Lab2 oskhi827

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Question 1

```
#, out.width='.49\\linewidth', fig.width=5, fig.height=5,fig.show='hold',fig.align='center'
library(HMM)

states = c("1", "2", "3", "4", "5", "6", "7", "8", "9", "10")
symbols = c("1", "2", "3", "4", "5", "6", "7", "8", "9", "10")
#start_prob = rep(0, 10)
#start_prob[1] = 1
start_prob = NULL
sur_state = function(x){
  state = x%%10
  if (state ==0) {
    state=10
  }
  return(state)
}
trans_prob = matrix(data=0, nrow = 10, ncol=10)
for (i in 1:10) {
  trans_prob[i,i] = 0.5
  trans_prob[i,sur_state(i+1)] = 0.5
}
emmis_prob = matrix(data=0, nrow = 10, ncol=10)
for (i in 1:10) {
  for (j in -2:2) {
    emmis_prob[i,sur_state(i+j)] = 0.5
  }
}
HMM = initHMM(states, symbols, start_prob, trans_prob, emmis_prob)
```

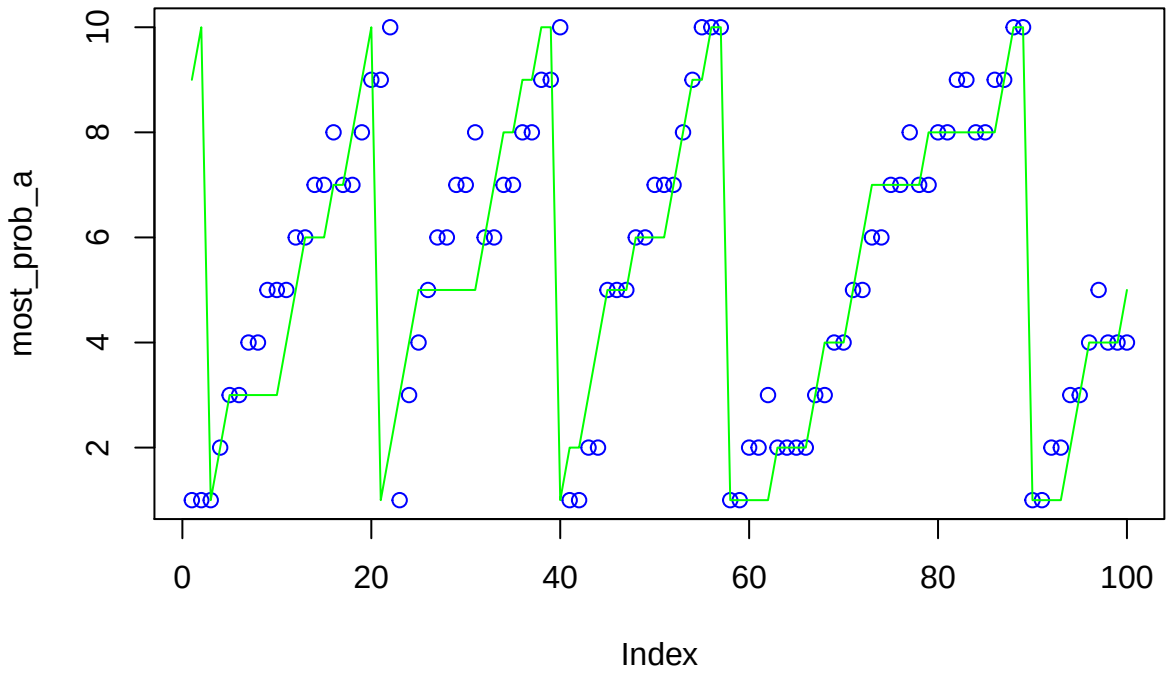
Question 2

```
N = 100
sim = simHMM(HMM, N)
```

Question 3 & 4

```
observed = sim$observation

# filterd alpha --Alpha uses all observations up to point t to estimate Zt
alpha_log = forward(HMM, observed)
alpha = exp(alpha_log)
most_prob_a = apply(alpha, MARGIN = 2, which.max)
plot(most_prob_a, col="blue")
lines(sim$states, col="green")
```



