENDS, THE GOVERNMENT'S BUDGET CONSTRAINT, AND REVENUE SMOOTHING*

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Received November 1987, final version received January 1988

The requirement that the government's budget be balanced in present value terms is shown to be equivalent to the condition that government expenditures inclusive of interest, tax receipts and seignorage be cointegrated. The condition is in fact stronger, requiring that the deficit inclusive of interest be stationary. Stationarity of the net-of-interest deficit is neither necessary nor sufficient for intertemporal budget balance. However, it does have implications for Barro's tax smoothing hypothesis. Data for the period 1890–1986 are consistent with intertemporal budget balance but not with tax smoothing.

1. Introduction

The large fiscal deficits experienced by the U.S. during the 1980s have generated considerable interest in a number of issues related to the expenditure and tax policies of the government. One such issue involves the sustainability of a particular deficit process and the requirement that the government's budget be balanced in present value terms. The intertemporal budget constraint, however, imposes restrictions only on the long-run relationship between expenditures and revenues so that almost any short-run deficit path is consistent with a budget balanced in present value terms. This makes it difficult to test the hypothesis that the government's intertemporal budget is balanced. But it is clear that a present value budget constraint implies that expenditures (appropriately defined) cannot drift too far away from revenues (appropriately defined). This suggests that the recent literature on cointegrated processes may provide insights and testing procedures that can be brought to bear on the issue of intertemporal budget balance.

*Any opinions expressed here are those of the authors and are not necessarily those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

In this paper we derive the restrictions imposed by the present value budget constraint on the deficit process in an environment that contains both deterministic and stochastic elements. We show that budget balance requires that the process describing the evolution of the deficit *inclusive* of interest be stationary; that is, the condition requires that government expenditures (inclusive of interest), tax receipts, and revenues from seignorage be cointegrated. An equivalent test is that the deficit exclusive of interest be cointegrated with the stock of debt.

Our focus on the deficit inclusive of interest is in contrast to recent studies which have concentrated upon the deficit *exclusive* of interest. It turns out that stationarity of the deficit exclusive of interest is neither necessary nor sufficient to ensure intertemporal budget balance.

However, stationarity of the deficit exclusive of interest is of interest for another issue, one related to positive theories which view the deficit as generated by fluctuating expenditures and optimally determined taxes [Barro (1979)]. If taxes are set on the basis of the expected permanent level of government expenditures, then taxes will exhibit nonstationary behavior even if expenditures are stationary. But if expenditures are also nonstationary, taxes will be nonstationary even if the government maintains a balanced budget each period (that is, even if the government does not smooth taxes). Thus, evidence on the random walk nature of the tax process cannot be considered sufficient for the tax smoothing hypothesis. Instead, it is necessary to examine the joint behavior of expenditures and taxes.

We set up a simple model to obtain the conditions that the tax smoothing hypothesis imposes on the time path of these variables. It turns out that expenditures and taxes must be cointegrated, but the cointegrating vector will not generally be $[1 \ -1]$. In other words, the deficit exclusive of interest must be nonstationary, except under very special (and unlikely) conditions.

The empirical results we present (using data over the period 1890–1986) suggest that the government's budget is consistent with intertemporal budget balance. Specifically, tax revenues and expenditures inclusive of interest payments are cointegrated, with a cointegrating vector that is close to $[1 \ -1]$. However, the results of adding the real value of changes in high powered money – which is the measure of seignorage we use – to this set of variables are ambiguous. Our results also imply rejection of the tax smoothing hypothesis. Specifically, we cannot reject stationarity of the difference between expenditures and taxes.

The paper is organized as follows: Section 2 sets out the basic framework and derives the restrictions on the stochastic behavior of government expenditures and revenues implied by the intertemporal budget constraint. Section 3 examines the implications of Barro's tax smoothing hypothesis for the stochastic process followed by the deficit. Section 4 briefly reviews previous tests of intertemporal budget balance and tax smoothing. The data we use in the empirical analysis are described in section 5, while section 6 reports the results of the tests for intertemporal budget balance and Barro's tax hypothesis. Conclusions are summarized in section 7.

2. The framework

In this section, we derive the restrictions on the joint process of expenditures, tax revenues and seignorage that are required to ensure the government's intertemporal budget is balanced. These restrictions are derived by Sargent (1987) for stationary expenditure and revenue processes. We generalize his results to deal with nonstationary processes. While most recent discussions of the government's intertemporal budget have focussed on expenditures net-of-interest payments, we show that intertemporal budget balance implies that expenditures including interest payments will be cointegrated with tax and seignorage revenues.

If the government issues one-period debt, the real value of the outstanding stock of debt, b_t, evolves according to

$$\begin{aligned} b_{t+1} &= (1+r)b_t + g_t - \tau_t - s_t \\ &= (1+r)b_t + d_t, \end{aligned} \tag{1}$$

where r is the real rate of interest (assumed to be constant), g_t is real expenditures net of interest, τ_t is real tax revenues, $s_t = (M_{t+1} - M_t)/P_t$ equals real revenue from seignorage when M_t is the nominal supply of high powered money, P_t is the price level, and $d_t = g_t - \tau_t - s_t$ is the net-of-interest deficit. Taking expected values of (1), solving for b_t, and recursively eliminating future values of the stock of debt by substitution yields the intertemporal budget constraint faced by the government:

$$\begin{aligned} b_t = &-\mathrm{E}_t \sum_{j=0} (1+r)^{-(j+1)}(g_{t+j} - \tau_{t+j} - s_{t+j}) \\ &+ \lim_{j\to\infty} \mathrm{E}_t(1+r)^{-(j+1)} b_{t+j+1}. \end{aligned} \tag{2}$$

In (2), $\mathrm{E}_t(\ )$ denotes the mathematical expectation conditional on time t information.

