# Estimating Extreme Risks and Dependence of Stock Indices

Research Project for the Course *Quantitative Risk Management*
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*Use* R or Python *for the programing and* Latex *to write the project* (output of Python for latex does not count. It should be a report you write in Latex). The report *cannot exceed* 15 pages, any longer report will not be considered. A report consists in a written text with tables and figures, **not** a computer listing. The code is not part of the report. The latter can, optionally, be put in an appendix. The report itself should be written as a presentation to somebody who does not know the problem. Thus, it should be self-contained.

Work can be done by a group of 2 or 3 (maximum), but the writing of the project must be individual (Do not forget to put your name on your report and also to indicate the names of the other students who worked with you). I will control that you wrote your report personally. If not, the exam will be failed.

The goal of this project is to learn how to use on data the tools of Extreme Value Theory (EVT) to deduce the risk contained in heavy tailed distributions. The analysis will be done on financial returns of stock indices. Results in the tables should be displayed in percentage, whenever possible. We provide as a help to you two sets of slides presenting succinctly EVT.

## I – Empirical/descriptive analysis

Get from Bloomberg price data (or any other good sources like Yahoo finance that you will make explicit in your report) for daily closing prices of three stock indices, from 1987 until end of 2022:

- S&P 500 (USA), the FTSE (UK) and the DAX (GE)

Do the following analysis on the data:

1. Show the plot of the daily prices, $p_i$, of the S&P 500, then describe what you see.
2. Transform your three data sets to consider now the daily log return:
$$x_i = \log p_i - \log p_{i-1},$$
   and give a plot of this new data set $(x_i\ , i = 1, \dots, n)$, for the S&P 500. What do you notice? Why did we perform this transformation? We consider the obtained returns as realization of three random variables (rv)'s: $U$ the rv for the S&P 500 returns, $V$ the rv for the FTSE returns, and $W$ the rv for the DAX.
3. Present your empirical analysis done on these later data sets in two tables, one for the basic statistics containing:
   a. Descriptive statistics (number of observations, $\mu, \sigma$, skewness, kurtosis)
   b. Rank statistics (Maximum, Median, Minimum)

   The second one for the risk:

c. What is the historical (i.e. empirical) Value-at- Risk (VaR) of *the losses* (negative returns $L = -X$) at 99.5% threshold and the Tail Value-at-Risk (TVaR, Expected Shortfall) at the 99% threshold? Compute the VaR and TVaR given by the Gaussian Model (using $\mu$ and $\sigma$ you computed in the descriptive statistics). Be carefull that we require a different threshold for VaR and TVaR as it is the case between Solvency II and Swiss Solvency Test (SST).
d. Discuss the results you have put in these two tables. Do you see fundamental differences in the descriptive statistics? And in the risk?

Then:

e. Build a portfolio, $Z = U + V + W$, with the three indices[1] and compute the empirical VaR of the losses at 99.5% and the TVaR at 99%. Allocate the capital according to: $C(L|P) = \mathbb{E}[L] - \mathbb{E}[L|Z \leq F_Z^{-1}(\alpha)]$, to the three indices in the portfolio.
f. What is the diversification benefit that you get for each index?
g. Compute the yearly return, $R_y$, of each index by multiplying the daily return by the number of business days : $R_y = \mathbb{E}[X] * 252$ (252 is approximately the number of business days in a year). Compute also the yearly risk measure: $\rho_y = \rho(X) * \sqrt{252}$. What is the RoRAC (return on risk adjusted capital) for each index standalone and for the portfolio and for each index in the portfolio? Discuss the difference between the two (in the portfolio and standalone).

## II – EVT model on the losses

Use different standard EVT methods, described in the notes given to you (EVT Tools), for modelling your loss sample, namely the block maxima method, the MEP one and, the Hill and QQ-estimators if the model distribution belongs to the Fréchet MDA (maximum Domain of Attraction).

For each case, briefly explain the method and how you apply it, discussing the choices of parameters you make.

Deduce the VaR of the loss at 99.5% for the various models- Summarize all the results in the same table and compare and discuss what you obtained with the empirical VaR. Which methods gives you the best approximation for the empirical results?

## III - Bivariate analysis

Consider now the returns of the indices two by two: $(U, V), (V, W)$, and $(U, W)$ and develop a bivariate analysis of the pair constituted by these financial indices:

- draw the **ranked** scatter plot for the three pairs, do you see a difference?

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[1] be sure to consider the same time period for the three indices and that the returns are synchronous (sometimes the holidays are not the same).

- compute the Spearman and the Pearson correlation, what do you obtain?
- analyze their type of dependence,
- suggest copulas to model the dependence, then fit them on your data
- present the fitting results in a table and discuss them,
- To illustrate the result, you can compute the probability of $\mathbb{P}(Z \geq k \mid Y = k)$ for the second index, where $k = VaR_{\alpha}(Y)$ with $\alpha = 99\%$ (be careful, this is another threshold than in the previous question) using the copula you found and compare it with the independent case and the Spearman correlation case. Display the results in a table and discuss it.

*Warning:* be sure to consider the same time period for the pair of indices and that the returns are synchronous (sometimes the holidays are not the same between the various indices). Explain how you ensure synchronicity.