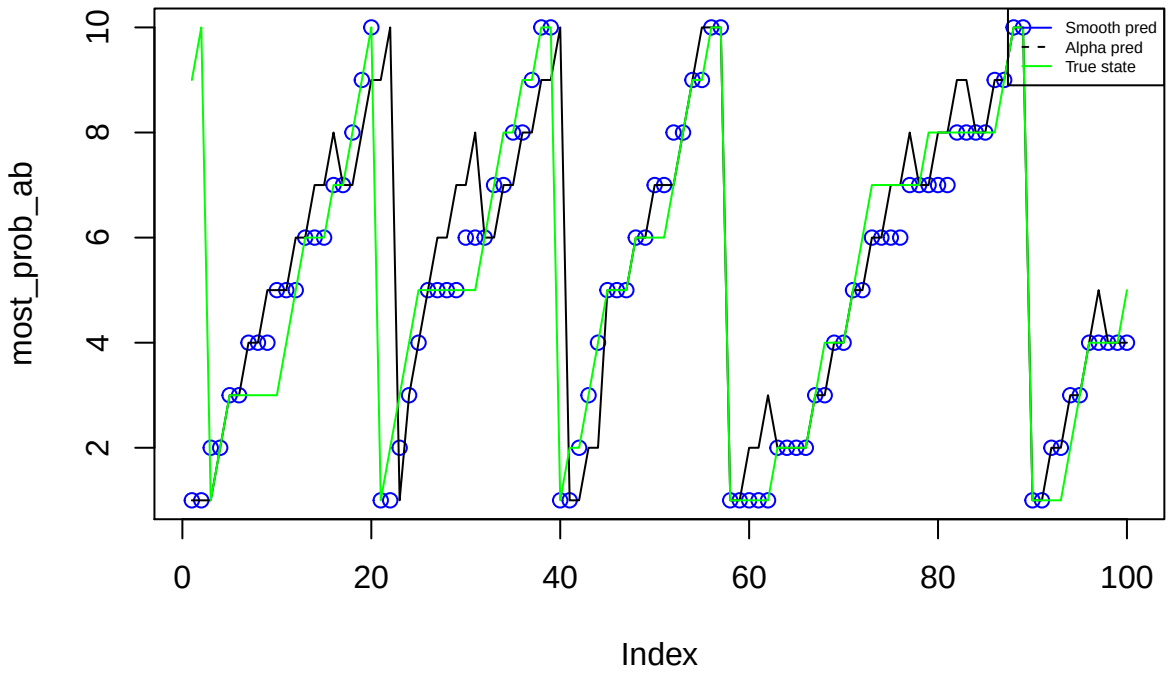

```
fi_acc = sum(sim$states==most_prob_a)/100
cat("Filtered accuracy:", fi_acc)
```

```
## Filtered accuracy: 0.44
```

```
# Beta.
beta_log = backward(HMM, observed)
beta = exp(beta_log)

# smoothed alpha*beta -- Alpha beta uses all observations (to T) to estimate Zt. "which is better"
alpha_beta = alpha*beta
most_prob_ab = apply(alpha_beta, MARGIN = 2, which.max)
```

```
plot(most_prob_ab, col="blue",)
lines(most_prob_a)
lines(sim$states, col="green")
legend("topright", c("Smooth pred", "Alpha pred", "True state"),
       col=c("blue", "black", "green"), lty=1:2, cex=0.5)
```




```
sm_acc = sum(sim$states==most_prob_ab)/100
cat("Smoothing accuracy:", sm_acc)
```