For (2) to impose a constraint analogous to the intertemporal budget constraint faced by a private individual, it must hold that[1]

$$\lim_{j\to\infty} \mathrm{E}_t(1+r)^{-(j+1)} b_{t+j+1} = 0. \tag{3}$$

If this condition is satisfied, then (2) requires that the government run future surpluses equal, in expected present value terms, to its current outstanding

[1] McCallum (1984) shows that a violation of (3) is inconsistent with optimizing behavior by the holders of the government's debt.

debt. When (3) holds, we will say that intertemporal budget balance is satisfied. In order to test whether (3) holds, we must determine the manner in which intertemporal budget balance imposes restrictions on the stochastic processes followed by g_t, τ_t and s_t.[2]

In order to derive explicit restrictions on g_t, τ_t and s_t that will ensure (3) is satisfied, define $x_t' = (g_t, \tau_t, s_t)$, and assume the evolution of x_t is described by

$$\begin{bmatrix} (1-L)g_t \\ (1-L)\tau_t \\ (1-L)s_t \end{bmatrix} = (1-L)x_t = \mu + C(L)\varepsilon_t, \tag{4}$$

where L is the lag operator, μ is a 3×1 vector of constants, $C(L)$ is a 3×3 matrix of polynomials in L, and ε_t is a 3×1 vector of white noise innovations. The matrix $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is assumed to be square summable and normalized so that $\mathrm{E}(\varepsilon_t \varepsilon_t') = I$. In specifying the x_t process by eq. (4), we are assuming that g_t, τ_t and s_t need to be differenced at most once to induce stationarity.[3] Eq. (4) allows for quite general joint processes for expenditures, tax revenues, and seignorage. If the polynomial $C(L)$ has a root at $L = 1$ [i.e., $C(1) = 0$], then both sides of (4) can be divided by $(1 - L)$ to express the level of x_t as a stationary process (trend-stationary if $\mu \neq 0$).[4] We wish to determine whether (1)–(3) imply testable restrictions on μ and $C(L)$.

Define $\alpha' = (1 \ \ -1 \ \ -1)$ so that $\alpha' x_t$ is the deficit exclusive of interest payments. The process describing the evolution of the first difference of the net-of-interest deficit is, from (4),

$$\begin{aligned} (1-L)\,d_t &= (1-L)\alpha' x_t \\ &= \alpha'(1-L)x_t \\ &= \alpha'\mu + \alpha' C(L)\varepsilon_t \\ &= \alpha'\mu + \sum \alpha' C_i L^i \varepsilon_{t-i}. \end{aligned} \tag{5}$$

Since the flow budget constraint given by eq. (1) shows that the evolution of the stock of debt is driven by the net-of-interest deficit, we can use (5) to derive an explicit expression for the limiting behavior of the discounted value

[2] Tests of intertemporal government budget balance are closely related to tests for speculative bubbles in models of asset pricing and hyperinflations. This connection is made explicit in Hamilton and Whiteman (1986). A survey and assessment of various bubbles tests can be found in Mattey and Meese (1986).

[3] This assumption is consistent with the empirical results reported in section 6.

[4] A process is trend stationary if it is stationary after the removal of a deterministic trend, and nontrend stationary if it is nonstationary even after the removal of a deterministic trend.

of the government's outstanding stock of debt. This will allow us to determine the restrictions on the d_t process necessary to ensure intertemporal budget balance.

Let d_0 and b_0 denote the initial values of the net-of-interest deficit and stock of debt respectively. Consider the impact of a shock $\varepsilon_1 = 1$ in period 1. The expected value of the stock of debt then evolves as

$$
\begin{aligned}
b_1 &= (1+r)b_0 + d_0, \\
b_2 &= (1+r)^2 b_0 + (1+r)d_0 + [d_0 + \alpha'\mu + \alpha' c_0], \\
b_3 &= (1+r)^3 b_0 + (1+r)^2 d_0 + (1+r)[d_0 + \alpha'\mu + \alpha' c_0] \\
&\quad + [d_0 + 2\alpha'\mu + \alpha' c_0 + \alpha' c_1],
\end{aligned}
$$

so that

$$
\begin{aligned}
\mathrm{E}_1 b_j = {} & (1+r)^j b_0 + (1+r)^{j-1}\Big(1 + (1+r)^{-1} \\
& + (1+r)^{-2} + \cdots + (1+r)^{-j+1}\Big) d_0 \\
& + (1+r)^{j-2}\Big(1 + 2(1+r)^{-1} + 3(1+r)^{-2} \\
& + \cdots + (j-1)(1+r)^{-j+2}\Big)\alpha'\mu \\
& + (1+r)^{j-2}\alpha'\Big(c_0 + c_1(1+r)^{-1} \\
& + c_2(1+r)^{-2} + \cdots + c_{j-2}(1+r)^{-j+2}\Big) \\
& + (1+r)^{j-3}\alpha'\Big(c_0 + c_1(1+r)^{-1} + \cdots + c_{j-3}(1+r)^{-j+3}\Big) \\
& \vdots \\
& + (1+r)\alpha'\Big(c_0 + c_1(1+r)^{-1}\Big) + \alpha'\Big(c_0 + c_1 + c_2 + \cdots + c_j\Big),
\end{aligned}
$$

or

$$
\begin{aligned}
\mathrm{E}_1 b_j = {} & (1+r)^j b_0 + (1+r)^{j-1}\left[\frac{1-(1+r)^j}{1-(1+r)^{-1}}\right] d_0 \\
& + (1+r)^{j-2}\left[\frac{1-(1+r)^{-j+2}}{\left(1-(1+r)^{-1}\right)^2} - \frac{(j-1)(1+r)^{1-j}}{1-(1+r)^{-1}}\right]\alpha'\mu \\
& + \sum_{i=2}^{j-1}(1+r)^{j-1}\left(\sum_{s=0}^{j-1}\alpha' c_s(1+r)^{-s}\right) + \sum_{s=0}^{j} c_s. \qquad (6)
\end{aligned}
$$

