Quantum Computing Hands On with Qiskit

Oskari Kerppo

Faculty of Information Technology
University of Jyväskylä

Seminar at Turku, March 28th 2024

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Table of Contents

Table of Contents

QUANTUM INFORMATION AND COMPUTATION

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

https://www.vttresearch.com/en/news-and-ideas/finland-launches-20-qubit-quantum-computer-development-towards-more-powerful-quantum

What is quantum computing?

What QC is:

What QC is not:

What is quantum computing?

What QC is:

- A model of computation where information is encoded into quantum mechanical systems

What QC is not:

What is quantum computing?

What QC is:

- A model of computation where information is encoded into quantum mechanical systems
- A quantum computer can utilize phenomena such as entanglement and superposition to speed up certain specific calculations

What QC is not:

What QC is:

- A model of computation where information is encoded into quantum mechanical systems
- A quantum computer can utilize phenomena such as entanglement and superposition to speed up certain specific calculations

What QC is not:

- Magic

QUANTUM INFORMATION AND COMPUTATION

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

What is quantum computing?

What QC is:

- A model of computation where information is encoded into quantum mechanical systems
- A quantum computer can utilize phenomena such as entanglement and superposition to speed up certain specific calculations

What QC is not:

- Magic
- A computer that tries all possible solutions simultaneously

What can a QC do?

In the near term (NISQ era):

In the long term (error correction):

What can a QC do?

In the near term (NISQ era):

- Simulate quantum chemistry, applications in materials science, data science, cryptography, finance, etc.

In the long term (error correction):

What can a QC do?

In the near term (NISQ era):

- Simulate quantum chemistry, applications in materials science, data science, cryptography, finance, etc.
- Algorithms: Variational Quantum Eigensolve (VQE), Quantum Approximate Optimization Algorithm (QAOA)

In the long term (error correction):

What can a QC do?

In the near term (NISQ era):

- Simulate quantum chemistry, applications in materials science, data science, cryptography, finance, etc.
- Algorithms: Variational Quantum Eigensolve (VQE), Quantum Approximate Optimization Algorithm (QAOA)

In the long term (error correction):

- Full utilization of the power of QC, breaking RSA, optimize AI/ML algorithms

What can a QC do?

In the near term (NISQ era):

- Simulate quantum chemistry, applications in materials science, data science, cryptography, finance, etc.
- Algorithms: Variational Quantum Eigensolve (VQE), Quantum Approximate Optimization Algorithm (QAOA)

In the long term (error correction):

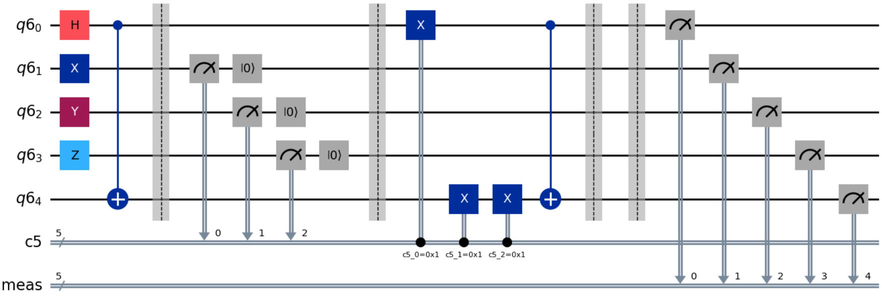
- Full utilization of the power of QC, breaking RSA, optimize AI/ML algorithms
- Shor's algorithm, Grover's algorithm, etc.

Who should study QC?

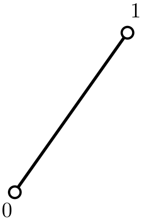
The quantum revolution is coming. To build a quantum computer we need physicists, software developers, engineers, etc.

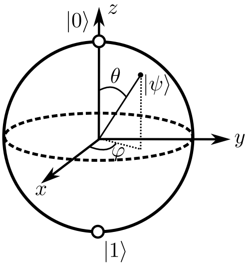
Table of Contents

primer



A bit vs. qubit





The bit

The bit has two extremal states, the 0 and 1 states. Geometrically the bit state space is a line segment, with the convex extreme points corresponding to the extremal states. Bit states in between 0 and 1 are probabilistic mixtures of the extremal states.