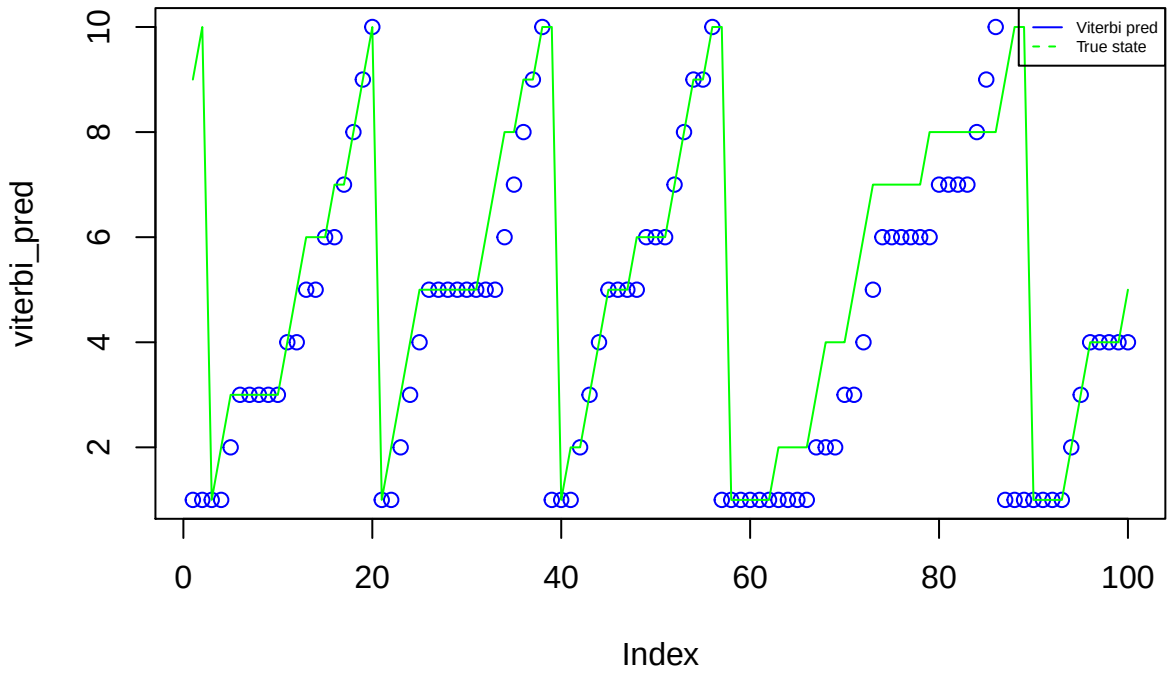
```
## Smoothing accuracy: 0.66
```

```
# Normalize
norm_fact =  apply(alpha, 2, sum)
filtered = apply(alpha, 1, "/", norm_fact )
filtered = t(filtered)

norm_fact = apply(alpha_beta, 2, sum)
smoothing = apply(alpha_beta, 1, "/", norm_fact)
smoothing = t(smoothing)

# Most prob path
viterbi_pred = viterbi(HMM, observed)
plot(viterbi_pred, col="blue",)
lines(sim$states, col="green")
```

```
legend("topright", c("Viterbi pred", "True state"),
       col=c("blue", "green"), lty=1:2, cex=0.5)
```



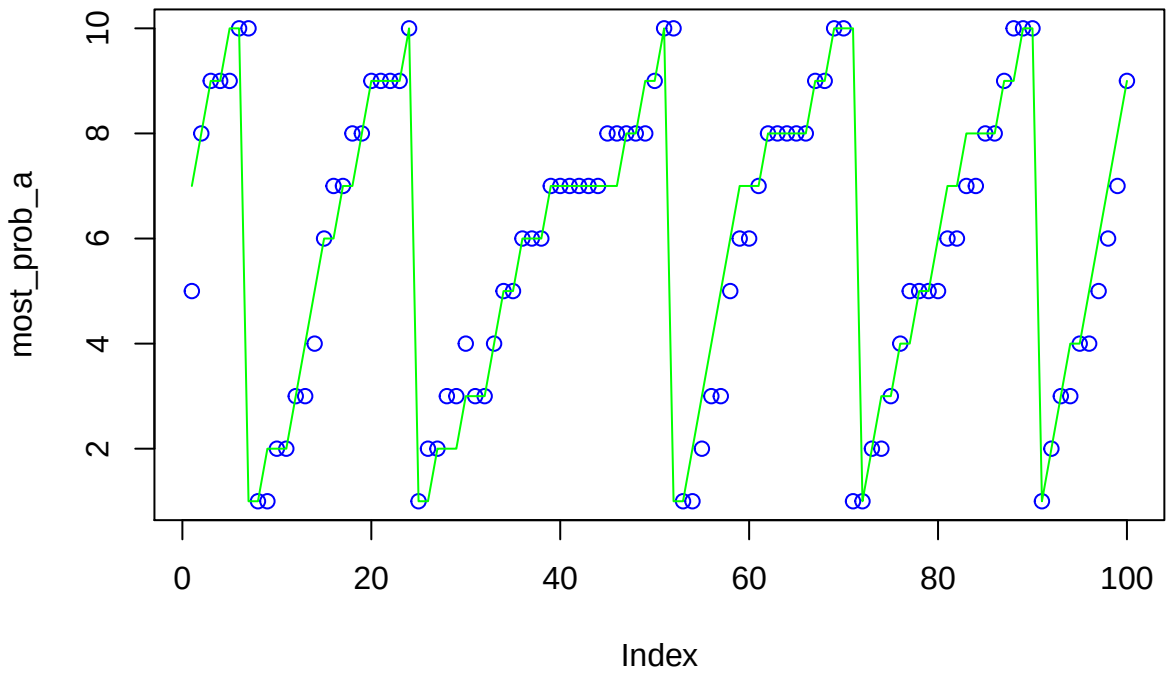

```
vi_acc = sum(sim$states==viterbi_pred)/100
cat("Most prob path accuracy:", vi_acc)
```

```
## Most prob path accuracy: 0.52
```