Dividing b_j by $(1+r)^j$ and letting j go to infinity yields

$$\mathrm{E}_1 \lim_{j \to \infty} (1+r)^{-j} b_j = b_0 + \frac{1}{r} d_0 + \frac{1}{r^2} \alpha' \mu + \frac{1}{r(1+r)} \alpha' C\big((1+r)^{-1}\big), \tag{7}$$

where

$$\alpha' C\big((1+r)^{-1}\big) = \sum_{i=0}^{\infty} (1+r)^{-i} \alpha' C_i .$$

The first three terms on the RHS of (7) represent the impact of the initial conditions (b_0 and d_0) and the deterministic trend ($\alpha'\mu$) on the limiting behavior of the discounted stock of debt. The last term on the RHS equals the effect of the stochastic shock at time 1 on the expected value of discounted debt as $j \to \infty$.

From (7), necessary and sufficient conditions for intertemporal budget balance to hold ($\mathrm{E}_t \lim (1+r)^{-j} b_{t+j} = 0$) are given by

$$\alpha' \mu = -r(d_0 + r b_0), \tag{8a}$$

$$\alpha' C\big((1+r)^{-1}\big) = 0. \tag{8b}$$

Condition (8a) restricts the deterministic trend in the deficit process. If the initial net-of-interest surplus, $-d_0$, is not sufficient to finance interest payments on the initial outstanding stock of debt ($d_0 + r b_0 > 0$), then the surplus must trend upwards at the rate $r(d_0 + r b_0)$ in order to prevent the outstanding stock of debt from growing without limit.

The stochastic component in the limiting behavior of the discounted stock of debt equals zero if and only if $\alpha' C(L)$ has a root at $(1+r)^{-1} < 1$. Eq. (8b) requires that the discounted effect on the net-of-interest deficit of any realization of ε_t equals zero. A positive shock, for example, must simultaneously lower expected future deficits by an amount equal in *present discounted terms* to the initial increase.

Under the assumption that (8a) and (8b) hold so that the government's intertemporal budget is balanced, eq. (2) becomes

$$b_t = -\mathrm{E}_t \sum_{j=0}^{\infty} (1+r)^{-(j+1)} d_{t+j}. \tag{2'}$$

First differencing this equation yields

$$(1-L)b_t = -\mathrm{E}_t\sum_0^\infty(1+r)^{-(j+1)}d_{t+j} + \mathrm{E}_{t-1}\sum_0^\infty(1+r)^{-(j+1)}d_{t-1+j}$$

$$= -\sum_0^\infty(1+r)^{-(j+1)}(\mathrm{E}_t d_{t+j} - \mathrm{E}_{t-1}d_{t+j})$$

$$-\mathrm{E}_{t-1}\sum_0^\infty(1+r)^{-(j+1)}(1-L)d_{t+j}. \tag{9}$$

Using (5), the first term on the RHS of (9) can be shown to equal

$$-\sum_{j=0}^\infty(1+r)^{-(j+1)}\alpha' C_j\varepsilon_t = -(1+r)^{-1}\alpha' C\big((1+r)^{-1}\big)\varepsilon_t = 0,$$

where the last equality follows from (8b). The second term is equal to

$$-\mathrm{E}_{t-1}\sum(1+r)^{-(j+1)}\big(\alpha'\mu + \alpha' C(L)\varepsilon_{t+j}\big)$$

$$= -\frac{\alpha'\mu}{r} + \frac{(1+r)^{-1}L^{-1}\alpha' C(L)}{1-(1+r)^{-1}L^{-1}}\varepsilon_{t-1},$$

where use has been made of Hansen and Sargent's (1980) formula for calculating the expected present value of a stationary moving average process. Eq. (9) can now be written as

$$(1-L)b_t = -\alpha'\mu/r + D(L)\varepsilon_t, \tag{9$'$}$$

where $D(L)\varepsilon_t$ is a stationary process. Intertemporal budget balance therefore implies that the first difference of the stock of debt is a stationary stochastic process.

Conversely, suppose the first difference of the stock of debt is a stationary process given by $(1-L)b_t = A + D(L)\varepsilon_t$. The deficit exclusive of interest, d_t, is just $(L^{-1}-(1+r))b_t$, so

$$(1-L)d_t = (1-L)\big(L^{-1}-(1+r)\big)b_t$$

$$= \big(L^{-1}-(1+r)\big)(1-L)b_t$$

$$= \big(L^{-1}-(1+r)\big)\big(A + D(L)\varepsilon_t\big)$$

$$= \mu + C(L)\varepsilon_t,$$

where

$$\mu = (L^{-1} - (1+r))A \quad \text{and} \quad C(L) = (L^{-1} - (1+r))D(L).$$

Hence, $C((1+r)^{-1}) = 0$, and $\mu = rA$ where A is the initial deterministic level of $(1-L)b_t$. Thus, stationarity of $(1-L)b_t$ implies (8a) and (8b), the conditions required for intertemporal budget balance.