A bit string of length n represents the combined state of n bits. There are 2^n different bit strings of length n. It takes just n numbers to uniquely represent a given bit string of lenght n.

The qubit

A qubit is a two-level system described by a vector in a 2-dimensional complex vector space: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α, β are complex numbers with $|\alpha|^2 + |\beta|^2 = 1$ and $\{|0\rangle, |1\rangle\}$ is a basis (the computational basis).

Suppose $\alpha = e^{i\delta}\cos\frac{\theta}{2}$ and $\beta = e^{i(\delta+\varphi)}\sin\frac{\theta}{2}$. Then $|\psi\rangle = e^{i\delta}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$. The global phase $e^{i\delta}$ is not physically observable, so the state of a qubit is completely determined by $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$, so two angles deteminining a point on the surface of a sphere!

Information processing on a qubit

The state of a qubit can be altered by applying gates[1] to it. Common gates:

- Bit-flip: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. How does X act on $|\psi\rangle$?
- $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
- Phase-flip: $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Hadamard gate (entangling gate): $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

[1] A gate is described by a unitary operator. In essence they are genralizations of rotations.

QUANTUM INFORMATION AND COMPUTATION
JYVÄSKYLÄN YLIOPISTO UNIVERSITY OF JYVÄSKYLÄ

State of two qubits

How do we represent the state of multiple qubits? This is done by taking the tensor product (Kronecker product). If $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $\varphi = \gamma|0\rangle + \delta|1\rangle$, then

$$|\psi\rangle \otimes |\varphi\rangle = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle.$$

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ is a basis for two-qubits ($\mathbb{C}^2 \otimes \mathbb{C}^2$)! It requires 2^n complex numbers to fully specify the state of an n-qubit state!

Two-qubit gates

A two-qubit gate is defined by how it acts on the basis states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

- Controlled NOT: $CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
- SWAP$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

QUANTUM INFORMATION AND COMPUTATION

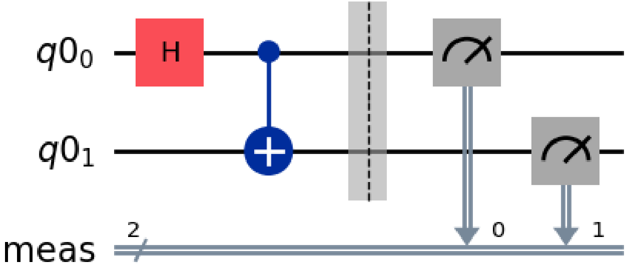
JYVÄSKYLÄN YLIOPISTO UNIVERSITY OF JYVÄSKYLÄ

Measurement

Typically, at the end of a circuit a quantum computer will measure the qubits in the computational basis, so $\{|0\rangle, |1\rangle\}$ for a single qubit, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ for two qubits, etc. The probability of obtaining a given outcome is given by the corresponding amplitude of the state vector. Hence, a QC will not give the same outcome everytime a circuit is run. Instead, circuits are run many times and the outcomes statistics are collected.

Example: entangled state

Let's calculate what the following circuit does

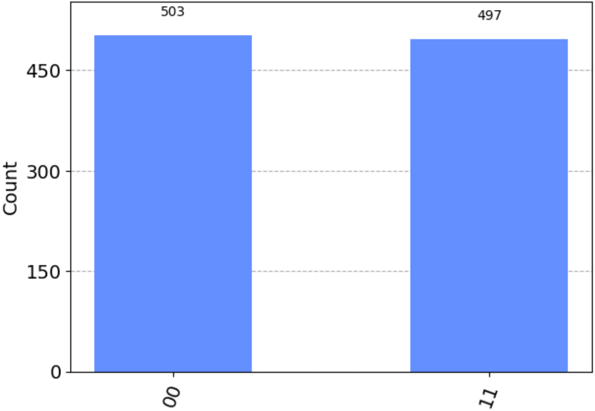


Example cont.

The initial state is $|00\rangle$. After applying the Hadamard gate the state of qubit $q0_0$ becomes $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. So the combined state is $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$. Applying the CNOT gate to this state we obtain

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

Outcome statistics



Example cont.

Is the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ entangled? Can you write this state as a product $|\psi\rangle \otimes |\varphi\rangle$ for some single qubit states $|\psi\rangle$ and $|\varphi\rangle$?

Table of Contents