Question 5

```
N = 100
sim = simHMM(HMM, N)
observed = sim$observation

# filterd alpha --Alpha uses all observations up to point t to estimate Zt
alpha_log = forward(HMM, observed)
alpha = exp(alpha_log)
most_prob_a = apply(alpha, MARGIN = 2, which.max)
plot(most_prob_a, col="blue")
lines(sim$states, col="green")
```

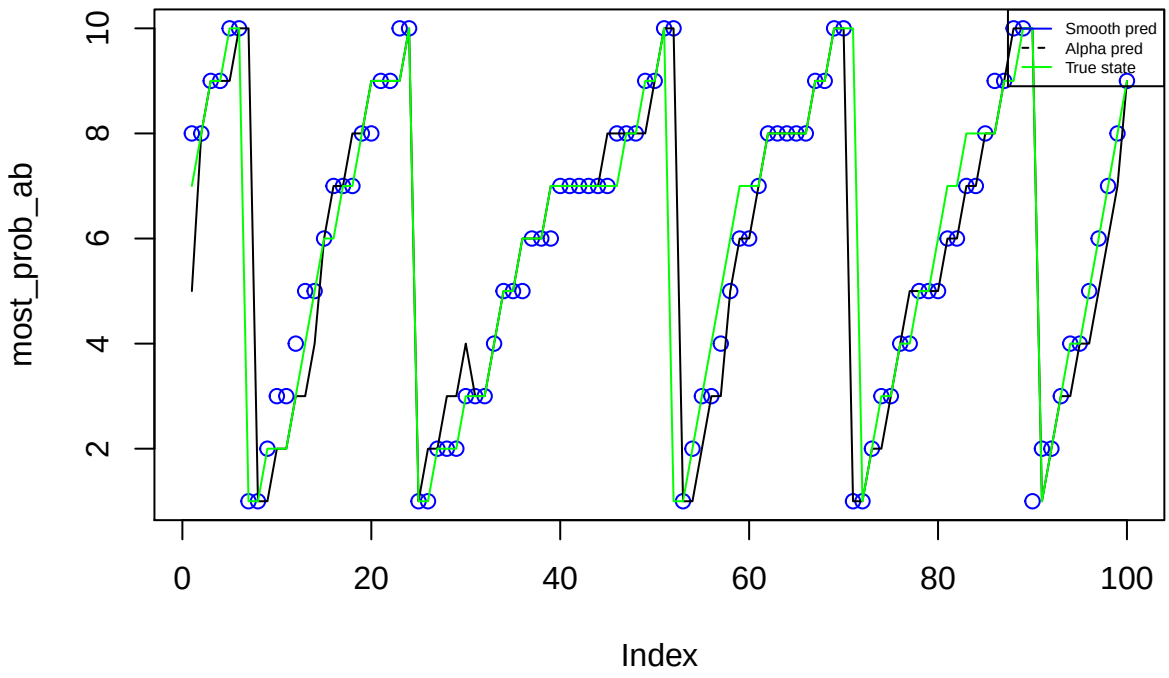



```
fi_acc = sum(sim$states==most_prob_a)/100
cat("Filtered accuracy:", fi_acc)
```

```
## Filtered accuracy: 0.63
```

```
# Beta.
beta_log = backward(HMM, observed)
beta = exp(beta_log)

# smoothed alpha*beta -- Alpha beta uses all observations (to T) to estimate Zt. "which is better"
alpha_beta = alpha*beta
most_prob_ab = apply(alpha_beta, MARGIN = 2, which.max)
plot(most_prob_ab, col="blue",)
lines(most_prob_a)
lines(sim$states, col="green")
legend("topright", c("Smooth pred", "Alpha pred", "True state"),
       col=c("blue", "black", "green"), lty=1:2, cex=0.5)
```