From the flow budget constraint, (1), $(1-L)b_{t+1} = rb_t + d_t$ is just the deficit *inclusive* of interest payments. Intertemporal budget balance therefore implies that the inclusive of interest deficit is stationary. Equivalently expressed, the stock of debt and the net-of-interest deficit are cointegrated with cointegrating vector $(r\ 1)$.[5]

The stationarity of the deficit inclusive of interest can be related to McCallum's (1984) recent discussion of the requirements for intertemporal budget balance. McCallum concludes that a constant positive deficit is inconsistent with a rational expectations equilibrium if deficit is defined *exclusive* of interest payments. However, a constant deficit *inclusive* of interest payments is consistent with equilibrium. In our notation, McCallum's assumption of a constant positive net-of-interest deficit implies that $\mu = 0$ and $C(L) = 0$ in eq. (4), so that the process for the deficit is simply $(1-L)d_t = 0$, given some positive initial deficit $d_0 > 0$. This process for the deficit violates the requirements of intertemporal budget balance, since the sum of the first two terms on the RHS of eq. (7) above does not go to 0 as $j \to \infty$ unless $d_0 = -rb_0 < 0$. So a constant deficit net-of-interest is inconsistent with intertemporal budget balance, but a constant surplus (equal to interest payments on the stock of debt) is possible. If $d_0 \neq -rb_0$, the net-of-interest deficit includes a deterministic trend $[\alpha'\mu = -r(d_0 + rb_0) \neq 0]$, but the deficit inclusive of interest is a constant equal to $\alpha'\mu/r$.

To summarize the results of this section, stationarity of the deficit inclusive of interest payments is both necessary and sufficient for the government's intertemporal budget to be balanced. Equivalently expressed, expenditures including interest $(g_t + rb_t)$, tax revenues, and seignorage revenue will be cointegrated with known cointegrating vector $(1\ -1\ -1)$ if and only if the intertemporal budget constraint is satisfied. This focus on the deficit inclusive of interest contrasts with the more common recent focus on the deficit exclusive of interest. Stationarity of the net-of-interest deficit is neither necessary nor sufficient for intertemporal budget balance. However, as we show in

[5] That intertemporal budget balance implies $(1-L)b_t$ is stationary when $(1-L)d_t$ is stationary also follows from the results of Hamilton and Whiteman (1985). Campbell and Shiller (1986) derive a test for present value models of the form $Y_t = \theta(1-\delta)\delta^i \mathrm{E}_t y_{t+i}$ by showing that Y_t and y_t will be cointegrated with cointegrating vector $(1\ \theta)$ if Y_t and y_t are stationary in first differences. In terms of the notation of (2′), $Y_t - \theta y_t$ is just $b_t + r^{-1}d_t$, or r^{-1} times the deficit inclusive of interest.

the next section, stationarity (or nonstationarity) of the deficit exclusive of interest does have implications for another interesting economic hypothesis.

3. Tax smoothing

The government's intertemporal budget constraint provides an accounting framework for analyzing expenditures and revenues. Barro (1979) has provided an economic theory of the determinants of the government's deficit. If a government must rely on distortionary taxes, Barro shows that the tax rate should be set on the basis of permanent (noninterest) government expenditures with transitory expenditure fluctuations financed by issuing debt. We recast Barro's model within the framework of section 2 and show that the deficit net-of-interest will generally be nonstationary under Barro's tax smoothing hypothesis.

To derive the restrictions on the deficit process implied by tax smoothing, we assume that the distortionary costs of revenue generation are quadratic in total revenues: $\beta_1\tau_t + (\beta_2/2)\tau_t^2$. Suppose further that the government sets the path for τ to minimize the present discounted value of distortions, subject to a given stochastic process for government expenditures (net-of-interest) and the intertemporal budget constraint (2′), rewritten here as eq. (10):[6]

$$\mathrm{E}_t\sum(1+r)^{-j}\tau_{t+j} = b_t + \mathrm{E}_t\sum(1+r)^{-j}g_{t+j}. \tag{10}$$

In keeping with Barro's specification, we do not deal here with the revenue obtained from currency issue. The first-order conditions for this problem imply

$$\mathrm{E}_t\tau_{t+1} = \tau_t. \tag{11}$$

Thus, τ will follow a martingale process.

Substituting (11) into the budget constraint (10) implies optimal tax revenues are given by

$$\tau_t = \left(\frac{r}{1+r}\right)\left[b_t + \mathrm{E}_t\sum(1+r)^{-j}g_{t+j}\right]. \tag{12}$$

To derive a more explicit expression for τ_t, assume that the expenditures process is stationary in first differences and given by

$$(1-L)g_t = g(L)\varepsilon_t. \tag{13}$$

[6] We are assuming $\mathrm{E}_t\lim_{j\to\infty}(1+r)^{-j}b_{t+j} = 0$. As the previous section demonstrates, this will place restrictions on the g and τ processes.

Using (13) to evaluate (12) yields the following expression for the first difference of τ_t:

$$(1-L)\tau_t = g\big((1+r)^{-1}\big)\varepsilon_t. \tag{14}$$

Eqs. (13) and (14) characterize the evolution of expenditures and revenues under Barro's tax smoothing hypothesis. The deficit exclusive of interest evolves according to

$$(1-L)(g_t-\tau_t) = \left[g(L) - g\big((1+r)^{-1}\big)\right]\varepsilon_t = B(L)\varepsilon_t, \tag{15}$$

where $B(L) = g(L) - g((1+r)^{-1})$.[7]

Barro's tax smoothing hypothesis imposes cross equation restrictions on the joint process of expenditures and revenue. If expenditures are not trend stationary, then g_t and τ_t are cointegrated, but the cointegrating vector is *not* $(1 \;\; -1)$. The cointegrating vector is, instead,

$$\hat{\alpha}' = (1 \quad -\delta),$$

where $\delta = g(1)/g((1+r)^{-1})$. This can perhaps best be seen by using (13) and (14) to write the levels of g and τ as

$$g_t = g_0 + g(1)\phi_t + g^*(L)\varepsilon_t, \tag{16}$$

$$\tau_t = \tau_0 + g\big((1+r)^{-1}\big)\phi_t, \tag{17}$$

where $\phi_t = \sum\varepsilon_t$ is a random walk with innovation ε_t and $g^*(L) = (1-L)^{-1}[g(L) - g(1)]$. As these expressions make clear, the nonstationarity of g_t [when $g(1) \neq 0$] and τ arises from a single common stochastic trend (ϕ).

The net-of-interest deficit will be stationary if and only if $g(1) = g((1+r)^{-1})$, that is, if the sum of the moving average coefficients in the process for $(1-L)g_t$ equals the discounted sum of the same moving average coefficients. If this condition is satisfied, δ will equal 1 even in the presence of tax smoothing. Consequently, a finding that the difference between g_t and τ_t is stationary is inconsistent with tax smoothing only if we can show that $g(1)$ is not equal to $g((1+r)^{-1})$.

4. Previous tests

In recent years, both Hamilton and Flavin (1986) and Hakkio and Rush (1986) have attempted to test whether the deficit process is consistent with intertemporal budget balance [i.e., $E_t \lim(1+r)^{-j}b_{t+j} = 0$ in eq. (2)]. Using

[7] To verify that the intertemporal budget constraint is satisfied, note that $B((1+r)^{-1}) = g((1+r)^{-1}) - g((1+r)^{-1}) = 0$.

annual U.S. data for 1962–1984, Hamilton and Flavin (1986) test for unit roots in the real deficit and the real stock of outstanding debt. They conclude that, for both processes, the hypothesis of a unit root can be rejected. This result is consistent with the government's intertemporal budget being balanced in expected value.

Hamilton and Flavin's results on the stationarity of the deficit and the stock of debt are, however, fairly weak. They reject the hypothesis of a unit root only because they adopt a 10 percent significance level. At a more conventional 5 percent level, the unit root hypothesis cannot be rejected. In addition, tests of long-run behavior on the basis of 22 years of annual data may be expected to have low power.

Hakkio and Rush (1986) test for a unit root in the net-of-interest deficit using quarterly data from 1962–1986. In the notation of section 2, they test for $\alpha'C(1) = 0$. They conclude that $\alpha'C(1) = 0$ from which they infer that the government's intertemporal budget is balanced. However, $\alpha'C(1) = 0$ is neither necessary nor sufficient to ensure $\mathrm{E}_t \lim_{j\to\infty}(1+r)^{-j}b_{t+j} = 0$. Instead, as we have shown in section 3, the stationarity of the deficit provides suggestive evidence against the tax smoothing hypothesis.

Previous tests of the tax smoothing hypothesis[8] have generally examined the behavior of tax revenues (or some measure of the tax rate), following Barro's original focus on fiscal policy alone. However, as discussed above, a finding that tax revenues are nontrend stationary can be used to support the tax smoothing hypothesis only if government expenditures are stationary. In particular, if expenditures are nonstationary, taxes will also be nonstationary even in the absence of smoothing. In this case, an appropriate test would involve showing that the difference between tax receipts and government expenditures is nonstationary; in other words, $[1 \ -1]$ should not be a cointegrating vector for g_t and τ_t, except in the special case discussed above.

Thus, our discussion suggests the following tests. To test whether the government's present value constraint is being violated, we need to test for cointegration between government expenditures inclusive of interest, tax revenues and revenues from seignorage (given that the univariate processes for these variables are nonstationary). Testing for tax smoothing (in the sense of Barro) requires an examination of both the moving average process for the first difference of government expenditures and the vector that cointegrates expenditures exclusive of interest and tax revenues (assuming one exists, of course).

5. Variable selection and data

Our data set consists of annual observations for the period from 1890 to 1986. Data on government expenditures net-of-interest and on interest payments by the government for 1890–1928 were obtained from table 3 of

[8] See, for example, Sahasakul (1986) and Kochin, Benjamin and Meador (1985).

Kremers (1985). From 1929 onwards, data on these two variables are from the National Income and Product Accounts (NIPA). Data on privately held interest bearing debt for 1890–1983 are also from Kremers, and we have followed his method to construct values for the remaining years. Note that this measure excludes debt held by the monetary authorities.

We have made the following adjustments to obtain a closer correspondence between the theoretical concepts and the measures available to us. First, a substantial portion of the interest payments made by the Treasury to the Fed are returned to the Treasury, and prior to 1982, these appeared as government interest expenditures. Therefore, it is necessary that the government interest payments variable be adjusted by this amount. Further, these payments by the Fed to the Treasury also showed up in Treasury receipts, and accordingly must be subtracted from there as well. The net effect of this adjustment is to leave the deficit inclusive of interest unchanged, but to increase the magnitude of the exclusive of interest deficit.