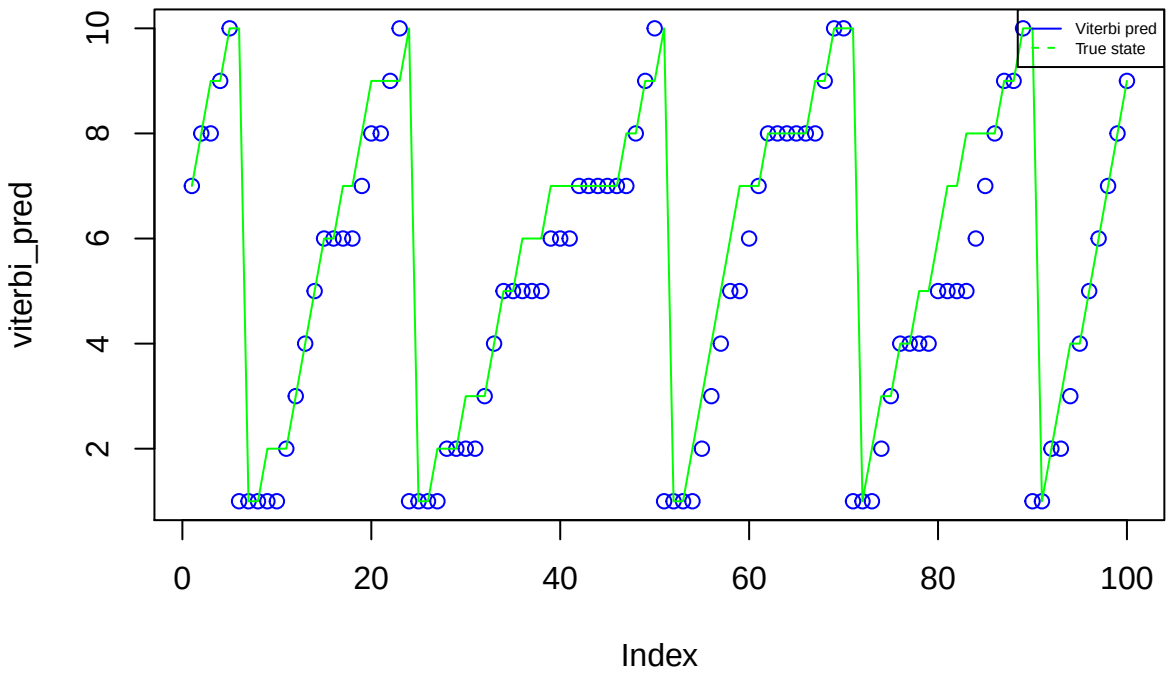

```
sm_acc = sum(sim$states==most_prob_ab)/100
cat("Smoothing accuracy:", sm_acc)
```

```
## Smoothing accuracy: 0.73
```

```
# Normalize
norm_fact =  apply(alpha, 2, sum)
filtered = apply(alpha, 1, "/", norm_fact )
filtered = t(filtered)

norm_fact = apply(alpha_beta, 2, sum)
smoothing = apply(alpha_beta, 1, "/", norm_fact)
smoothing = t(smoothing)

# Most prob path
viterbi_pred = viterbi(HMM, observed)
plot(viterbi_pred, col="blue",)
lines(sim$states, col="green")
legend("topright", c("Viterbi pred", "True state"),
       col=c("blue", "green"), lty=1:2, cex=0.5)
```




```
vi_acc = sum(sim$states==viterbi_pred)/100
cat("Most prob path accuracy:", vi_acc)
```

```
## Most prob path accuracy: 0.55
```

The filtered prediction uses all obervations up to the time t to predict Xt, where the smoothed prediciton uses all observed values (X0...XT) in its prediction. Smoothed prediction uses more information and therfore it should in general be more accurate.

In general the smoothed is also better than most prob path, Constrain, valid path, prob i sell or ecual accurate

Question 6

```
library(entropy)
entr = apply(filtered, 2, entropy.empirical)
entropy.empirical(rep(0.1,10)) # Max entrpy value!
```

```
## [1] 2.302585
```

In the first iterations the entrpy drops since we get more information, and therfore we get a better prediction. But after a couple of iterations the entropy increases again. Therfore we do not always get a better prediction with more observations. For some t the entropy is 0, and then we are sure of our predicton. But then the entropy increases agin. depending on the combination of observations we could get a better prediction.

Question 7

```
# Last prediction distribution of Z
z_T_prob = filtered[,dim(filtered)[2]]

z_101_prob = z_T_prob%*%HMM$transProbs
z_101_prob
```

```
##       to
##        1 2 3 4 5 6 7 8   9  10
##   [1,] 0 0 0 0 0 0 0 0 0.5 0.5
```

```
z_101_pred = which.max(z_101_prob)
cat("Prediction of state in timestep 101 is: ", z_101_pred)
```

```
## Prediction of state in timestep 101 is:  9
```