Since inflation erodes the real value of outstanding government debt, the Treasury's interest payments must be adjusted accordingly. We therefore subtract the product of the inflation rate (from period $t-1$ to t) and the stock of debt at the end of period $t-1$ from interest payments in period t.

We use the real value of the change in high powered money as a measure of the seignorage obtained by the government. This is preferable to measures of the adjusted monetary base (used by Hakkio and Rush, for example), which are adjusted for changes in reserve requirements over the length of the data series and therefore do not reflect seignorage obtained by the government. The series we use is from table 4.8 of Friedman and Schwartz (1982). Since their series ends in 1975 we have used their method to extend the series through 1986.

To obtain real values, all variables in nominal terms are divided by the GNP deflator. The deflator is obtained as the ratio of nominal to real GNP, where data on both variables are from Kendrick (1961) for 1890–1908 and from NIPA for the remaining years.[9]

6. Empirical results

In table 1 we present the results of the Dickey–Fuller test for a unit root. The test involves regressing the first difference of each series on its lagged level and lagged differences. The test statistic is the 't ratio' for the lagged level of

[9]A final issue concerns the difference between the market and par values of debt. Unfortunately, we were unable to obtain a market value series that covered our entire sample period. However, we were able to obtain a series on the market value of debt constructed by Seater (1981) that covers the period 1919–1975. For this shorter sample period, the results that we obtained when using his series were almost identical to those obtained when we used our measure of debt and did not appear to be any different from those obtained for the full sample.

Table 1

Dickey–Fuller tests, sample 1890–1986.

	Unit root tests				Tests of intertemporal budget balance[a]	
	g_t	$g_t + rb_t$	τ_t	s_t	$g_t + rb_t - \tau_t$	$g_t + rb_t - \tau_t - s_t$
Constant	-23.46	-35.92	-8.91	—	—	—
	(-1.78)	(-1.73)	(-1.31)	—	—	—
Trend	1.03	1.51	0.43	—	—	—
	(2.57)	(2.45)	(2.04)	—	—	—
Lagged level	-0.11[b]	-0.15[b]	-0.03[b]	-0.09[c]	-0.53[d]	-0.58[d]
	(-2.23)	(-2.01)	(-0.98)	(-1.52)	(-5.87)	(-6.25)
No. of lags of differenced variable	3	3	3	2	1	1

[a] If a trend is included it has a negative (but insignificant) coefficient.
[b] Critical value of the test statistic for sample size 100 is -3.45 at the 5% significance level and -3.15 at 10%.
[c] Critical value of the test statistic for sample size 100 is -1.95 at the 5% significance level and -1.61 at 10%.
[d] Significant at 1%.

the variable, except that the statistic has a nonstandard distribution under the null that the series contains a unit root. Critical values for the computed statistic are tabulated in Fuller (1976).

We first present the results for real government expenditures net-of-interest (g_t), real government expenditures including interest payments ($g_t + rb_t$), real government receipts (τ_t) and the real change in high powered money (s_t). In no case can we reject at even the 10 percent level the hypothesis that the variable has a unit root. We have also carried out tests to determine whether the first differences of these variables are nontrend stationary (not shown here) and are able to reject the unit root hypothesis at the 1 percent level in all cases. The next two variables are measures of the deficit inclusive of interest, $g_t + rb_t - \tau_t$ and $g_t + rb_t - \tau_t - s_t$. In both cases the unit root hypothesis is rejected quite comfortably. Note the absence of a trend term in both regressions.

The rejection of a unit root in $g_t + rb_{t-1} - \tau_t - s_t$ provides evidence that the expenditure and revenue processes are consistent with the present value budget constraint. However, the results are not totally conclusive since the finding that both $g_t + rb_t - \tau_t$ and $g_t + rb_t - \tau_t - s_t$ are stationary is not consistent with the finding that s_t is nonstationary. This contradiction suggests that the tests are not very powerful. In particular, it is quite possible that the vectors that we have imposed are not cointegrating vectors, but the tests are unable to distinguish them from the 'correct' ones.

Accordingly, we next present results for the case where the cointegrating vectors are estimated directly from the data, using techniques discussed in Engle and Granger (1987) and Engle and Yoo (1987). The Engle–Granger tests are based on the result that if two processes which are integrated of order 1 are cointegrated, then the residuals obtained from regressing one on the other should be stationary. The test for residual stationarity that we use here is the same as the Dickey–Fuller test in table 1, except that the critical values are different.

Table 2 presents the results for intertemporal budget balance. An examination of the residuals from the regression of $g_t + rb_t$ on τ_t shows that the two variables are cointegrated, with cointegrating vector [1 −1.06]. The reverse regression leads to the same result; we can reject the hypothesis of no cointegration at the 1 percent level. However, we cannot reject the unit root hypothesis for the residuals obtained from the regression of either $g_t + rb_t$ or τ_t on s_t. Similarly, regressing s_t on either of these variables also leads to nonstationary residuals.

Table 2 also presents regressions of each of these variables on the other two. We can reject the hypothesis that the residuals obtained from the regression of $g_t + rb_t$ on τ_t and s_t have a unit root at the 1 percent level, implying that the three variables are cointegrated. (Critical values for this test are presented in Engle and Yoo.) However, s_t has an estimated coefficient of 4.9 in this regression, instead of the unit coefficient that was imposed in the Dickey–Fuller test. Regressing τ_t on the other two variables also suggests that the three variables are cointegrated, but here the coefficient on s_t is −3.4. Finally, we are unable to reject the unit root hypothesis for the residuals obtained from the regression of s_t on τ_t and $g_t + rb_t$.

Table 3 presents the results from our tests of Barro's tax smoothing hypothesis. We first examine whether the difference between g_t and τ_t is stationary. The Dickey–Fuller test (not shown here) allows us to reject the null hypothesis of a unit root in the process for $g_t - \tau_t$ at the 1 percent significance level. In the first part of the table we show the results from the Engle–Granger test. Regressing g_t on τ_t (or vice-versa) leads to stationary residuals, with a cointegrating vector that is close to (1 −1).

Part B of the table presents tests for the null hypothesis that $g(1) = g((1+r)^{-1})$, for alternative values of the real interest rate. The estimated moving average process is

$$(1-L)g_t = \underset{(2.8)}{7.64} + \varepsilon_t + \underset{(5.1)}{0.49}\,\varepsilon_{t-1} - \underset{(-3.3)}{0.33}\,\varepsilon_{t-3}$$

$$- \underset{(-3.7)}{0.40}\,\varepsilon_{t-4} - \underset{(-1.6)}{0.17}\,\varepsilon_{t-5},$$

where the t-values are shown in parentheses. For this process we cannot reject

Table 2

Engle–Granger tests for intertemporal budget balance.

	Dependent variable								
	$g_t + rb_t$			τ_t			s_t		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Constant	—	106.59 (3.96)	−16.16 (−1.73)	18.24 (2.03)	124.62 (4.80)	27.76 (3.07)	2.50 (2.66)	3.15 (3.04)	3.18 (3.53)
$g_t + rb_t$	—	—	—	0.85 (31.09)	—	0.92 (29.65)	0.02 (6.42)	—	0.05 (5.56)
τ_t	1.06 (42.04)	—	0.98 (29.65)	—	—	—	—	0.02 (4.72)	−0.04 (−3.69)
s_t		16.46 (6.42)	4.93 (5.56)	—	11.70 (4.72)	−3.41 (−3.69)		—	
Residual regressions									
Test statistic[a]	−6.16	0.62	−6.89	−5.59	−0.91	−6.25	−2.76	−2.79	−2.07
No. of lags of differenced variable	1	4	1	1	1	1	2	2	4

[a] Critical values:

	1%	5%	10%
For two-variable regression (100 obs.)	4.07	3.37	3.03
For three-variable regression	4.45	3.93	3.59

the null that the residuals are white noise at the 99 percent significance level. The table shows that we can reject the hypothesis that $g(1) = g((1+r)^{-1})$ at the 1 percent level of significance for values of the real interest rate from 1 to 5 percent. Thus, our finding that $(1\ -1.05)$ is the estimated cointegrating vector for g_t and τ_t is not consistent with tax smoothing.[10]

While the single-equation tests presented here allow us to determine whether certain linear combinations of the three variables under scrutiny are stationary, they do not provide a direct method of determining the number of the underlying independent nontrend stationary processes. A procedure recently developed by Stock and Watson (1987) provides a means of testing the

[10] Strictly speaking, Barro's hypothesis has to do with tax rates and not tax revenues. However – given our result that revenues and expenditures are cointegrated with cointegrating vector close to $[1\ -1]$ – a finding that the tax rate and expenditures are not cointegrated would require what appear to be implausible restrictions on the behavior of the tax base over time. However, the particular numerical restrictions imposed on the cointegrating vector by the estimated government expenditure process may be sensitive to the information set used to generate forecasts of future expenditures.

Table 3

Tests for the tax smoothing hypothesis.

A. Engle–Granger tests

	Dependent variables g_t (1)	Dependent variables τ_t (1)
g_t		0.91 (49)
τ_t	1.05 (49)	
Residual regressions		
Test statistic	-5.53[a]	-5.28[a]
No. of lags of differenced variable	1	1

B. Testing for $g(1) = g((1+r)^{-1})$

r	$g(1)$	$g((1+r)^{-1})$	$g(1)/g((1+r)^{-1})$	Significance level of F-statistic
0.01	0.558	0.588	0.949	0.01
0.02	0.558	0.616	0.906	0.01
0.03	0.558	0.643	0.868	0.01
0.04	0.558	0.668	0.835	0.01
0.05	0.558	0.692	0.806	0.01

[a] See table 2 for critical values.

hypothesis that an $n \times 1$ vector x has $k \leq n$ distinct unit roots versus the alternative that x has only $m < k$ unit roots.

Stock and Watson's test involves transforming x so that the first $n-k$ components correspond to the stationary components and the last k correspond to the integrated components. Under the hypothesis that x has k trends, the eigenvalues of the first-order autoregression of the k integrated components should be equal to 1. Under the alternative that there are only $m < k$ common trends, only m of the eigenvalues should be equal to 1. Under this alternative, the $(m+1)$st largest eigenvalue, λ_{m+1}, should be less than 1. The actual test statistic is given by $T(\lambda_{m+1} - 1)$, where T is the number of observations. Critical values from a Monte Carlo experiment are reported in Stock and Watson.

The first part of table 4 presents the results of our tests for intertemporal budget balance. The first system we estimate consists of $g_t + rb_t$ and τ_t. The computed test statistic for the null that our system is driven by two indepen-

Table 4

Stock–Watson tests.

System	Eigenvalues	Test statistic[a]		Normalized cointegrating vector
	A. For intertemporal budget balance			
(1) $g_t + rb_t$	1.02	(a) 2 vs. 1	$= -21.4$	0.91
τ_t	0.78	(b) 2 vs. 0	$= 2.2$	-1.00
(2) $g_t + rb_t$	1.01	(a) 3 vs. 2	$= -22.8$	0.91
τ_t	0.95	(b) 3 vs. 1	$= -4.8$	-1.00
s_t	0.76	(c) 3 vs. 0	$= 1.0$	0.08
		Conditional on 1 cointegrating vector		
	1.00	(a) 2 vs. 1	$= -15.2$	
	0.84	(b) 2 vs. 0	$= -0.5$	
	B. For the tax smoothing hypothesis			
(3) g_t	1.03	(a) 2 vs. 1 roots	$= -17.9$	0.93
τ_t	0.81	(b) 2 vs. 0 roots	$= 2.3$	-1.00

[a] Critical values:

	5%	10%
3 vs. 2 unit roots:	-26.0	-22.2
3 vs. 1 unit roots:	-11.1	-9.2
3 vs. 0 unit roots:	-2.5	-1.8
2 vs. 1 unit roots:	-17.5	-15.6
2 vs. 0 unit roots:	-3.8	-3.2

dent random trends versus the alternative that both variables are driven by a single random trend is significant at 5 percent, and the cointegrating vector is close to [1 −1]. Notice also that the null of two independent random trends cannot be rejected against the alternative that the underlying processes are stationary.

Next, we add s_t to this system. We are unable to reject the hypothesis of three independent random trends against the alternative of two independent random trends at the 5 percent significance level, but can do so at the 10 percent level. The cointegrating vector now is [0.91 −1 0.08], where the variables are ordered $g_t + rb_t$, τ_t and s_t.

For this three-variable system, we are unable to reject the null of three independent trends against the alternatives of either a single random trend or of no random trend. A perhaps more appropriate way to proceed, once we have rejected the hypothesis of three independent trends versus two, is to estimate a system that omits the linear combination of variables for which the random walk hypothesis can be rejected. This is a test for the existence of a second cointegrating vector conditional on the existence of the first one. The table shows that we are unable to reject the hypothesis of two independent random trends at the 10 percent level.

The second part of the table presents the results of our test for the tax smoothing hypothesis. For the two-variable system consisting of g_t and τ_t, we are able to reject the null of two independent random trends in favor of a single random trend at the 5 percent significance level. Once again, the cointegrating vector is close to $[1 \ -1]$.

Taken together, the results of this section support the proposition that the government's budget is balanced over time but do not support the tax smoothing hypothesis. On the basis of the Dickey–Fuller tests we cannot reject the hypotheses that the processes for government spending (including and excluding interest), tax revenues and high powered money contain unit roots. The results also show that the difference between government expenditures inclusive of interest and tax revenues is stationary, as is the deficit inclusive of interest. This result is troublesome, since we have previously shown that high powered money is nonstationary. This apparent puzzle is perhaps a reflection of the monetary policy regime shifts that have occurred during our sample period. Providing a more adequate treatment of regime shifts is an area for future research.

The Dickey–Fuller tests for the deficit measures can be interpreted as tests to determine whether the vectors we have imposed on the spending and revenue variables are in fact cointegrating vectors. The alternative is to estimate the cointegrating vectors from the data. We do so in two different ways, first using tests developed by Granger and Engle and then those developed by Stock and Watson. We find that expenditures inclusive of interest payments and tax receipts are cointegrated, with a cointegrating vector that is close to $[1 \ -1]$. We also find a cointegrating vector for a system that contains high powered money in addition to these variables. Thus, the evidence suggests that the government's budget is balanced over time.

While all our tests support the hypothesis of cointegration between expenditures inclusive of interest and the revenue variables, they lead to different coefficients on the seignorage variable. The Dickey–Fuller tests suggest that a regression of expenditures on tax revenues and high powered money should lead to a coefficient of 1 on the last variable. However, the Engle–Granger tests lead to a coefficient substantially greater than 1, while the Stock–Watson tests suggest a coefficient close to 0.

Our results are not favorable to the tax smoothing hypothesis. In particular, data for net-of-interest government expenditures and tax revenues are consistent with the assertion that tax smoothing considerations have not played a role in generating the observed tax revenue process over the last century. The Dickey–Fuller tests show that the difference between expenditures exclusive of interest and taxes is stationary. This result appears to be confirmed by the

Engle–Granger and Stock–Watson tests, both of which lead to a cointegrating vector close to [1 −1]. This is consistent with tax smoothing only if the process governing government expenditures satisfies very specific restrictions that are, in fact, rejected by the data.

7. Conclusions

In this paper we have shown that recent developments in the theory of cointegration can be used to investigate two aspects of the government's financing decision. The first is the issue of whether the federal government's budget is balanced over time. We show that intertemporal budget balance implies cointegration between expenditures *inclusive* of interest, tax revenues and seignorage. Equivalently, the condition is that the deficit inclusive of interest be stationary. We have used a series of tests on data over the period 1890–1986 to show that these conditions are satisfied.

Our focus on expenditures inclusive of interest is in contrast to other recent studies on this issue, which have generally examined the relationship between net-of-interest expenditures and revenues, that is, the behavior of the net-of-interest deficit. However, the relationship between net-of-interest expenditures and tax revenues does provide evidence on Barro's tax smoothing hypothesis. We show that if the government sets taxes to minimize distortions that are quadratic in tax revenues, the difference between these two variables generally will not be stationary. Our empirical tests, however, suggest that this difference is stationary, implying rejection of the tax smoothing hypothesis.

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